DESIGN OF LC HARMONIC NOTCH FILTER FOR RIPPLE REDUCTION IN STEP-DOWN DC-DC BUCK CONVERTER

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1. Research Background
   • Motivation, objectives and achievements
   • Characteristics of an adaptive feedback network

2. Analysis of Power-Stage of DC-DC Converter
   • Operating regions of 2\textsuperscript{nd}-order systems
   • Phase margin of power-stage of DC-DC converter

3. Ripple Reduction for DC-DC Converters
   • Linear swept frequency modulation
   • Passive and active LC Harmonic Notch filters

4. Conclusions
1. Research Background

Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:

\[ \text{SNR} = \frac{\text{Signal power}}{\text{Noise power}} \]

Performance of a device

Figure of Merit:

\[ F = \frac{\text{Output SNR}}{\text{Input SNR}} \]

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

DC-DC converters

- Overshoot,
- Ringing
- Ripple
1. Research Background

Effects of Ripple and Ringing on Electronic Systems

- **Ringing** is overshoot/undershoot voltage or current when it’s seen on time domain.
- **Ripple** is wasted power, and has many undesirable effects in a DC circuit.

Ringing does the following things:
- Causes EMI noise,
- Increases current flow,
- Consumes the power,
- Decreases the performance,
- Damages the devices.

Ripple does the following things:
- Heats components,
- Increases noise,
- Creates the distortion,
- Causes digital circuits to operate improperly.
1. Research Background

Objectives of Study

• **Ringing test** for **power-stage of DC-DC buck converters**.

• **Ripple reduction** using linear swept frequency modulation and LC harmonic notch filters

• **Proposed design** of an **active inductor** for the harmonic notch filter for DC-DC buck converter using a general impedance converter

• **Measurement of self-loop function** in **power-stage** of DC-DC buck converter
1. Research Background

Achievements of Study

Alternating current conservation for deriving self-loop function

\[ L(\omega) = -\frac{V_{\text{inc}}}{V_{\text{trans}}} \]

Implemented circuit

Stability test for power-stage

Derivation of self-loop function

Phase margin of power stage

Incident current

Balun transformer

Transmitted current

LC notch filter

Magnitude (dB)

Phase margin

49 degrees

Without notch harmonic filter

With notch harmonic filter

Phase margin

131°

46 degrees

Phase margin

134°
1. Research Background

Approaching Methods

Simplified power-stage

![Simplified power-stage diagram](image)

Balun transformer

Measurement set up

Design of DC-DC buck converter

![Design of DC-DC buck converter](image)
1. Research Background
Characteristics of Adaptive Feedback Network

Block diagram of a typical adaptive feedback system

Adaptive feedback is used to control the output source along with the decision source (DC-DC Buck converter).
Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network.

→ Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).
1. Research Background

Self-loop Function in A Transfer Function

Linear system

\[ H(\omega) \]

\[ \begin{align*}
    \text{Input} & \quad V_{in}(\omega) \quad \rightarrow \\
    \text{Output} & \quad V_{out}(\omega) \quad \rightarrow \quad H(\omega)
\end{align*} \]

Transfer function

\[ H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)} \]

- Polar chart \( \rightarrow \) Nyquist chart
- Magnitude-frequency plot
- Angular-frequency plot
- Magnitude-angular diagram \( \rightarrow \) Nichols diagram

Model of a linear system

\[ H(\omega) = \frac{b_0 (j\omega)^n + \ldots + b_{n-1} (j\omega) + b_n}{a_0 (j\omega)^n + \ldots + a_{n-1} (j\omega) + a_n} \]

- \( A(\omega) \) : Open loop function
- \( H(\omega) \) : Transfer function
- \( L(\omega) \) : Self-loop function

Variable: angular frequency \((\omega)\)

Bode plots
1. Research Background

Alternating Current Conservation

Transfer function

\[ H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + \frac{Z_{in}}{Z_{out}}} \]

\[ \Rightarrow L(\omega) = \frac{Z_{in}}{Z_{out}} \]

Self-loop function

\[ \frac{V_{inc}}{Z_{in}} = -\frac{V_{trans}}{Z_{out}} \Rightarrow L(\omega) = -\frac{V_{inc}}{V_{trans}} = \frac{Z_{in}}{Z_{out}} \]

10 mH inductance

Simplified linear system

Derivation of self-loop function
1. Research Background
Limitations of Conventional Methods

- **Middlebrook’s measurement of loop gain**
  → Applying only in feedback systems (DC-DC converters).

- **Replica measurement of loop gain**
  → Using two identical networks (not real measurement).

- **Nyquist’s stability condition**
  → Theoretical analysis for feedback systems (Lab tool).

- **Nichols chart of loop gain**
  → Only used in feedback control theory (Lab tool).
1. Research Background
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2. Analysis of Power-Stage of DC-DC Converter
   • Operating regions of 2\textsuperscript{nd}-order systems
   • Phase margin of power-stage of DC-DC converter

3. Ripple Reduction for DC-DC Converters
   • Linear swept frequency modulation
   • Passive and active LC Harmonic Notch filters

4. Conclusions
2. Analysis of Power-Stage of DC-DC Converter

Simulations of Second-order Transfer Function

- **Under-damping:**
  \[ H_1(\omega) = \frac{1}{(j\omega)^2 + j\omega + 1}; \]

- **Critical damping:**
  \[ H_2(\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1}; \]

- **Over-damping:**
  \[ H_3(\omega) = \frac{1}{(j\omega)^2 + 3j\omega + 1}; \]

**Nyquist chart of transfer function**

**Bode plot of transfer function**

- Under-damping
- Critical damping
- Over-damping
## 2. Analysis of Power-Stage of DC-DC Converter Behaviors of Second-order Transfer Function

Second-order transfer function: 

\[ H(\omega) = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega} \]

<table>
<thead>
<tr>
<th>Case</th>
<th>Over-damping</th>
<th>Critically damping</th>
<th>Under-damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta ((\Delta))</td>
<td>(\frac{1}{a_0} &lt; \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 &gt; 0)</td>
<td>(\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 = 0)</td>
<td>(\frac{1}{a_0} &gt; \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 &lt; 0)</td>
</tr>
<tr>
<td>Module</td>
<td>(\frac{1}{a_0} \left(\frac{a_1}{2a_0}\right)^2 \right) \Rightarrow \Delta = a_1^2 - 4a_0 &gt; 0)</td>
<td>(\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 = 0)</td>
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</tr>
<tr>
<td>(</td>
<td>H(\omega)</td>
<td>)</td>
<td>(\sqrt{1 + \left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}} \right) \Rightarrow \Delta = a_1^2 - 4a_0 &gt; 0)</td>
</tr>
<tr>
<td>Angular (\theta(\omega))</td>
<td>(-\arctan\left(\frac{\omega}{a_1} \right) - \frac{1}{a_0} \right) \Rightarrow \Delta = a_1^2 - 4a_0 &gt; 0)</td>
<td>(-2 \arctan\left(\frac{2a_1\omega}{a_1} \right) \Rightarrow \Delta = a_1^2 - 4a_0 = 0)</td>
<td>(-\arctan\left(\frac{\omega}{a_1} \right) - \frac{1}{a_0} \right) \Rightarrow \Delta = a_1^2 - 4a_0 &lt; 0)</td>
</tr>
<tr>
<td>(\omega_{cut} = \frac{a_1}{2a_0})</td>
<td>(</td>
<td>H(\omega_{cut})</td>
<td>&lt; \frac{2a_0}{a_1} \right) \Rightarrow \theta(\omega_{cut}) &gt; -\frac{\pi}{2})</td>
</tr>
</tbody>
</table>
2. Analysis of Power-Stage of DC-DC Converter

Simulations of Second-order Self-loop Function

- **Under-damping:** 
  \[ L_1(\omega) = (j\omega)^2 + j\omega; \]

- **Critical damping:** 
  \[ L_2(\omega) = (j\omega)^2 + 2j\omega; \]

- **Over-damping:** 
  \[ L_3(\omega) = (j\omega)^2 + 3j\omega; \]

**Nyquist chart of self-loop function**

**Bode plot of self-loop function**

**Phase margin**
- **52 degrees**
- **128°**
- **103.7°**
- **92°**
2. Analysis of Power-Stage of DC-DC Converter Behaviors of Second-order Self-loop Function

Second-order self-loop function: \[ L(\omega) = j\omega \left[ a_0 j\omega + a_1 \right] \]

<table>
<thead>
<tr>
<th>Case</th>
<th>Over-damping</th>
<th>Critical damping</th>
<th>Under-damping</th>
</tr>
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<tbody>
<tr>
<td>Delta ((\Delta))</td>
<td>(\Delta = a_1^2 - 4a_0 &gt; 0)</td>
<td>(\Delta = a_1^2 - 4a_0 = 0)</td>
<td>(\Delta = a_1^2 - 4a_0 &lt; 0)</td>
</tr>
<tr>
<td>(</td>
<td>L(\omega)</td>
<td>)</td>
<td>(\omega \sqrt{(a_0 \omega)^2 + a_1^2})</td>
</tr>
<tr>
<td>(\theta(\omega))</td>
<td>(\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1})</td>
<td>(\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1})</td>
<td>(\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1})</td>
</tr>
</tbody>
</table>

- \(\omega_1 = \frac{a_1}{2a_0} \sqrt{5} - 2\)
  - \(|L(\omega_1)| > 1\)
  - \(\pi - \theta(\omega_1) > 76.3^\circ\)
  - \(|L(\omega_1)| = 1\)
  - \(\pi - \theta(\omega_1) = 76.3^\circ\)
  - \(|L(\omega_1)| < 1\)
  - \(\pi - \theta(\omega_1) < 76.3^\circ\)

- \(\omega_2 = \frac{a_1}{2a_0}\)
  - \(|L(\omega_2)| > \sqrt{5}\)
  - \(\pi - \theta(\omega_2) > 63.4^\circ\)
  - \(|L(\omega_2)| = \sqrt{5}\)
  - \(\pi - \theta(\omega_2) = 63.4^\circ\)
  - \(|L(\omega_2)| < \sqrt{5}\)
  - \(\pi - \theta(\omega_2) < 63.4^\circ\)

- \(\omega_3 = \frac{a_1}{a_0}\)
  - \(|L(\omega_3)| > 4\sqrt{2}\)
  - \(\pi - \theta(\omega_3) > 45^\circ\)
  - \(|L(\omega_3)| = 4\sqrt{2}\)
  - \(\pi - \theta(\omega_3) = 45^\circ\)
  - \(|L(\omega_3)| < 4\sqrt{2}\)
  - \(\pi - \theta(\omega_3) < 45^\circ\)
2. Analysis of Power-Stage of DC-DC Converter

Summary of Operating Regions of 2\textsuperscript{nd}-Order System

- **Over-damping:**
  - Phase margin is 88 degrees.

- **Critical damping:**
  - Phase margin is 76.3 degrees.

- **Under-damping:**
  - Phase margin is 52 degrees.
2. Analysis of Power-Stage of DC-DC Converter
Behaviors of power-stage of DC-DC Converter

Parallel RLC low-pass filter

Transfer function & self-loop function:

\[ H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega}; \]

\[ L(\omega) = a_0 (j\omega)^2 + a_1 j\omega; \]

Where: \( a_0 = LC; \quad a_1 = \frac{L}{R}; \)

\( \omega_0 = \frac{1}{\sqrt{LC}}; \)

\( |Z_L| = \omega_0 L; \quad |Z_C| = \frac{1}{\omega_0 C}; \)

Operating regions

- **Over-damping:** \( \frac{1}{LC} < \left( \frac{R}{2L} \right)^2 \iff |Z_L| = |Z_C| < R / 2 \)

- **Critical damping:** \( \frac{1}{LC} = \left( \frac{R}{2L} \right)^2 \iff |Z_L| = |Z_C| = R / 2 \)

- **Under-damping:** \( \frac{1}{LC} > \left( \frac{R}{2L} \right)^2 \iff |Z_L| = |Z_C| > R / 2 \)

Balanced charging-discharging time condition:

\( |Z_L| = |Z_C| = 2R \)
2. Analysis of Power-Stage of DC-DC Converter

Phase Margin of Power-Stage of DC-DC Converter

Simplified power-stage

\[ L_1 = 220 \, \text{uH}, \ C_1 = 100 \, \text{uF}, \ R_1 = 5 \, \Omega, \]

Implemented circuit

Bode plot of transfer function

Nichols plot of self-loop function

Measured results

Phase margin 46 degrees

Under-damping region
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4. Conclusions
3. Ripple Reduction for DC-DC Converters

Ripple Reduction using Linear Swept Frequency Modulation

Block diagram of DC-DC buck converter

Without frequency modulation

With frequency modulation

Linear swept frequency modulation

Without frequency modulation

With frequency modulation
3. Ripple Reduction for DC-DC Converters

Ripple Reduction using LC Harmonic Notch Filter

Schematic diagram of DC-DC buck converter

Switching Stage

Power Stage

DC Input

PWM CONTROL BLOCK

D1

L1

C1

R2

Cf

R1

C2

RL

DC Output

5.0 V

Transfer function & self-loop function

\[ H(\omega) = \frac{b_0(j\omega)^2 + 1}{a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega + 1} \]

\[ L(\omega) = a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega \]

Where,

\[ b_0 = L_2 C_2; a_0 = L_1 C_1 L_2 C_2; \]

\[ a_1 = \frac{L_1 L_2 C_2}{R_L}; a_2 = L_1 C_1 + L_2 C_2 + L_1 C_2; a_3 = \frac{L_1}{R_L}; \]
3. Ripple Reduction for DC-DC Converters
Passive and Active LC Harmonic Notch Filters

Schematic diagram of DC-DC converter

Approximated value of inductor
\[ Z_L = \frac{R_2 R_3}{R_1} \frac{Z_{out}}{Z_C} = \frac{R_2 R_3}{R_1} sCZ_{out} \]

Passive component parameters
\[ L_1 = 220 \text{ uH}, \quad C_1 = 100 \text{ uF}, \quad R_1 = 4 \text{ k}\Omega, \quad R_2 = 2 \text{ k}\Omega, \quad R_L = 5 \text{ } \Omega, \quad C_2 = 65 \text{ nF}, \quad L_2 = 12 \text{ uH}, \quad R_3 = 15 \text{ k}\Omega, \quad C_1 = 100 \text{ pF}, \quad R_4 = 1 \text{ k}\Omega, \quad R_5 = 1 \text{ k}\Omega, \quad \text{and } f_n \text{ at } 180 \text{ kHz.} \]
3. Ripple Reduction for DC-DC Converters

Implemented Circuit for DC-DC Converter

### Design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage (Vin)</td>
<td>12 V</td>
</tr>
<tr>
<td>Output Voltage (Vo)</td>
<td>5.0 V</td>
</tr>
<tr>
<td>Output Current (Io)</td>
<td>1 A</td>
</tr>
<tr>
<td>Clock Frequency (Fck)</td>
<td>180 kHz</td>
</tr>
<tr>
<td>Output Ripple</td>
<td>&lt; 10 mVpp</td>
</tr>
</tbody>
</table>

### Measurement set up

Implemented circuit
3. Ripple Reduction for DC-DC Converters

Measurement Results of Proposed Design Circuit

- **Bode plot of transfer function**
- **Nichols plot of self-loop function**

○ Reduce the cut-off frequency of the power-stage
  → Reduce the ripple caused by high-order harmonic signals

○ Improvement of phase margin of the power-stage
  → Reduce the overshoot caused by the passive components
3. Ripple Reduction for DC-DC Converters

Ripple Reduction using Harmonic Notch Filter

Schematic diagram of DC-DC Buck converter

Implemented circuit

**Without notch filter**

**With notch filter**

**Output spectrum**

Output ripple

Ripple Reduction 5 mVpp

Output spectrum

Conventional buck converter

Output spectrum

Spectrum reduction

-35 dBV → -38 dBV at 180 kHz

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4. Conclusions
### 4. Comparison

<table>
<thead>
<tr>
<th>Features</th>
<th>This work</th>
<th>Replica measurement</th>
<th>Middlebrook’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main objective</td>
<td>Self-loop function</td>
<td>Loop gain</td>
<td>Loop gain</td>
</tr>
<tr>
<td>Transfer function accuracy</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Ringing Test</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Operating region accuracy</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Phase margin accuracy</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Passive networks</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
4. Discussions

- Loop gain is independent of frequency variable.
- Loop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.

Nichols chart isn’t used widely in practical measurements (only used in control theory).

https://www.mathworks.com/help/control/ref/nichols.html

(Time limitations)
4. Conclusions

This work:

• **Investigation of behaviors** of power-stage in DC-DC converters based on alternating current conservation

• **Proposed designs of passive and active LC harmonic notch filters** for ripple reduction
  → **Phase margin improvement** from 46 degrees into 49 degrees
  → **Ripple reduction** from 11 mVpp into 5 mVpp

Future of work:

• **Stability test** for dynamic loads, and **parasitic components** in printed circuit boards
References


Thank you very much!
谢谢