ICSICT-2020
2020 IEEE 15th International Conference on Solid-State and Integrated Circuit Technology
Nov. 3-6, 2020
Wyndham Grand Plaza Royale Colorful Hotel,Kunming,China

MEASUREMENTS OF SELF-LOOP FUNCTIONS IN HIGH-ORDER PASSIVE AND ACTIVE LOW-PASS FILTERS

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Outline

1. **Research Background**
   - Motivation, objectives and achievements
   - Self-loop function in a transfer function
2. **Stability Test for Linear Non-Feedback Networks**
   - Alternating current conservation for passive networks
   - Ringing test for passive and active RLC low-pass filters
3. **Stability Test for Linear Feedback Networks**
   - Alternating current conservation for active networks
   - Ringing test for amplifiers with feedback loops
4. **Conclusions**

1. Research Background

Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:

$$\mathrm{SNR} = \frac{\text{Signal power}}{\text{Noise power}}$$

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Performance of a device

Figure of Merit:

$$\mathrm{F} = \frac{\text{Output SNR}}{\text{Input SNR}}$$

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

Linear networks

- Overshoot,
- Ringing
- Oscillation noise

1. Research Background

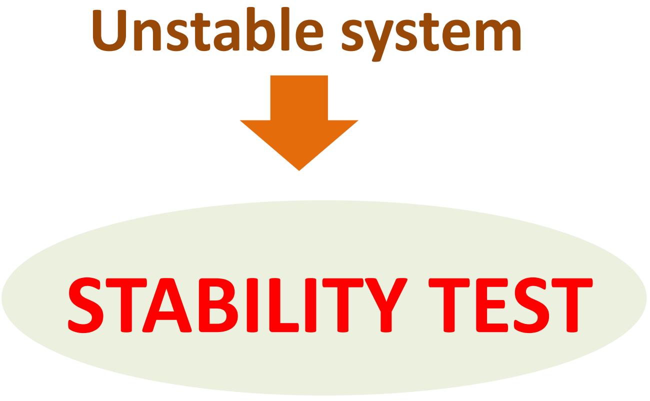
Effects of Ringing on Electronic Systems

Ringing represents a distortion of a signal.

Ringing is overshoot/undershoot voltage or current when it's seen on time domain.

Ringing does the following things:

- **Causes EMI noise,**
- **Increases current flow,**
- **Consumes the power,**
- **Decreases the performance, and**
- **Damages the devices.**



Unstable system

STABILITY TEST

1. Research Background

Objectives and Achievements

Objectives

○ **Investigation of operating regions of linear negative feedback networks**

→ Over-damping (high delay in rising time)

→ Critical damping (max power propagation)

→ Under-damping (overshoot and ringing)

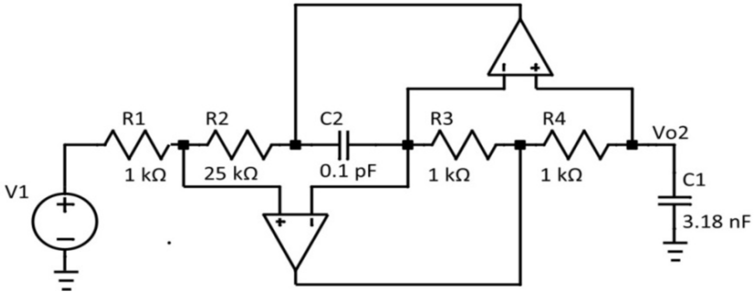
Achievements

○ **Measurement of self-loop function and stability test for both passive and active low-pass filters.**

1. Research Background

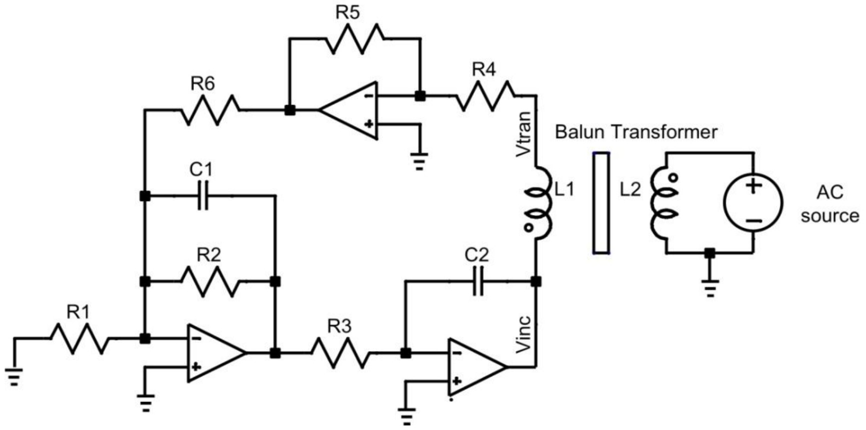
Approaching Methods

2nd-order ladder LPF

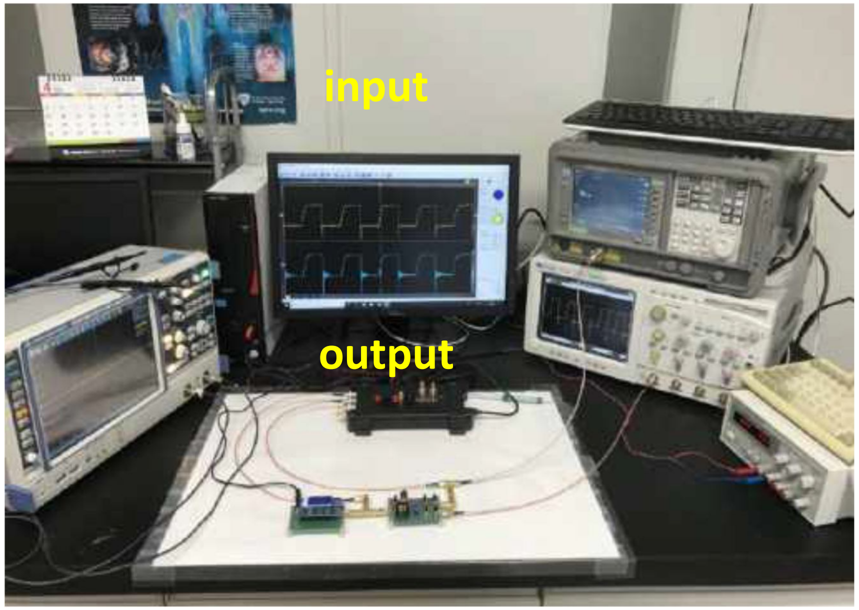


Balun transformer

2nd-order Tow-Thomas LPF



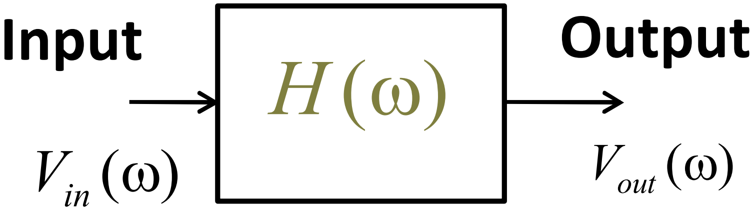
Implemented circuit



1. Research Background

Self-loop Function in A Transfer Function

Linear system



Input

$H(\omega)$

Output

$V_{in}(\omega)$

$V_{out}(\omega)$

Model of a linear system

$$H(\omega) = \frac{b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n}$$

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

$A(\omega)$ **: Open loop function**

$H(\omega)$ **: Transfer function**

$L(\omega)$ **: Self-loop function**

Variable: angular frequency (ω)

- **Polar chart → Nyquist chart**
- **Magnitude-frequency plot**
- **Angular-frequency plot**

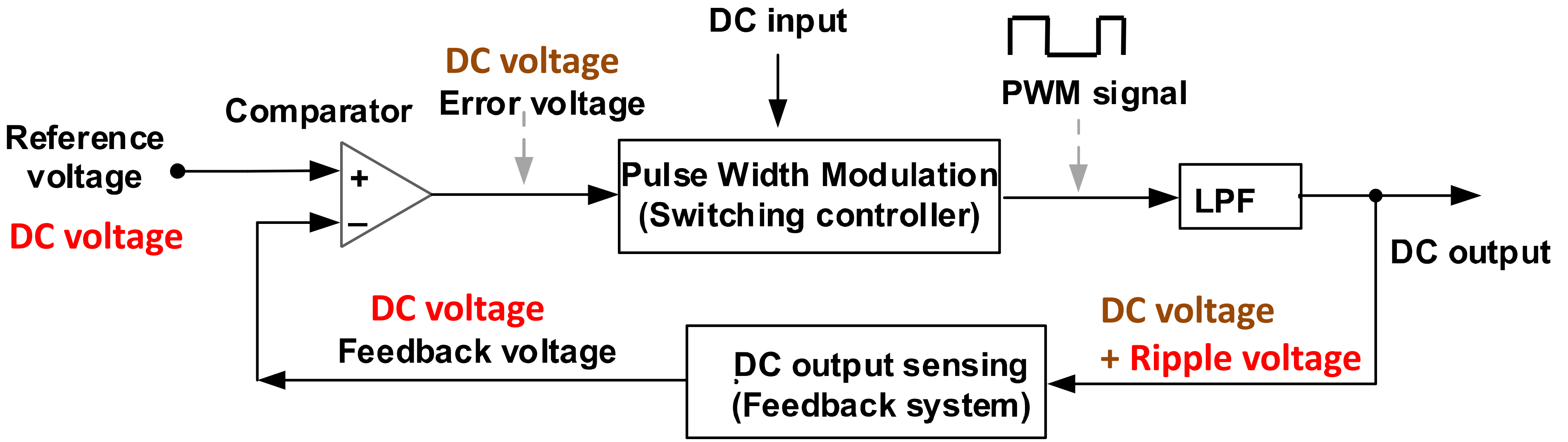
Bode plots

- **Magnitude-angular diagram → Nichols diagram**

1. Research Background

Characteristics of Adaptive Feedback Network

Block diagram of a typical adaptive feedback system



Adaptive feedback is used to control the output source along with the decision source (**DC-DC Buck converter**).

Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network.

→ Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).

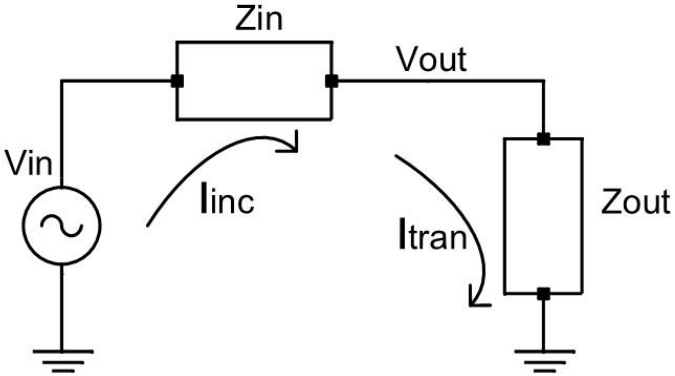
1. Research Background

Alternating Current Conservation

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + \frac{Z_{in}}{Z_{out}}}$$

$$\Rightarrow \boxed{L(\omega) = \frac{Z_{in}}{Z_{out}};}$$

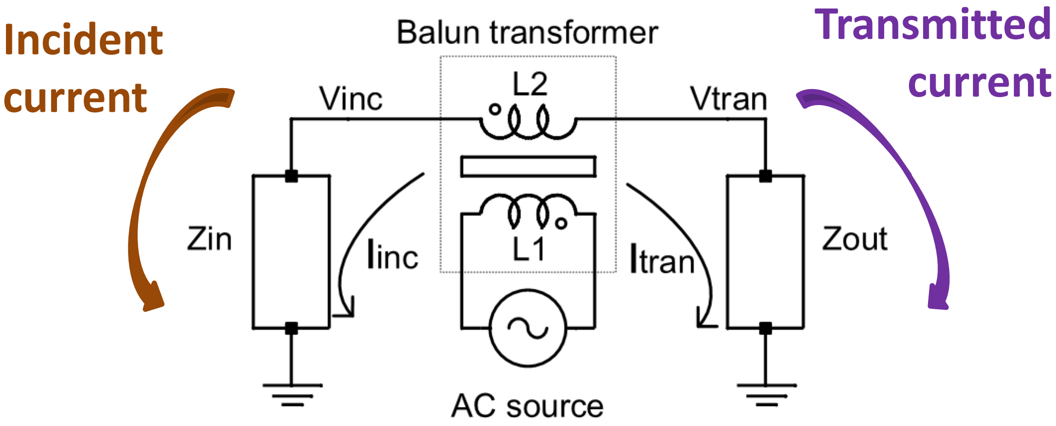


Simplified linear system

Self-loop function

$$\frac{V_{inc}}{Z_{in}} = -\frac{V_{trans}}{Z_{out}} \Rightarrow \boxed{L(\omega) = -\frac{V_{inc}}{V_{trans}} = \frac{Z_{in}}{Z_{out}}}$$

10 mH inductance



Derivation of self-loop function

1. Research Background

Limitations of Conventional Methods

○ **Middlebrook’s measurement of loop gain**

→**Applying only in feedback systems (DC-DC converters).**

○ **Replica measurement of loop gain**

→**Using two identical networks (not real measurement).**

○ **Nyquist’s stability condition**

→ **Theoretical analysis for feedback systems (Lab tool).**

○ **Nichols chart of loop gain**

→ **Only used in feedback control theory (Lab tool).**

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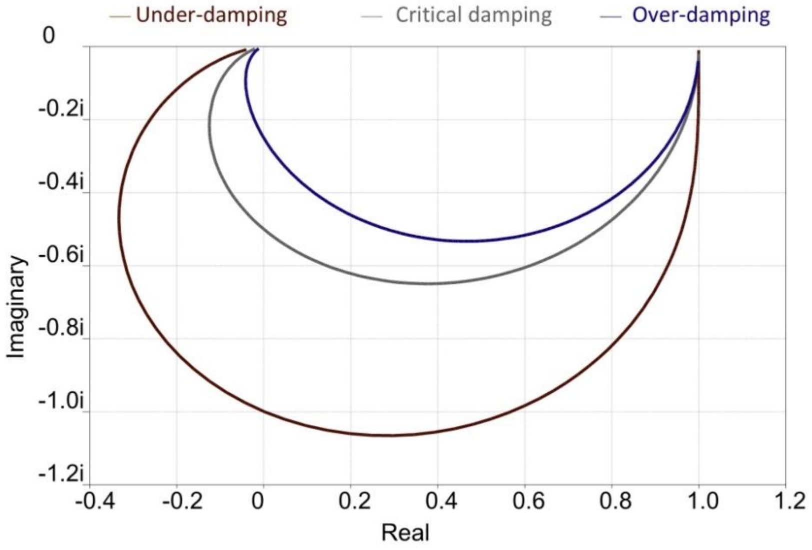
   - Alternating current conservation for active networks
   - Ringing test for amplifiers with feedback loops
4. Conclusions

2. Stability Test for Linear Non-Feedback Networks

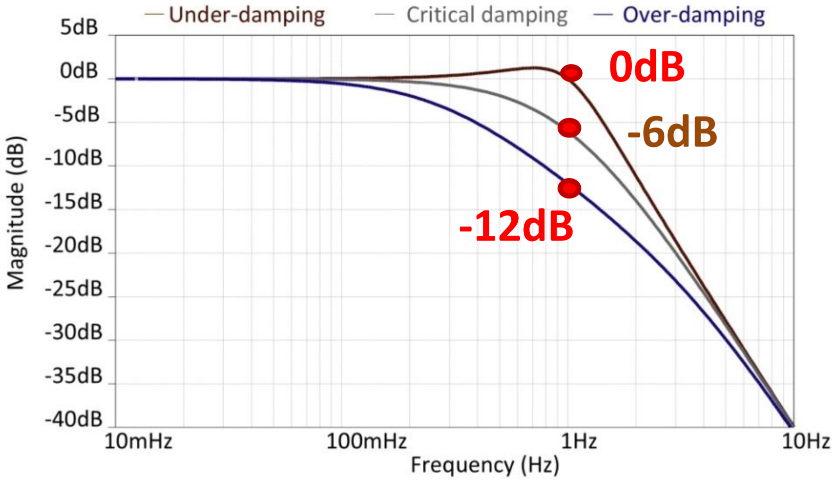
Behaviors of 2nd-Order Transfer Function

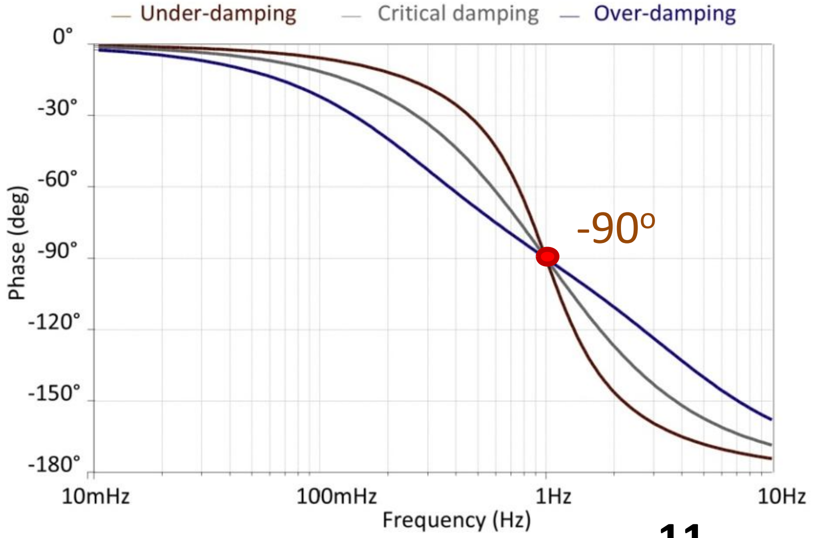
- **Under-damping:** $H_1(\omega) = \dfrac{1}{(j\omega)^2 + j\omega + 1};$
- **Critical damping:** $H_2(\omega) = \dfrac{1}{(j\omega)^2 + 2j\omega + 1};$
- **Over-damping:** $H_3(\omega) = \dfrac{1}{(j\omega)^2 + 3j\omega + 1};$

Nyquist chart of transfer function



Bode plot of transfer function





2. Stability Test for Linear Non-Feedback Networks

Characteristics of 2nd-order Transfer Function

Second-order transfer function: $H(\omega)=\frac{1}{1+a_0(j\omega)^2+a_1j\omega}$

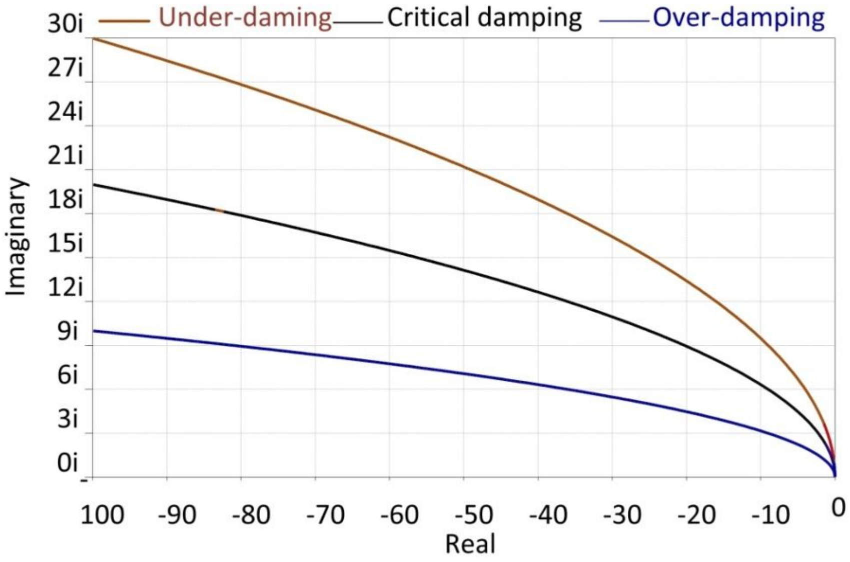
Case	Over-damping		Critical damping		Under-damping	
Delta (Δ)	$\frac{1}{a_0}<\left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta=a_1^2-4a_0>0$		$\frac{1}{a_0}=\left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta=a_1^2-4a_0=0$		$\frac{1}{a_0}>\left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta=a_1^2-4a_0<0$	
Module $\lvert H(\omega)\rvert$	$\frac{\frac{1}{a_0}}{\sqrt{\omega^2+\left(\frac{a_1}{2a_0}-\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}\right)^2}\sqrt{\omega^2+\left(\frac{a_1}{2a_0}+\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}\right)^2}}$		$\boxed{\frac{\frac{1}{a_0}}{\left[\omega^2+\left(\frac{a_1}{2a_0}\right)^2\right]}=\frac{1}{2}=-6dB}$		$\frac{\frac{1}{a_0}}{\sqrt{\left(\omega-\sqrt{\frac{1}{a_0}-\left(\frac{a_1}{2a_0}\right)^2}\right)^2+\left(\frac{a_1}{2a_0}\right)^2}\sqrt{\left(\omega+\sqrt{\frac{1}{a_0}-\left(\frac{a_1}{2a_0}\right)^2}\right)^2+\left(\frac{a_1}{2a_0}\right)^2}}$	
Angular $\theta(\omega)$	$-\arctan\left(\frac{\omega}{\frac{a_1}{2a_0}-\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}}\right)-\arctan\left(\frac{\omega}{\frac{a_1}{2a_0}+\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}}\right)$		$-2\arctan\left(\frac{2a_0\omega}{a_1}\right)$		$-\arctan\left(\frac{\omega-\sqrt{\frac{1}{a_0}-\left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)-\arctan\left(\frac{\omega+\sqrt{\frac{1}{a_0}-\left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)$	
$\omega_{cut}=\frac{a_1}{2a_0}$	$\lvert H(\omega_{cut})\rvert<\frac{2a_0}{a_1}$	$\theta(\omega_{cut})>-\frac{\pi}{2}$	$\lvert H(\omega_{cut})\rvert=\frac{2a_0}{a_1}$	$\theta(\omega_{cut})=-\frac{\pi}{2}$	$\lvert H(\omega_{cut})\rvert>\frac{2a_0}{a_1}$	$\theta(\omega_{cut})<-\frac{\pi}{2}$

2. Stability Test for Linear Non-Feedback Networks

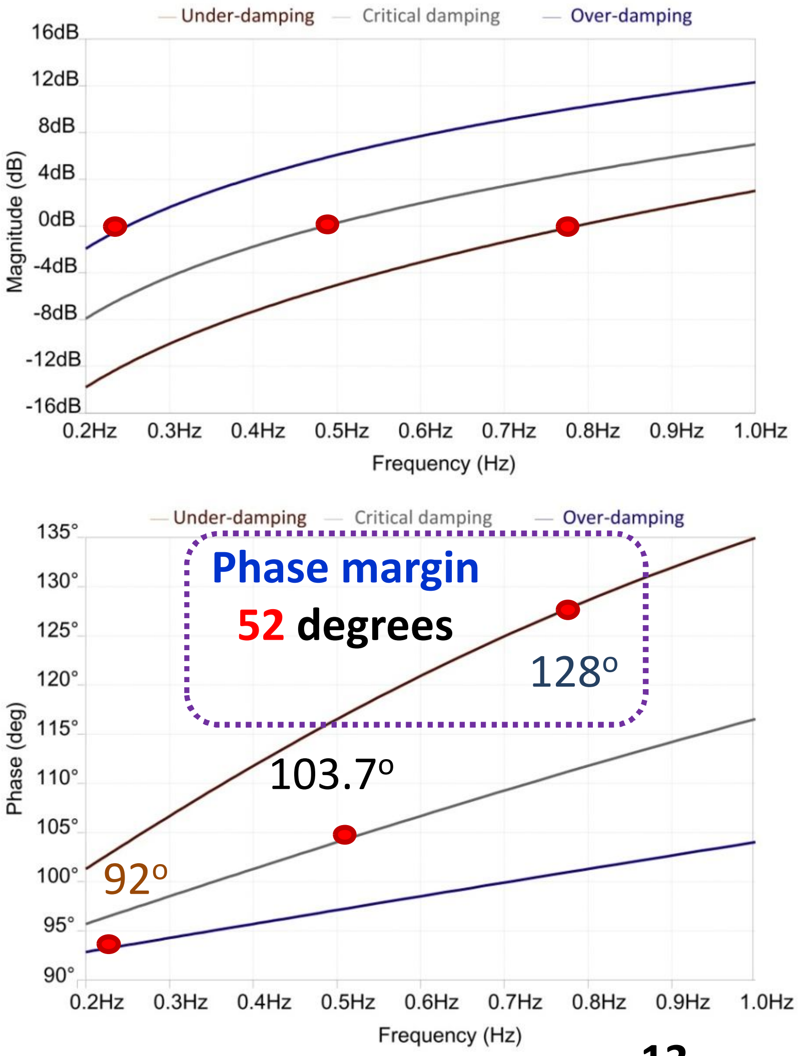
Behaviors of 2nd-Order Self-loop Function

- **Under-damping:** $L_1(\omega) = (j\omega)^2 + j\omega;$
- **Critical damping:** $L_2(\omega) = (j\omega)^2 + 2j\omega;$
- **Over-damping:** $L_3(\omega) = (j\omega)^2 + 3j\omega;$

Nyquist chart of self-loop function



Bode plot of self-loop function



Phase margin
52 degrees

2. Stability Test for Linear Non-Feedback Networks

Characteristics of 2nd-order Self-loop Function

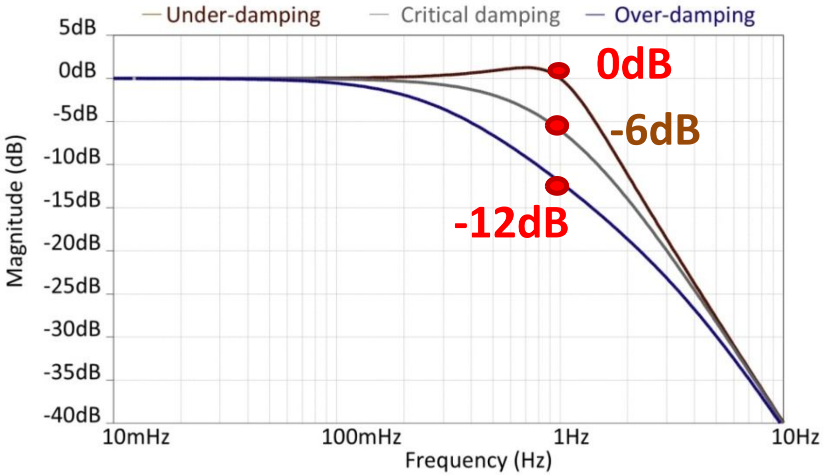
Second-order self-loop function: $L(\omega) = j\omega\left[a_0 j\omega + a_1\right]$

Case	**Over-damping**		**Critical damping**		**Under-damping**	
Delta (Δ)	$\Delta = a_1^{\,2} - 4a_0 > 0$		$\Delta = a_1^{\,2} - 4a_0 = 0$		$\Delta = a_1^{\,2} - 4a_0 < 0$	
$\lvert L(\omega)\rvert$	$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$	
$\theta(\omega)$	$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$	
$\omega_1 = \frac{a_1}{2a_0}\sqrt{\sqrt{5}-2}$	$\lvert L(\omega_1)\rvert > 1$	$\pi - \theta(\omega_1) > 76.3^o$	$\lvert L(\omega_1)\rvert = 1$	$\pi - \theta(\omega_1) = 76.3^o$	$\lvert L(\omega_1)\rvert < 1$	$\pi - \theta(\omega_1) < 76.3^o$
$\omega_2 = \frac{a_1}{2a_0}$	$\lvert L(\omega_2)\rvert > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^o$	$\lvert L(\omega_2)\rvert = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^o$	$\lvert L(\omega_2)\rvert < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^o$
$\omega_3 = \frac{a_1}{a_0}$	$\lvert L(\omega_3)\rvert > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^o$	$\lvert L(\omega_3)\rvert = 4\sqrt{2}$	$\pi - \theta(\omega_3) = 45^o$	$\lvert L(\omega_3)\rvert < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^o$

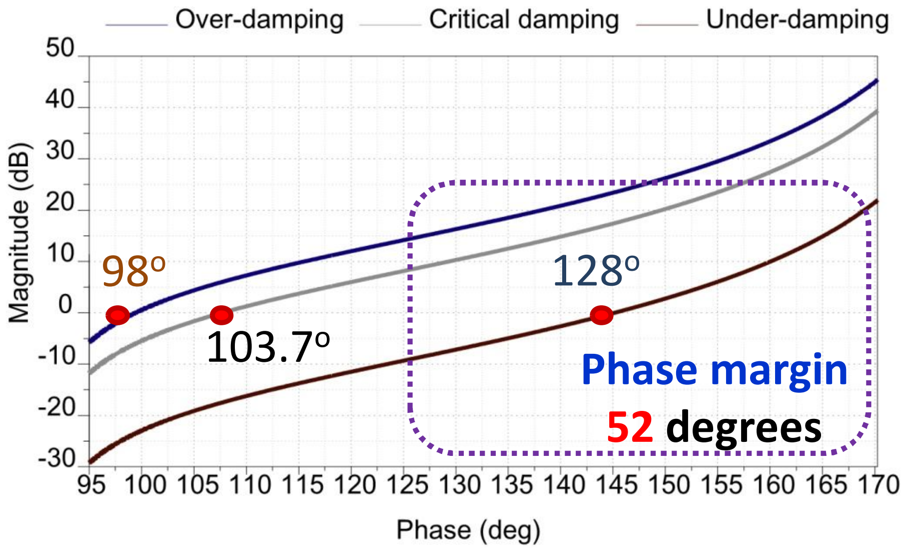
2. Stability Test for Linear Non-Feedback Networks

Summary of Operating Regions of 2nd-Order System

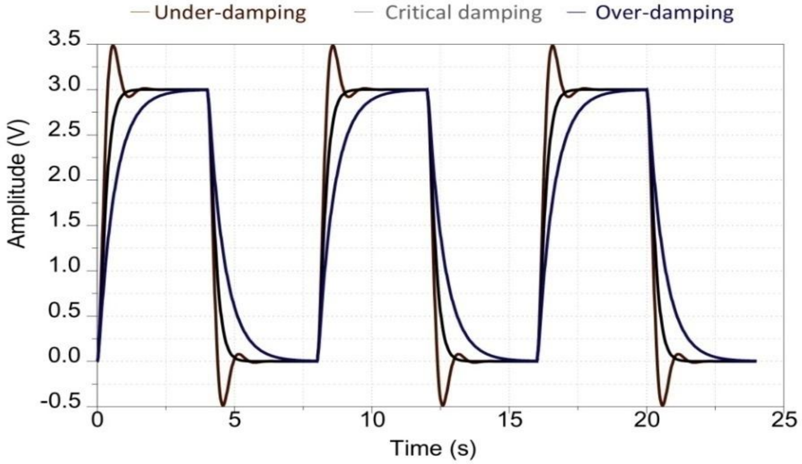
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Operating regions

Over-damping:

→Phase margin is 88 degrees.

Critical damping:

→Phase margin is 76.3 degrees.

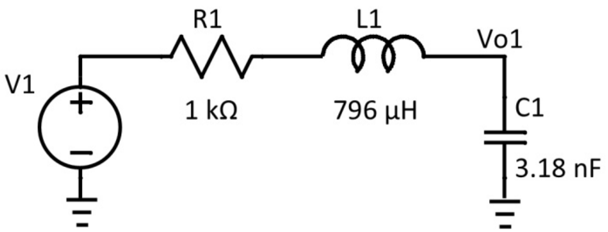
Under-damping:

→Phase margin is 52 degrees.

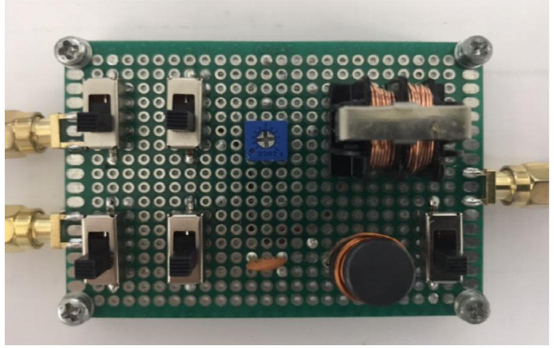
2. Stability Test for Linear Non-Feedback Networks

Stability Test for 2nd-Order Passive RLC LPF

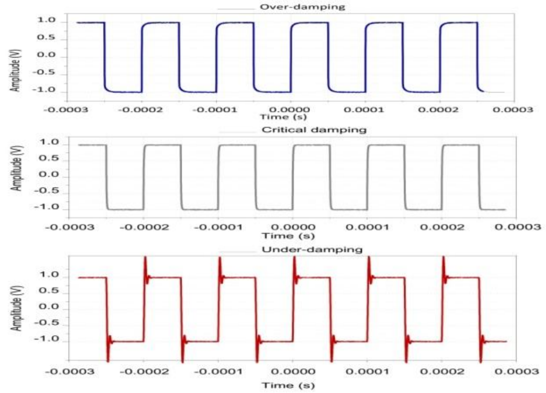
Passive RLC Low-pass Filter



Implemented circuit



Transient responses



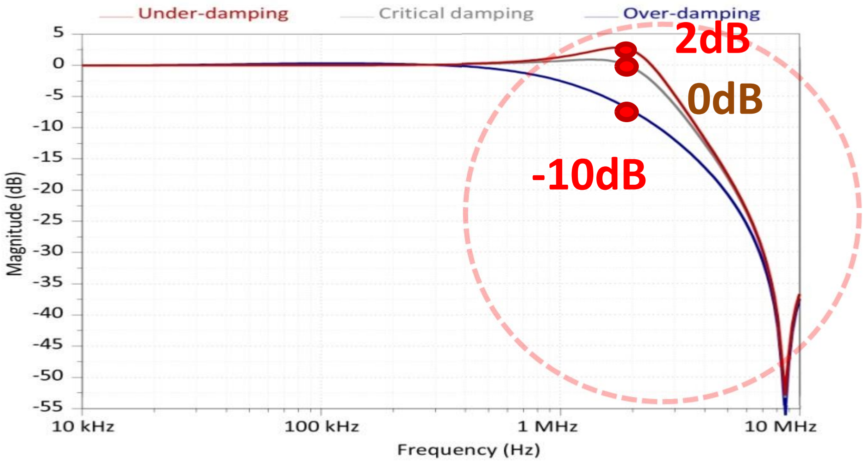
Transfer function

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

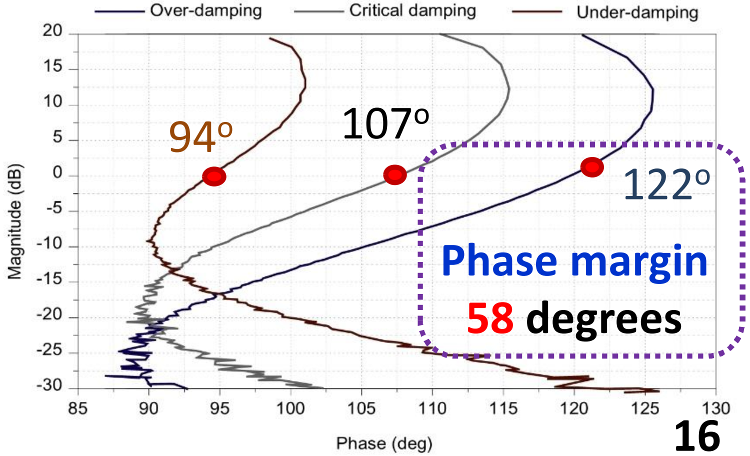
Self-loop function

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Bode plot of transfer function



Nichols plot of self-loop function

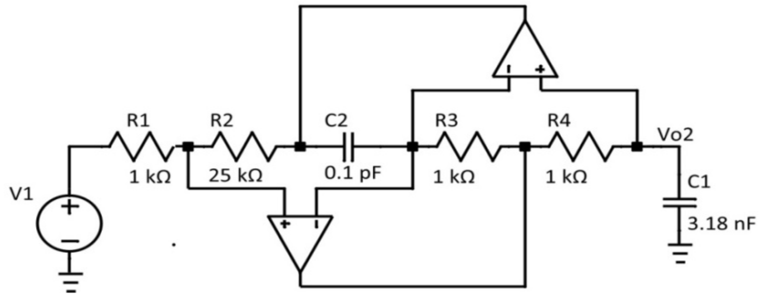


Phase margin
58 degrees

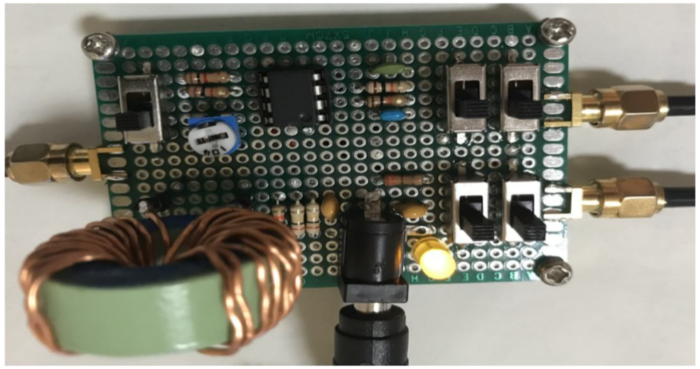
2. Stability Test for Linear Non-Feedback Networks

Stability Test for 2nd-Order Active Ladder LPF

Active ladder low-pass filter



Implemented circuit



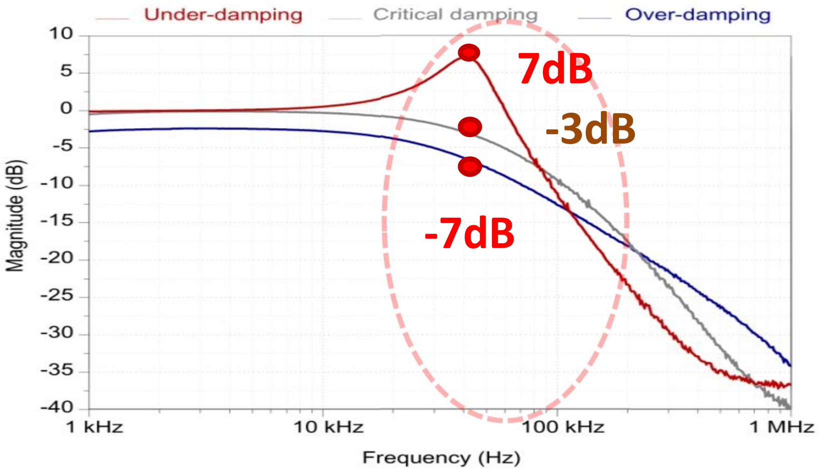
Transfer function

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{a_0\left(j\omega\right)^2 + a_1 j\omega + 1};$$

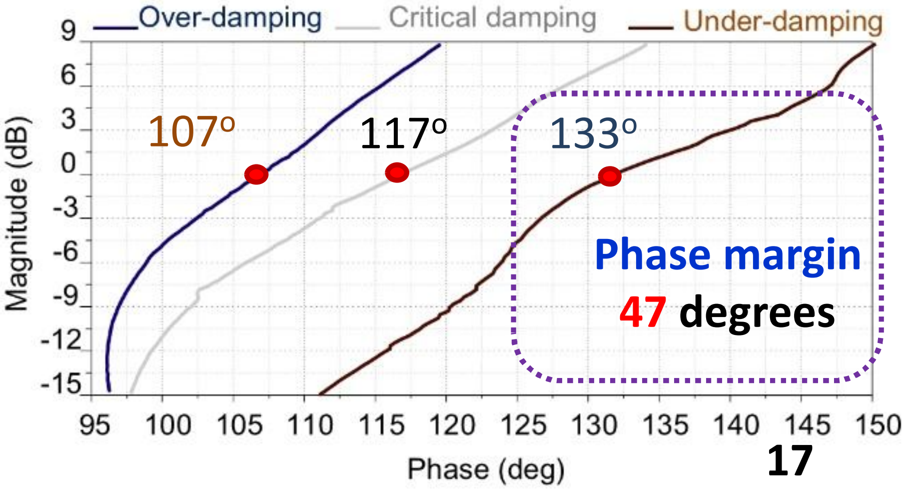
Self-loop function

$$L(\omega) = a_0\left(j\omega\right)^2 + a_1 j\omega;$$

Bode plot of transfer function



Nichols plot of self-loop function



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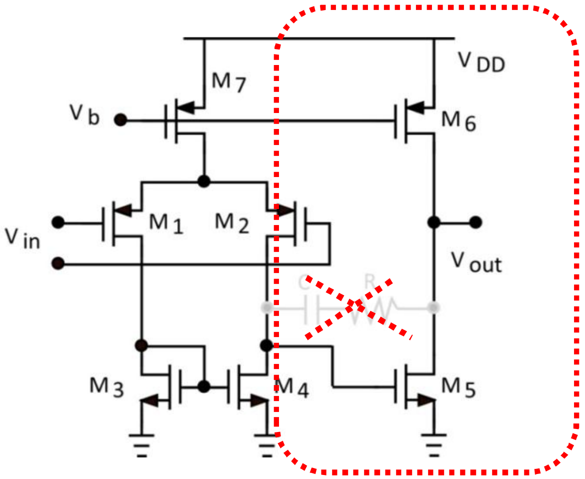
- **Alternating current conservation for active networks**
- **Ringing test for amplifiers with feedback loops**
4. Conclusions

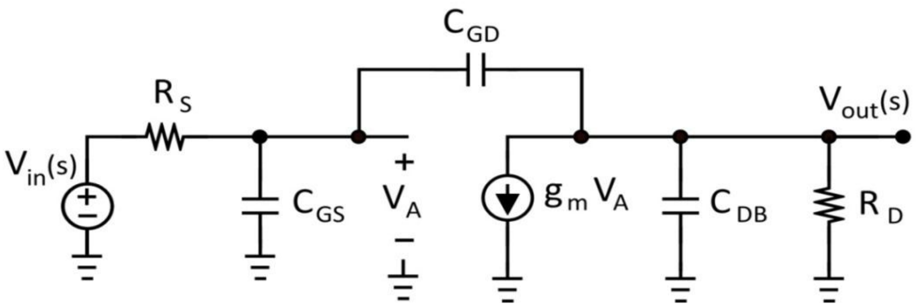
3. Stability Test for Linear Feedback Networks

Analysis of Op Amp without Miller's Capacitor

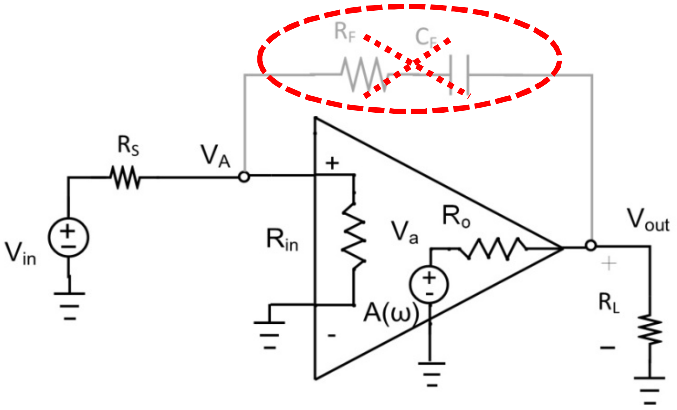
Without frequency compensation



Small signal model



Simplified model



Transfer function H(ω) and self-loop function L(ω)

$$H(\omega) = \frac{b_0 j\omega + b_1}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega$$

Where,

$$b_0 = R_D R_S \left[(C_{GD} + C_{DB})(C_{GS} + C_{GD}) - C_{GD}^2 \right]$$

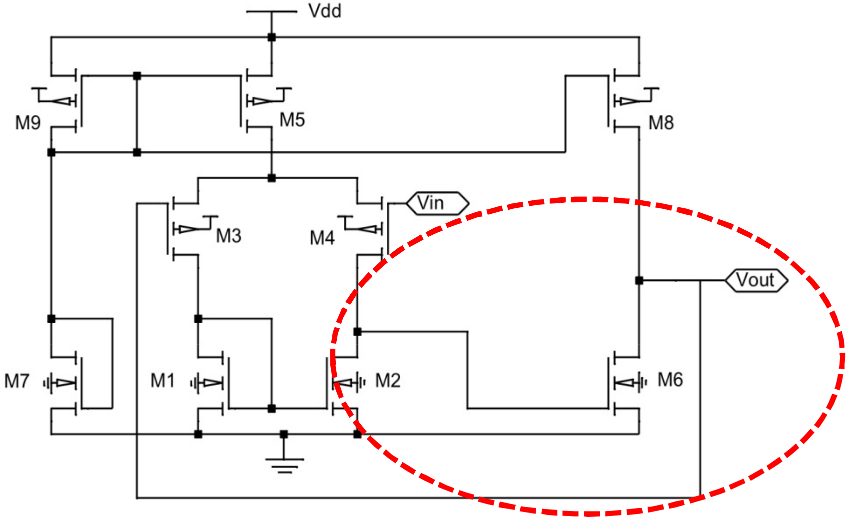
$$b_1 = \left[R_D (C_{GD} + C_{DB}) + R_S (C_{GS} + C_{GD}) + R_D R_S g_m C_{GD} \right]$$

$$a_0 = R_D C_{GD};\ a_1 = -R_D g_m;$$

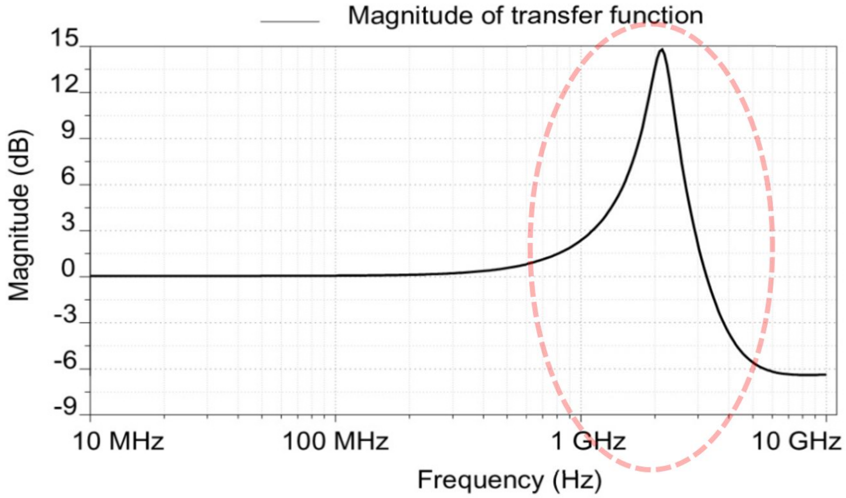
3. Stability Test for Linear Feedback Networks

Unity-Gain Amplifier without Miller's Capacitor

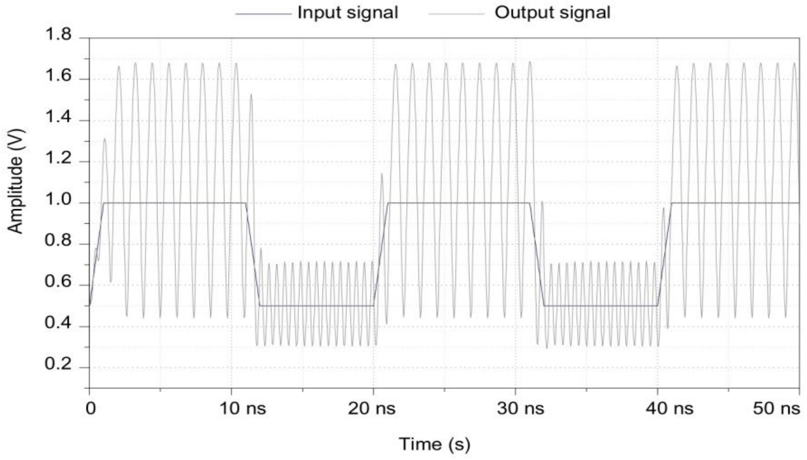
Unity-Gain Amplifier



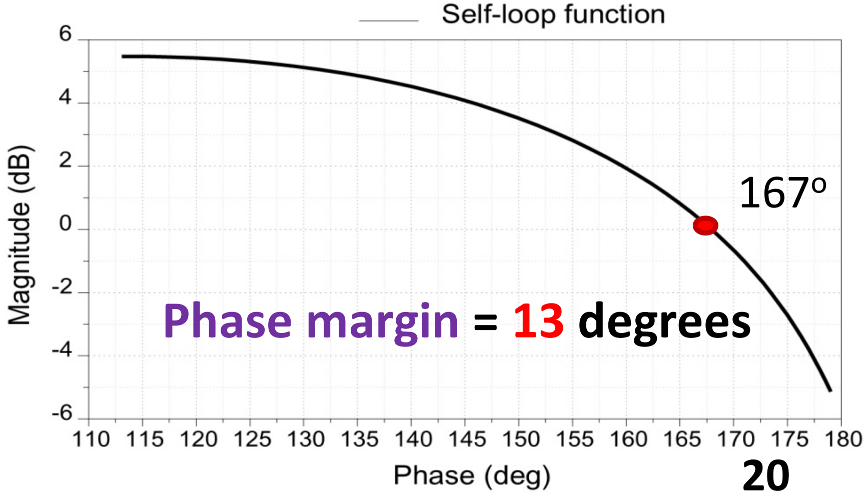
Bode plot of transfer function H(ω)



Transient response



Nichols plot of self-loop function L(ω)

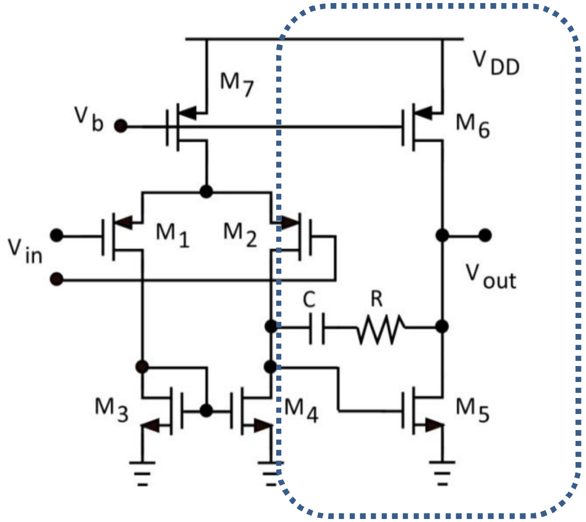


Phase margin = **13** degrees

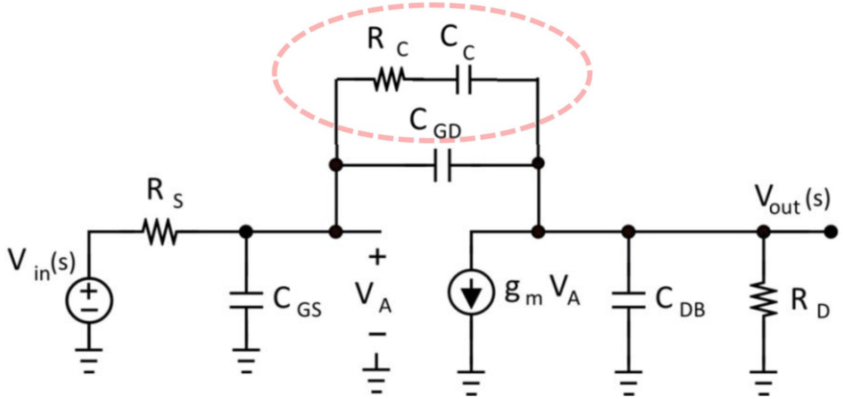
3. Stability Test for Linear Feedback Networks

Two-stage Op Amp with Frequency Compensation

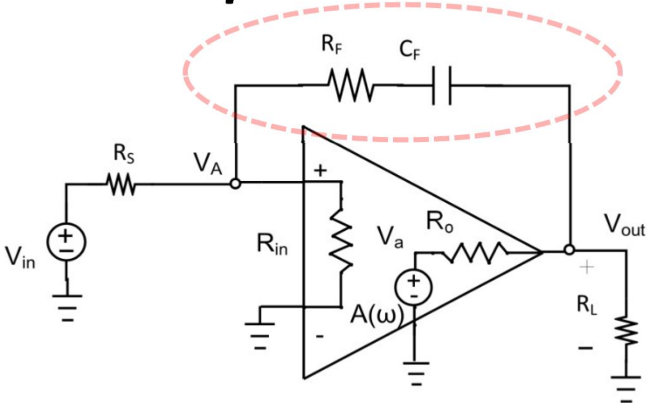
With Miller's capacitor and resistor



Small signal model



Simplified model



Transfer function H(ω)

$$H(\omega) = \frac{b_0 (j\omega)^3 + b_1 (j\omega)^2 + b_2 j\omega + b_3}{a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega + 1};$$

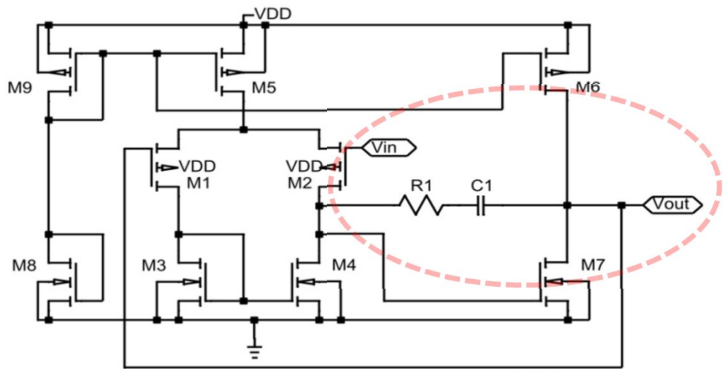
Self-loop function L(ω)

$$L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega$$

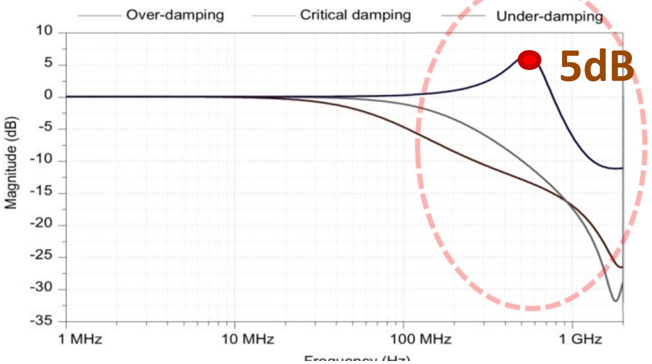
3. Stability Test for Linear Feedback Networks

Stability Test for Op Amp with Miller's Capacitor

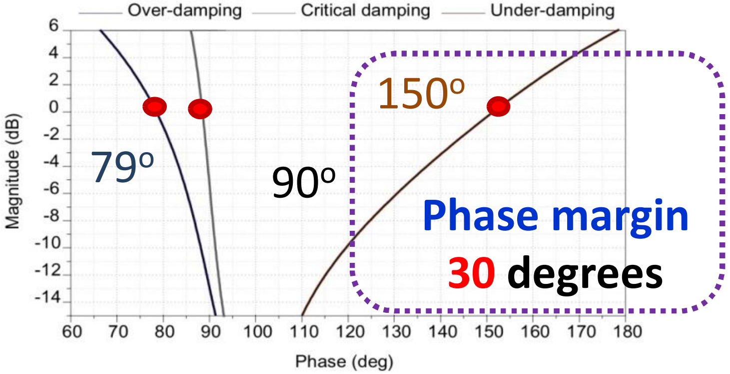
Unity-gain amplifier with Miller's capacitor



Bode plot of transfer function

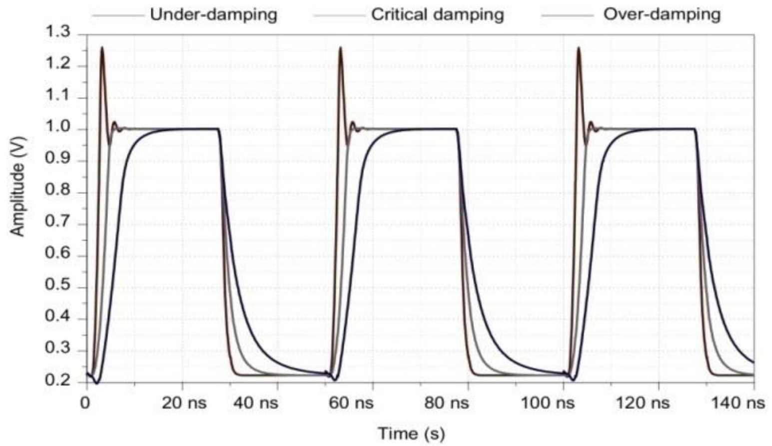


Nichols plot of self-loop function



Phase margin
30 degrees

Simulated transient response



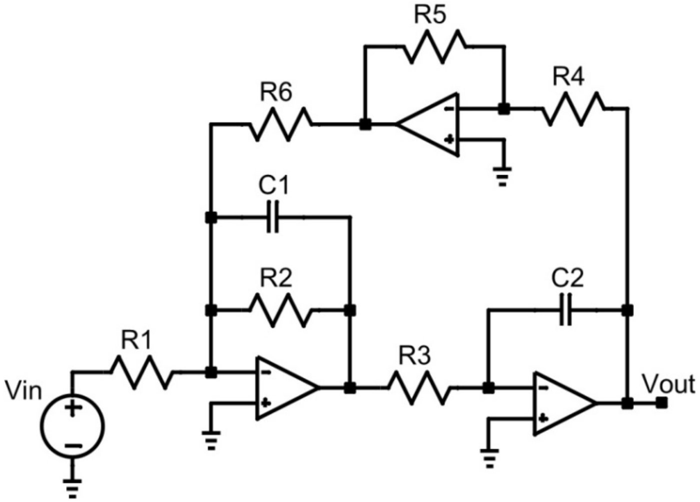
Operating regions

Under-damping:
R1= 2 kΩ, C1 = 1 pF
Critical damping:
R1 = 3.5 kΩ, C1 = 0.2 pF
Over-damping:
R1 = 3.5 kΩ, C1 = 0.8 pF

3. Stability Test for Linear Feedback Networks

Analysis of 2nd-order Tow-Thomas LPF

2nd-order Tow-Thomas LPF

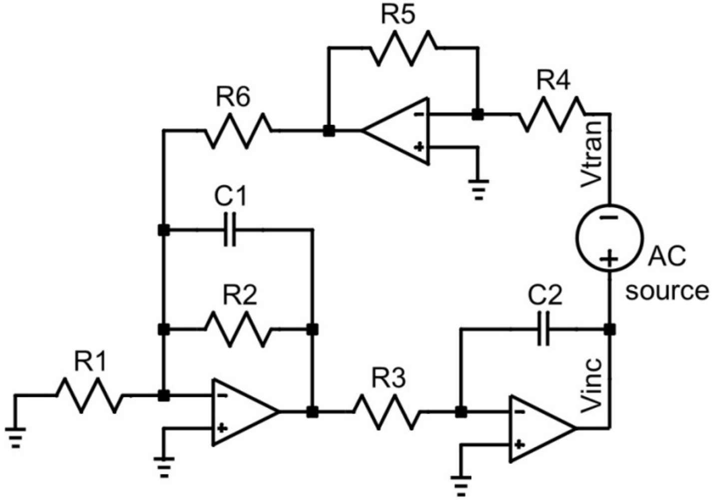


Transfer function H(ω) and self-loop function L(ω)

$$H(\omega) = \frac{\frac{R_4R_6}{R_1R_5}}{\left(j\omega\right)^2 \frac{R_3R_4R_6}{R_5} C_1C_2 + j\omega \frac{R_3R_4R_6}{R_5R_2} C_2 + 1}$$

$$L(\omega) = \left(j\omega\right)^2 \frac{R_3R_4R_6}{R_5} C_1C_2 + j\omega \frac{R_3R_4R_6}{R_5R_2} C_2$$

Derivation of self-loop function



Based on alternating current conservation principle, self-loop function

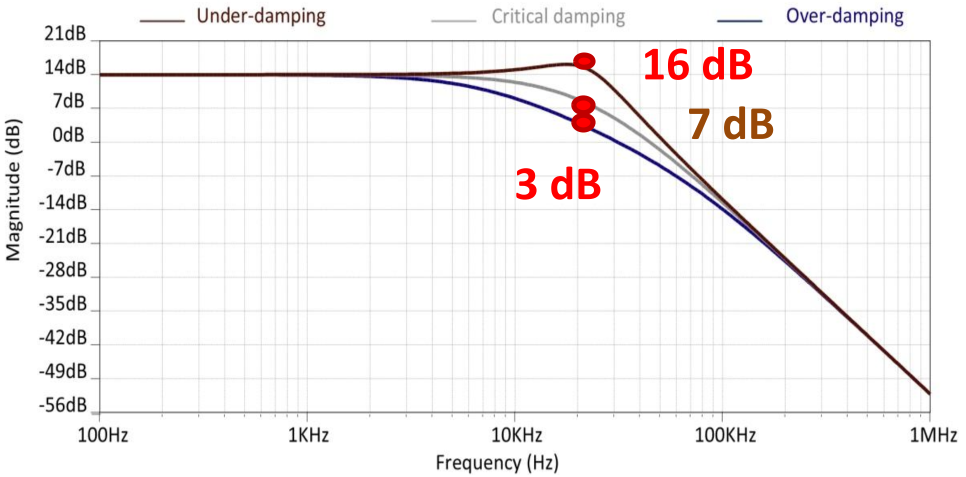
$$\frac{V_{inc}}{A(\omega)} = -\frac{L(\omega)}{A(\omega)} V_{trans}$$

$$\boxed{\Rightarrow L(\omega) = -\frac{V_{inc}}{V_{trans}}}$$

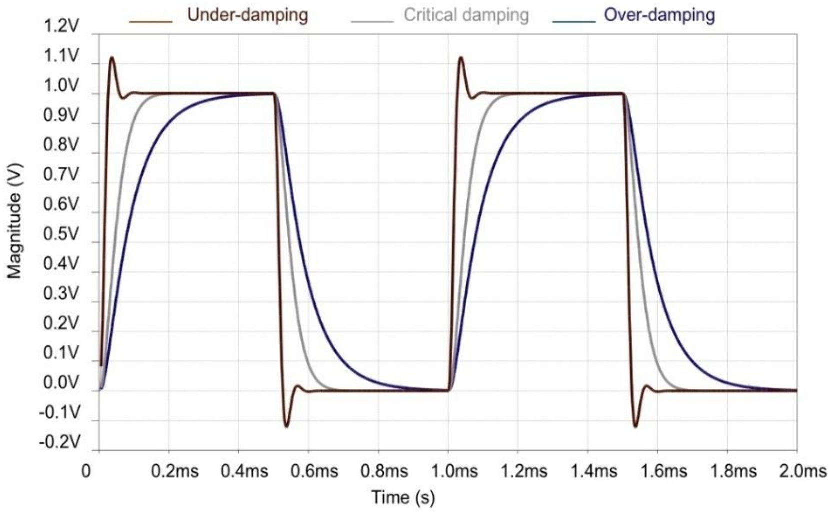
3. Stability Test for Linear Feedback Networks

Simulation Results of 2nd-Order Tow-Thomas LPF

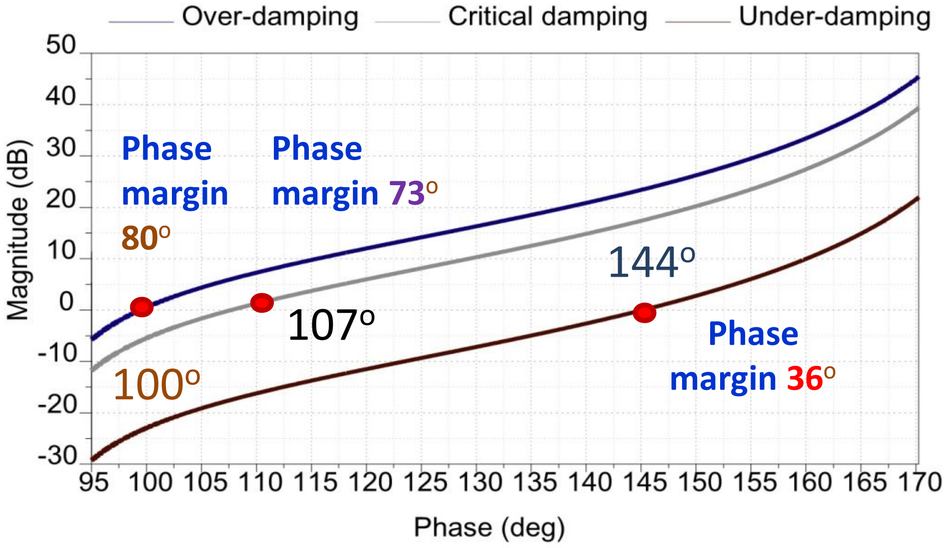
Bode plot of transfer function



Transient response



Nichols plot of self-loop function



Component parameters

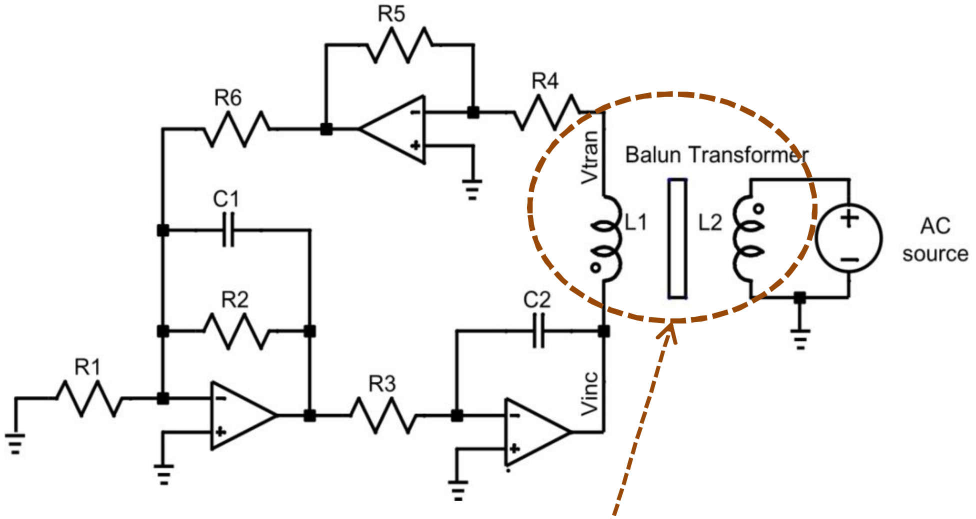
GBW = 10MHz, Ao = 100000,
fo = 25kHz, C1 =1 nF, C2 = 100 pF,
R1= R4 = R5 = 1kΩ,
R3 = 100 kΩ, R6 = 5 kΩ.

Under-damping: R2 = 10 kΩ,
Critical damping: R2 = 3.5 kΩ,
Over-damping: R2 = 10 kΩ

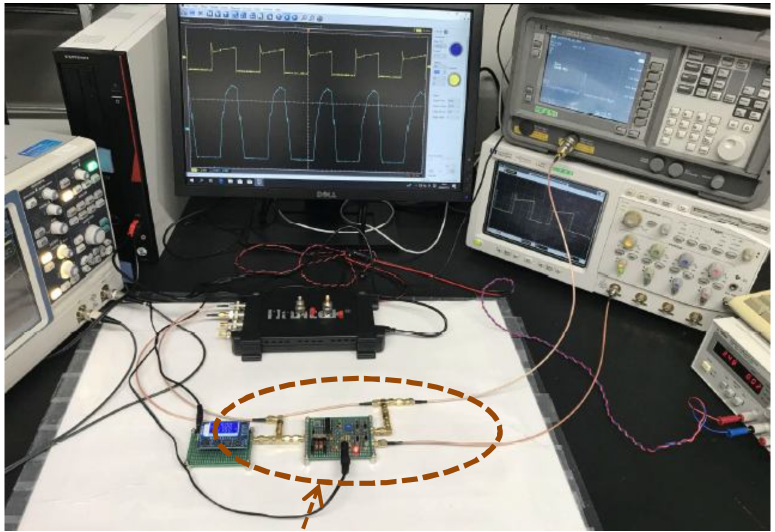
3. Stability Test for Linear Feedback Networks

Implemented Circuit of 2nd-Order Tow-Thomas LPF

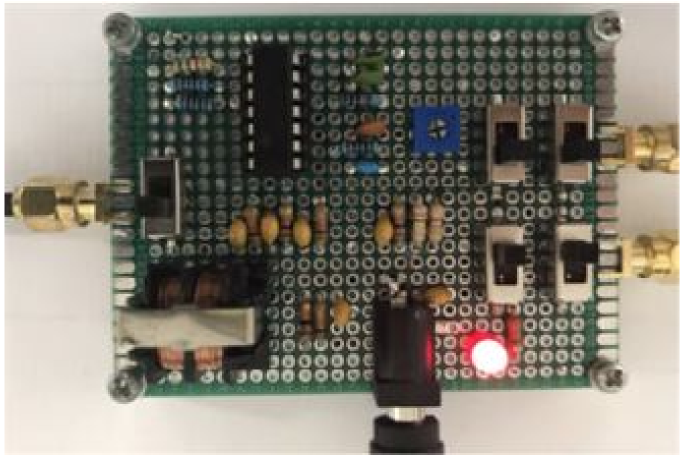
Measurement of self-loop function



Balun transformer
(10 mH inductance)



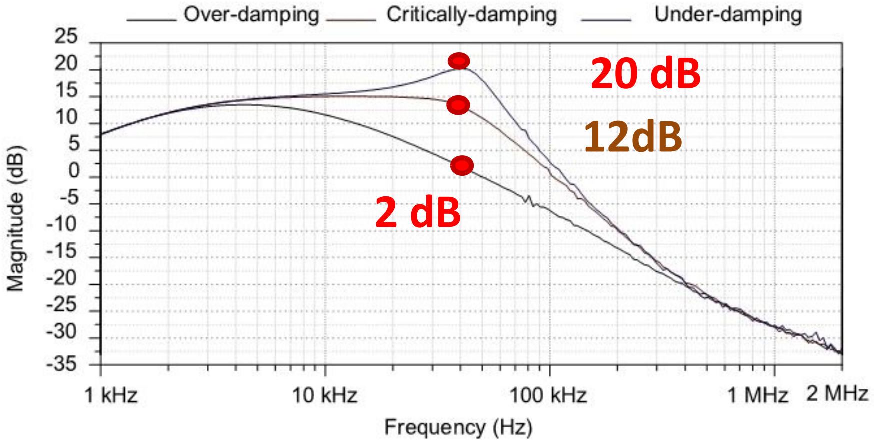
Device under test



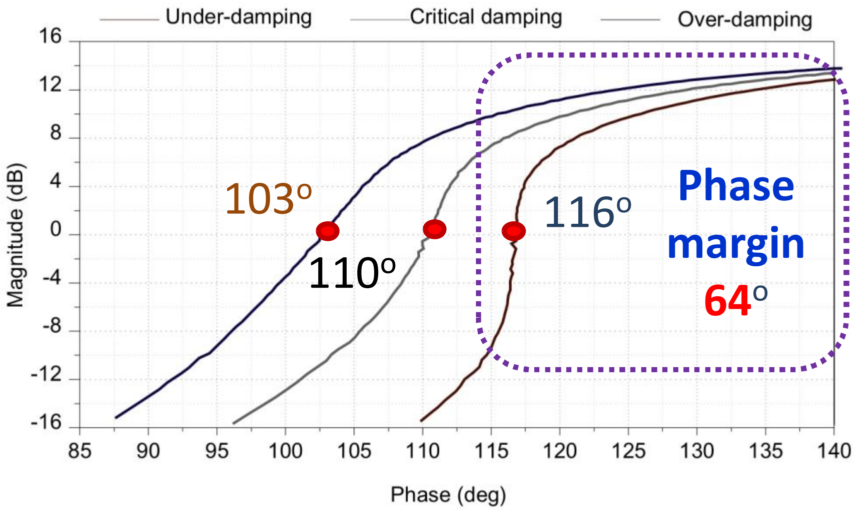
3. Stability Test for Linear Feedback Networks

Measurement Results of 2nd-order Tow-Thomas LPF

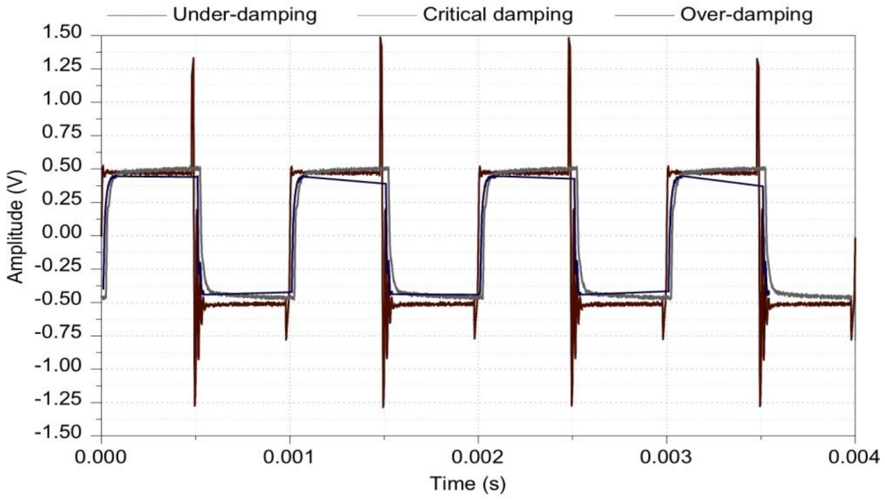
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Over-damping:

→Phase margin is 77 degrees.

Critical damping:

→Phase margin is 70 degrees.

Under-damping:

→Phase margin is 64 degrees.

Outline

1. Research Background
   - Motivation, objectives and achievements
   - Self-loop function in a transfer function
2. Stability Test for Linear Non-Feedback Networks
   - Alternating current conservation for passive networks
   - Ringing test for passive and active RLC low-pass filters
3. Stability Test for Linear Feedback Networks
   - Alternating current conservation for active networks
   - Ringing test for amplifiers with feedback loops
4. **Conclusions**

4. Comparison

Features	This work	Replica measurement	Middlebrook’s method
Main objective	Self-loop function	Loop gain	Loop gain
Transfer function accuracy	Yes	No	No
Ringing Test	Yes	Yes	Yes
Operating region accuracy	Yes	No	No
Phase margin accuracy	Yes	No	No
Passive networks	Yes	No	No

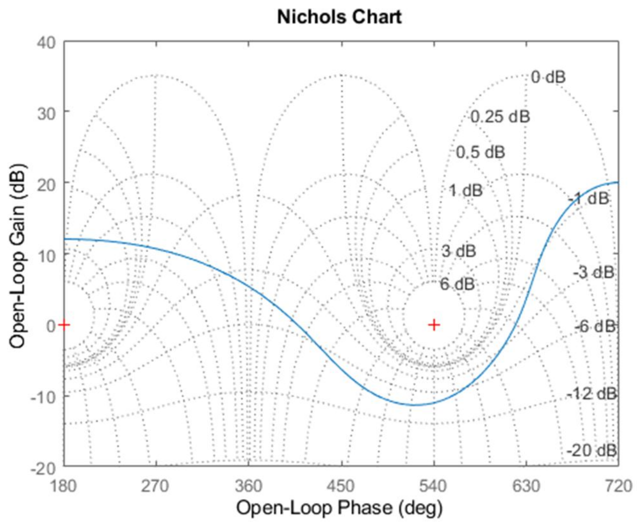
4. Discussions

- **Loop gain is independent of frequency variable.**

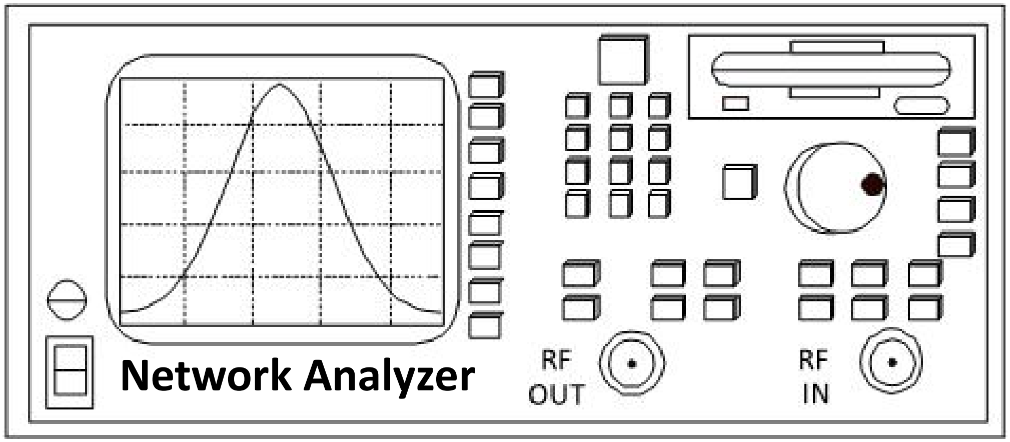
→Loop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.

Nichols chart isn't used widely in practical measurements (only used in control theory).



https://www.mathworks.com/help/control/ref/nichols.html



➡ **(Technology limitations)**

4. Conclusions

This work:

- **Proposal of alternating current conservation for deriving self-loop function in a transfer function**

  → Observation of self-loop function can help us optimize the behavior of a high-order system.

- **Implementations of circuits and measurements of self-loop functions for passive & active low-pass filters**
  →Theoretical concepts of stability test are verified by laboratory simulations and practical experiments.

Future of work:

- **Stability test for parasitic components in transmission lines, printed circuit boards, physical layout layers**

References

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[7] M. Tran, A. Kuwana, H. Kobayashi, "*Design of Active Inductor and Stability Test for Ladder RLC Low Pass Filter Based on Widened Superposition and Voltage Injection*", 8th IIAE Int. Conf. IAE, Shimane, Japan, March, 2020.

Thank you very much!

谢 谢

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