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STABILITY TEST FOR HIGH-ORDER LADDER LOW-PASS FILTERS

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Outline

1. Research Background
   - Motivation, objectives and achievements
   - Self-loop function in a transfer function
2. Ringing Test for Passive Networks
   - Stability test for RLC low-pass filters
3. Ringing Test for Ladder Low-Pass Filters
   - Stability test for active high-order ladder low-pass filters
4. Conclusions

1. Research Background

Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:

$$SNR = \frac{\text{Signal power}}{\text{Noise power}}$$

Performance of a device

Figure of Merit:

$$F = \frac{\text{Output SNR}}{\text{Input SNR}}$$

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

Linear networks

- Overshoot,
- Ringing
- Oscillation noise

1. Research Background

Effects of Ringing on Electronic Systems

Ringing represents a distortion of a signal.

Ringing is overshoot/undershoot voltage or current when it's seen on time domain.

Ringing does the following things:

- Causes EMI noise,
- Increases current flow,
- Consumes the power,
- Decreases the performance, and
- Damages the devices.

Unstable system

↓

STABILITY TEST

1. Research Background

Objectives of Study

- Derivation of transfer function in electronic systems using superposition theorem

○ Investigation of operating regions of linear negative feedback networks

→ Over-damping (high delay in rising time)

→ Critical damping (max power propagation)

→ Under-damping (overshoot and ringing)

- Stability test for electronic networks based on alternating current conservation

1. Research Background

Achievements of Study

Superposition formula for multi-source networks

$$V_O(t)\sum_{i=1}^{n}\frac{1}{Z_i}+V_O(t)\sum_{i=1}^{n}\frac{1}{Z_{si}+\dfrac{1}{\sum_{k=1}^{n}\dfrac{1}{Z_{pik}}}}=\sum_{i=1}^{n}\left(\frac{V_i(t)}{Z_i}+I_{ai}(t)-I_{gi}(t)\right)$$

Self-loop function

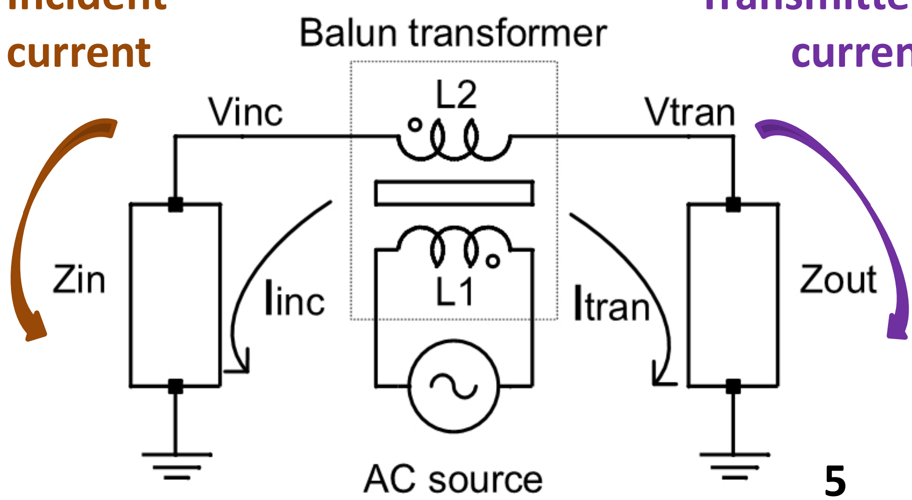
$$L(\omega)=-\frac{V_{inc}}{V_{trans}}$$

10 mH inductance

Alternating current conservation

Incident current

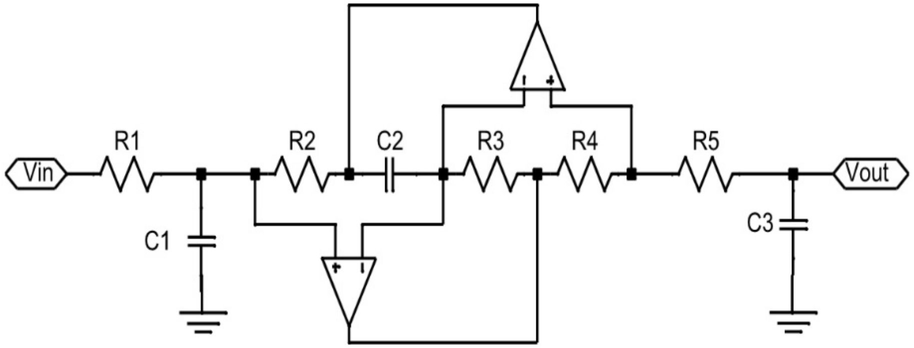
Transmitted current



1. Research Background

Approaching Methods

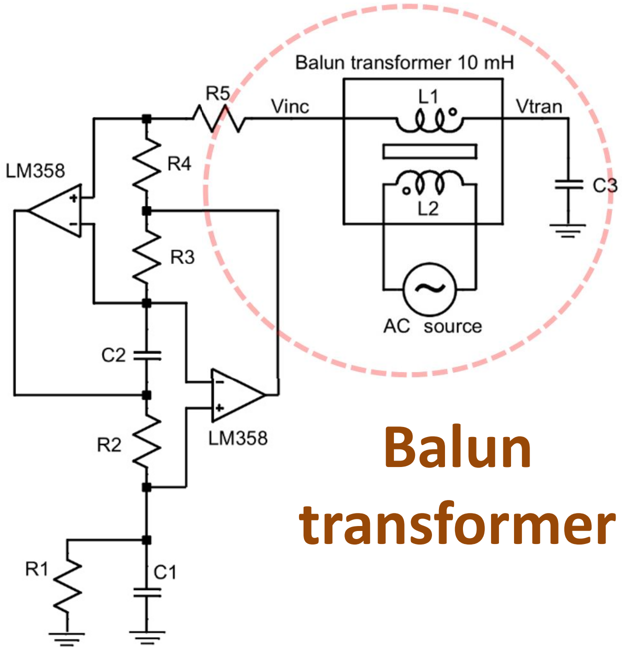
3rd-order ladder LPF



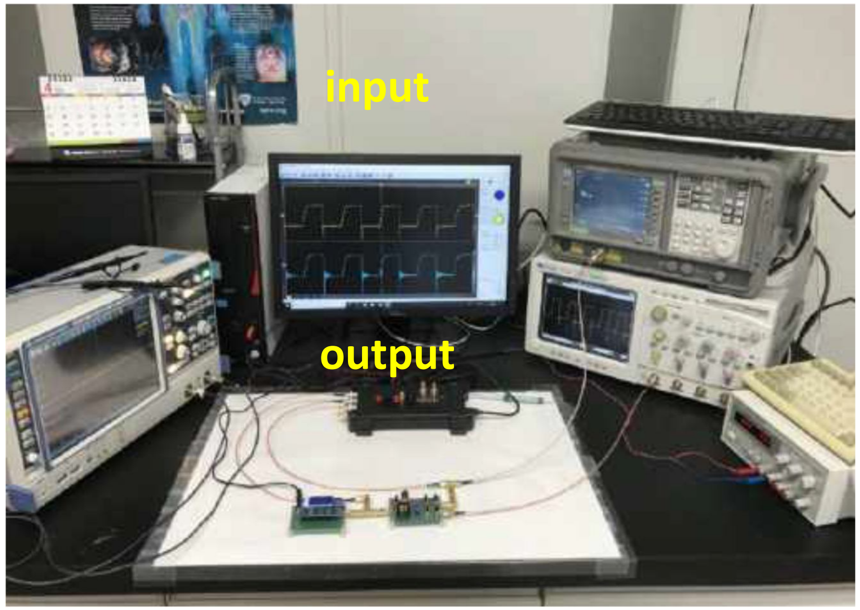
Balun transformer

Implemented circuit

Derivation of self-loop function



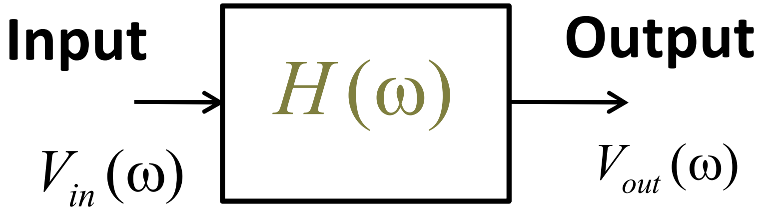
Balun transformer



1. Research Background

Self-loop Function in A Transfer Function

Linear system



Model of a linear system

$$H(\omega) = \frac{b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n}$$

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

$A(\omega)$ **: Open loop function**

$H(\omega)$ **: Transfer function**

$L(\omega)$ **: Self-loop function**

Variable: angular frequency (ω)

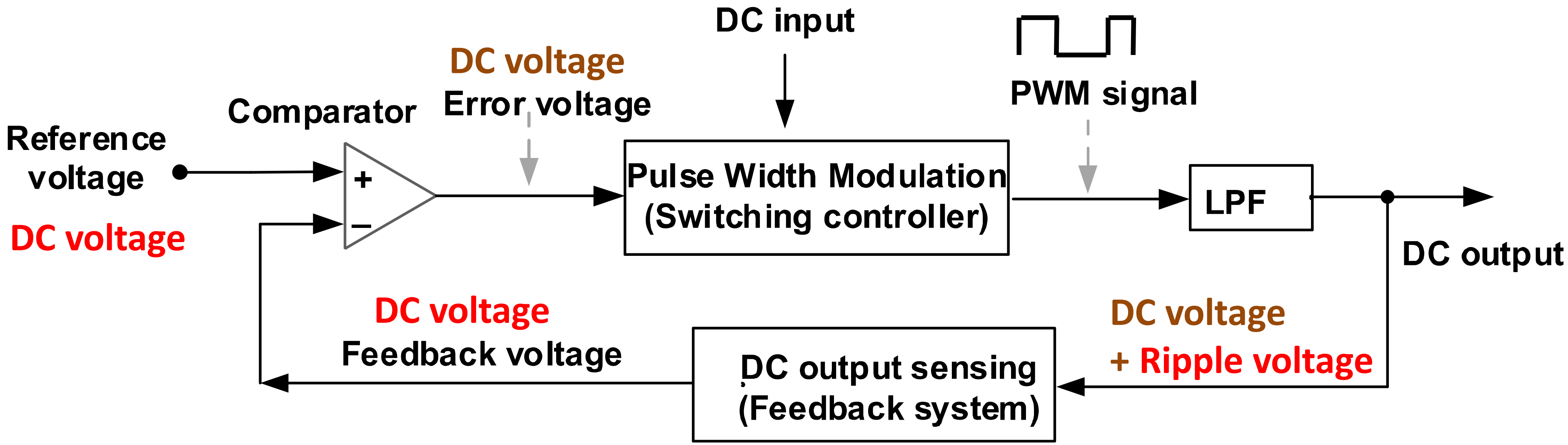
- **Polar chart → Nyquist chart**
- **Magnitude-frequency plot**
- **Angular-frequency plot**

  (Magnitude-frequency plot and Angular-frequency plot) → **Bode plots**
- **Magnitude-angular diagram → Nichols diagram**

1. Research Background

Characteristics of Adaptive Feedback Network

Block diagram of a typical adaptive feedback system



Adaptive feedback is used to control the output source along with the decision source (DC-DC Buck converter).

Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network.

→ Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).

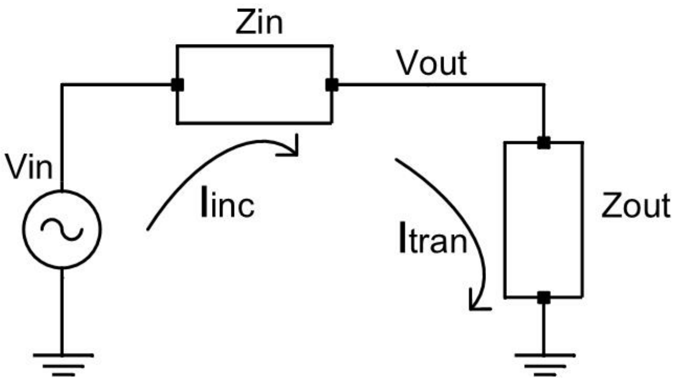
1. Research Background

Alternating Current Conservation

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + \dfrac{Z_{in}}{Z_{out}}}$$

$$\Rightarrow L(\omega) = \frac{Z_{in}}{Z_{out}};$$



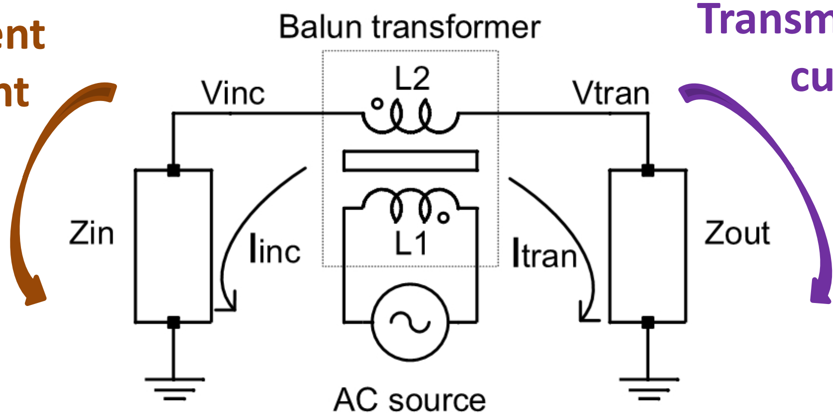
Simplified linear system

Self-loop function

$$\frac{V_{inc}}{Z_{in}} = -\frac{V_{trans}}{Z_{out}} \Rightarrow L(\omega) = -\frac{V_{inc}}{V_{trans}} = \frac{Z_{in}}{Z_{out}}$$

10 mH inductance

Incident current



Transmitted current

Derivation of self-loop function

1. Research Background

Limitations of Conventional Methods

- **Middlebrook's measurement of loop gain**

→ **Applying only in feedback systems (DC-DC converters).**

- **Replica measurement of loop gain**

→ **Using two identical networks (not real measurement).**

- **Nyquist's stability condition**

→ **Theoretical analysis for feedback systems (Lab tool).**

- **Nichols chart of loop gain**

→ **Only used in feedback control theory (Lab tool).**

- **Conventional superposition**

→ **Solving for every source (several times).**

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- Self-loop function in a transfer function

2. Ringing Test for Passive Networks
- **Stability test for RLC low-pass filters**

3. Ringing Test for Ladder Low-Pass Filters
- Stability test for active high-order ladder low-pass filters

4. Conclusions

2. Ringing Test for Passive Networks

Characteristics of 2nd-order Transfer Function

Second-order transfer function: $$H(\omega) = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega}$$

Case	Over-damping	Critical damping	Under-damping
Delta (Δ)	$\frac{1}{a_0} < \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 > 0$	$\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 = 0$	$\frac{1}{a_0} > \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 < 0$
Module $\lvert H(\omega)\rvert$	$\frac{\frac{1}{a_0}}{\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}}$	$\frac{\frac{1}{a_0}}{\left[\omega^2 + \left(\frac{a_1}{2a_0}\right)^2\right]} = \frac{1}{2} = -6dB$	$\frac{\frac{1}{a_0}}{\sqrt{\left(\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}\sqrt{\left(\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}}$
Angular $\theta(\omega)$	$-\arctan\left(\frac{\omega}{\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}}\right) - \arctan\left(\frac{\omega}{\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}}\right)$	$-2\arctan\left(\frac{2a_0\omega}{a_1}\right)$	$-\arctan\left(\frac{\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right) - \arctan\left(\frac{\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)$
$\omega_{cut} = \frac{a_1}{2a_0}$	$\lvert H(\omega_{cut})\rvert < \frac{2a_0}{a_1}$; $\theta(\omega_{cut}) > -\frac{\pi}{2}$	$\lvert H(\omega_{cut})\rvert = \frac{2a_0}{a_1}$; $\theta(\omega_{cut}) = -\frac{\pi}{2}$	$\lvert H(\omega_{cut})\rvert > \frac{2a_0}{a_1}$; $\theta(\omega_{cut}) < -\frac{\pi}{2}$

2. Ringing Test for Passive Networks

Characteristics of 2nd-order Self-loop Function

Second-order self-loop function: $L(\omega) = j\omega\left[a_0 j\omega + a_1\right]$

Case	Over-damping		Critical damping		Under-damping	
Delta (Δ)	$\Delta = a_1^2 - 4a_0 > 0$		$\Delta = a_1^2 - 4a_0 = 0$		$\Delta = a_1^2 - 4a_0 < 0$	
$\lvert L(\omega)\rvert$	$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$	
$\theta(\omega)$	$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$	
$\omega_1 = \frac{a_1}{2a_0}\sqrt{\sqrt{5}-2}$	$\lvert L(\omega_1)\rvert > 1$	$\pi - \theta(\omega_1) > 76.3^o$	$\lvert L(\omega_1)\rvert = 1$	$\pi - \theta(\omega_1) = 76.3^o$	$\lvert L(\omega_1)\rvert < 1$	$\pi - \theta(\omega_1) < 76.3^o$
$\omega_2 = \frac{a_1}{2a_0}$	$\lvert L(\omega_2)\rvert > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^o$	$\lvert L(\omega_2)\rvert = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^o$	$\lvert L(\omega_2)\rvert < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^o$
$\omega_3 = \frac{a_1}{a_0}$	$\lvert L(\omega_3)\rvert > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^o$	$\lvert L(\omega_3)\rvert = 4\sqrt{2}$	$\pi - \theta(\omega_3) = 45^o$	$\lvert L(\omega_3)\rvert < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^o$

2. Ringing Test for Passive Networks

Operating Regions of 2nd-Order System

- **Under-damping:**

$$L_1(\omega) = (j\omega)^2 + j\omega;$$

$$H_1(\omega) = \frac{1}{(j\omega)^2 + j\omega + 1};$$

- **Critical damping:**

$$L_2(\omega) = (j\omega)^2 + 2j\omega;$$

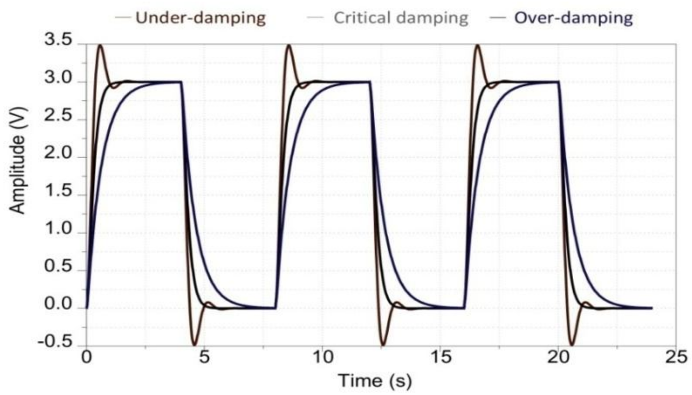
$$H_2(\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1};$$

- **Over-damping:**

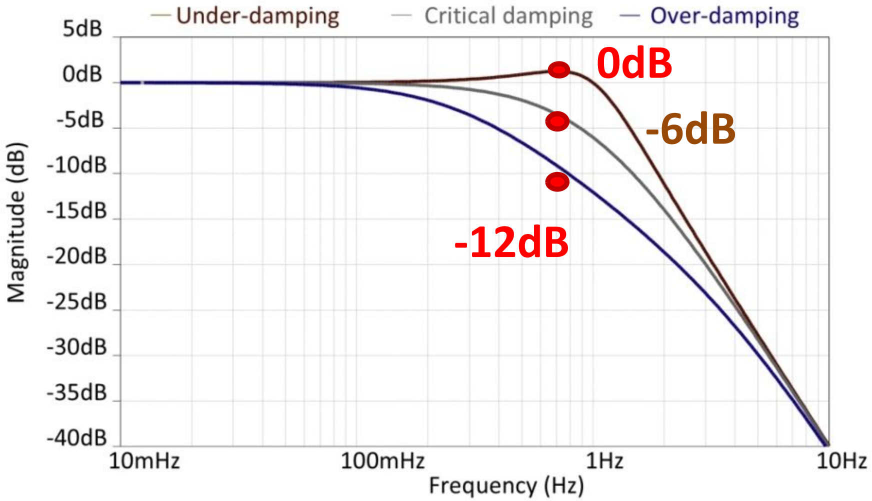
$$L_3(\omega) = (j\omega)^2 + 3j\omega;$$

$$H_3(\omega) = \frac{1}{(j\omega)^2 + 3j\omega + 1};$$

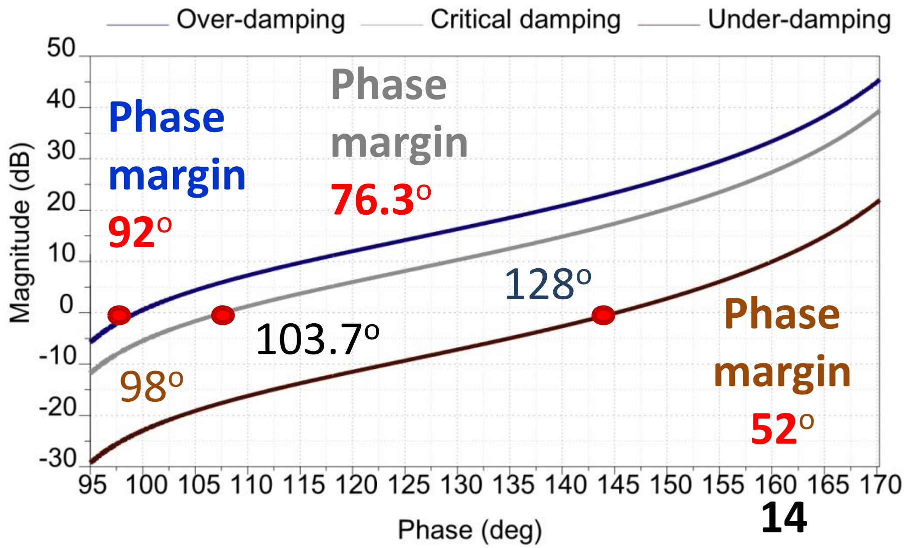
Transient response



Bode plot of transfer function



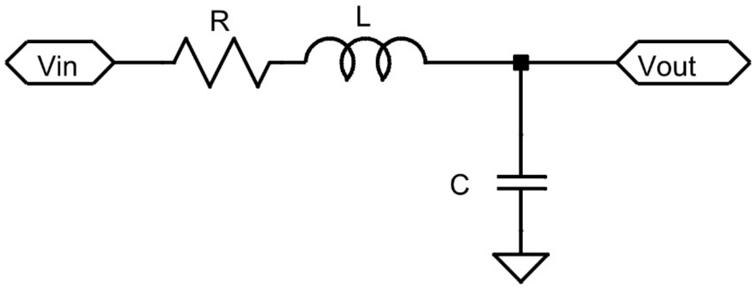
Nichols plot of self-loop function



2. Ringing Test for Passive Networks

Stability Test for Passive 2rd-Order RLC LPF

Passive RLC Low-pass Filter



Transfer function

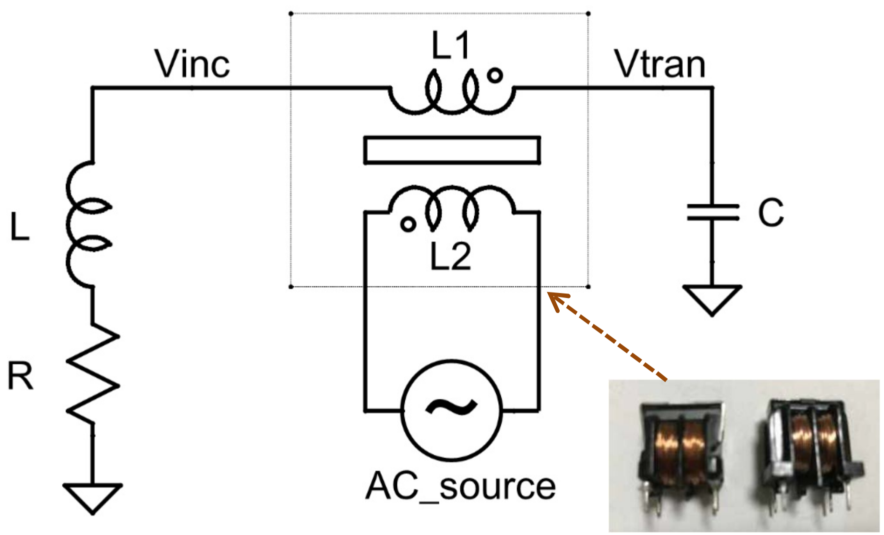
$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

Self-loop function

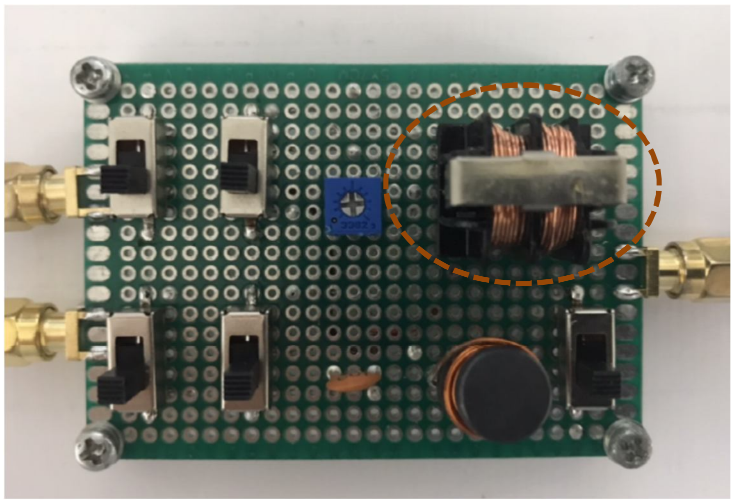
$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

where, $a_0 = LC; a_1 = RC;$

Derivation of self-loop function



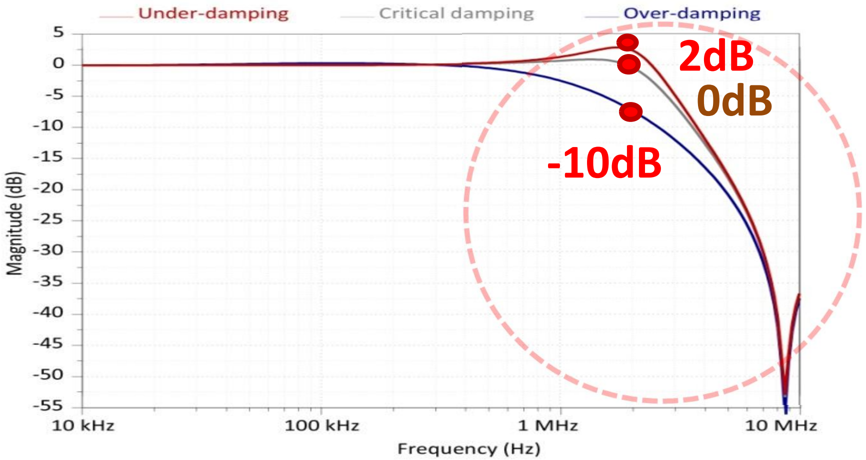
Implemented circuit



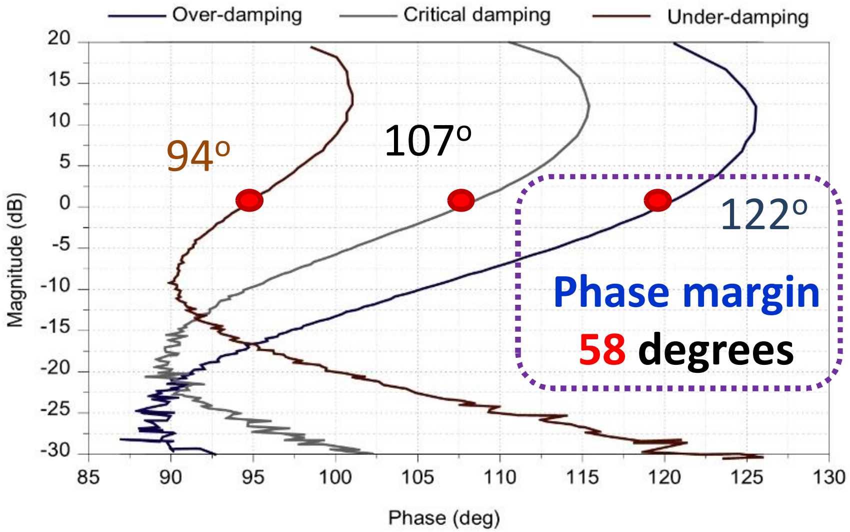
2. Ringing Test for Passive Networks

Stability Test for 2rd-Order Passive RLC LPF

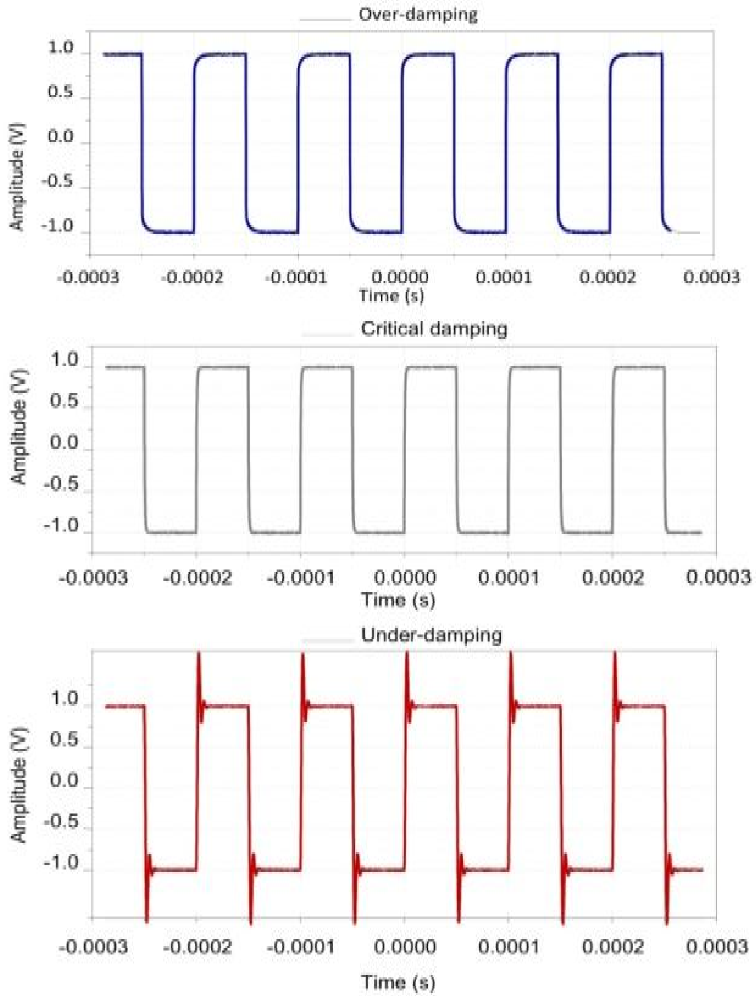
Bode plot of transfer function



Nichols plot of self-loop function



Transient responses



Outline

1. Research Background

- Motivation, objectives and achievements
- Self-loop function in a transfer function

2. Ringing Test for Passive Networks

- Stability test for RLC low-pass filters

3. **Ringing Test for Ladder Low-Pass Filters**

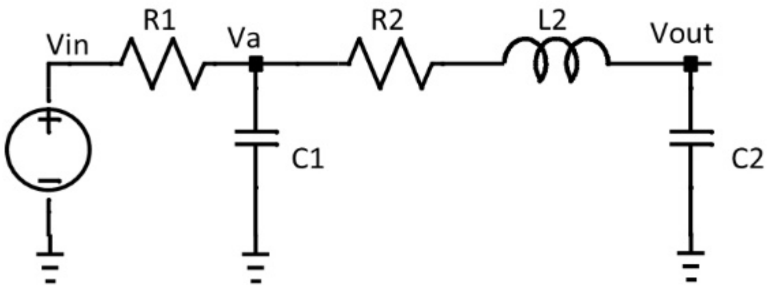
- **Stability test for active high-order ladder low-pass filters**

4. Conclusions

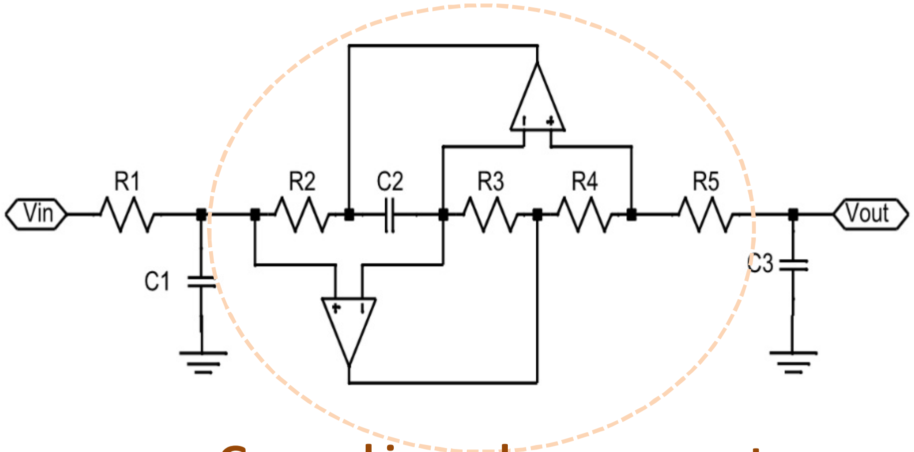
3. Ringing Test for Ladder Low-Pass Filters

Analysis of Active 3rd-Order Ladder LPF

Passive 3rd-order ladder LPF



Active 3rd-order ladder LPF



General impedance converter

Transfer function & self-loop function

$$H_{out}(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{a_0(j\omega)^3 + a_1(j\omega)^2 + a_2 j\omega + 1};$$

$$L(\omega) = j\omega\left[a_0(j\omega)^2 + a_1 j\omega + a_2\right]$$

where, $b_0 = L_2C_2;\ b_1 = R_2C_2;$

$$a_0 = R_1C_1L_2C_2;\ \ a_1 = R_1C_1R_2C_2 + L_2C_2;$$

$$a_2 = R_1\left(C_1 + C_2\right) + R_2C_2;$$

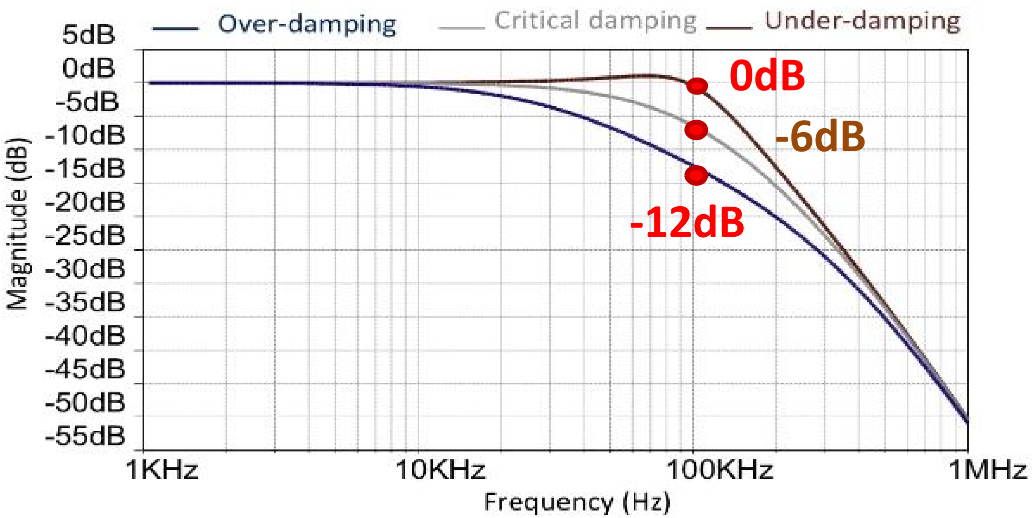
R1 = 100 Ω, R2 = 50 kΩ, R3 = R4 = 50 kΩ, C1 = 5 nF, C2 = 10 nF, C3 = 3.18 nF, at f_0 = 100 kHz.

- **Over-damping (R5 = 0.5 kΩ),**
- **Critical damping (R5 = 1 kΩ), and**
- **Under-damping (R5 = 2 kΩ).**

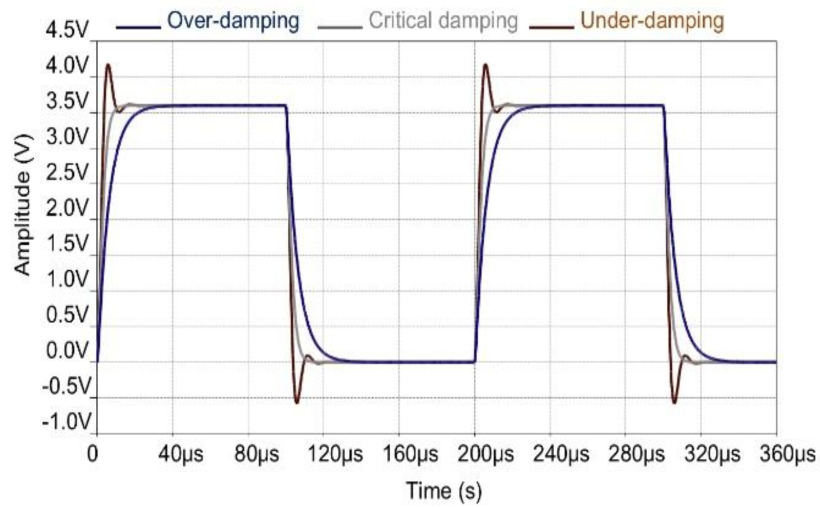
3. Ringing Test for Ladder Low-Pass Filters

Simulation Results of 3rd-Order Ladder LPF

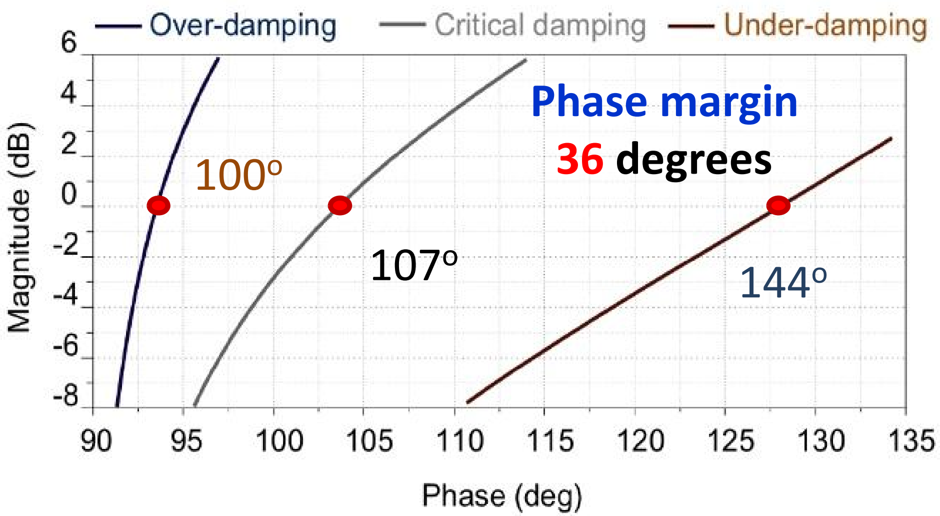
Bode plot of transfer function



Transient response



Nichols plot of self-loop function



Over-damping:

→ **Phase margin is 80 degrees.**

Critical damping:

→ **Phase margin is 73 degrees.**

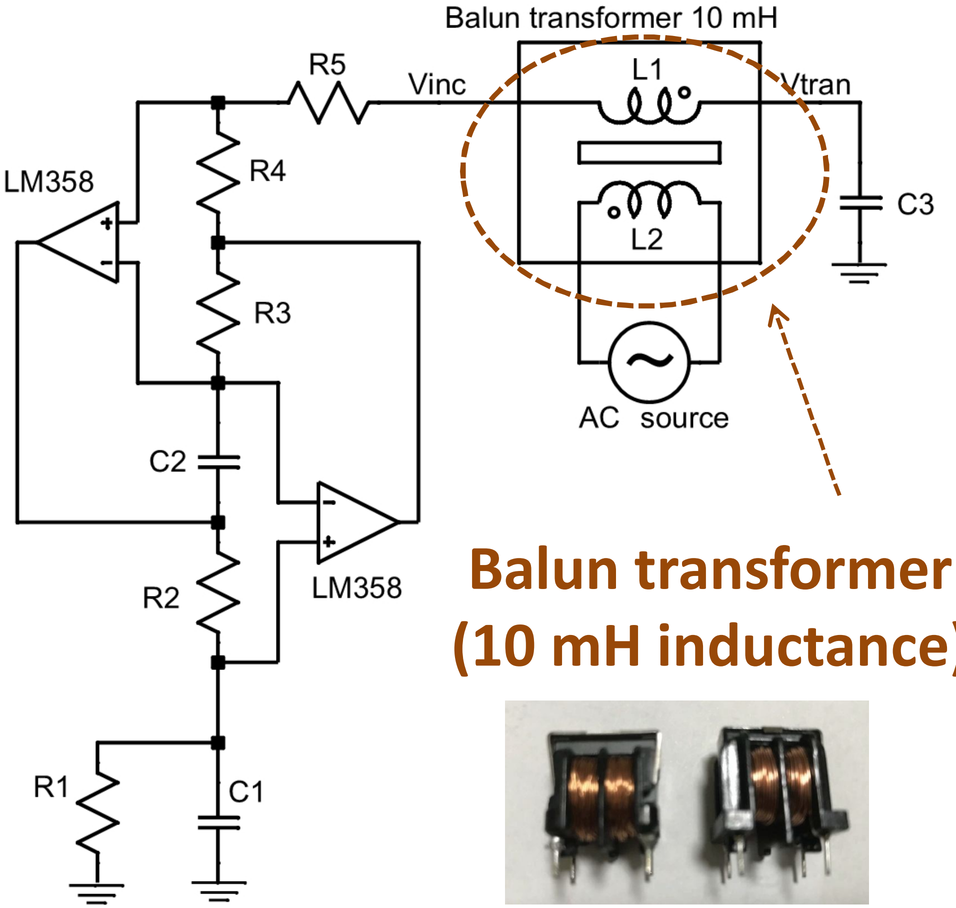
Under-damping:

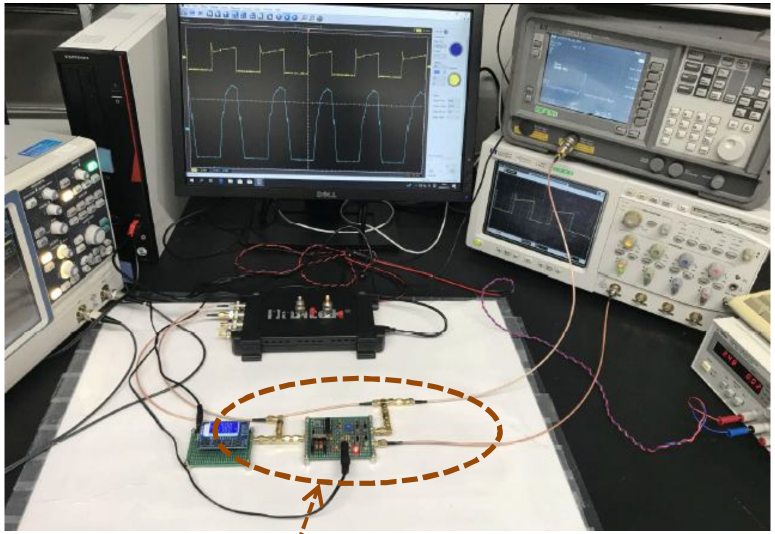
→ **Phase margin is 36 degrees.**

3. Ringing Test for Ladder Low-Pass Filters

Implemented Circuit of 3rd-Order Ladder LPF

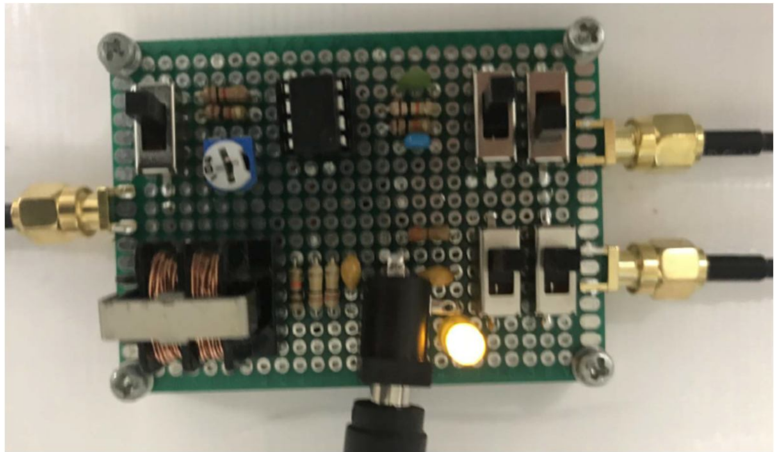
Measurement of self-loop function





Device under test

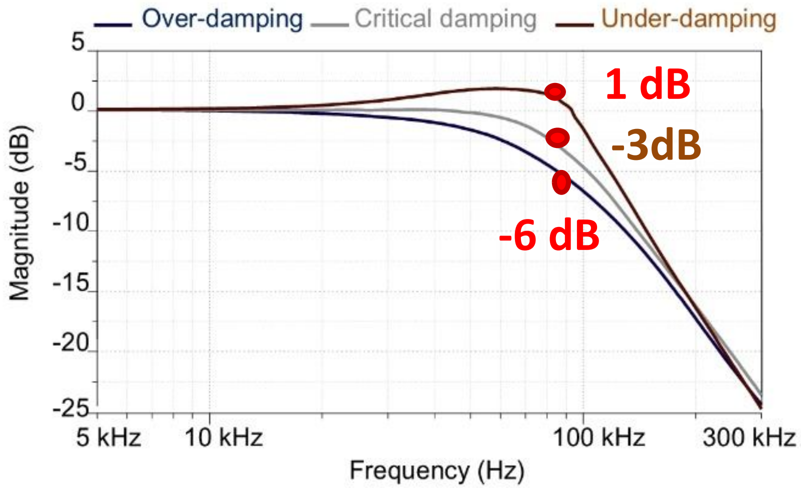
Balun transformer (10 mH inductance)



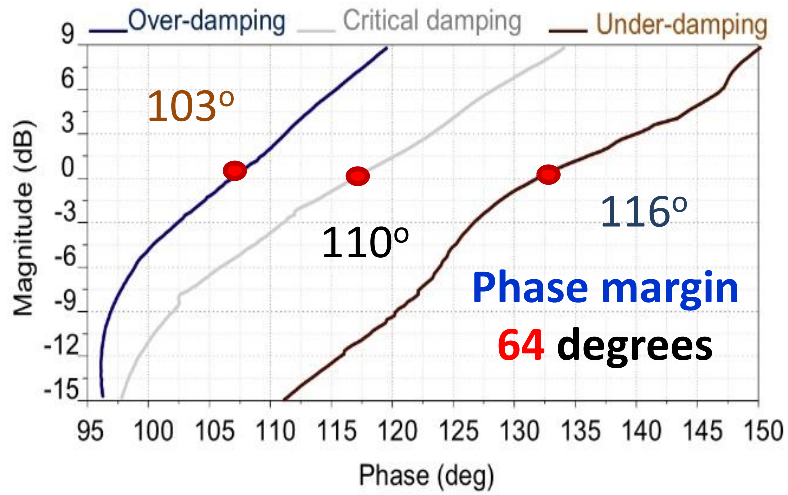
3. Ringing Test for Ladder Low-Pass Filters

Measurement Results of 3rd-order Ladder LPF

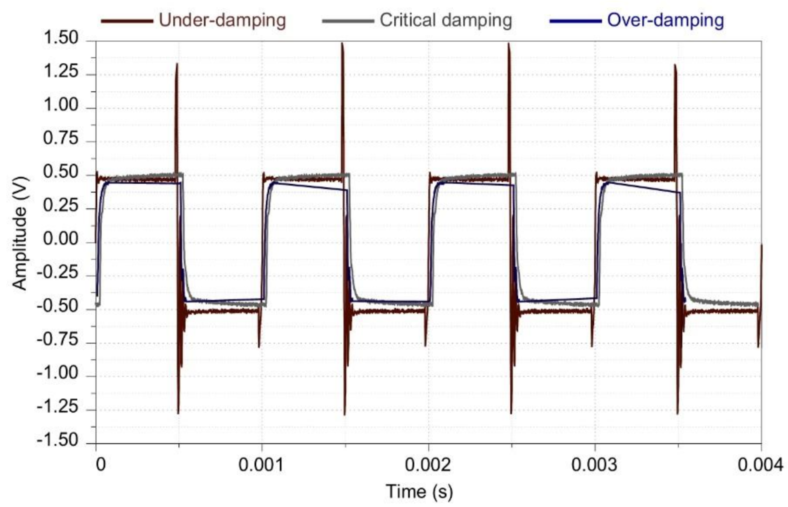
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Over-damping:

→Phase margin is 77 degrees.

Critical damping:

→Phase margin is 70 degrees.

Under-damping:

→Phase margin is 64 degrees.

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4. **Conclusions**

4. Comparison (Self-loop function)

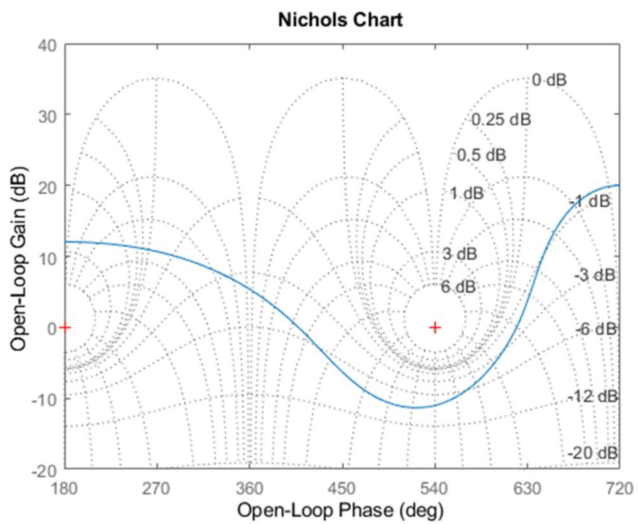
Features	Alternating current conservation	Replica measurement	Middlebrook's method
Main objective	Self-loop function	Loop gain	Loop gain
Transfer function accuracy	Yes	No	No
Ringing Test	Yes	Yes	Yes
Operating region accuracy	Yes	No	No
Phase margin accuracy	Yes	No	No
Passive networks	Yes	No	No

4. Discussions (Self-loop function)

- Loop gain is independent of frequency variable.

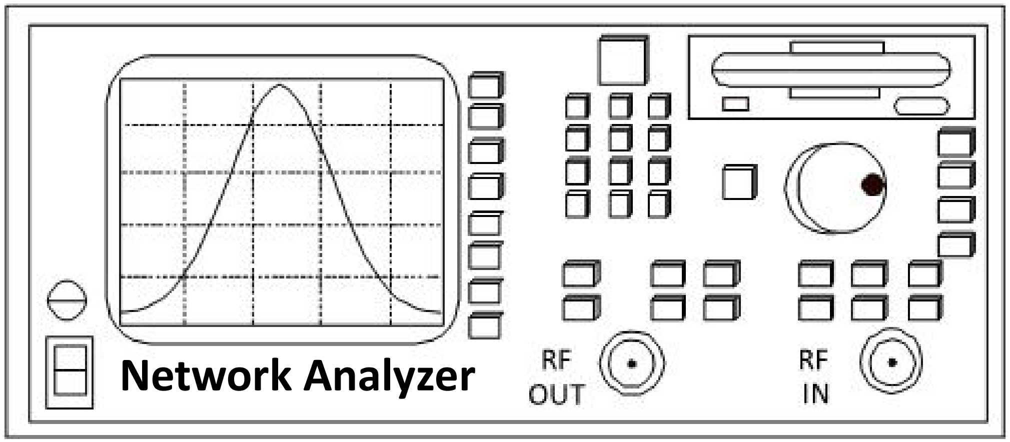
→Loop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.



https://www.mathworks.com/help/control/ref/nichols.html

Nichols chart isn't used widely in practical measurements (only used in control theory).



➡ (Technology limitations)

4. Conclusions

This work:

- **Proposal of alternating current conservation for deriving self-loop function in a transfer function**

  → Observation of self-loop function can help us optimize the behavior of a high-order system.

- **Implementations of circuits and measurements of self-loop functions for passive & active low-pass filters**

  →Theoretical concepts of stability test are verified by laboratory simulations and practical experiments.

Future of work:

- **Stability test for parasitic components in transmission lines, printed circuit boards, physical layout layers**

References

[1] H. Kobayashi, N. Kushita, M. Tran, K. Asami, H. San, A. Kuwana "*Analog - Mixed-Signal - RF Circuits for Complex Signal Processing*", 13th IEEE Int. Conf. ASIC, Chongqing, China, Nov, 2019.

[2] M. Liu, I. Dassios, F. Milano, "*On the Stability Analysis of Systems of Neutral Delay Differential Equations*", Circuits, Systems, and Signal Processing, Vol. 38(4), 1639-1653, 2019.

[3] X. Peng, H. Yang, "*Impedance-based stability criterion for the stability evaluation of grid-connected inverter systems with distributed parameter lines*", CSEE J. Power and Energy Systems, pp 1-13, 2020.

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Thank you very much!

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