ICSICT-2020
2020 IEEE 15th International Conference on Solid-State and Integrated Circuit Technology
Nov. 3-6, 2020
Wyndham Grand Plaza Royale Colorful Hotel,Kunming,China

DESIGN OF LC HARMONIC NOTCH FILTER FOR RIPPLE REDUCTION IN STEP-DOWN DC-DC BUCK CONVERTER

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Outline

1. Research Background

- **Motivation, objectives and achievements**
- **Characteristics of an adaptive feedback network**

2. Analysis of Power-Stage of DC-DC Converter

- **Operating regions of 2^{nd}-order systems**
- **Phase margin of power-stage of DC-DC converter**

3. Ripple Reduction for DC-DC Converters

- **Linear swept frequency modulation**
- **Passive and active LC Harmonic Notch filters**

4. Conclusions

1. Research Background

Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:

$$\text{SNR} = \frac{\text{Signal power}}{\text{Noise power}}$$

Performance of a device

Figure of Merit:

$$\text{F} = \frac{\text{Output SNR}}{\text{Input SNR}}$$

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

DC-DC converters

- Overshoot,
- Ringing
- Ripple

1. Research Background

Effects of Ripple and Ringing on Electronic Systems

- Ringing is overshoot/undershoot voltage or current when it's seen on time domain.
- Ripple is wasted power, and has many undesirable effects in a DC circuit.

Ringing does the following things:

- Causes EMI noise,
- Increases current flow,
- Consumes the power,
- Decreases the performance,
- Damages the devices.

Ripple does the following things:

- Heats components,
- Increases noise,
- Creates the distortion,
- Causes digital circuits to operate improperly.

1. Research Background

Objectives of Study

- **Ringing test** for **power-stage of DC-DC buck converters.**
- **Ripple reduction** using linear swept frequency modulation and LC harmonic notch filters
- **Proposed design** of an **active inductor** for the harmonic notch filter for DC-DC buck converter using **a general impedance converter**
- **Measurement of self-loop function** in **power-stage** of DC-DC buck converter

1. Research Background

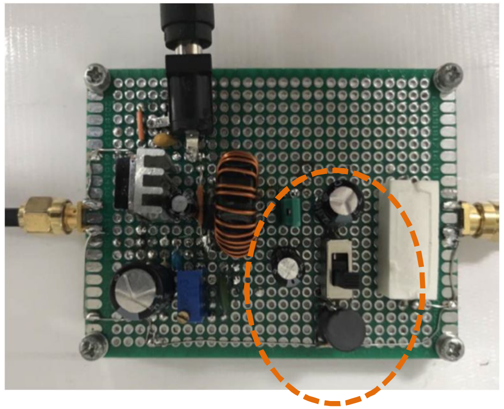
Achievements of Study

Alternating current conservation for deriving self-loop function

$$L(\omega) = -\frac{V_{inc}}{V_{trans}}$$

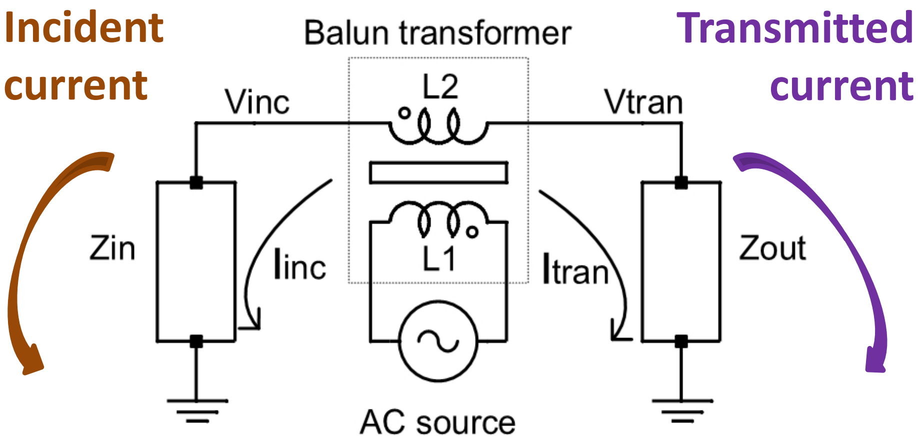
Stability test for power-stage

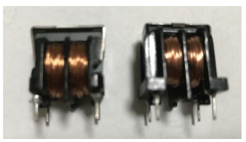
Implemented circuit



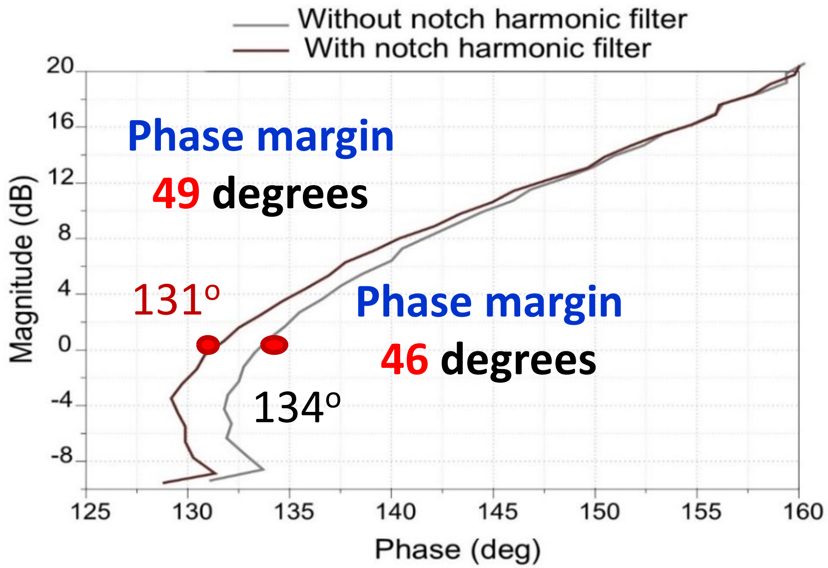
LC notch filter

Derivation of self-loop function





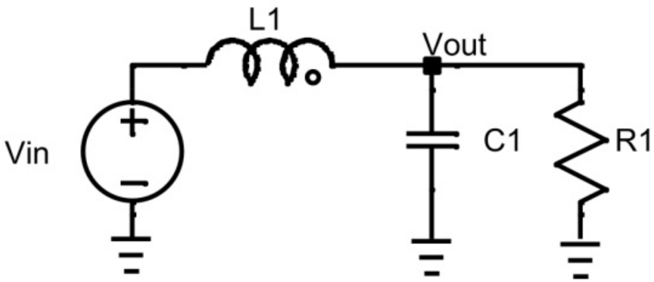
Phase margin of power stage



1. Research Background

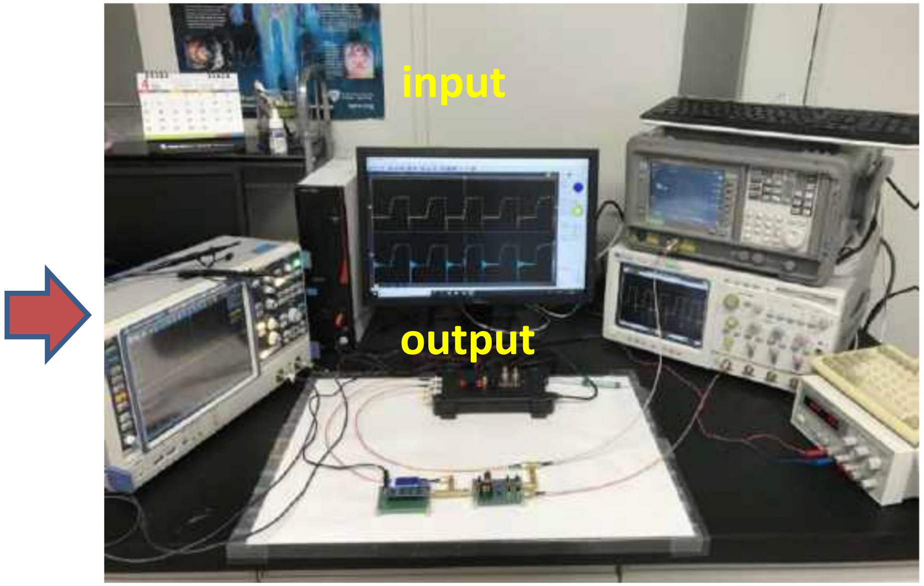
Approaching Methods

Simplified power-stage

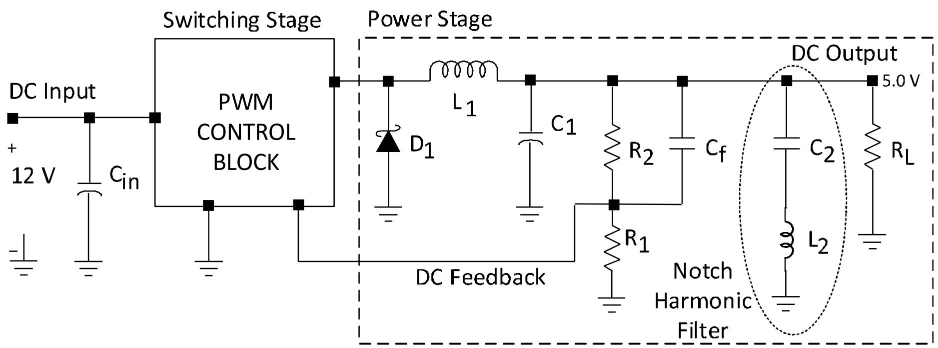


Balun transformer

Measurement set up



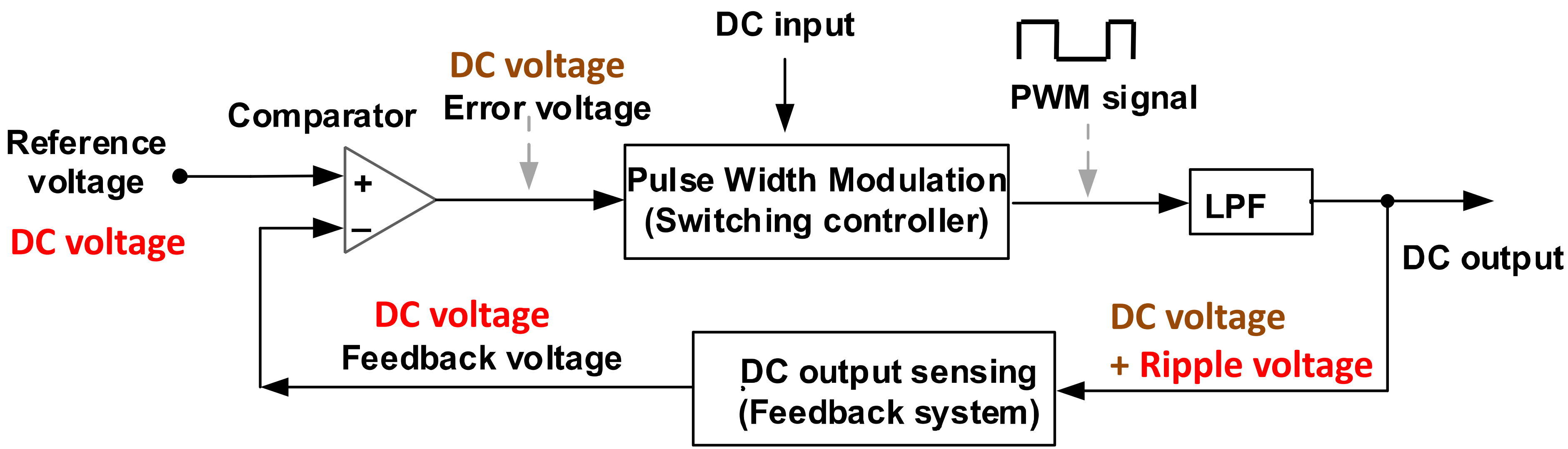
Design of DC-DC buck converter



1. Research Background

Characteristics of Adaptive Feedback Network

Block diagram of a typical adaptive feedback system



Adaptive feedback is used to control the output source along with the decision source (DC-DC Buck converter).

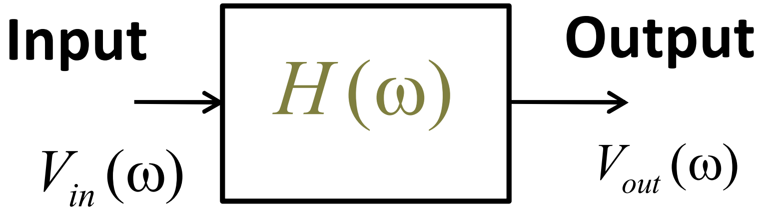
Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network.

→ Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).

1. Research Background

Self-loop Function in A Transfer Function

Linear system



Model of a linear system

$$H(\omega) = \frac{b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n}$$

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

$A(\omega)$: **Open loop function**

$H(\omega)$: **Transfer function**

$L(\omega)$: **Self-loop function**

Variable: angular frequency (ω)

- **Polar chart → Nyquist chart**
- **Magnitude-frequency plot** } **Bode plots**
- **Angular-frequency plot** } **Bode plots**
- **Magnitude-angular diagram → Nichols diagram**

1. Research Background

Alternating Current Conservation

Transfer function

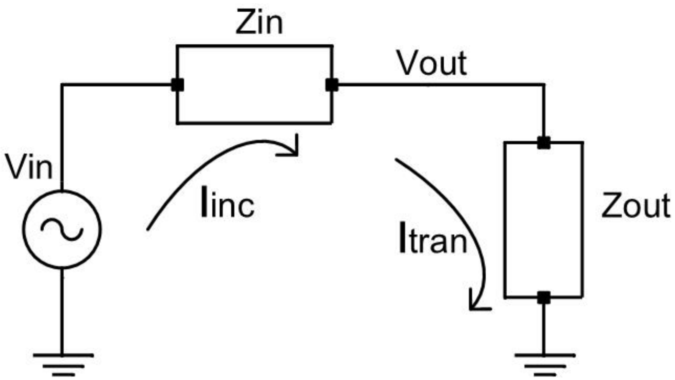
$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + \dfrac{Z_{in}}{Z_{out}}}$$

$$\Rightarrow L(\omega) = \frac{Z_{in}}{Z_{out}};$$

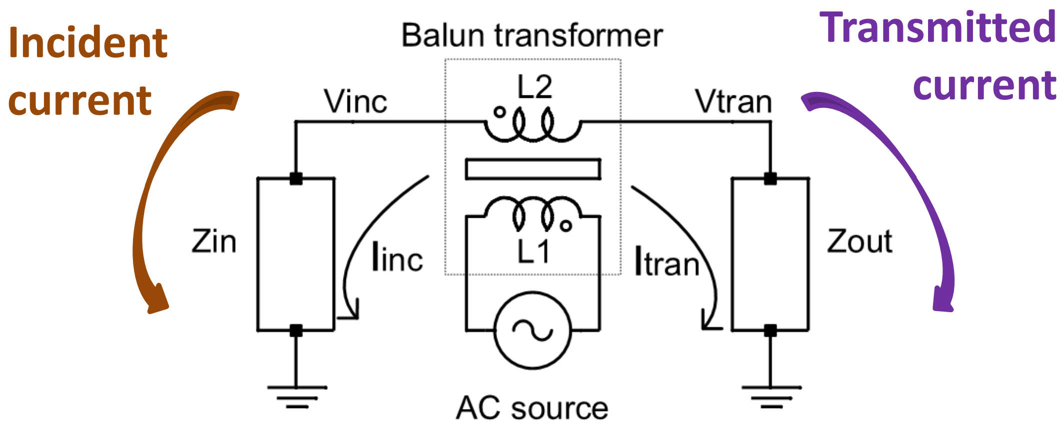
Self-loop function

$$\frac{V_{inc}}{Z_{in}} = -\frac{V_{trans}}{Z_{out}} \Rightarrow L(\omega) = -\frac{V_{inc}}{V_{trans}} = \frac{Z_{in}}{Z_{out}}$$

10 mH inductance



Simplified linear system



Derivation of self-loop function

1. Research Background

Limitations of Conventional Methods

- **Middlebrook's measurement of loop gain**

→ **Applying only in feedback systems (DC-DC converters).**

- **Replica measurement of loop gain**

→ **Using two identical networks (not real measurement).**

- **Nyquist's stability condition**

→ **Theoretical analysis for feedback systems (Lab tool).**

- **Nichols chart of loop gain**

→ **Only used in feedback control theory (Lab tool).**

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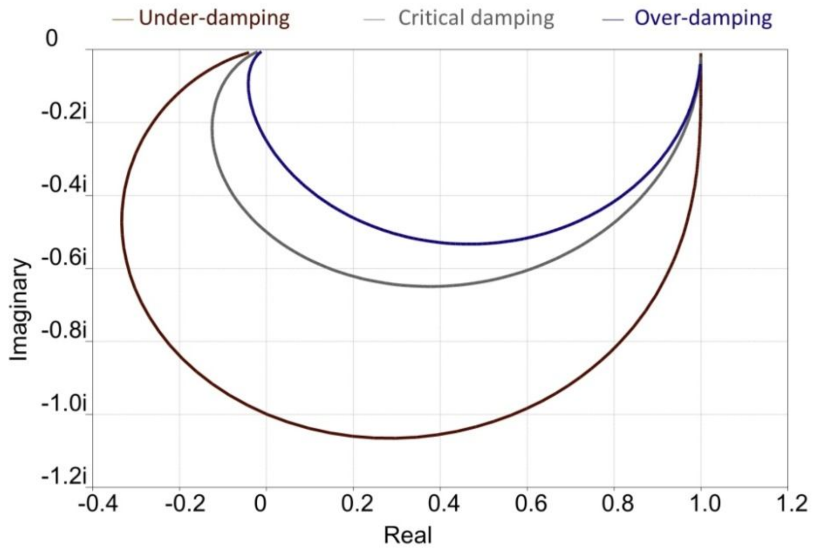
   - Passive and active LC Harmonic Notch filters
4. Conclusions

2. Analysis of Power-Stage of DC-DC Converter

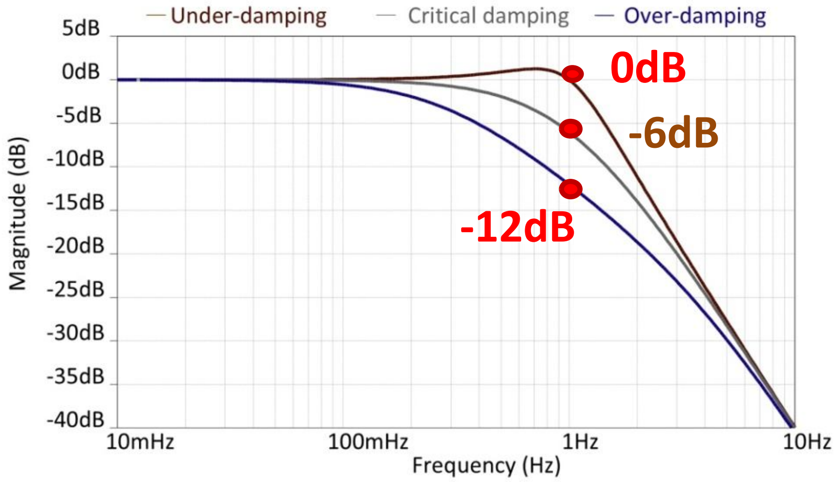
Simulations of Second-order Transfer Function

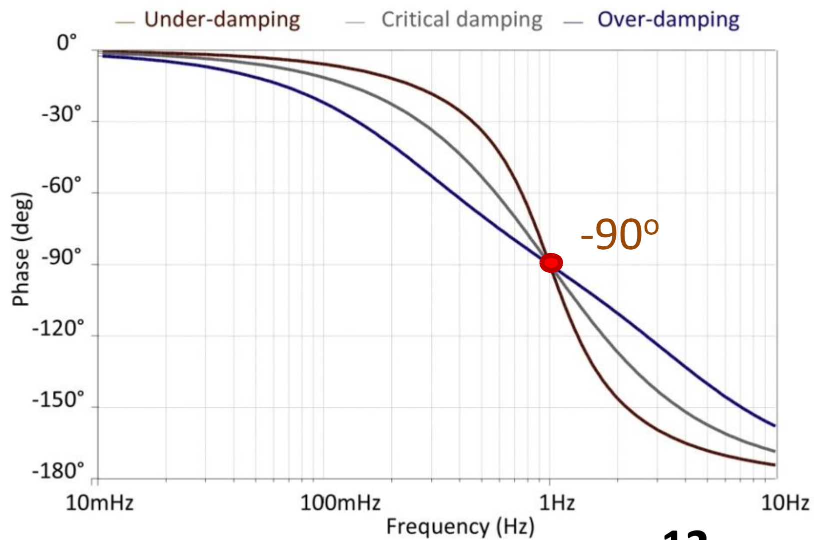
- **Under-damping:** $H_1(\omega) = \dfrac{1}{(j\omega)^2 + j\omega + 1};$
- **Critical damping:** $H_2(\omega) = \dfrac{1}{(j\omega)^2 + 2j\omega + 1};$
- **Over-damping:** $H_3(\omega) = \dfrac{1}{(j\omega)^2 + 3j\omega + 1};$

Nyquist chart of transfer function



Bode plot of transfer function





2. Analysis of Power-Stage of DC-DC Converter

Behaviors of Second-order Transfer Function

Second-order transfer function: $H(\omega) = \dfrac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega}$

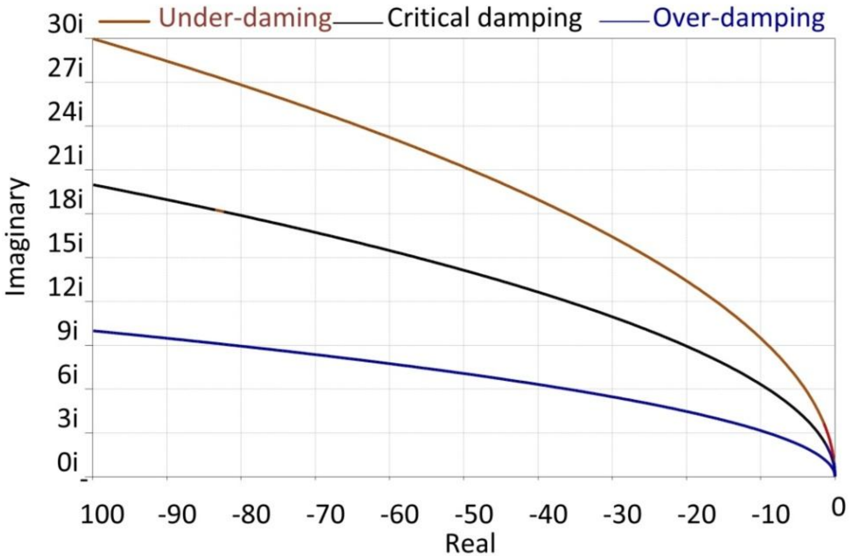
Case	Over-damping	Critically damping	Under-damping
Delta (Δ)	$\frac{1}{a_0} < \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 > 0$	$\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 = 0$	$\frac{1}{a_0} > \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 < 0$
Module $\lvert H(\omega)\rvert$	$\dfrac{\frac{1}{a_0}}{\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}}$	$\dfrac{\frac{1}{a_0}}{\left[\omega^2 + \left(\frac{a_1}{2a_0}\right)^2\right]} = \frac{1}{2} = -6dB$	$\dfrac{\frac{1}{a_0}}{\sqrt{\left(\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}\sqrt{\left(\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}}$
Angular $\theta(\omega)$	$-\arctan\left(\dfrac{\omega}{\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}}\right) - \arctan\left(\dfrac{\omega}{\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}}\right)$	$-2\arctan\left(\dfrac{2a_0\omega}{a_1}\right)$	$-\arctan\left(\dfrac{\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right) - \arctan\left(\dfrac{\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)$
$\omega_{cut} = \dfrac{a_1}{2a_0}$	$\lvert H(\omega_{cut})\rvert < \dfrac{2a_0}{a_1}$ $\theta(\omega_{cut}) > -\dfrac{\pi}{2}$	$\lvert H(\omega_{cut})\rvert = \dfrac{2a_0}{a_1}$ $\theta(\omega_{cut}) = -\dfrac{\pi}{2}$	$\lvert H(\omega_{cut})\rvert > \dfrac{2a_0}{a_1}$ $\theta(\omega_{cut}) < -\dfrac{\pi}{2}$

2. Analysis of Power-Stage of DC-DC Converter

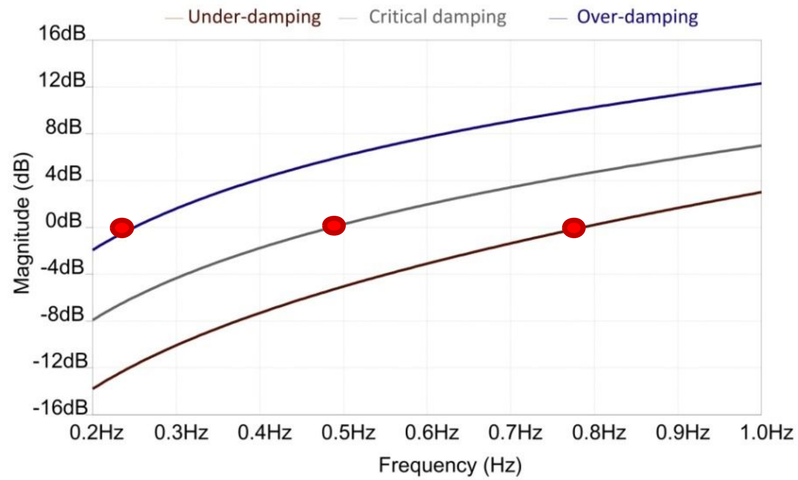
Simulations of Second-order Self-loop Function

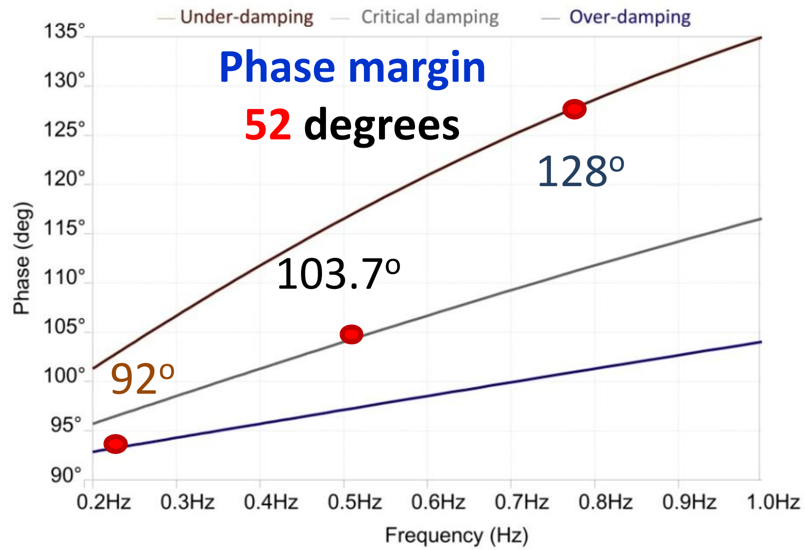
- **Under-damping:** $L_1(\omega) = (j\omega)^2 + j\omega;$
- **Critical damping:** $L_2(\omega) = (j\omega)^2 + 2j\omega;$
- **Over-damping:** $L_3(\omega) = (j\omega)^2 + 3j\omega;$

Nyquist chart of self-loop function



Bode plot of self-loop function





2. Analysis of Power-Stage of DC-DC Converter

Behaviors of Second-order Self-loop Function

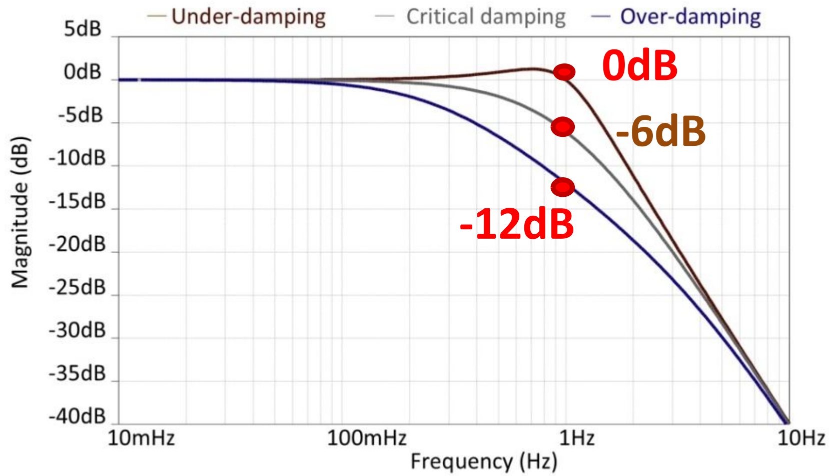
Second-order self-loop function: $L(\omega) = j\omega\left[a_0 j\omega + a_1\right]$

Case	**Over-damping**		**Critical damping**		**Under-damping**	
Delta (Δ)	$\Delta = a_1^{\ 2} - 4a_0 > 0$		$\Delta = a_1^{\ 2} - 4a_0 = 0$		$\Delta = a_1^{\ 2} - 4a_0 < 0$	
$\lvert L(\omega)\rvert$	$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$	
$\theta(\omega)$	$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$	
$\omega_1 = \frac{a_1}{2a_0}\sqrt{\sqrt{5}-2}$	$\lvert L(\omega_1)\rvert > 1$	$\pi - \theta(\omega_1) > 76.3^o$	$\lvert L(\omega_1)\rvert = 1$	$\pi - \theta(\omega_1) = 76.3^o$	$\lvert L(\omega_1)\rvert < 1$	$\pi - \theta(\omega_1) < 76.3^o$
$\omega_2 = \frac{a_1}{2a_0}$	$\lvert L(\omega_2)\rvert > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^o$	$\lvert L(\omega_2)\rvert = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^o$	$\lvert L(\omega_2)\rvert < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^o$
$\omega_3 = \frac{a_1}{a_0}$	$\lvert L(\omega_3)\rvert > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^o$	$\lvert L(\omega_3)\rvert = 4\sqrt{2}$	$\pi - \theta(\omega_3) = 45^o$	$\lvert L(\omega_3)\rvert < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^o$

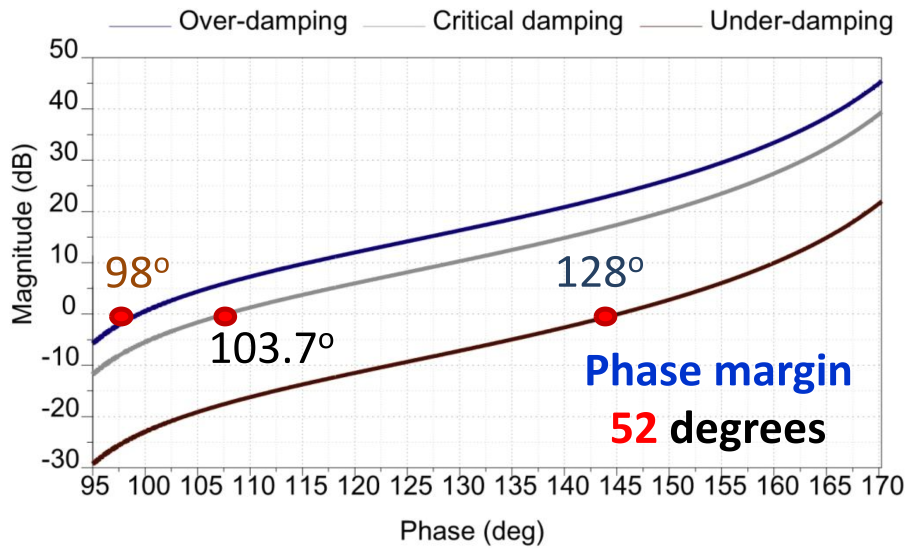
2. Analysis of Power-Stage of DC-DC Converter

Summary of Operating Regions of 2nd-Order System

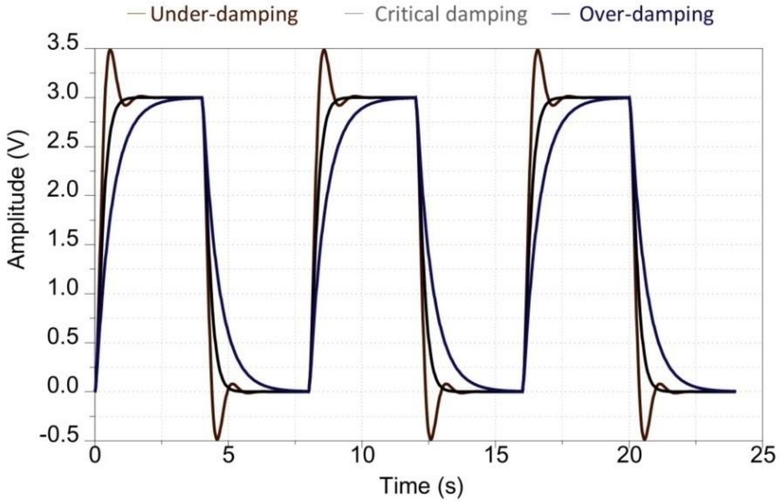
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Over-damping:

→Phase margin is 88 degrees.

Critical damping:

→Phase margin is 76.3 degrees.

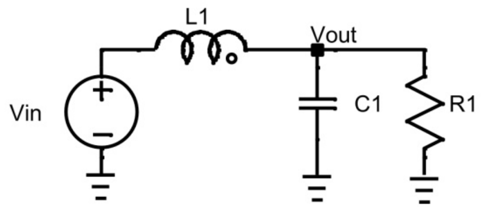
Under-damping:

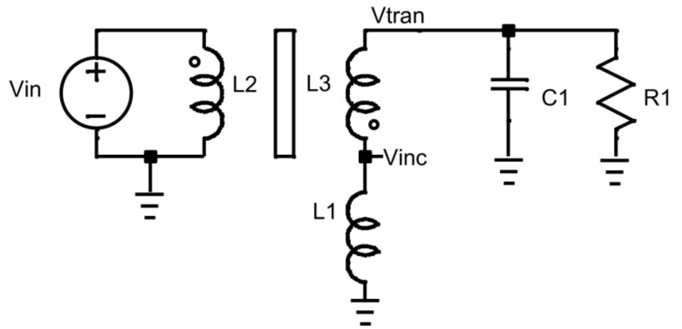
→Phase margin is 52 degrees.

2. Analysis of Power-Stage of DC-DC Converter

Behaviors of power-stage of DC-DC Converter

Parallel RLC low-pass filter





Transfer function & self-loop function:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Where: $a_0 = LC; \quad a_1 = \frac{L}{R};$

$$\omega_0 = 1/\sqrt{LC};$$

$$|Z_L| = \omega_0 L; \quad |Z_C| = 1/\omega_0 C;$$

Operating regions

- **Over-damping:** $\frac{1}{LC} < \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| < R/2$
- **Critical damping:** $\frac{1}{LC} = \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| = R/2$
- **Under-damping:** $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| > R/2$

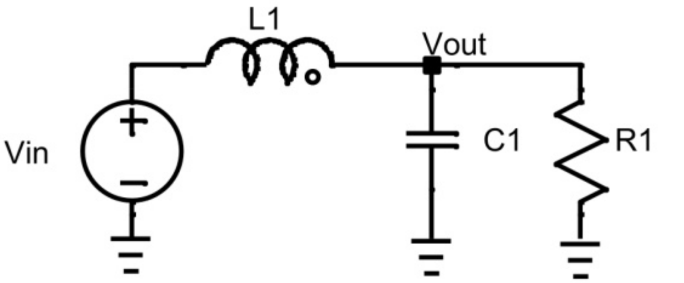
$$|Z_L| = |Z_C| = 2R$$

Balanced charging-discharging time condition

2. Analysis of Power-Stage of DC-DC Converter

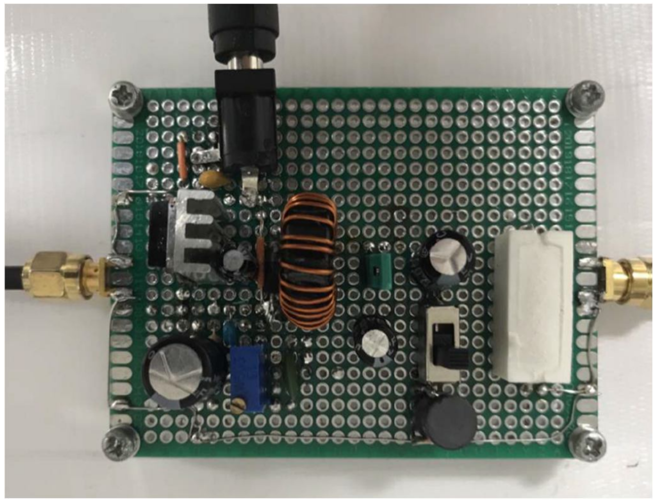
Phase Margin of Power-Stage of DC-DC Converter

Simplified power-stage



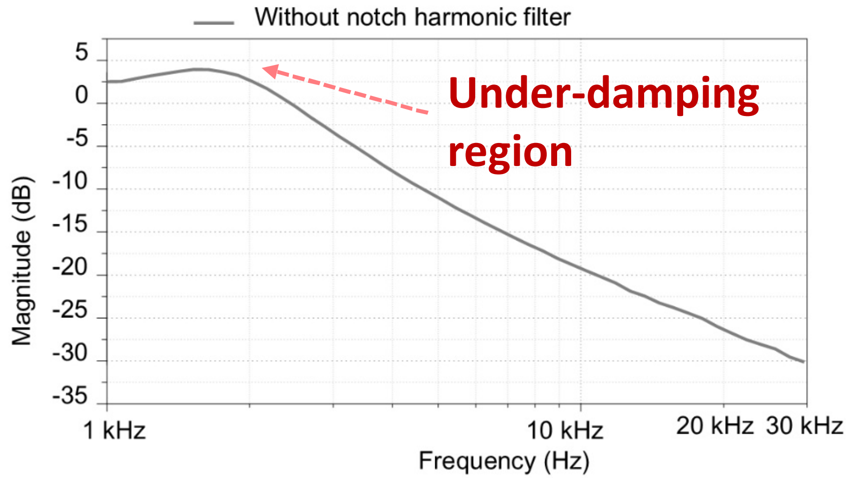
L_1 = 220 uH, C_1 = 100 uF, R_1 = 5 Ω,

Implemented circuit

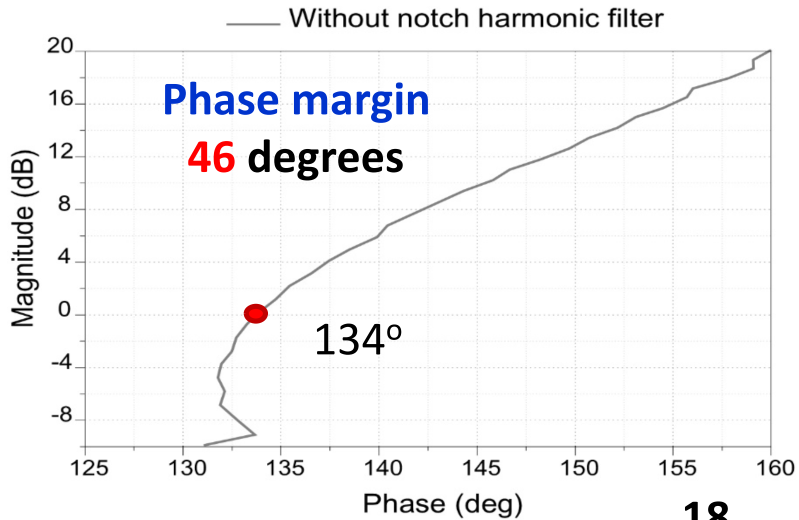


Measured results

Bode plot of transfer function



Nichols plot of self-loop function



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   - **Linear swept frequency modulation**
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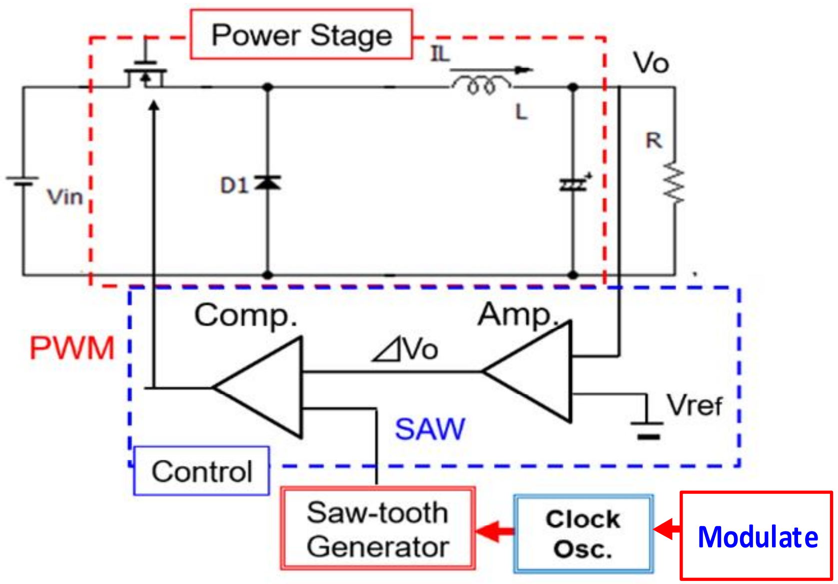
4. Conclusions

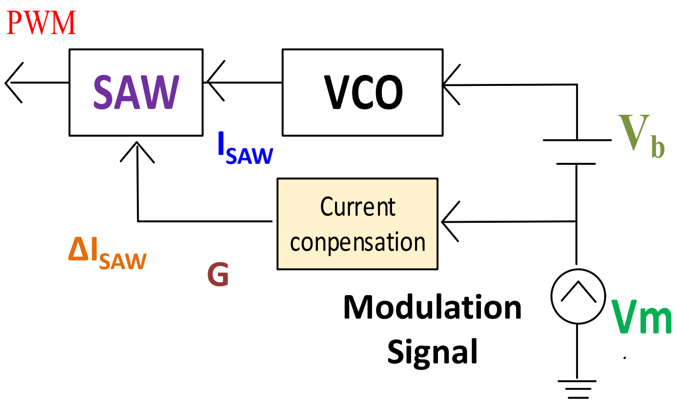
3. Ripple Reduction for DC-DC Converters

Ripple Reduction using Linear Swept Frequency Modulation

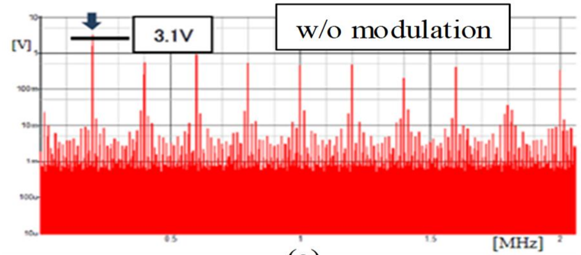
Block diagram of DC-DC buck converter



Linear swept frequency modulation

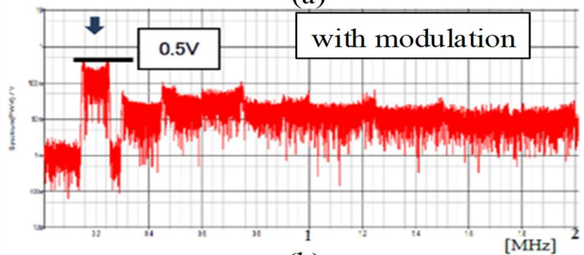


Without frequency modulation



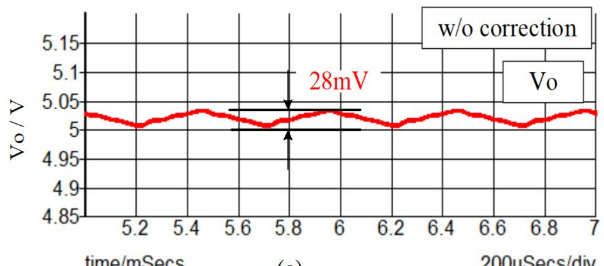
(a)

With frequency modulation



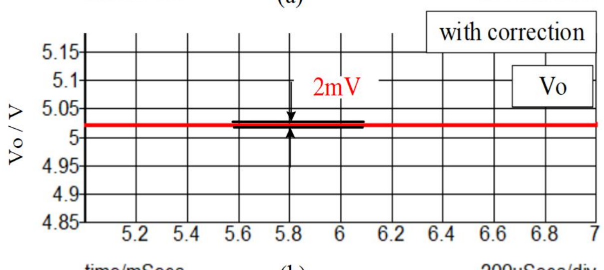
(b)

Without frequency modulation



(a)

With frequency modulation

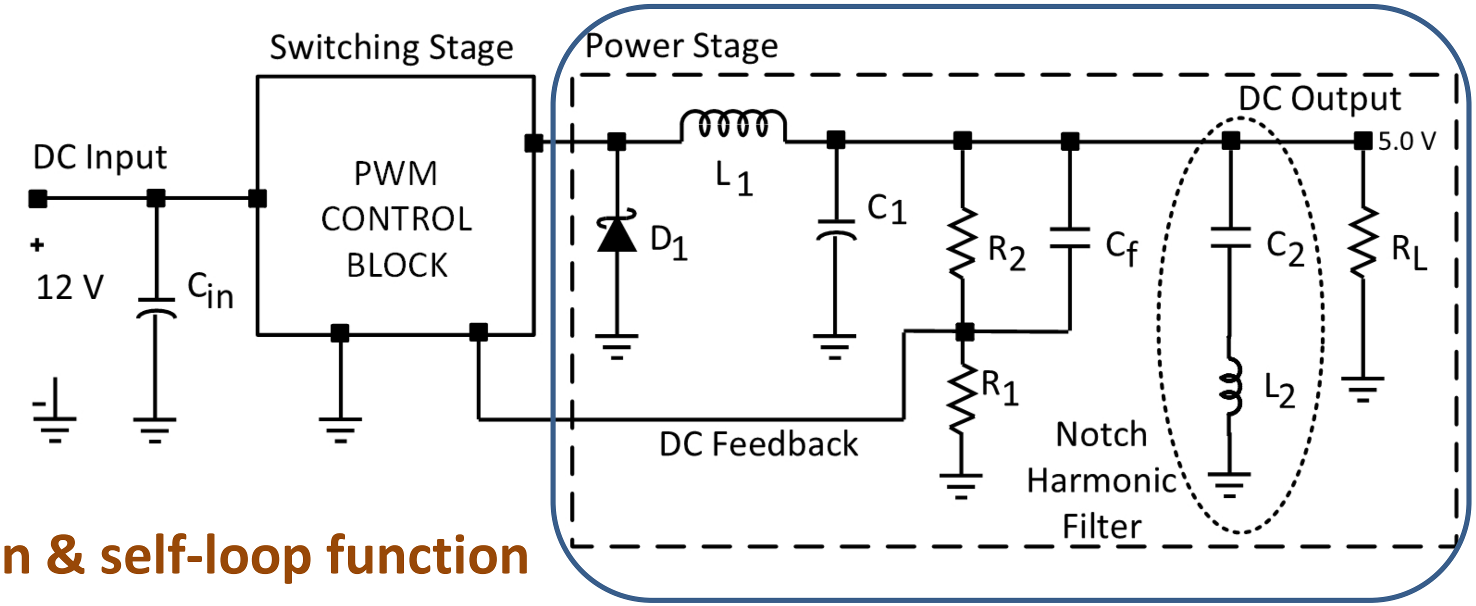


(b)

3. Ripple Reduction for DC-DC Converters

Ripple Reduction using LC Harmonic Notch Filter

Schematic diagram of DC-DC buck converter



Transfer function & self-loop function

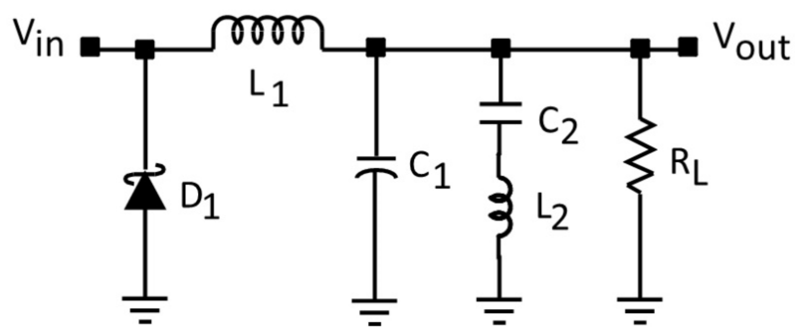
$$H(\omega) = \frac{b0(j\omega)^2 + 1}{a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega + 1}$$

$$L(\omega) = a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega$$

Where, $b_0 = L_2C_2; a_0 = L_1C_1L_2C_2;$

$$a_1 = \frac{L_1L_2C_2}{R_L}; a_2 = L_1C_1 + L_2C_2 + L_1C_2; a_3 = \frac{L_1}{R_L};$$

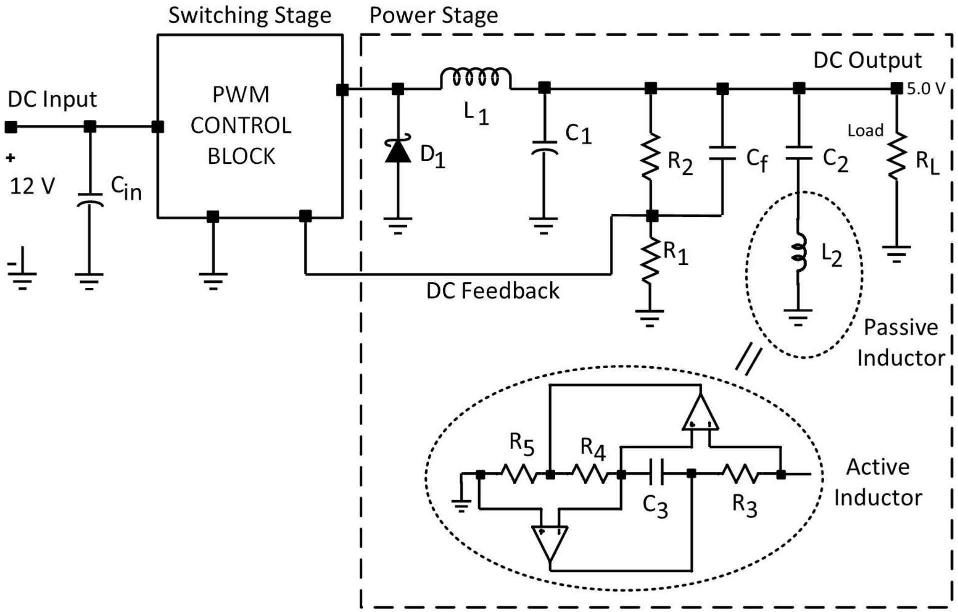
Simplified model of power-stage



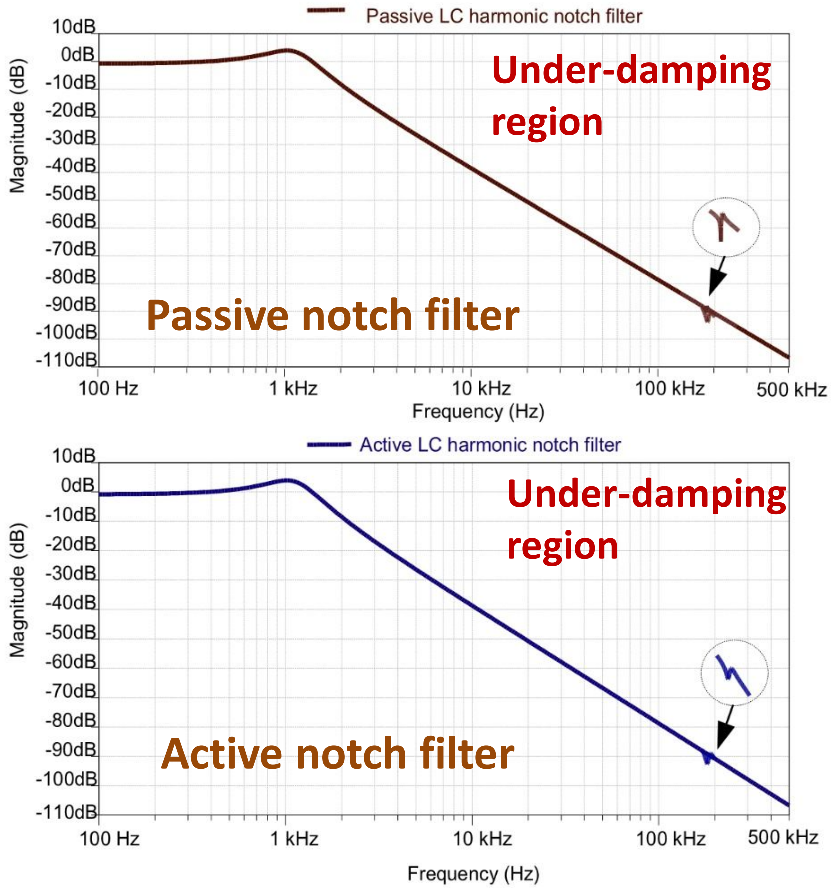
3. Ripple Reduction for DC-DC Converters

Passive and Active LC Harmonic Notch Filters

Schematic diagram of DC-DC converter



Behaviors of LC harmonic notch filters



Approximated value of inductor

$$Z_L = \frac{R_2}{R_1}\frac{R_3}{Z_C} Z_{out} = \frac{R_2 R_3}{R_1} sCZ_{out}$$

Passive component parameters

L_1 = 220 uH, C_1 = 100 uF, R_1 = 4 kΩ, R_2 = 2 kΩ, R_L = 5 Ω, C_2 = 65 nF, L_2 = 12 uH, R_3 = 15 kΩ, C_1 = 100 pF, R_4 = 1 kΩ, R_5 = 1 kΩ, and fn at 180 kHz.

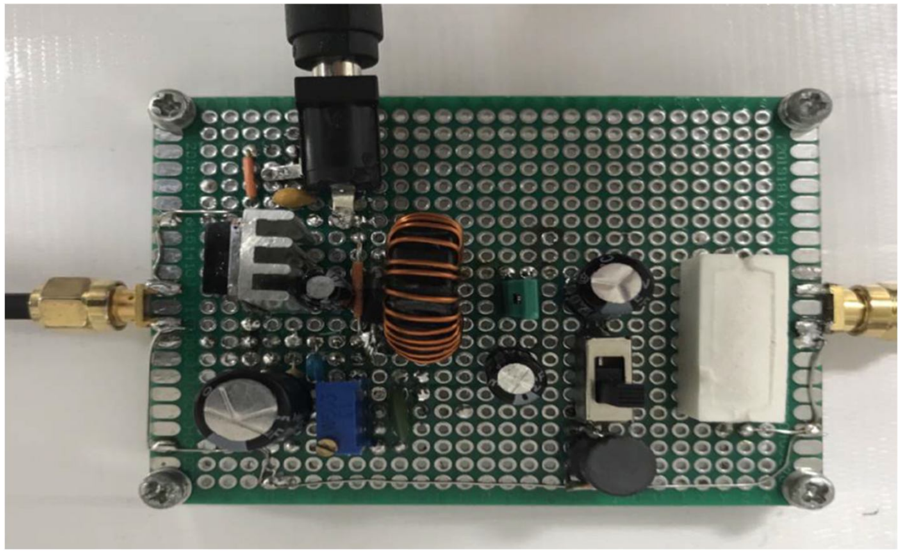
3. Ripple Reduction for DC-DC Converters

Implemented Circuit for DC-DC Converter

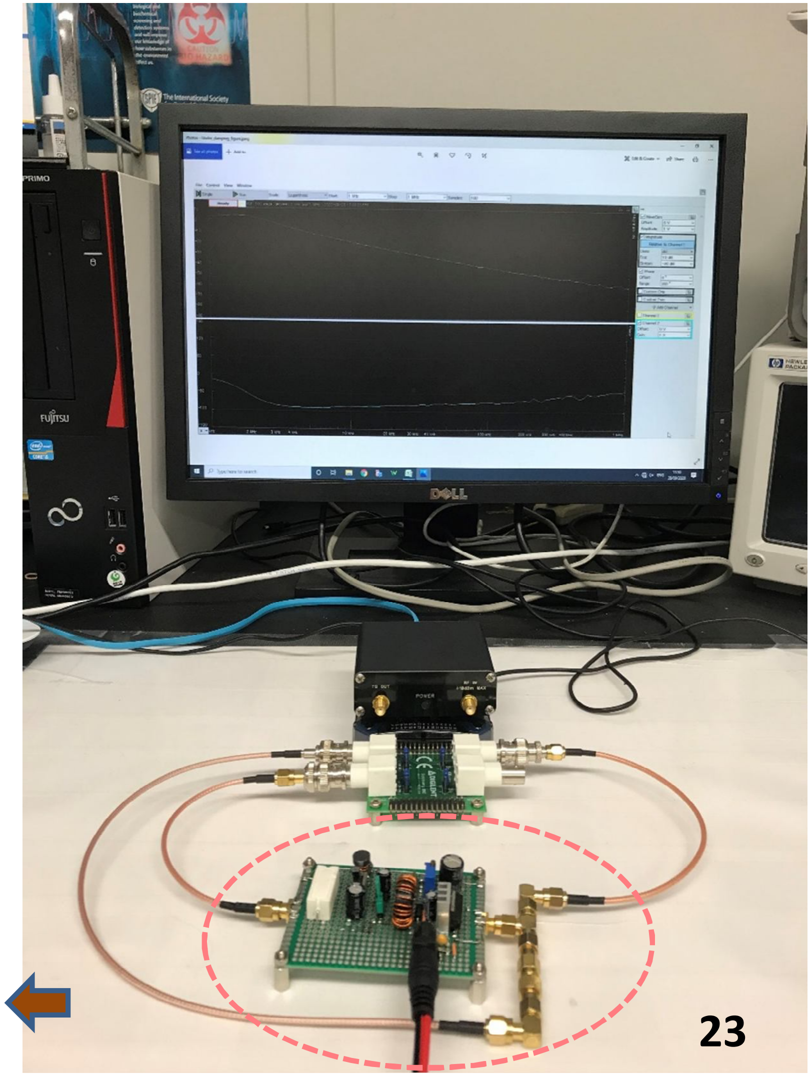
Design parameters

Input Voltage (Vin)	12 V
Output Voltage (Vo)	5.0 V
Output Current (Io)	1 A
Clock Frequency (Fck)	180 kHz
Output Ripple	< 10 mVpp

Implemented circuit



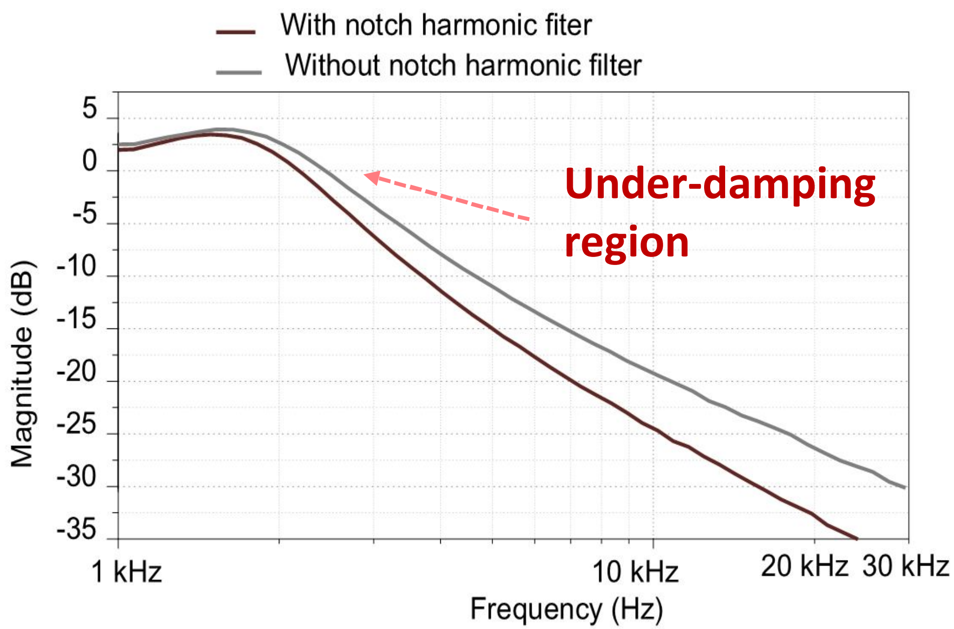
Measurement set up



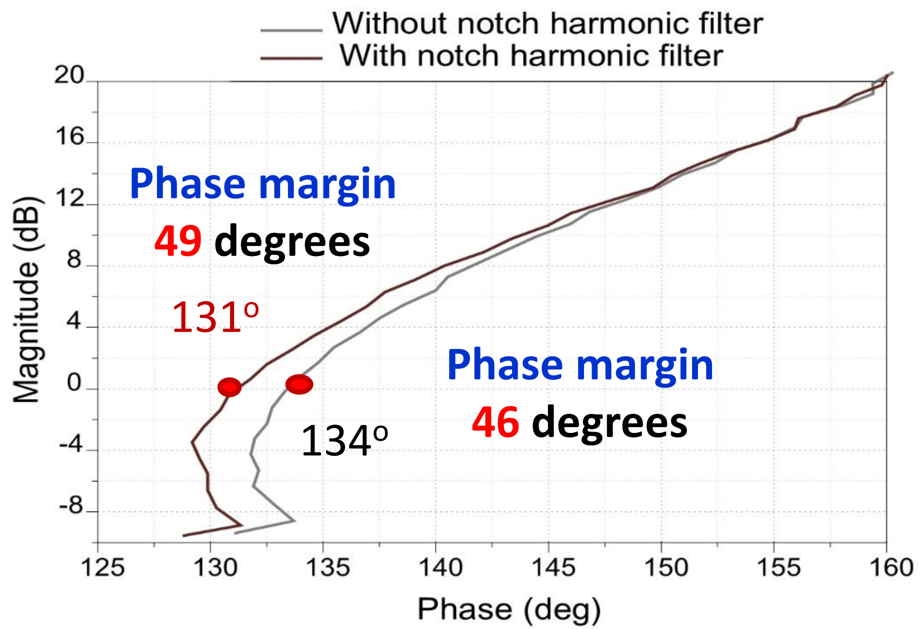
3. Ripple Reduction for DC-DC Converters

Measurement Results of Proposed Design Circuit

Bode plot of transfer function



Nichols plot of self-loop function



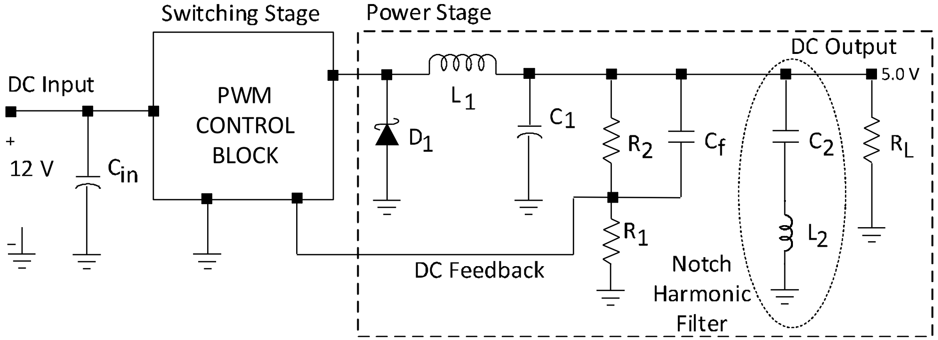
○ **Reduce the cut-off frequency of the power-stage**
→ **Reduce the ripple caused by high-order harmonic signals**

○ **Improvement of phase margin of the power-stage**
→ **Reduce the overshoot caused by the passive components**

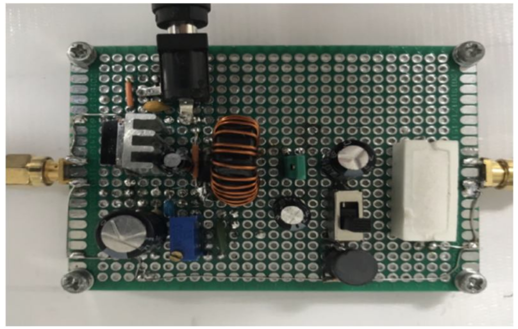
3. Ripple Reduction for DC-DC Converters

Ripple Reduction using Harmonic Notch Filter

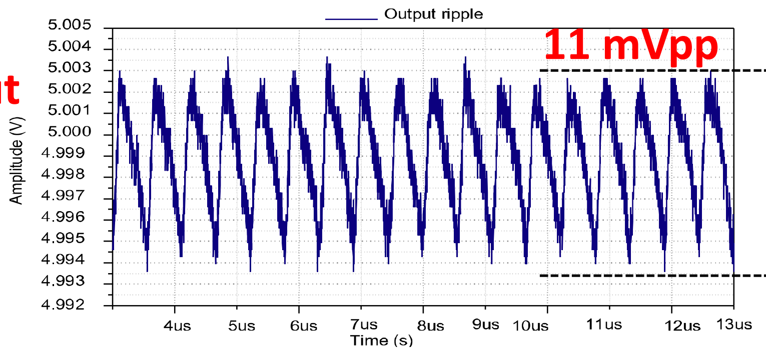
Schematic diagram of DC-DC Buck converter



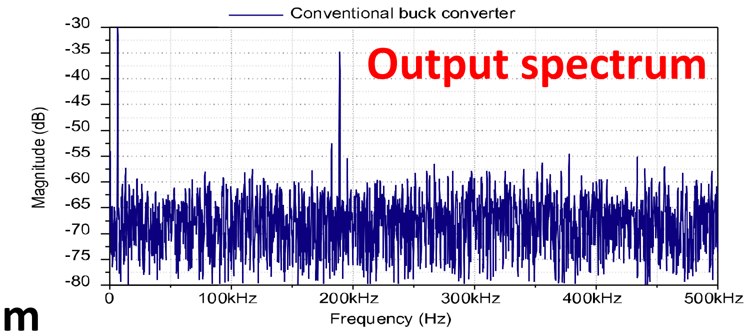
Implemented circuit



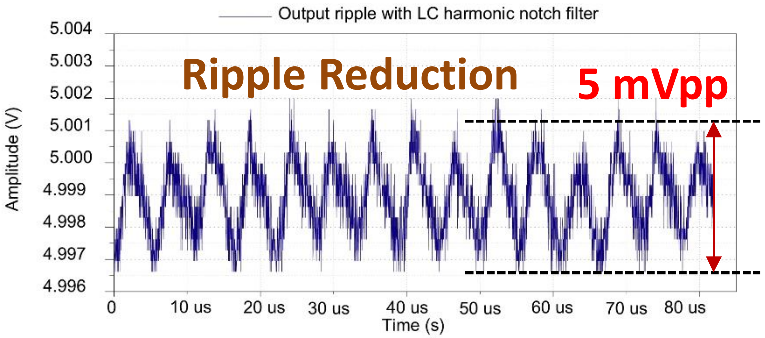
Without notch filter



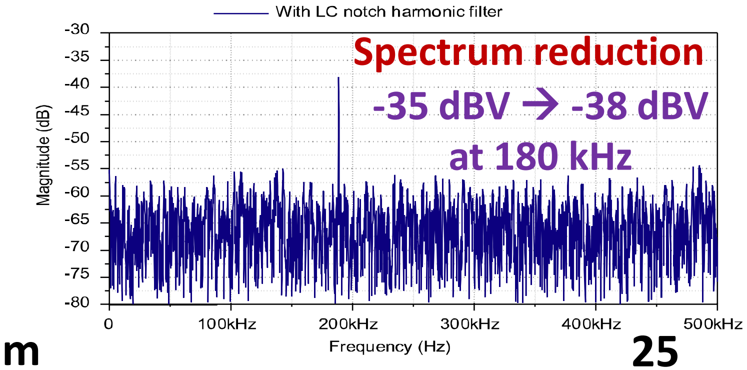
Output spectrum



With notch filter



Output spectrum



Outline

1. Research Background
   - Motivation, objectives and achievements
   - Characteristics of an adaptive feedback network
2. Analysis of Power-Stage of DC-DC Converter
   - Operating regions of power-stage of DC-DC converter
   - Passive parallel RLC network
3. Ripple Reduction for DC-DC Converters
   - Linear swept frequency modulation
   - Passive and active LC Harmonic Notch filters
4. Conclusions

4. Comparison

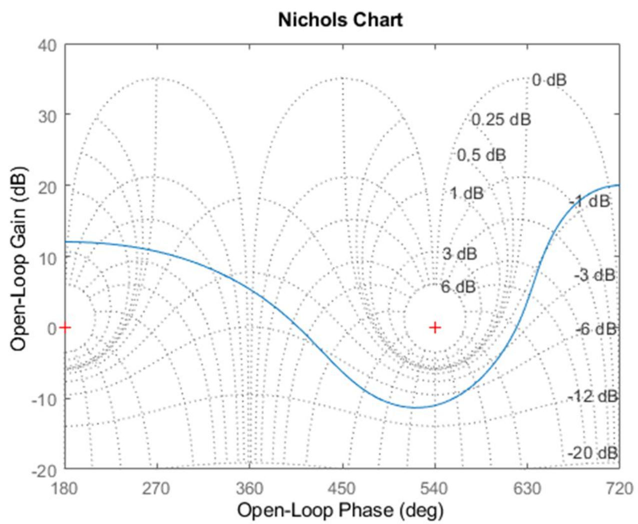
Features	This work	Replica measurement	Middlebrook's method
Main objective	Self-loop function	Loop gain	Loop gain
Transfer function accuracy	Yes	No	No
Ringing Test	Yes	Yes	Yes
Operating region accuracy	Yes	No	No
Phase margin accuracy	Yes	No	No
Passive networks	Yes	No	No

4. Discussions

- Loop gain is independent of frequency variable.

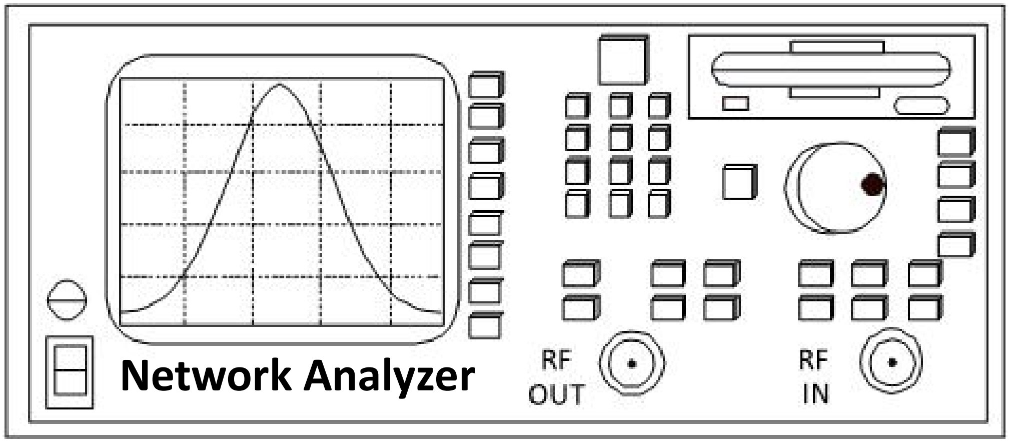
→Loop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.



https://www.mathworks.com/help/control/ref/nichols.html

Nichols chart isn't used widely in practical measurements (only used in control theory).



➡ (Technology limitations)

4. Conclusions

This work:

- **Investigation of behaviors of power-stage in DC-DC converters based on alternating current conservation**
- **Proposed designs of passive and active LC harmonic notch filters for ripple reduction**
  → **Phase margin improvement from 46 degrees into 49 degrees**
  → **Ripple reduction from 11 mVpp into 5 mVpp**

Future of work:

- **Stability test for dynamic loads, and parasitic components in printed circuit boards**

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Thank you very much!

谢谢

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