ICSICT-2020
2020 IEEE 15th International Conference on Solid-State and Integrated Circuit Technology
Nov. 3-6, 2020
Wyndham Grand Plaza Royale Colorful Hotel, Kunming, China

DESIGN OF SIXTH-ORDER PASSIVE QUADRATURE SIGNAL GENERATION NETWORK BASED ON POLYPHASE FILTER

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Outline

1. **Research Background**
   - Motivation, objectives and achievements
   - Review of characteristics of Low-IF receiver
2. **Investigation of Multi-Phase Networks**
   - Superposition theorem for multi-source networks
   - Design principle for passive polyphase filters
3. **Proposed Designs and Experimental Results**
   - Analysis of quadrature signal generation networks
   - Simulation and measurement results
4. **Conclusions**

1. Research Background

Motivation of Study

Performance of a system

Signal to Noise Ratio:

$$SNR = \frac{\text{Signal power}}{\text{Noise power}}$$

Performance of a device

Figure of Merit:

$$F = \frac{\text{Output SNR}}{\text{Input SNR}}$$

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

Multi-phase networks

- Image noise,
- I/Q mismatches
- DC offsets

1. Research Background

Objectives of Study

- **Derivation of transfer function in multi-source systems using superposition theorem**
- **Investigation of behaviors of high-order passive RC polyphase filter networks**
- **Analysis and design of quadrature signal generation network for measuring frequency response of polyphase and complex filters**
- **Implementation of 6th-oder polyphase signal generation based on polyphase filter circuits**

1. Research Background

Achievements of Study

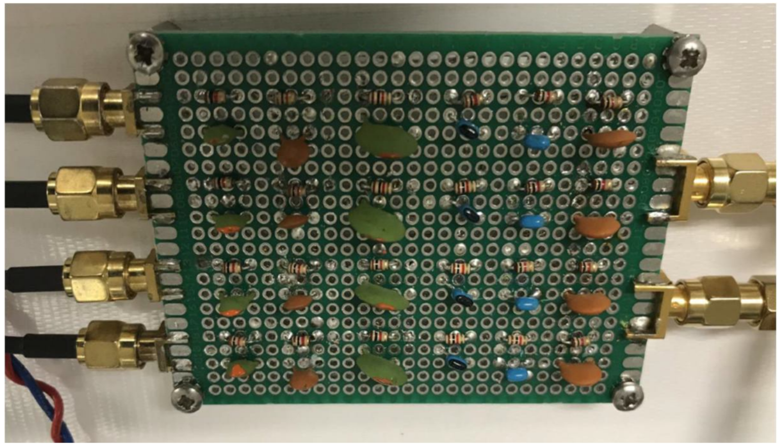
Superposition formula for multi-source networks

$$V_O(t)\sum_{i=1}^{n}\frac{1}{Z_i}+V_O(t)\sum_{i=1}^{n}\frac{1}{Z_{si}+\frac{1}{\sum_{k=1}^{n}\frac{1}{Z_{pik}}}}=\sum_{i=1}^{n}\left(\frac{V_i(t)}{Z_i}+I_{ai}(t)-I_{gi}(t)\right)$$

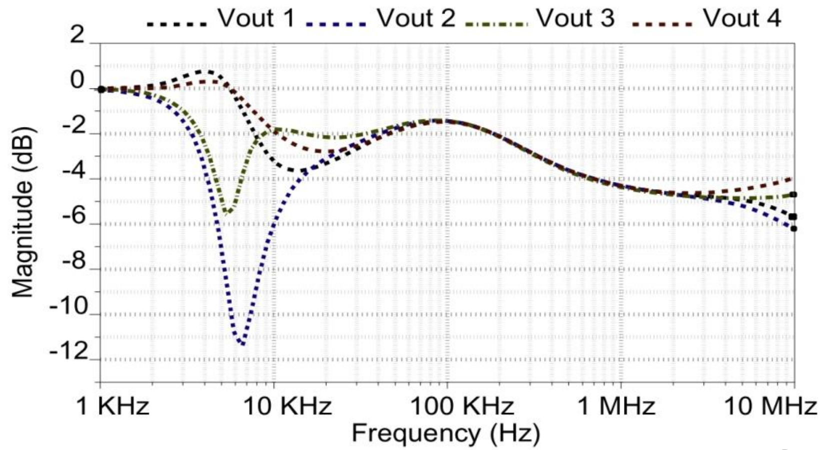
Conventional superposition

→ Solving for every source (several times).

Implemented circuit

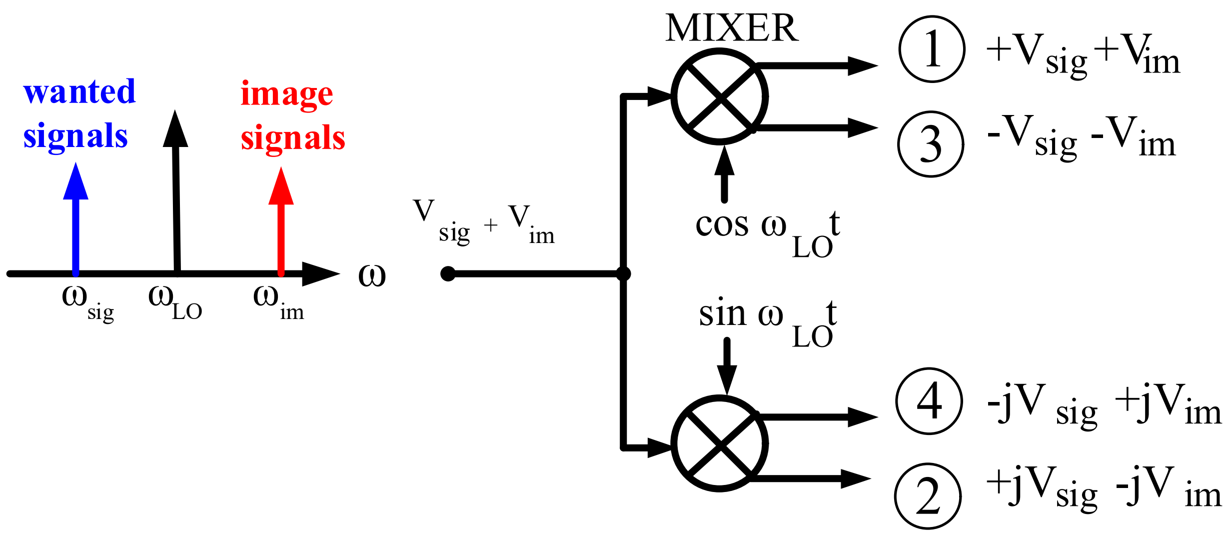


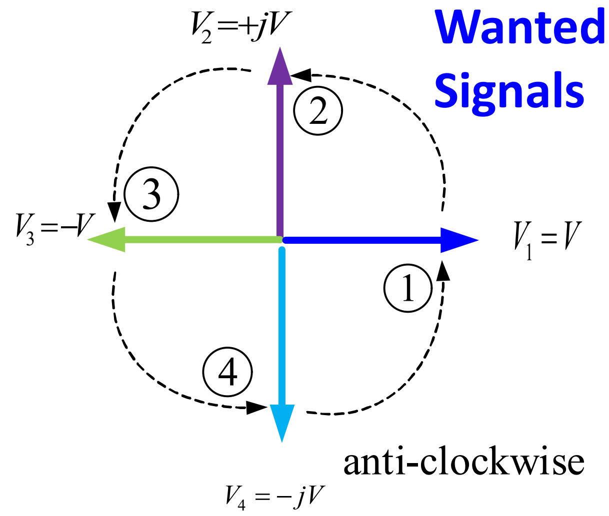
Experimental results

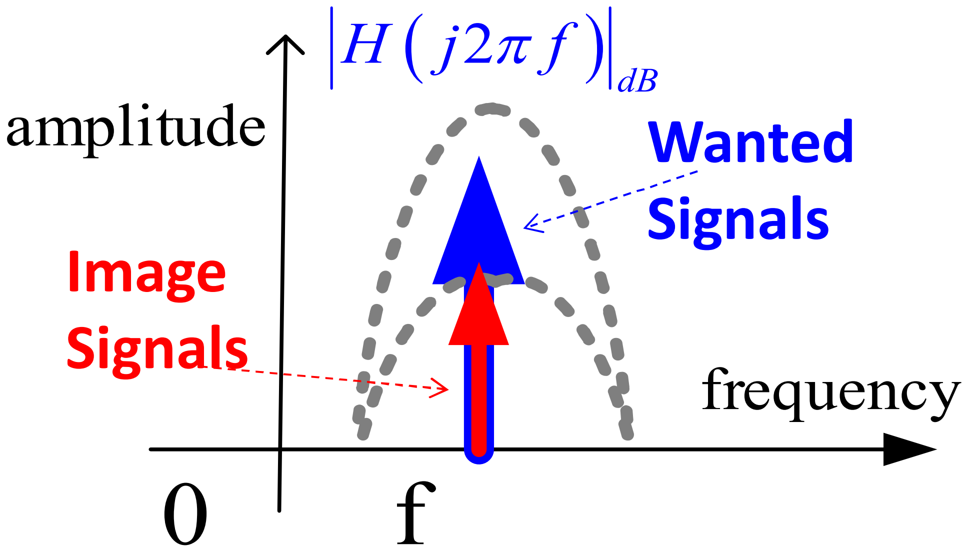


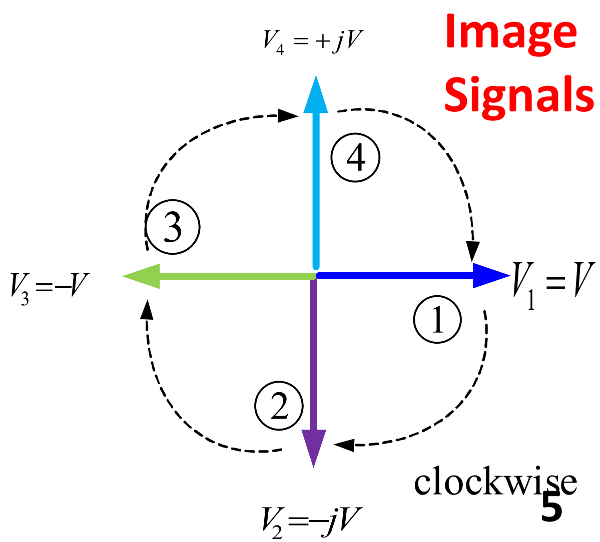
1. Research Background

Characteristics of Low-IF Receiver Signals





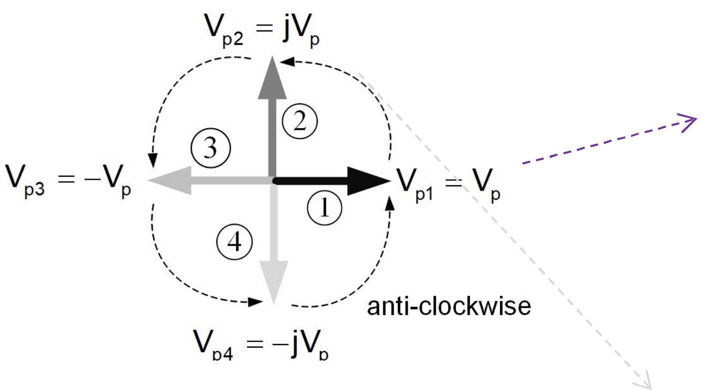


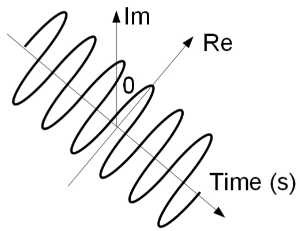


1. Research Background

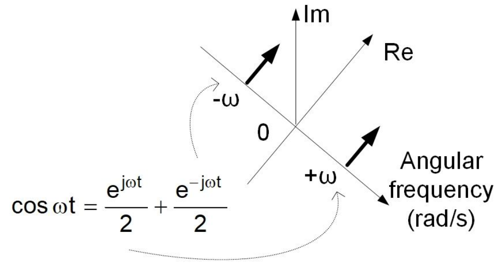
Positive Polyphase Signals on Frequency Domain

Positive polyphase signals

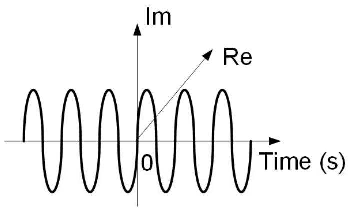




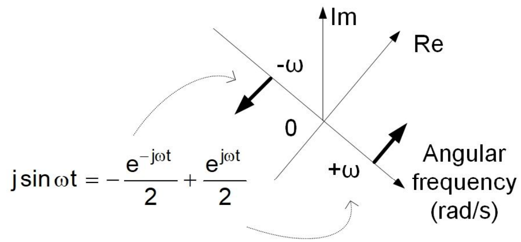
(a) Cosine wave on real axis



(b) Spectrum of cosine wave

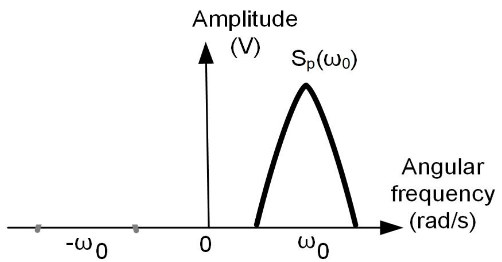


(c) Plus sine wave on imaginary axis

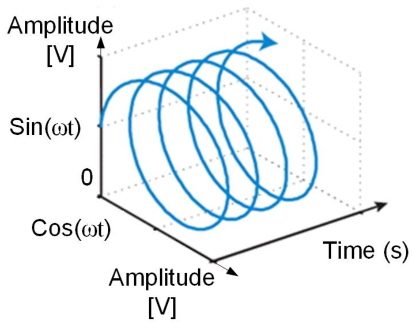


(d) Spectrum of plus sine wave

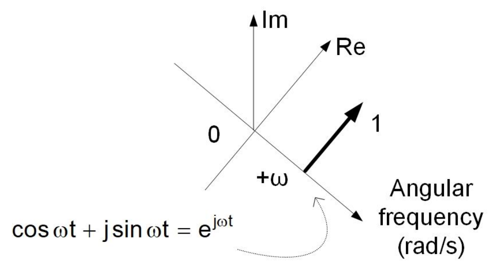
$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;+j;(+j)^2;(+j)^3\}V_{pos}(t)$$



(c) Angular frequency plane for polyphase signals



(e) Positive angular frequency wave

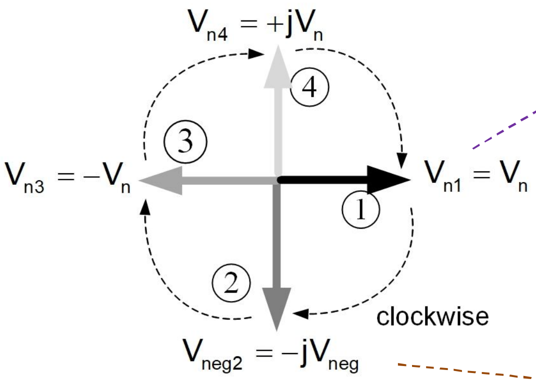


(f) Spectrum of positive angular frequency wave

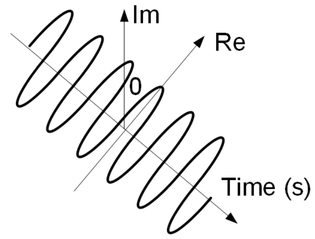
1. Research Background

Negative Polyphase Signals on Frequency Domain

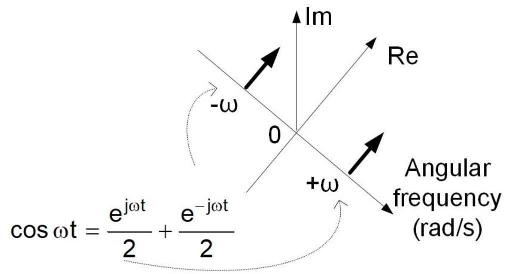
Negative polyphase signals



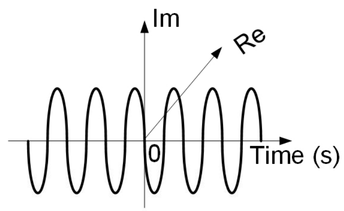
$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;-j;(-j)^2;(-j)^3\}V_{neg}(t)$$



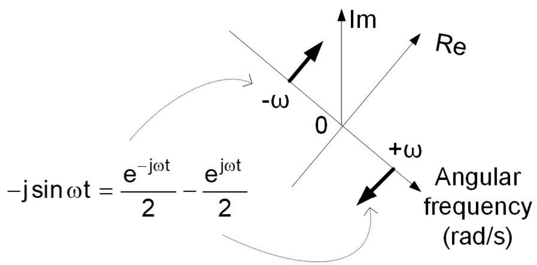
(a) Cosine wave on real plane



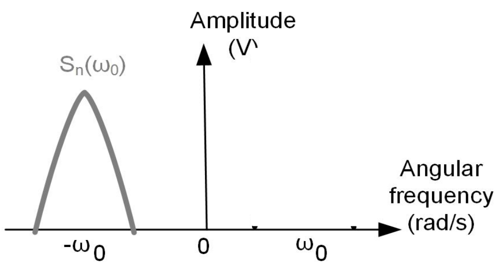
(b) Spectrum of cosine wave



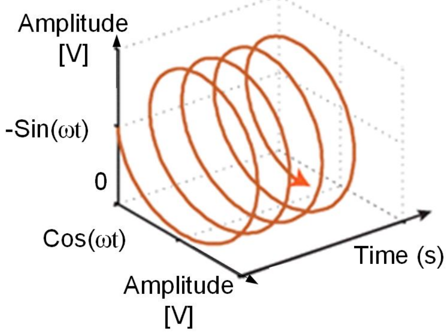
(c) Minus sine wave on imaginary plane



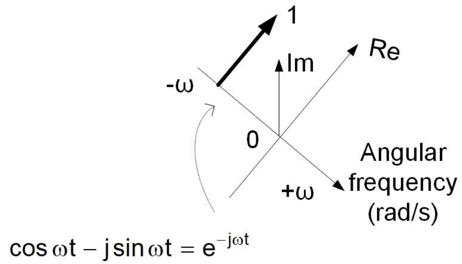
(d) Spectrum of minus sine wave



(c) Angular frequency plane for polyphase signals



(e) Negative angular frequency wave

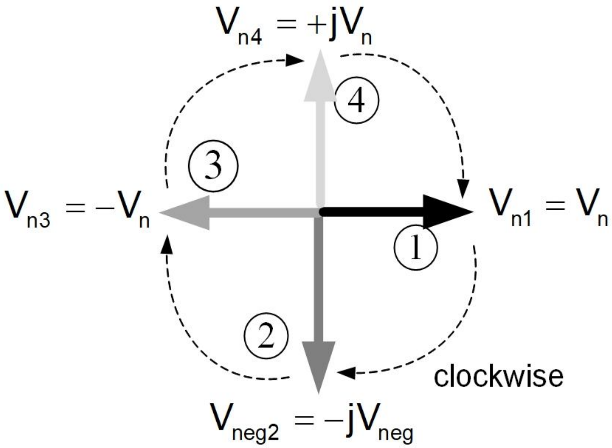


(f) Spectrum of negative angular frequency wave

1. Research Background

Polyphase Signals on Frequency Domain

Negative polyphase signals



Positive polyphase signals

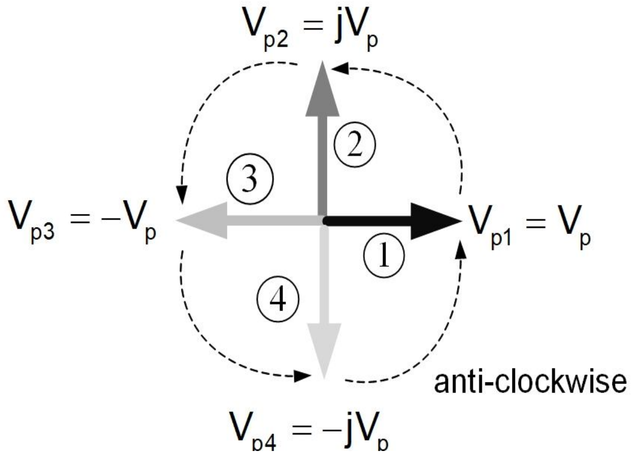
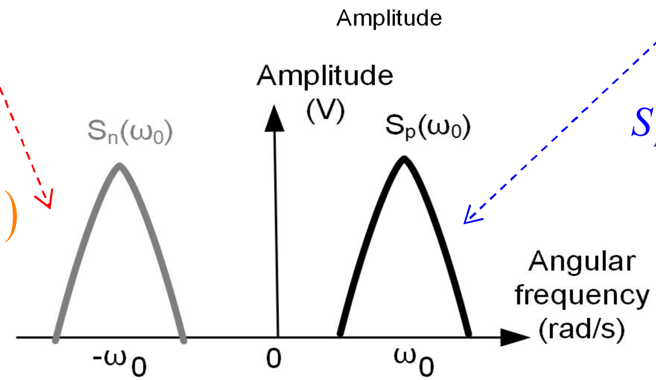


Image Signals

$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;-j;(-j)^2;(-j)^3\}V_{neg}(t)$$



(c) Angular frequency plane for polyphase signals

Wanted Signals

$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;+j;(+j)^2;(+j)^3\}V_{pos}(t)$$

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- **Superposition theorem for multi-source networks**
- **Design principle for passive polyphase filters**

3. Proposed Designs and Experimental Results
- Analysis of quadrature signal generation networks
- Simulation and measurement results

4. Conclusions

2. Investigation of Multi-Phase Networks

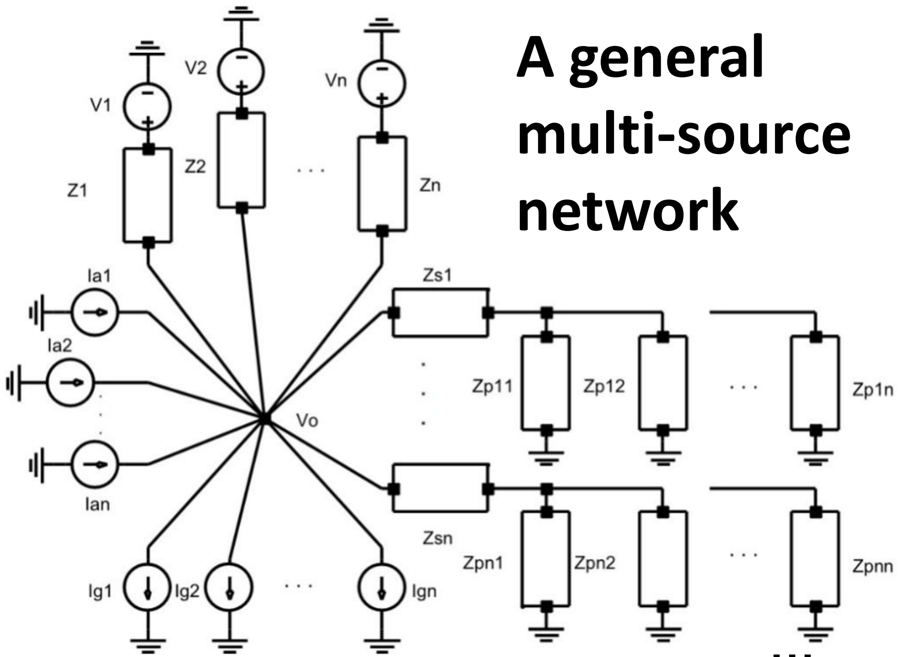
Superposition Theorem for Multi-Source Systems

Superposition formula:

$$V_O(t)\sum_{i=1}^{n}\frac{1}{Z_i}+V_O(t)\sum_{i=1}^{n}\frac{1}{Z_{si}+\frac{1}{\sum_{k=1}^{n}\frac{1}{Z_{pik}}}}=\sum_{i=1}^{n}\left(\frac{V_i(t)}{Z_i}+I_{ai}(t)-I_{gi}(t)\right)$$

$V_O(t)$: ***Voltage at one node***
$V_i(t)$: ***Input voltage sources***
$I_{ai}(t)$: ***Ahead-toward current sources***
$I_{gi}(t)$: ***Ground-toward current sources***
$Z_{i,\ si,\ pi,}(t)$: Impedances at each branch

○ **Multi-source systems, feedback networks (op amps, amplifiers), polyphase filters, complex filters…**



A general multi-source network

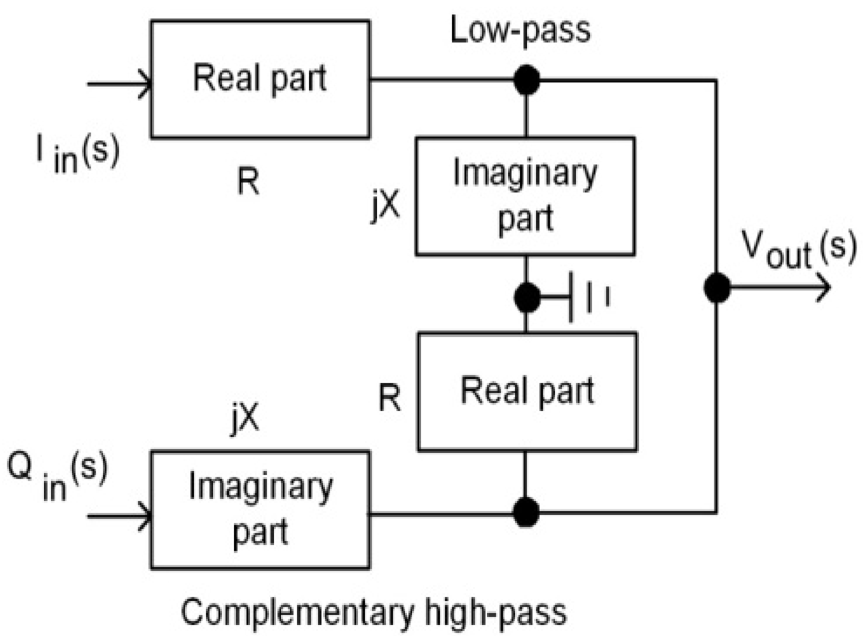
2. Investigation of Multi-Phase Networks

Design Principle for Polyphase Filter Networks

Complementation between low-pass and high-pass circuits

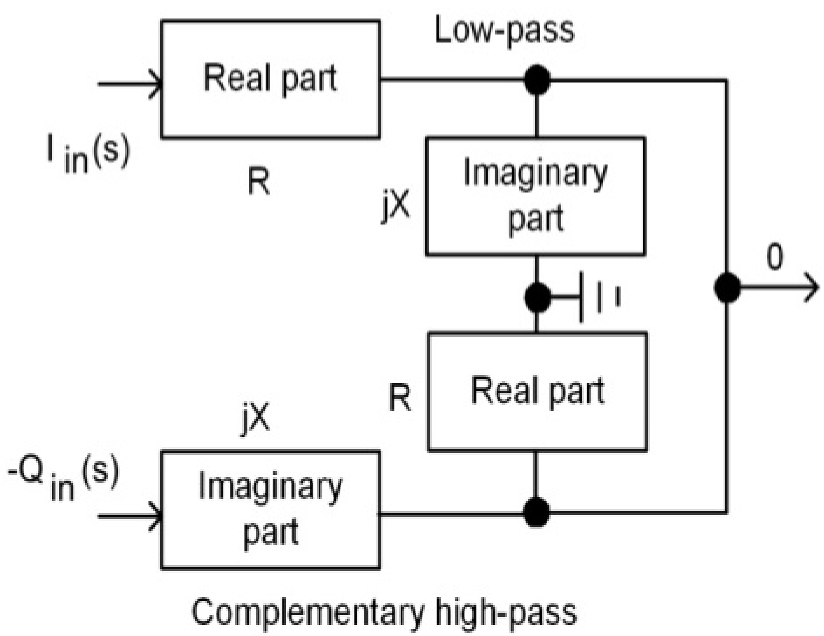
→ a passive polyphase filter

Wanted Signals



Pass-band filter (wanted signals)

Image Signals

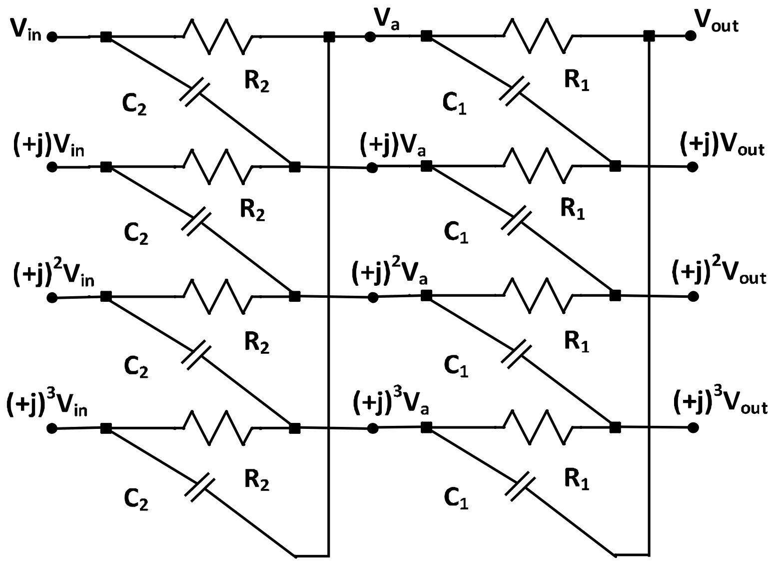


Notch-band filter (image signals)

2. Investigation of Multi-Phase Networks

Analysis of 2nd–Order Polyphase Filter

2nd-order RC polyphase filter



Apply superposition at each node

$$V_{out}\left(\frac{1}{Z_{C1}}+\frac{1}{R_1}\right)=\frac{V_a}{R_1}+\frac{(+j)^3V_a}{Z_{C1}};$$

$$V_a\left(\frac{1}{Z_{C2}}+\frac{1}{R_2}+\frac{2}{R_1+Z_{C1}}\right)=\frac{V_{in}}{R_2}+\frac{(+j)^3V_{in}}{Z_{C2}};$$

Transfer function for positive polyphase signal

$$H_P(\omega)=\frac{V_{out}}{V_{in}}=\frac{\left[1+(+j)^3b_1j\omega\right]\left[1+(+j)^3b_2j\omega\right]}{a_0(j\omega)^2+a_1j\omega+1};$$

Transfer function for negative polyphase signal

$$H_N(\omega)=\frac{V_{out}}{V_{in}}=\frac{\left[1+(-j)^3b_1j\omega\right]\left[1+(-j)^3b_2j\omega\right]}{a_0(j\omega)^2+a_1j\omega+1};$$

Here: $b_0=R_1C_1;b_1=R_2C_2;\ a_0=b_0b_1;\ a_1=b_0+b_1+2R_2C_1;$

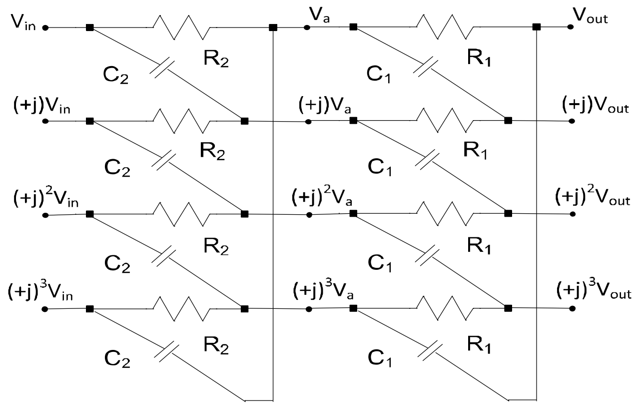
Image rejection ratio (IRR)

$$IRR(\omega)=\frac{|H_P(\omega)|}{|H_N(\omega)|}=\frac{|(1+b_1\omega)(1+b_2\omega)|}{|(1-b_1\omega)(1-b_2\omega)|};$$

2. Investigation of Multi-Phase Networks

Behaviors of 2nd–Order Polyphase Filter

2-order RC polyphase filter

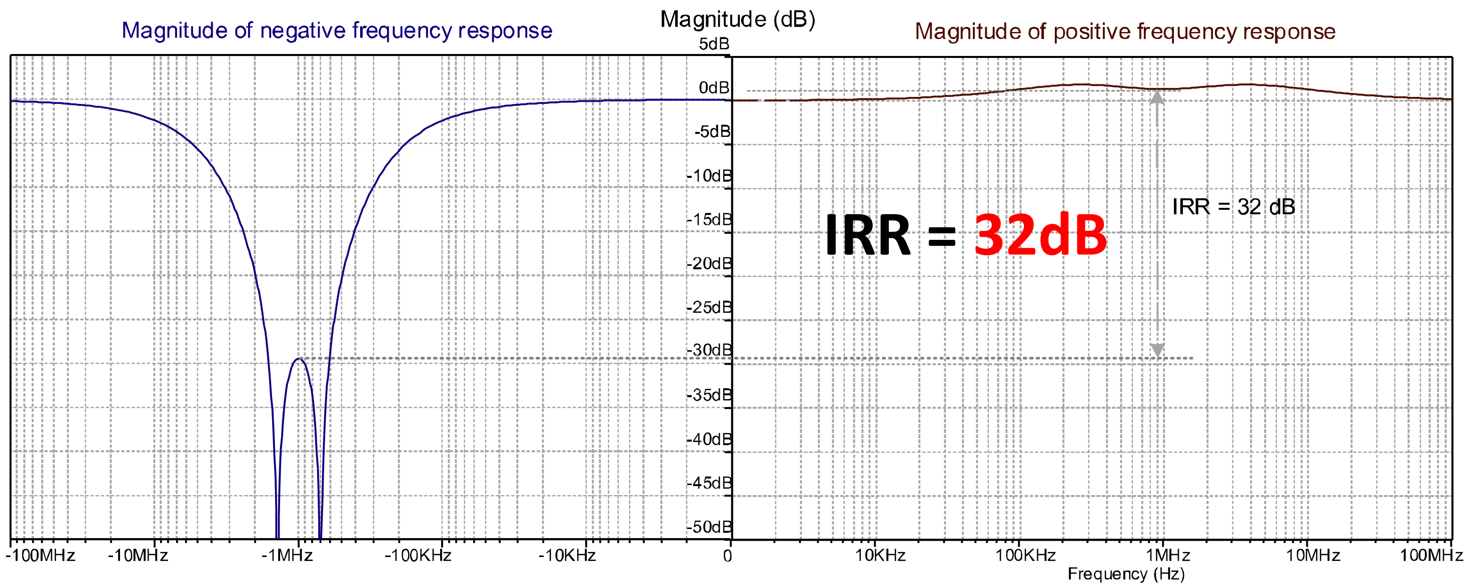


Transfer function in all frequency domain

$$\left|H(\omega)\right| = \frac{(1+b_1\omega)(1+b_2\omega)}{\sqrt{\left(1-a_0\omega^2\right)^2+\left(a_1\omega\right)^2}};\omega \in R$$

Here, R1 = 1 kΩ, C1 = 227 pF, R2 = 1 kΩ, C2 = 114 pF, at f_1 = 700 kHz, f_2 = 1.4 MHz,

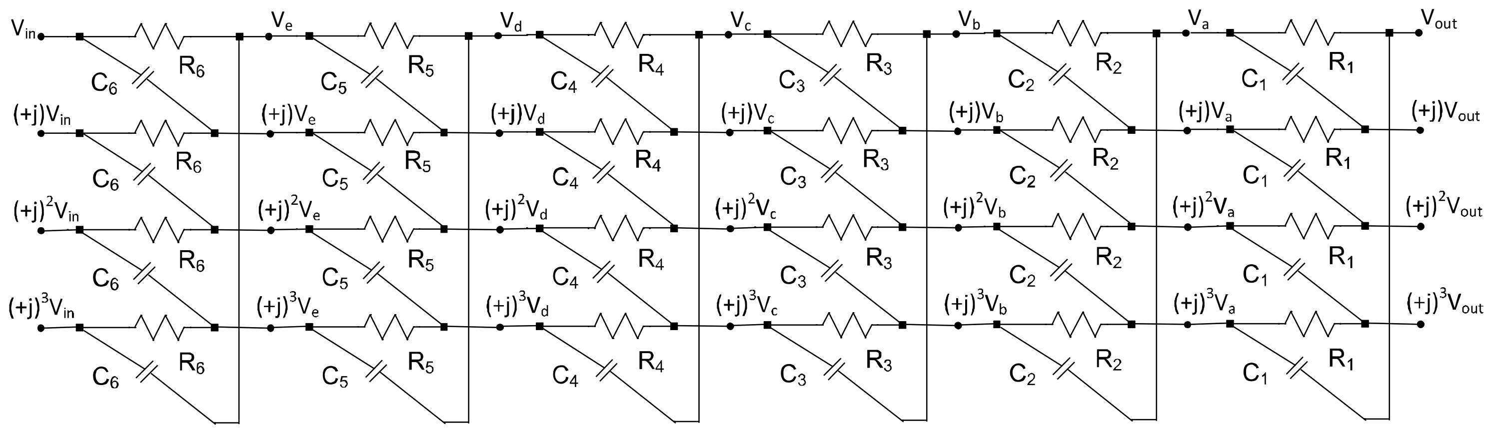
Bode plot of transfer function in all frequency domain



2. Investigation of Multi-Phase Networks

Transfer Function of 6th-Order RC Polyphase Filter

6th-order polyphase filter



Apply the superposition principle at nodes V_a , V_b, V_c, V_d, V_e, V_{out} , we get

$$V_{out}\left(\frac{1}{Z_{C1}}+\frac{1}{R_1}\right)=\frac{V_a}{R_1}+\frac{(+j)^3 V_a}{Z_{C1}};V_a\left(\frac{1}{Z_{C2}}+\frac{1}{R_2}+\frac{2}{M_A}\right)=\frac{V_a}{R_2}+\frac{(+j)^3 V_b}{Z_{C2}};$$

$$V_b\left(\frac{1}{Z_{C3}}+\frac{1}{R_3}+\frac{1}{M_{B1}}+\frac{1}{M_{B2}}\right)=\frac{V_c}{R_3}+\frac{(+j)^3 V_c}{Z_{C3}};V_c\left(\frac{1}{Z_{C4}}+\frac{1}{R_4}+\frac{1}{M_{C1}}+\frac{1}{M_{C2}}\right)=\frac{V_d}{R_4}+\frac{(+j)^3 V_d}{Z_{C4}};$$

$$V_d\left(\frac{1}{Z_{C5}}+\frac{1}{R_5}+\frac{1}{M_{D1}}+\frac{1}{M_{D2}}\right)=\frac{V_e}{R_5}+\frac{(+j)^3 V_e}{Z_{C5}};V_e\left(\frac{1}{Z_{C6}}+\frac{1}{R_6}+\frac{1}{M_{E1}}+\frac{1}{M_{E2}}\right)=\frac{V_{in}}{R_6}+\frac{(+j)^3 V_{in}}{Z_{C6}};$$

Transfer function

$$H(\omega)=\frac{[1+b_1\omega][1+b_2\omega][1+b_3\omega][1+b_4\omega][1+b_5\omega][1+b_6\omega]}{a_0(j\omega)^6+a_1(j\omega)^5+a_2(j\omega)^4+a_3(j\omega)^3+a_4(j\omega)^2+a_5 j\omega+1};$$

2. Investigation of Multi-Phase Networks

Characteristics of 6th-order RC Polyphase Filter

Behaviors of transfer function in all frequency domain

$$
\left|H_p(\omega)\right| = \begin{cases} \lim\limits_{\omega\to 0^+} \left|H_p(\omega)\right| = 1 \\ Max_1\left\{H_P(\omega)\right\} \; as \; \omega_{\max 1} \\ Min\left\{H_p(\omega)\right\} \quad as \; \omega_{\min} = \dfrac{1}{\sqrt[6]{b_1 b_2 b_3 b_4 b_5 b_6}}; \\ Max_2\left\{H_p(\omega)\right\} \; as \; \omega_{\max 2} \\ \lim\limits_{\omega\to +\infty} \left|H_p(\omega)\right| = 1 \end{cases}
$$

$$
\left|H_n(\omega)\right| = \begin{cases} \lim\limits_{\omega\to 0^-} H_n(\omega) = 1; Min_1\left\{H_n(\omega)\right\} = 0 \quad as \; \omega_1 = -\dfrac{1}{b_1} \\ Max_1\left\{H_n(\omega)\right\} \quad as \; \omega_{\max 1} = -\dfrac{1}{\sqrt{b_1 b_2}}; Min_2\left\{H_n(\omega)\right\} \quad as \; \omega_2 = -\dfrac{1}{b_2} \\ Max_2\left\{H_n(\omega)\right\} \quad as \; \omega_{\max 2} = -\dfrac{1}{\sqrt{b_2 b_3}}; Min_3\left\{H_n(\omega)\right\} \quad as \; \omega_3 = -\dfrac{1}{b_3} \\ Max_3\left\{H_n(\omega)\right\} \quad as \; \omega_{\max 3} = -\dfrac{1}{\sqrt{b_3 b_4}}; Min_4\left\{H_n(\omega)\right\} \quad as \; \omega_4 = -\dfrac{1}{b_4} \\ Max_4\left\{H_n(\omega)\right\} \quad as \; \omega_{\max 4} = -\dfrac{1}{\sqrt{b_4 b_5}}; Min_5\left\{H_n(\omega)\right\} \quad as \; \omega_5 = -\dfrac{1}{b_5} \\ Max_5\left\{H_n(\omega)\right\} \quad as \; \omega_{\max 5} = -\dfrac{1}{\sqrt{b_5 b_6}}; Min_6\left\{H_n(\omega)\right\} \quad as \; \omega_6 = -\dfrac{1}{b_6}; \lim\limits_{\omega\to -\infty} H_n(\omega) = 1 \end{cases} ;
$$

$$
\theta_P(\omega) = \begin{cases} \theta_{P1} = 0 \; ; \; \omega \to 0^+ \\ \theta_{P2} = \dfrac{-\pi}{4} \; ; \; \omega_{\max 1} \\ \theta_{P3} = \dfrac{-3\pi}{2} \; ; \; \omega_{\min} = \dfrac{1}{\sqrt[6]{b_1 b_2 b_3 b_4 b_5 b_6}} \\ \theta_{P4} = \dfrac{-11\pi}{4} \; ; \; \omega_{\max 2} \\ \theta_{P5} = -3\pi \; ; \; \omega \to +\infty \end{cases}
$$

Positive frequency

$$
\theta_n(\omega) = \begin{cases} \theta_1 = 0; \; \omega \to 0^-; \theta_2 = \dfrac{-5\pi}{8} \to \dfrac{3\pi}{8} \; ; \; \omega_1 = -\dfrac{1}{b_1} \\ \theta_3 = \dfrac{\pi}{4} \; ; \; \omega_{\max 1} = -\dfrac{1}{\sqrt{b_1 b_2}}; \theta_4 = \dfrac{\pi}{8} \to \dfrac{9\pi}{8} \; ; \; \omega_2 = -\dfrac{1}{b_2} \\ \theta_5 = \dfrac{15\pi}{16} \; ; \; \omega_{\max 2} = -\dfrac{1}{\sqrt{b_2 b_3}}; \theta_6 = \dfrac{3\pi}{4} \to \dfrac{7\pi}{4}; \; \omega_3 = -\dfrac{1}{b_3} \\ \theta_7 = \dfrac{25\pi}{16} \; ; \; \omega_{\max 3} = -\dfrac{1}{\sqrt{b_3 b_4}}; \theta_8 = \dfrac{11\pi}{8} \to \dfrac{19\pi}{8}; \; \omega_4 = -\dfrac{1}{b_4} \\ \theta_9 = \dfrac{37\pi}{16} \; ; \; \omega_{\max 4} = -\dfrac{1}{\sqrt{b_4 b_5}}; \theta_{10} = \dfrac{9\pi}{4} \to \dfrac{13\pi}{4}; \; \omega_5 = -\dfrac{1}{b_5} \\ \theta_{11} = \dfrac{37\pi}{16} \; ; \; \omega_{\max 5} = -\dfrac{1}{\sqrt{b_5 b_6}}; \theta_{12} = \dfrac{9\pi}{4} \to \dfrac{13\pi}{4}; \; \omega_5 = -\dfrac{1}{b_6}; \theta_{13} = \dfrac{5\pi}{2}; \; \omega \to -\infty \end{cases}
$$

Negative frequency

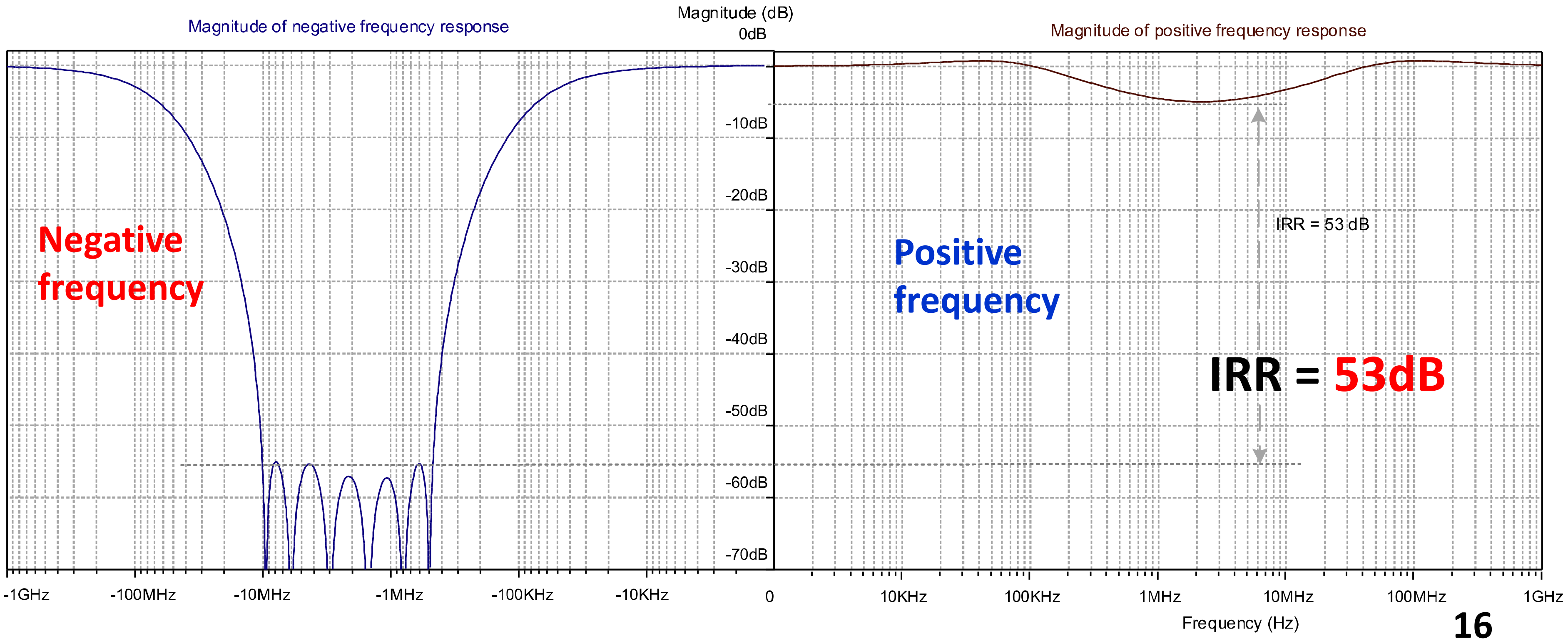
2. Investigation of Multi-Phase Networks

Bode Plot of 6^{th}-Order RC Polyphase Filter

Passive component parameters

R_1 = 1 kΩ, C_1 = 318 pF, R_2 = 1 kΩ, C_2 = 199 pF, R_3 = 1 kΩ, C_3 = 106 pF, R_4 = 1 kΩ, C_4 = 53.1 pF, R_5 = 1 kΩ, C_5 = 26.5 pF, R_6 = 1 kΩ, C_6 = 16.8 pF,
f_1 = 500 kHz, f_2 = 800 kHz, and f_3 = 1.5 MHz, f_4 = 3 MHz, f_5 = 6 MHz, f_6 = 9.5 MHz,

Simulation results of transfer function in all frequency domain



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3. Proposed Designs and Experimental Results

Design Principle for Quadrature Signal Generation

Complementation between low-pass and high-pass circuits

→ a passive quadrature signal generation network

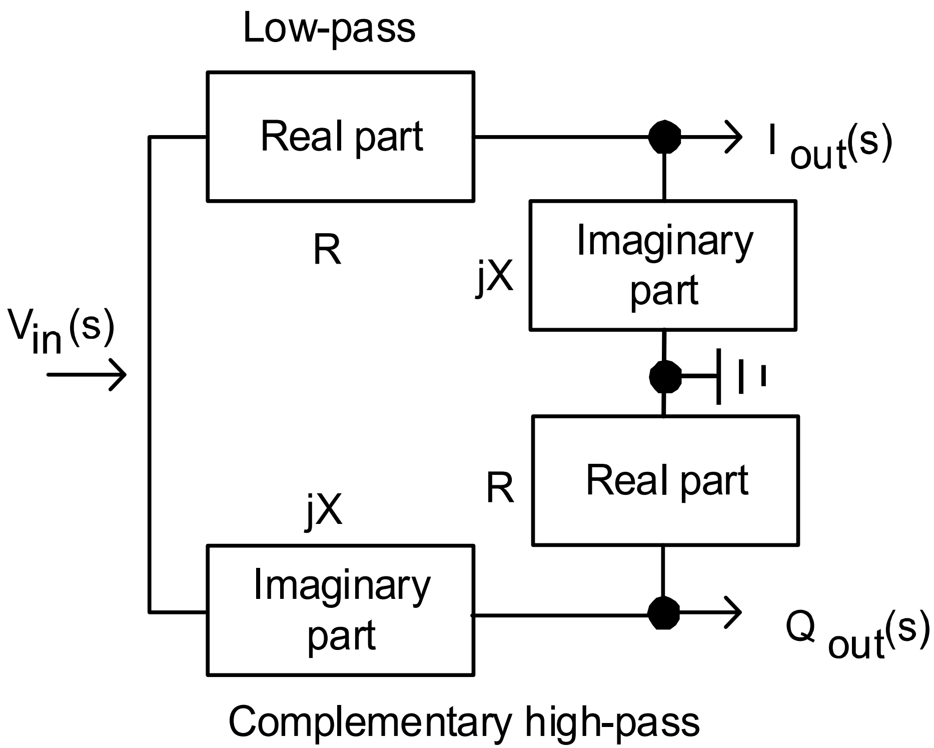
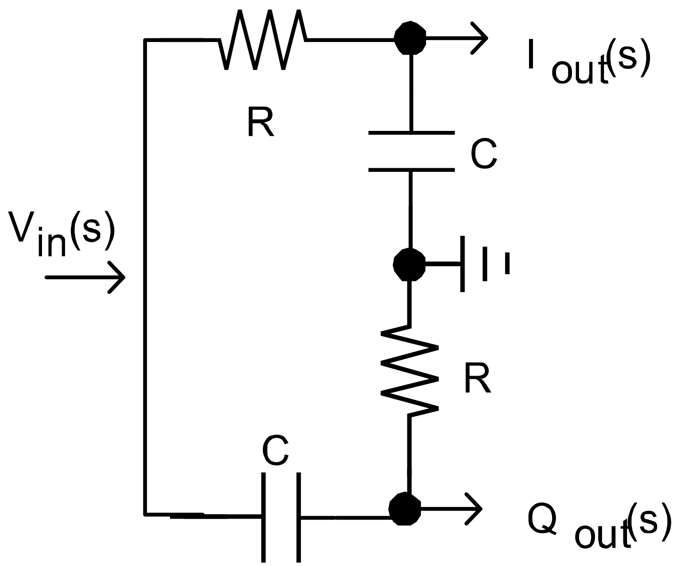


Diagram of a quadratic generator

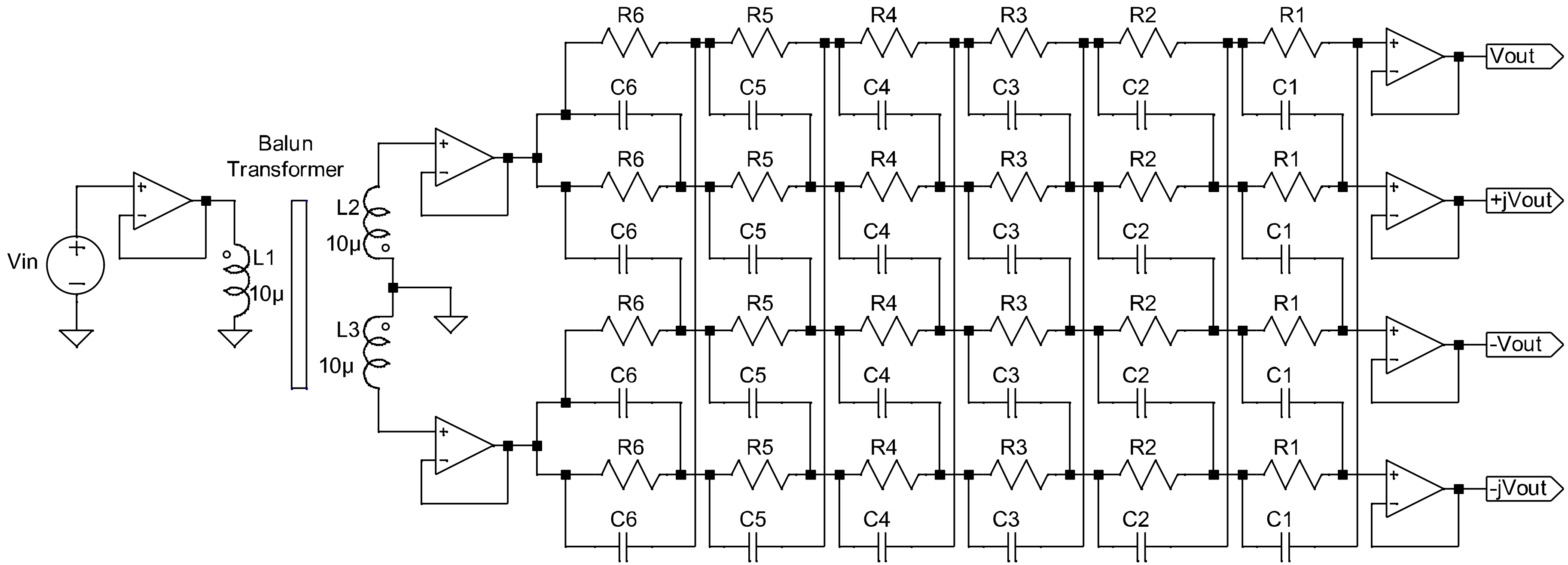


Circuit of a quadratic generator

3. Proposed Designs and Experimental Results

Design of 6th-Order Polyphase Signal Generation

Proposed design of 6th-order polyphase signal generation



Transfer function

$$H(\omega) = \frac{[1+b_1\omega][1+b_2\omega][1+b_3\omega][1+b_4\omega][1+b_5\omega][1+b_6\omega]}{a_0(j\omega)^6 + a_1(j\omega)^5 + a_2(j\omega)^4 + a_3(j\omega)^3 + a_4(j\omega)^2 + a_5 j\omega + 1};$$

3. Proposed Designs and Experimental Results

Behavior of 6th-order Quadrature Signal Generation

Transfer function

$$H(f) = \frac{\left(1+\frac{1}{5*10^5}f\right)\left(1+\frac{1}{8*10^5}f\right)\left(1+\frac{1}{1.5*10^6}f\right)\left(1+\frac{1}{3*10^6}f\right)\left(1+\frac{1}{6*10^6}f\right)\left(1+\frac{1}{9.5*10^6}f\right)}{1.54*10^{-38}(jf)^6+1.85*10^{-30}(jf)^5+2.88*10^{-23}(jf)^4+1.14*10^{-16}(jf)^3+1.35*10^{-10}(jf)^2+4.02*10^{-5}jf+1}$$

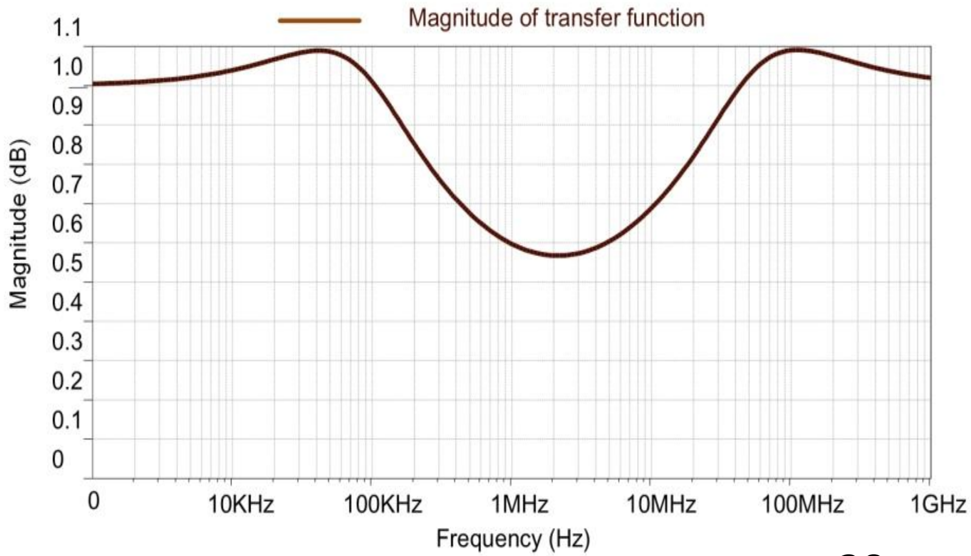
Behaviors of transfer function

$$\left|H(f)\right| = \begin{cases} \lim\limits_{f\to 0^+}\left|H(f)\right| = 1 \\ Max_1 = 1.09;\ f_{\max 1} = 4.08*10^4\,Hz \\ Min = 0.812;\ f_{\min} = 2.2*10^6\,Hz \\ Max_2 = 1.09;\ f_{\max 2} = 107*10^6\,Hz \\ \lim\limits_{f\to +\infty}\left|H(f)\right| = 1 \end{cases};$$

$$\theta(f) = \begin{cases} \theta_{P1} = 0;\ f\to 0^+ \\ \theta_{P2} = \frac{-\pi}{4};\ f_{\max 1} = 4.08*10^4\,Hz \\ \theta_{P3} = \frac{-3\pi}{2};\ f_{\min} = 2.2*10^6\,Hz \\ \theta_{P4} = \frac{-11\pi}{4};\ f_{\max 2} = 107*10^6\,Hz \\ \theta_{P5} = -3\pi;\ f\to +\infty \end{cases}$$

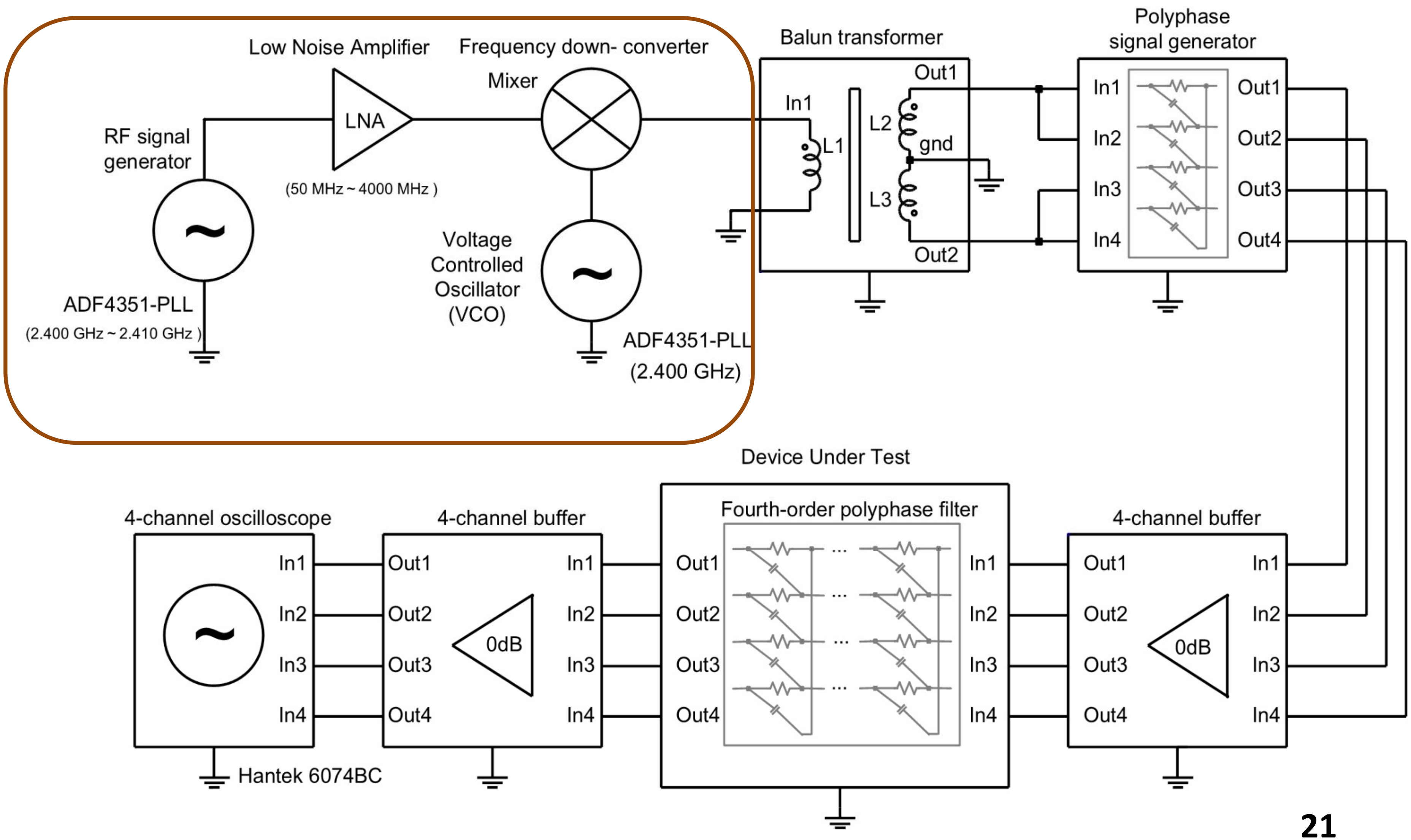
R_1 = 1 kΩ, C_1 = 318 pF, R_2 = 1 kΩ, C_2 = 199 pF, R_3 = 1 kΩ, C_3 = 106 pF, R_4 = 1 kΩ, C_4 = 53.1 pF, R_5 = 1 kΩ, C_5 = 26.5 pF, R_6 = 1 kΩ, C_6 = 16.8 pF

Simulation result of transfer function



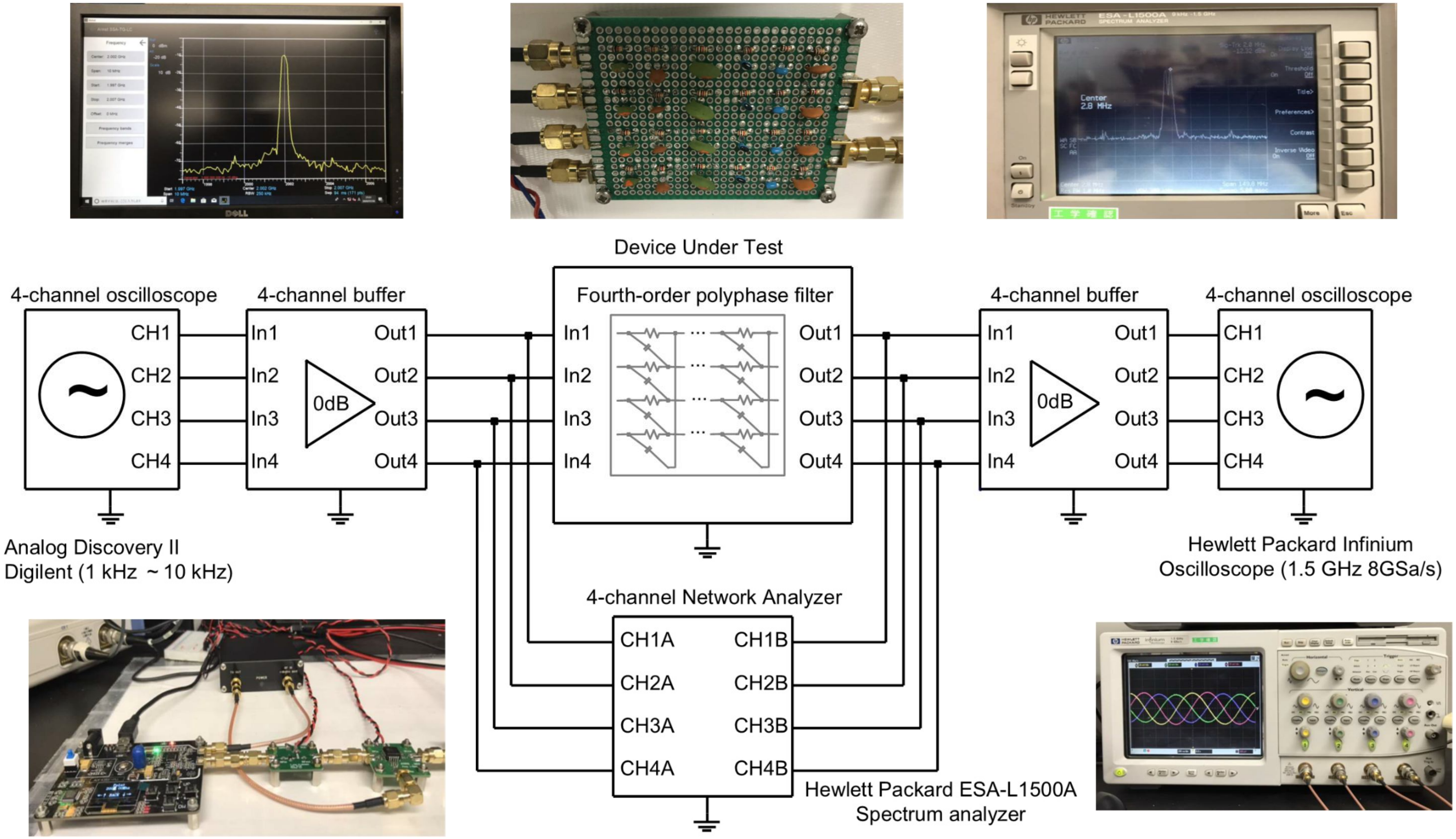
3. Proposed Designs and Experimental Results

Block Diagram of Measurement Set Up



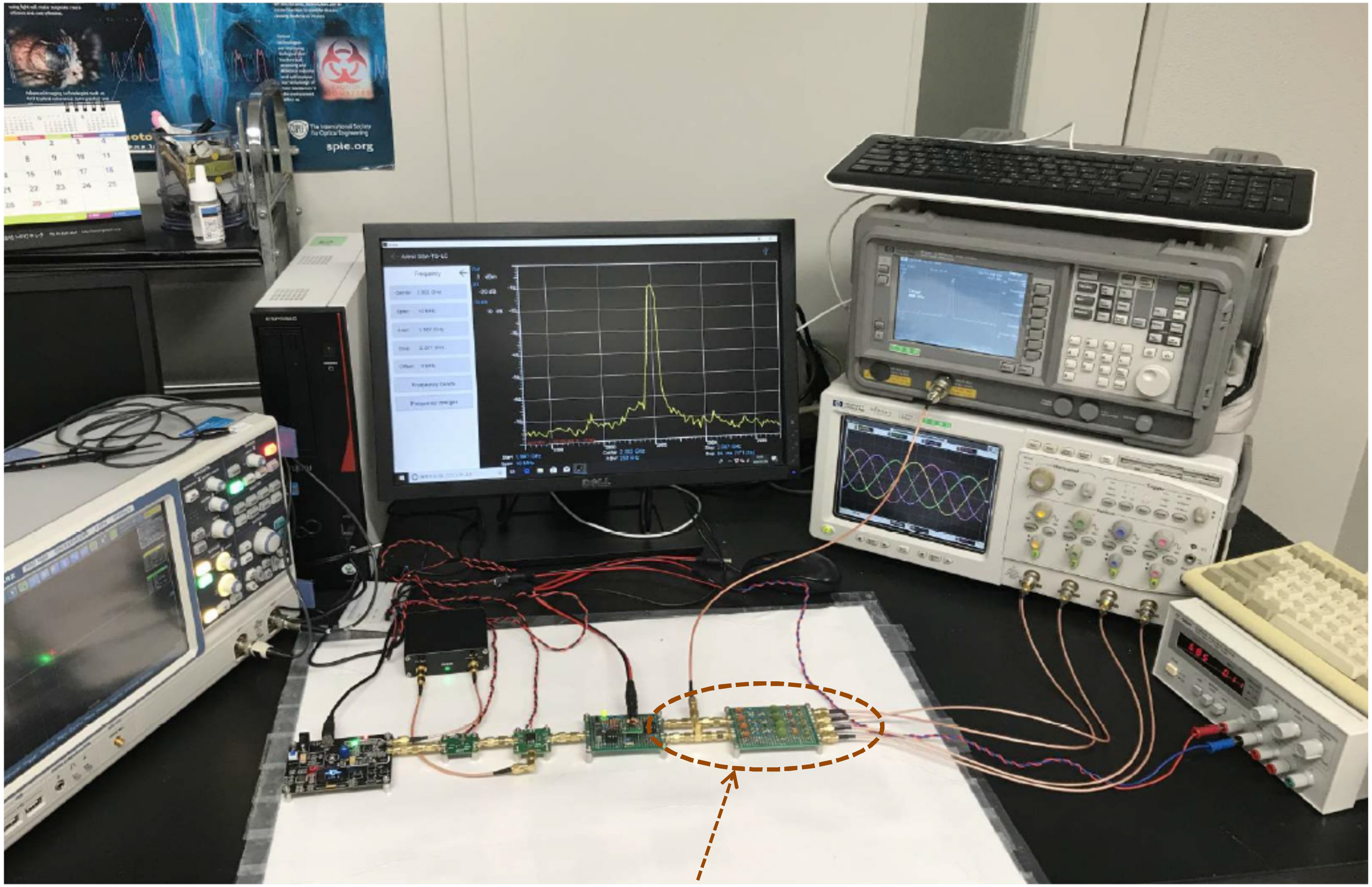
3. Proposed Designs and Experimental Results

Simplified Block Diagram of Measurement Set Up



3. Proposed Designs and Experimental Results

Measurement Set Up for Device Under Test

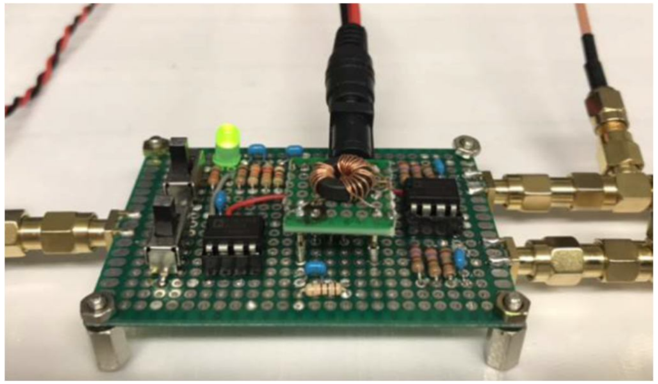


Device under test

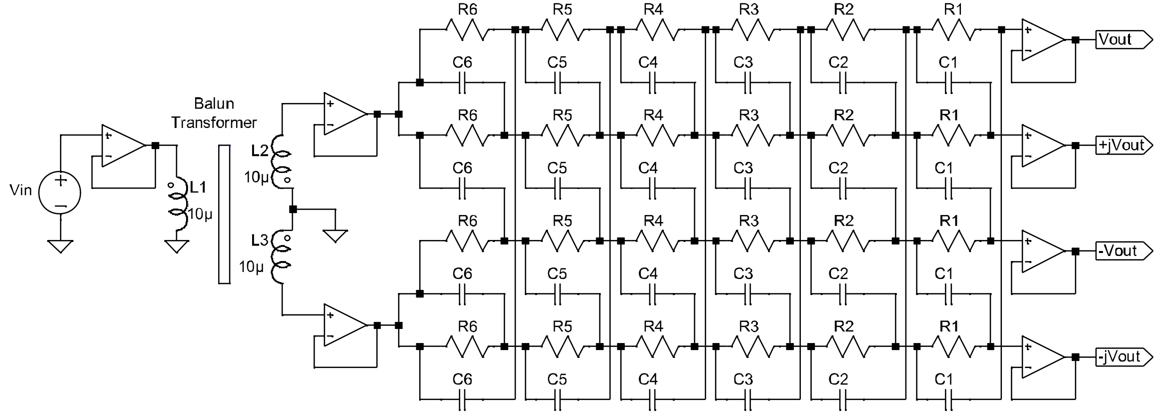
3. Proposed Designs and Experimental Results

Measurement Results of Implemented Circuit

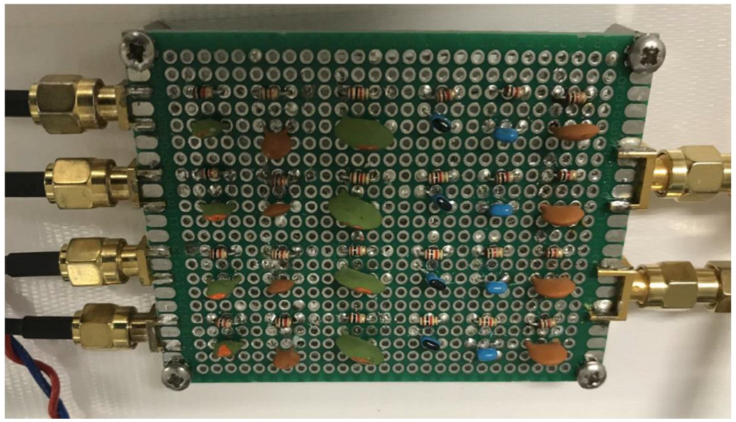
Passive balun transfer



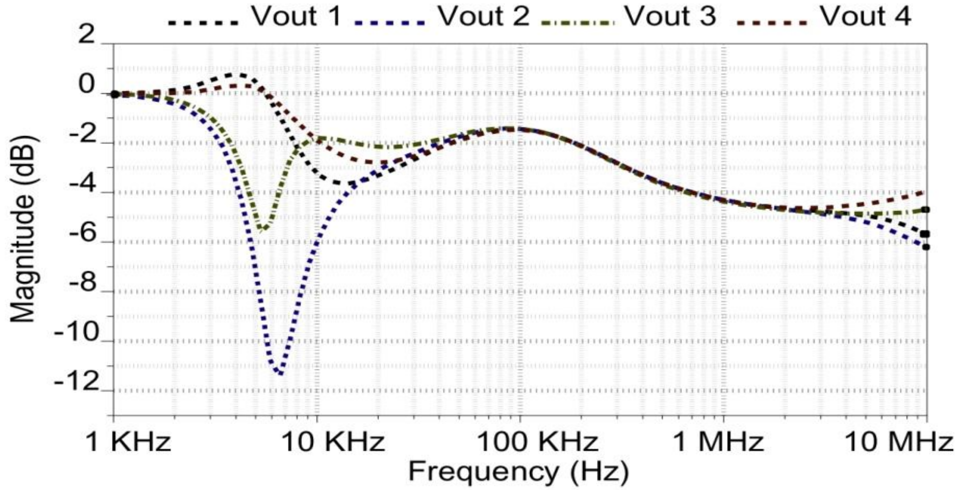
6^{th}-order polyphase signal generation



Implemented circuit



Measurement results



Outline

1. Research Background
   - Motivation, objectives and achievements
   - Review of characteristics of Low-IF receiver
2. Investigation of Multi-Phase Networks
   - Superposition theorem for multi-source networks
   - Design principle for passive polyphase filters
3. Proposed Designs and Experimental Results
   - Analysis of quadrature signal generation networks
   - Simulation and measurement results
4. Conclusions

4. Comparison

Features	Proposed formula	Conventional Superposition	Millan's theorem
Effects of all actuating sources	At one time	Several times	At one time
Transfer function accuracy	Yes	No	No
Single-input network analysis	Yes	Yes	Yes
Polyphase network analysis	Yes	No	No
Complex network analysis	Yes	No	No
Image rejection ratio accuracy	Yes	No	No

4. Discussions

Transfer function and image rejection ratio give useful information about the behaviors of polyphase filters and complex filters.

Fundamental network analysis theory for multi-source systems:

- Compute the effects of all sources at one time,
- Reduce the wasteful time,
- Decrease the hand calculation times,
- Get the transfer function faster, and
- Reduce the network complexity.

4. Conclusions

This work:

- **Proposal of superposition formula for multi-source network analysis**
- **Analysis of high-order passive RC poly-phase filters in all frequency domain**
- **Design of quadrature signal generation network**
- **Implementation and measurement of 6th-oder quadrature signal generation circuit**

Future of work:

- **Analysis of I/Q mismatches, DC offsets, and parasitic components in polyphase and complex filters**

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Thank you very much!

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