Virtual TJCAS 2020
Taiwan and Japan Conference on
Circuits and Systems

DERIVATION OF IMAGE REJECTION RATIO FOR HIGH-ORDER COMPLEX FILTERS

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(Nov. 7th, 2020)

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Outline

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1. Research Background

- Motivation, objectives and achievements
- Superposition formula for multi-source networks

2. Behaviors of High-Order Polyphase Filters

- Derivation of image rejection ratio for polyphase filters

3. Behaviors of High-Order Complex Filters

- Derivation of image rejection ratio for complex filters

4. Conclusions

1. Research Background

Motivation of Study

Performance of a system

Signal to Noise Ratio:

$$\mathrm{SNR} = \frac{\text{Signal power}}{\text{Noise power}}$$

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Performance of a device

Figure of Merit:

$$F = \frac{\text{Output SNR}}{\text{Input SNR}}$$

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

Multi-phase networks

- Image noise,
- I/Q mismatches
- DC offsets

1. Research Background

Objectives of Study

- Derivation of transfer function in multi-source systems using superposition theorem
- Investigation of behaviors of high-order passive RC polyphase filter networks
- Investigation of behaviors of high-order complex filter networks
- Derivation of image rejection ratio in low-IF receivers

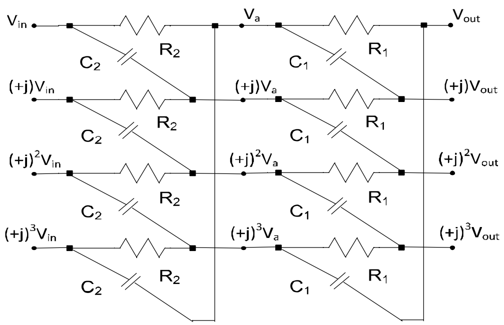
1. Research Background

Achievements of Study

Superposition formula for multi-source networks

$$V_O(t)\sum_{i=1}^{n}\frac{1}{Z_i}+V_O(t)\sum_{i=1}^{n}\frac{1}{Z_{si}+\cfrac{1}{\sum_{k=1}^{n}\frac{1}{Z_{pik}}}}=\sum_{i=1}^{n}\left(\frac{V_i(t)}{Z_i}+I_{ai}(t)-I_{gi}(t)\right)$$

2nd-order polyphase filter



4th-order complex filter

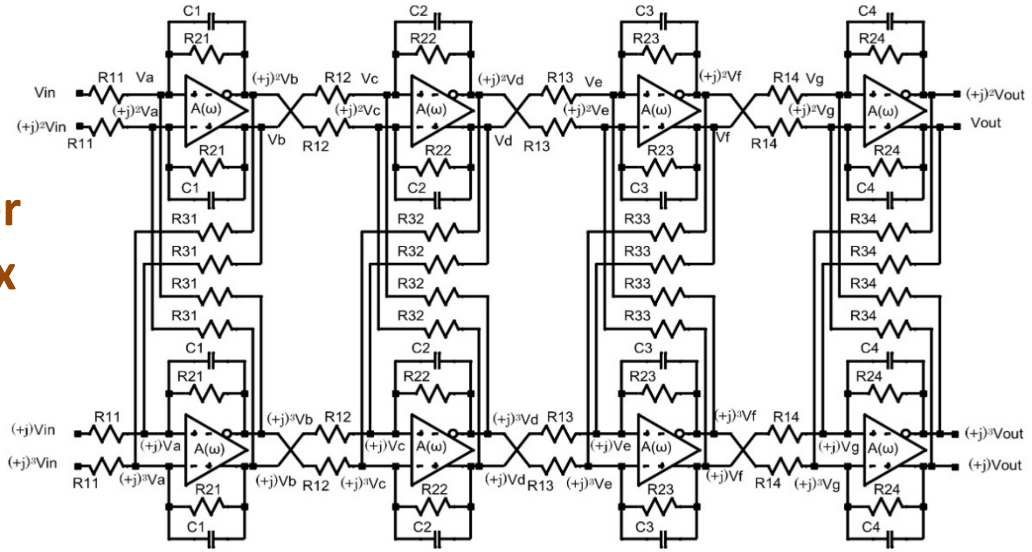


Image rejection ratio

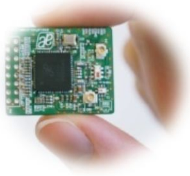
$$IRR(\omega)=\frac{\left|(1+b_1\omega)(1+b_2\omega)\right|}{\left|(1-b_1\omega)(1-b_2\omega)\right|};$$

Image rejection ratio

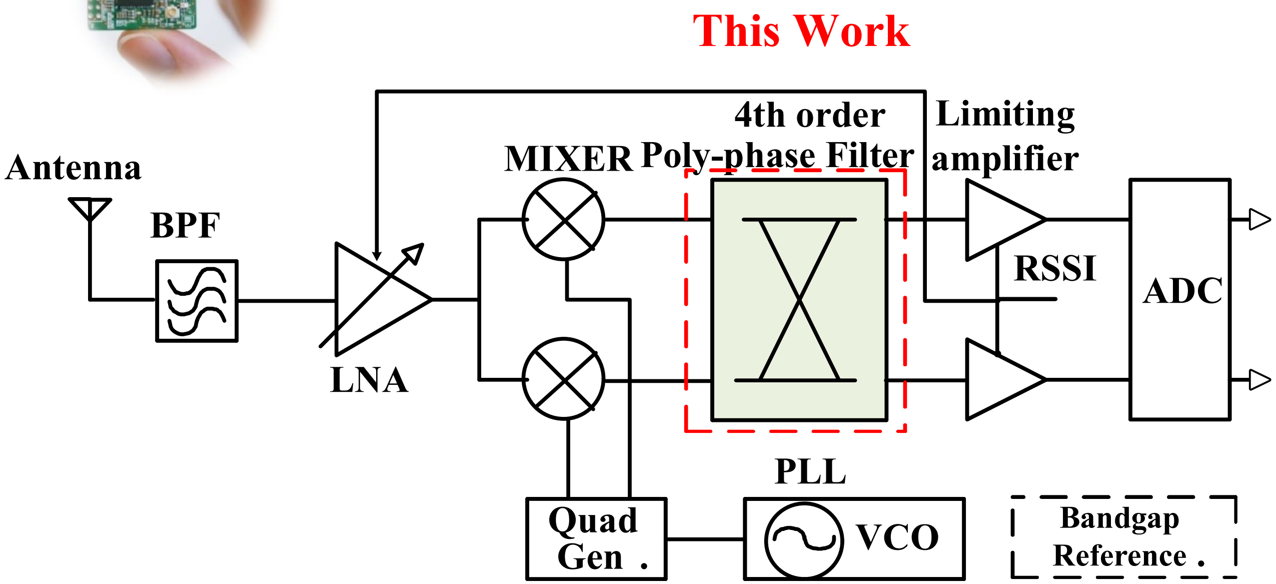
$$IRR(\omega)=\frac{\left[j\left(\frac{\omega}{\omega_{cut1}}+\frac{R_{21}}{R_{31}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut2}}+\frac{R_{22}}{R_{32}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut3}}+\frac{R_{23}}{R_{33}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut4}}+\frac{R_{24}}{R_{34}}\right)+1\right]}{\left[j\left(\frac{\omega}{\omega_{cut1}}-\frac{R_{21}}{R_{31}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut2}}-\frac{R_{22}}{R_{32}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut3}}-\frac{R_{23}}{R_{33}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut4}}-\frac{R_{24}}{R_{34}}\right)+1\right]}$$

1. Research Background

Low-IF Receiver System Architecture



Block diagram of low-IF receiver



Applications: Wi-Fi, WiMax, UWB, GSM, WCDMA, LTE , 4G, Cordless Phones, RFID, ZigBee, Bluetooth, TV Set Top Box, Sensing, Radar…

Merits

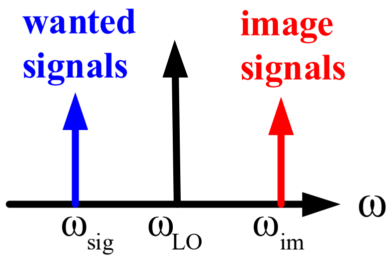
- Low-cost
- Small-size
- High-integration

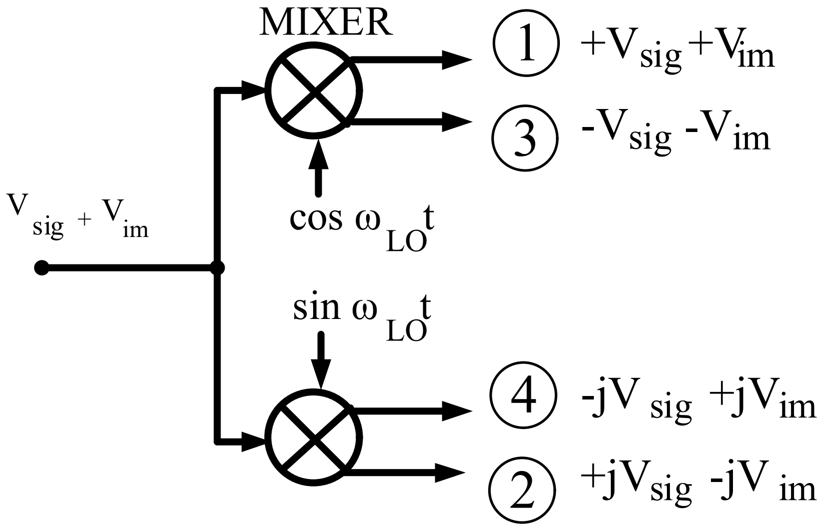
Demerits

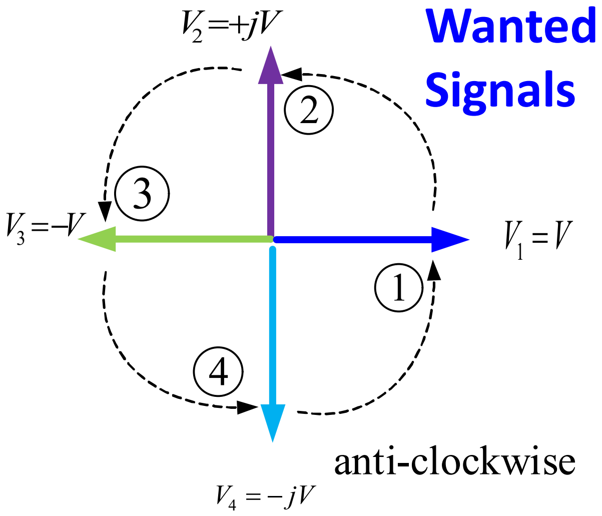
- Image Noises
- Power Loss
- Noise Figure

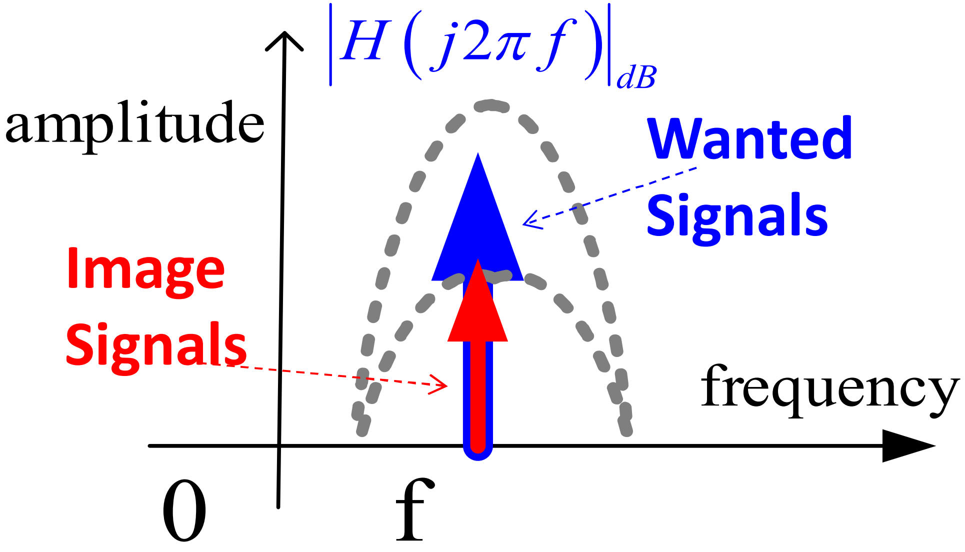
1. Research Background

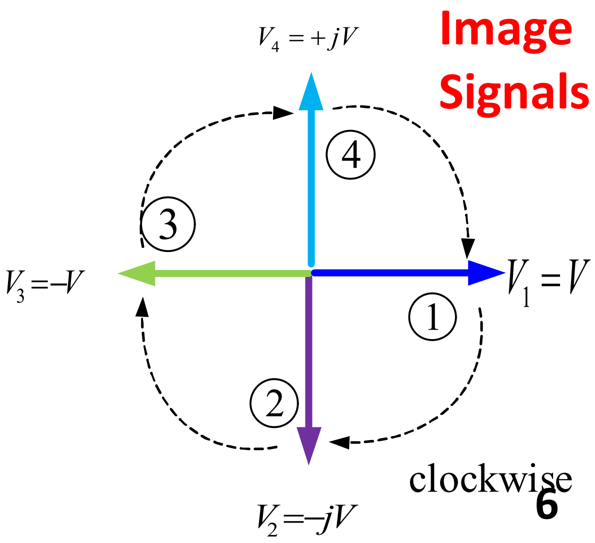
Characteristics of Low-IF Receiver Signals







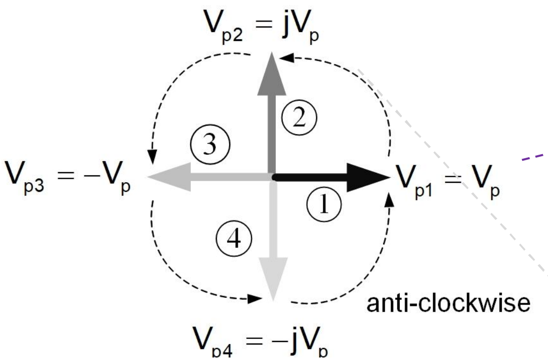




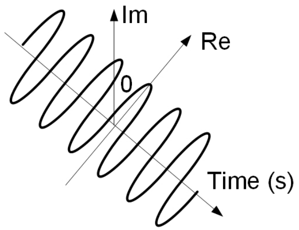
1. Research Background

Positive Polyphase Signals on Frequency Domain

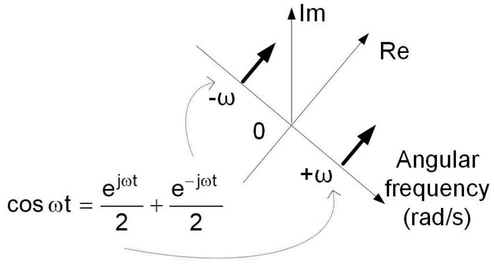
Positive polyphase signals



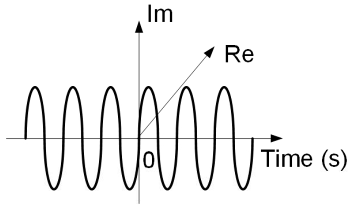
$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}$$
$$=\{1;+j;(+j)^2;(+j)^3\}V_{pos}(t)$$



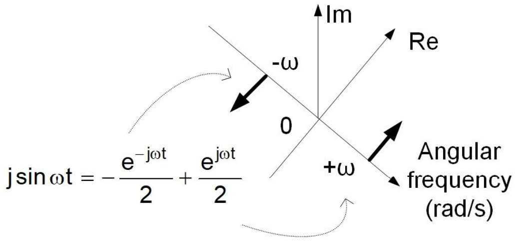
(a) Cosine wave on real axis



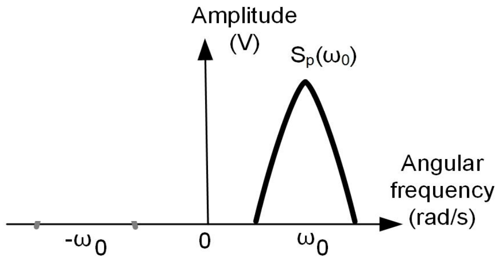
(b) Spectrum of cosine wave



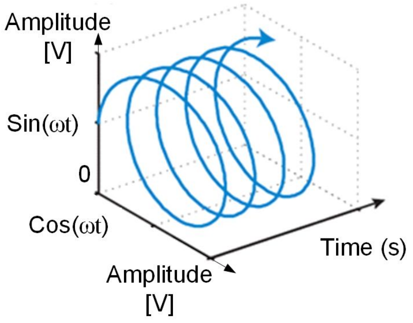
(c) Plus sine wave on imaginary axis



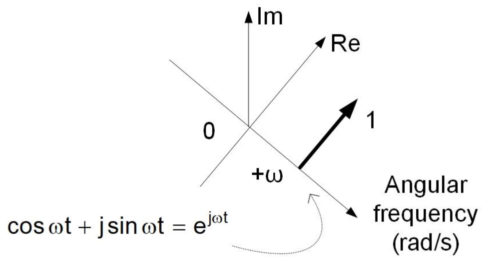
(d) Spectrum of plus sine wave



(c) Angular frequency plane for polyphase signals



(e) Positive angular frequency wave

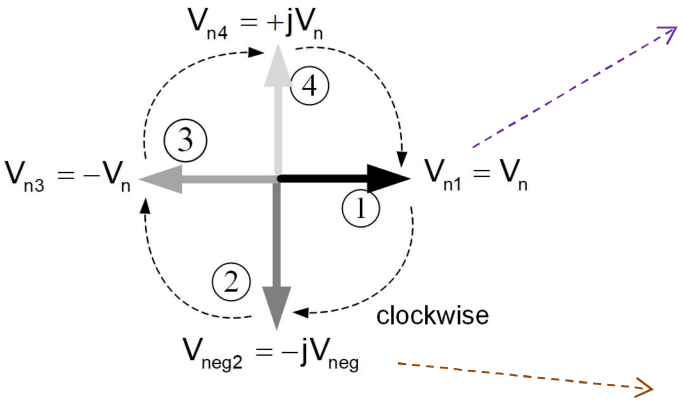


(f) Spectrum of positive angular frequency wave

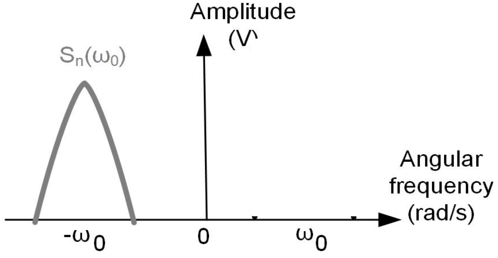
1. Research Background

Negative Polyphase Signals on Frequency Domain

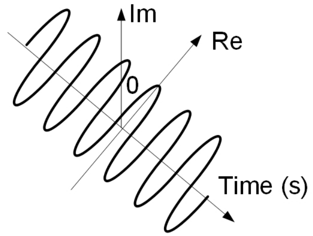
Negative polyphase signals



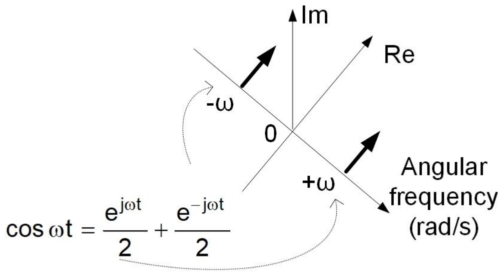
$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;-j;(-j)^2;(-j)^3\}V_{neg}(t)$$



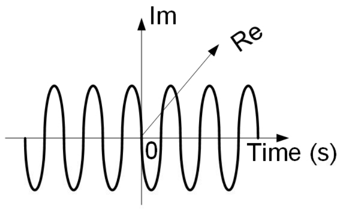
(c) Angular frequency plane for polyphase signals



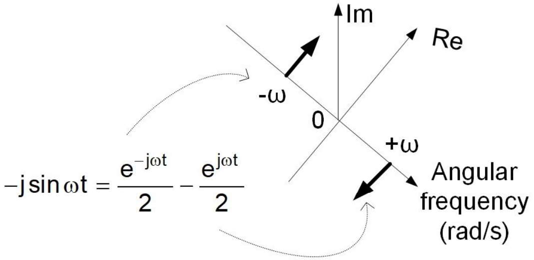
(a) Cosine wave on real plane



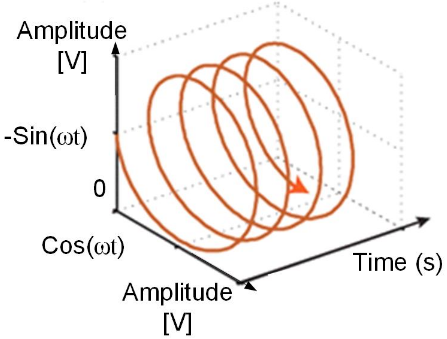
(b) Spectrum of cosine wave



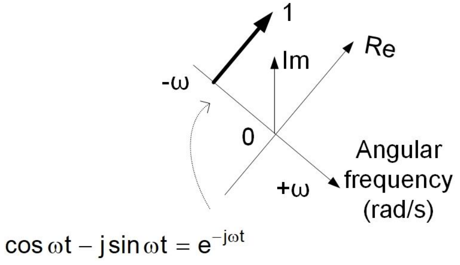
(c) Minus sine wave on imaginary plane



(d) Spectrum of minus sine wave



(e) Negative angular frequency wave

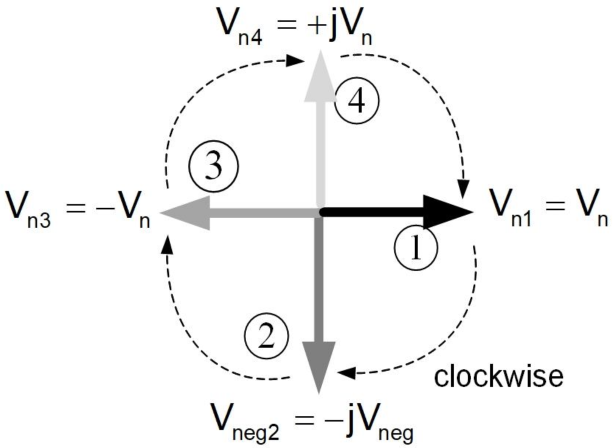


(f) Spectrum of negative angular frequency wave

1. Research Background

Polyphase Signals on Frequency Domain

Negative polyphase signals



Positive polyphase signals

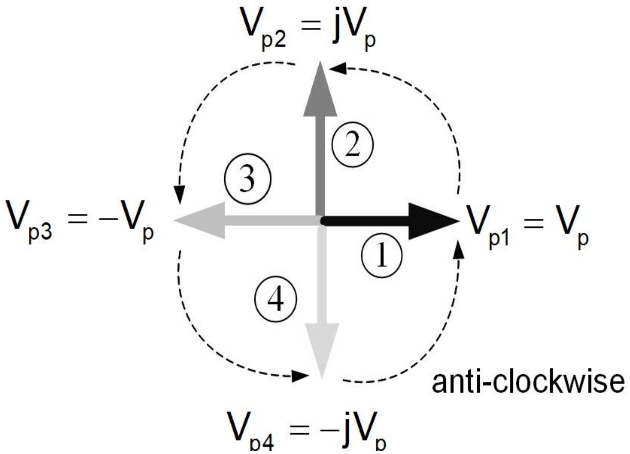
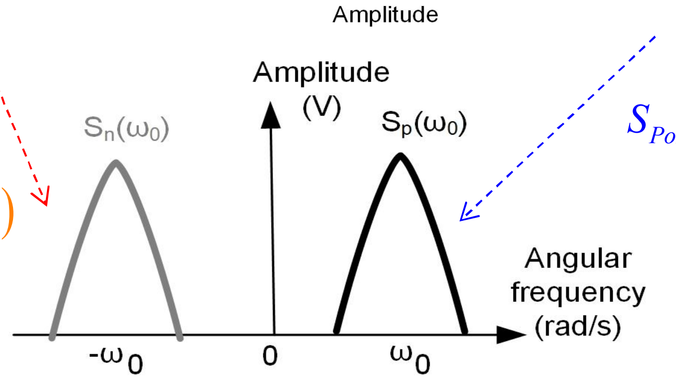


Image Signals

$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;-j;(-j)^2;(-j)^3\}V_{neg}(t)$$



(c) Angular frequency plane for polyphase signals

Wanted Signals

$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;+j;(+j)^2;(+j)^3\}V_{pos}(t)$$

1. Research Background

Superposition Theorem for Multi-Source Systems

Superposition formula:

$$V_O(t)\sum_{i=1}^{n}\frac{1}{Z_i}+V_O(t)\sum_{i=1}^{n}\frac{1}{Z_{si}+\frac{1}{\sum_{k=1}^{n}\frac{1}{Z_{pik}}}}=\sum_{i=1}^{n}\left(\frac{V_i(t)}{Z_i}+I_{ai}(t)-I_{gi}(t)\right)$$

$V_O(t)$: *Voltage at one node*

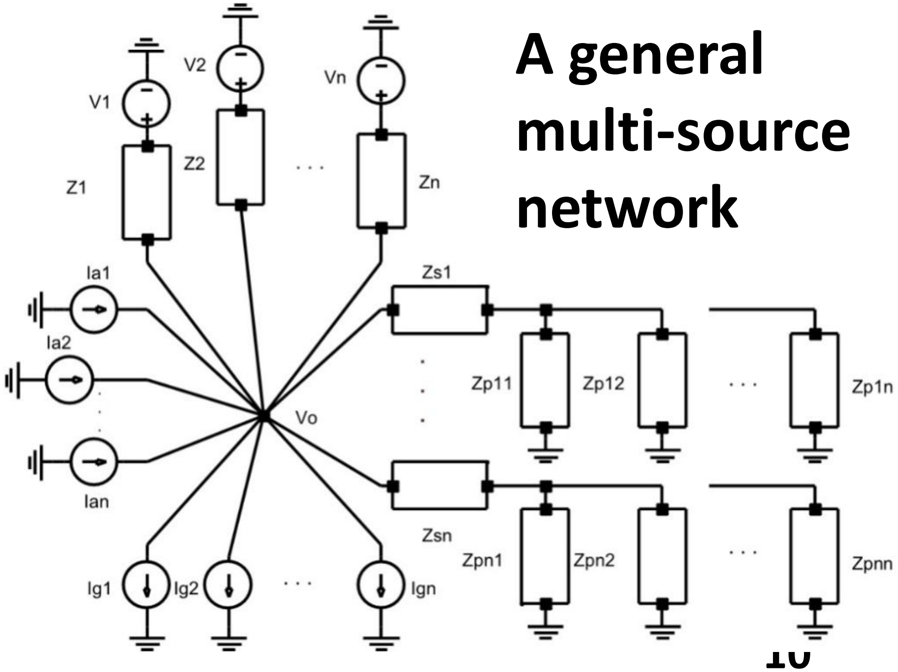
$V_i(t)$: *Input voltage sources*

$I_{ai}(t)$: *Ahead-toward current sources*

$I_{gi}(t)$: *Ground-toward current sources*

$Z_{i,\ si,\ pi,}(t)$: Impedances at each branch

- ○ **Multi-source systems, feedback networks (op amps, amplifiers), polyphase filters, complex filters…**



A general multi-source network

Outline

1. Research Background
   - Motivation, objectives and achievements
   - Superposition formula for multi-source networks
2. **Behaviors of High-Order Polyphase Filters**
   - **Derivation of image rejection ratio for polyphase filters**
3. Behaviors of High-Order Complex Filters
   - Derivation of image rejection ratio for complex filters
4. Conclusions

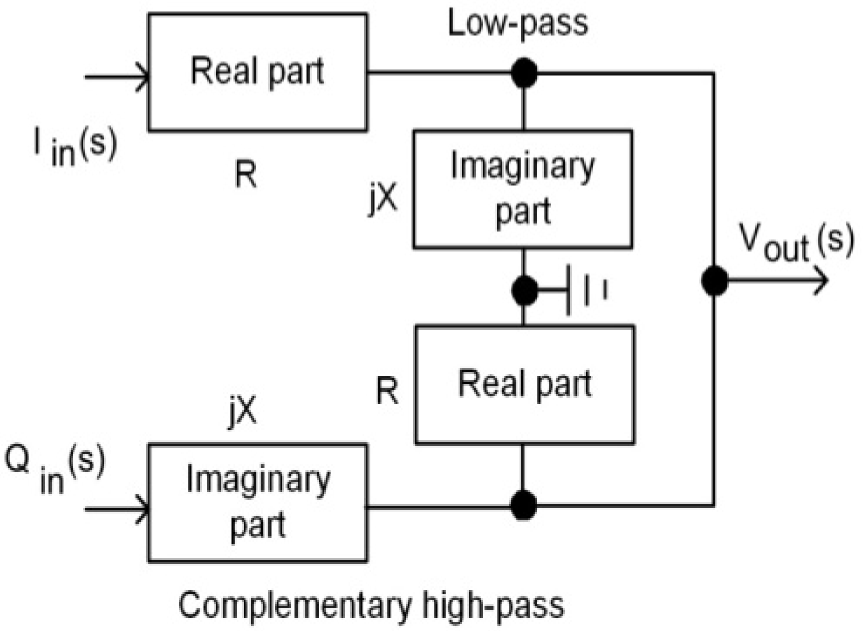
2. Investigation of Multi-Phase Networks

Design Principle for Polyphase Filter Networks

Complementation between low-pass and high-pass circuits

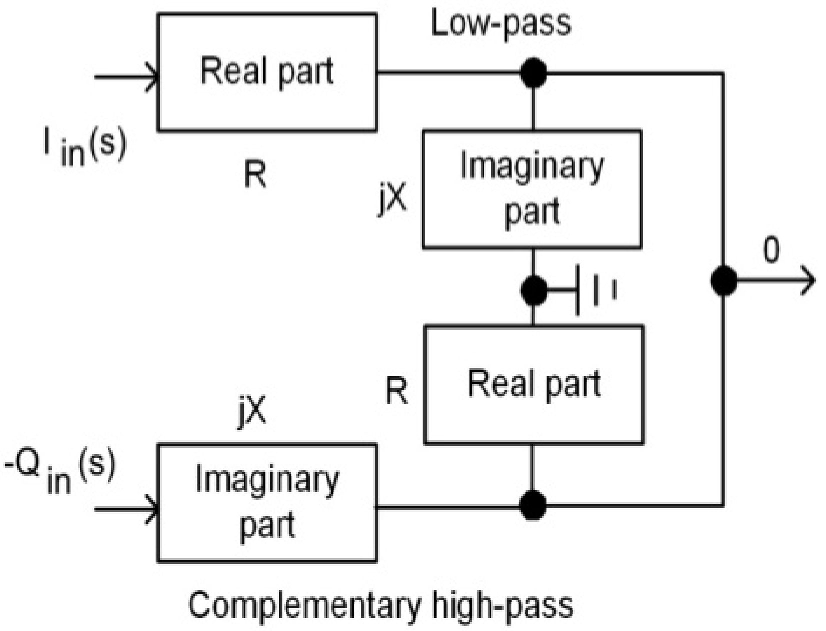
→ a passive polyphase filter

Wanted Signals



(c) Pass-band filter (wanted signals)

Image Signals

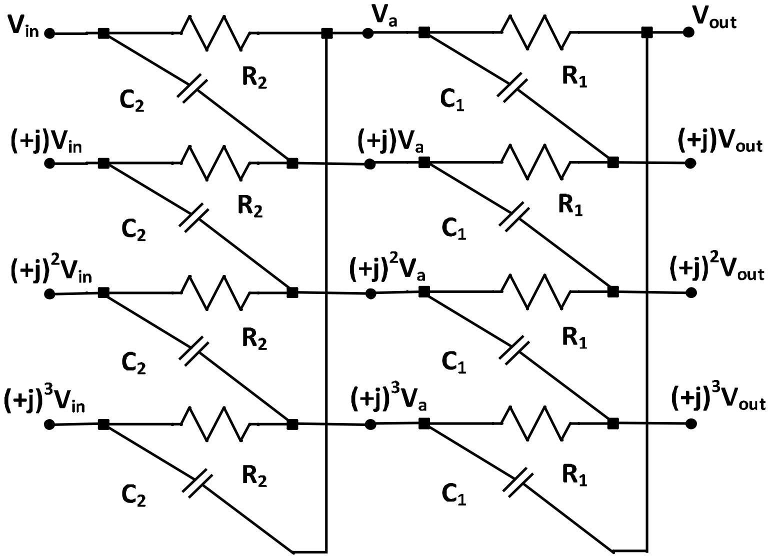


(d) Notch-band filter (image signal)

2. Investigation of Multi-Phase Networks

Analysis of 2nd–Order Polyphase Filter

Second-order RC polyphase filter



Apply superposition at each node

$$V_{out}\left(\frac{1}{Z_{C1}}+\frac{1}{R_1}\right)=\frac{V_a}{R_1}+\frac{(+j)^3 V_a}{Z_{C1}};$$

$$V_a\left(\frac{1}{Z_{C2}}+\frac{1}{R_2}+\frac{2}{R_1+Z_{C1}}\right)=\frac{V_{in}}{R_2}+\frac{(+j)^3 V_{in}}{Z_{C2}};$$

Transfer function for positive polyphase signal

$$H_P(\omega)=\frac{V_{out}}{V_{in}}=\frac{\left[1+(+j)^3 b_1 j\omega\right]\left[1+(+j)^3 b_2 j\omega\right]}{a_0 (j\omega)^2+a_1 j\omega+1};$$

Transfer function for negative polyphase signal

$$H_N(\omega)=\frac{V_{out}}{V_{in}}=\frac{\left[1+(-j)^3 b_1 j\omega\right]\left[1+(-j)^3 b_2 j\omega\right]}{a_0 (j\omega)^2+a_1 j\omega+1};$$

Here: $b_0=R_1C_1; b_1=R_2C_2;\ a_0=b_0b_1;\ a_1=b_0+b_1+2R_2C_1;$

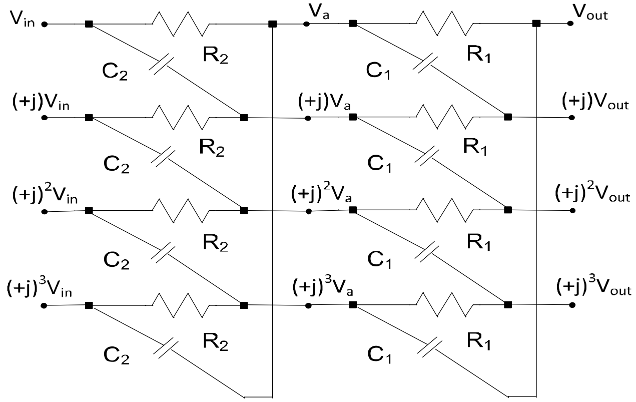
Image rejection ratio (IRR)

$$IRR(\omega)=\frac{\left|H_P(\omega)\right|}{\left|H_N(\omega)\right|}=\frac{\left|(1+b_1\omega)(1+b_2\omega)\right|}{\left|(1-b_1\omega)(1-b_2\omega)\right|};$$

2. Investigation of Multi-Phase Networks

Behaviors of 2nd–Order Polyphase Filter

2-order RC polyphase filter

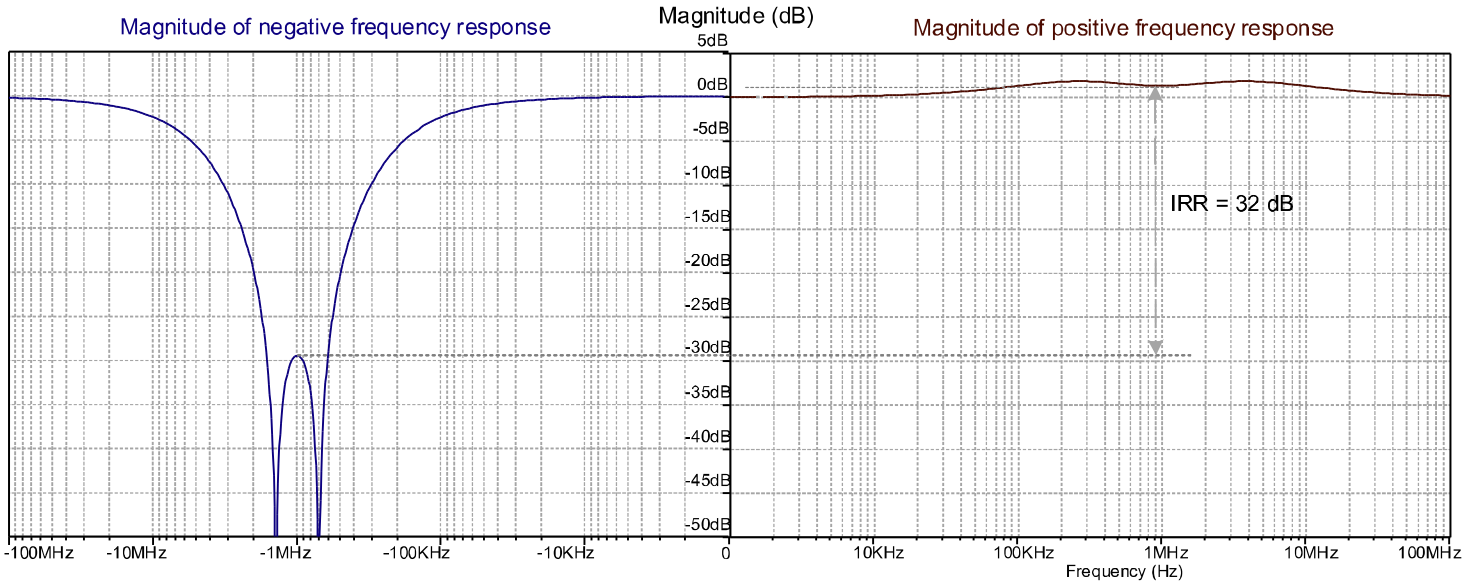


Transfer function in all frequency domain

$$\left|H(\omega)\right| = \frac{(1+b_1\omega)(1+b_2\omega)}{\sqrt{\left(1-a_0\omega^2\right)^2+\left(a_1\omega\right)^2}};\omega \in R$$

Here, R1 = 1 kΩ, C1 = 227 pF, R2 = 1 kΩ, C2 = 114 pF, at f_1 = 700 kHz, f_2 = 1.4 MHz,

Bode plot of transfer function in all frequency domain



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3. **Behaviors of High-Order Complex Filters**
   - **Derivation of image rejection ratio for complex filters**
4. Conclusions

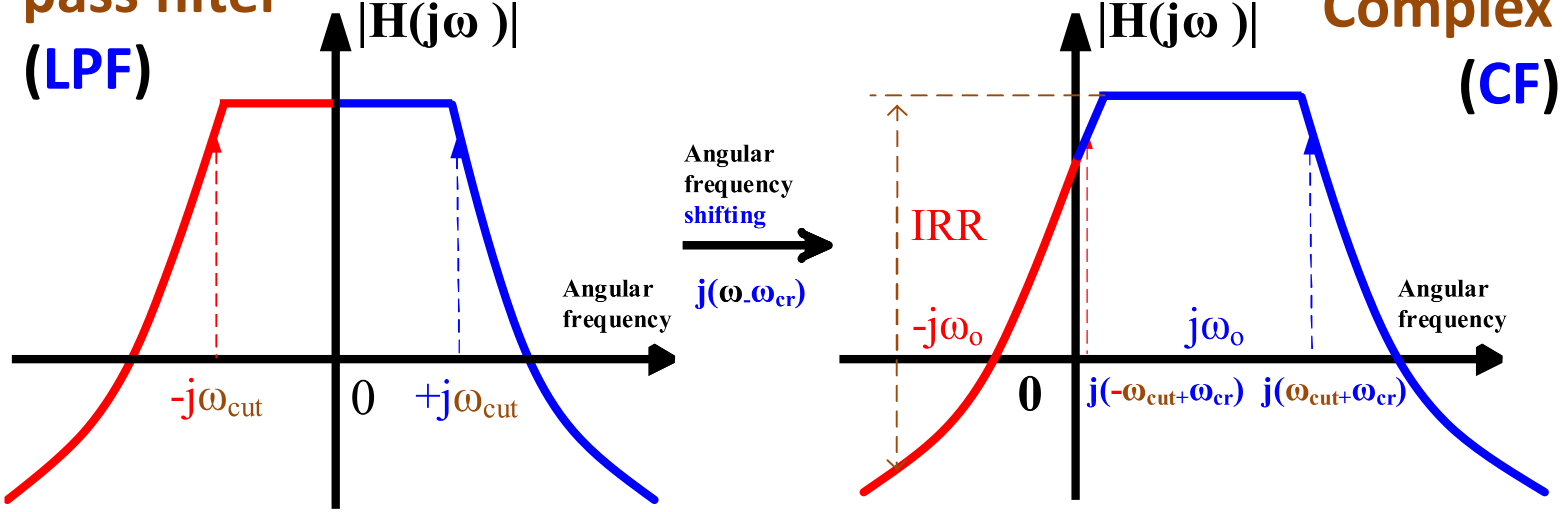
3. Behaviors of High-Order Complex Filters

Design Principle for Complex Filter Networks

Frequency shifting of real low-pass filter in all frequency domains → an active complex filter

Low-pass filter (LPF)

Complex filter (CF)



$$H_{LPF}(\omega) = -\frac{A_{21}}{\left(j\dfrac{\omega}{\omega_{cut}}+1\right)} \xrightarrow{\omega-\omega_{cr}} H_{CF}(\omega) = -\frac{A_{21}}{\left(j\dfrac{\omega-\omega_{cr}}{\omega_{cut}}+1\right)}$$

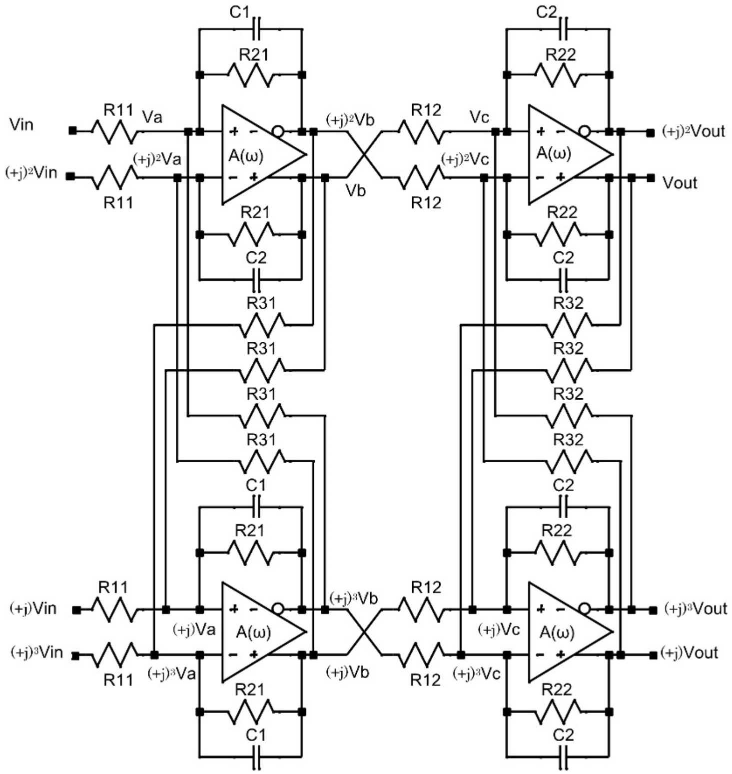
$$\omega_{cr} = \frac{1}{R_3 C_1}$$

ω_{cr} : cross angular frequency

3. Behaviors of High-Order Complex Filters

Behavior of 2th-order Complex Filter

2nd-order complex filter



Here, cut-off angular frequencies:

$$\omega_{cut1}=\frac{1}{R_{21}C_1};\omega_{cut2}=\frac{1}{R_{22}C_2};$$

Apply superposition at each node

$$V_a\left(\frac{1}{Z_{C1}}+\frac{1}{R_{21}}\right)=\frac{V_{in}}{R_{11}}+\frac{(+j)^3V_b}{R_{31}}+V_b\left(\frac{1}{Z_{C1}}+\frac{1}{R_{21}}\right);$$

$$V_C\left(\frac{1}{Z_{C2}}+\frac{1}{R_{22}}\right)=\frac{V_b}{R_{12}}+\frac{(+j)^3V_{out}}{R_{32}}+V_{out}\left(\frac{1}{Z_{C2}}+\frac{1}{R_{22}}\right);$$

$$V_b=\left[V_a-(+j)^2V_a\right]A(\omega);\ V_{out}=\left[V_c-(+j)^2V_c\right]A(\omega);$$

Transfer function for positive polyphase signals

$$H_P(\omega)=\frac{V_{out}}{V_{in}}=\frac{\dfrac{R_{21}}{R_{11}}}{\left[1+j\left(\dfrac{\omega}{\omega_{cut1}}-\dfrac{R_{21}}{R_{31}}\right)\right]}\frac{\dfrac{R_{22}}{R_{12}}}{\left[1+j\left(\dfrac{\omega}{\omega_{cut2}}-\dfrac{R_{22}}{R_{32}}\right)\right]};$$

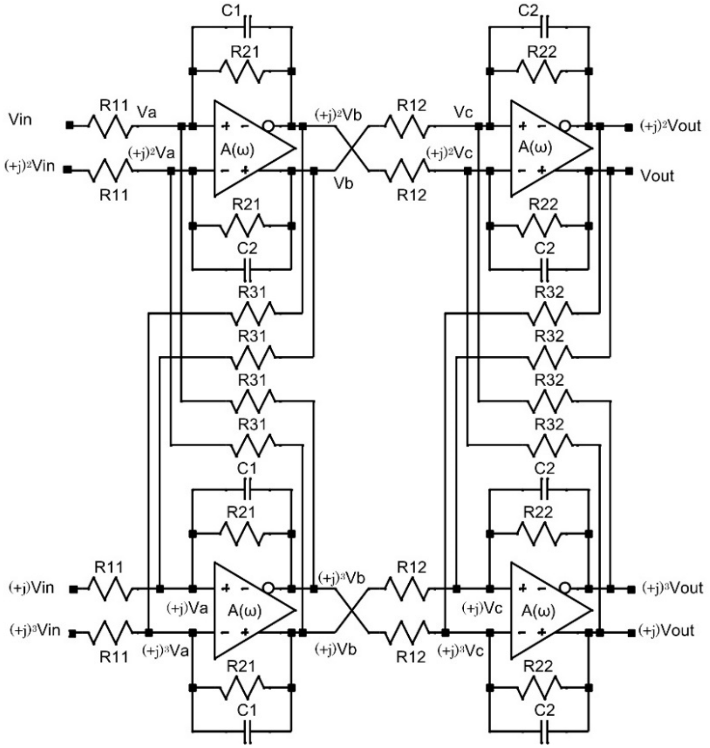
Transfer function for negative polyphase signals

$$H_N(\omega)=\frac{V_{out}}{V_{in}}=\frac{\dfrac{R_{21}}{R_{11}}}{\left[j\left(\dfrac{\omega}{\omega_{cut1}}+\dfrac{R_{21}}{R_{31}}\right)+1\right]}\frac{\dfrac{R_{22}}{R_{12}}}{\left[j\left(\dfrac{\omega}{\omega_{cut2}}+\dfrac{R_{22}}{R_{32}}\right)+1\right]}$$

3. Behaviors of High-Order Complex Filters

Behavior of 2th-order Complex Filter

2nd-order complex filter



Component parameters

Stage1		Stage2	
Element	**Value**	**Element**	**Value**
R11	2kΩ	R12	1kΩ
R21	7kΩ	R22	3.5kΩ
R31	2kΩ	R32	1kΩ
C1	86pF	C2	52pF

Bode plot of transfer function

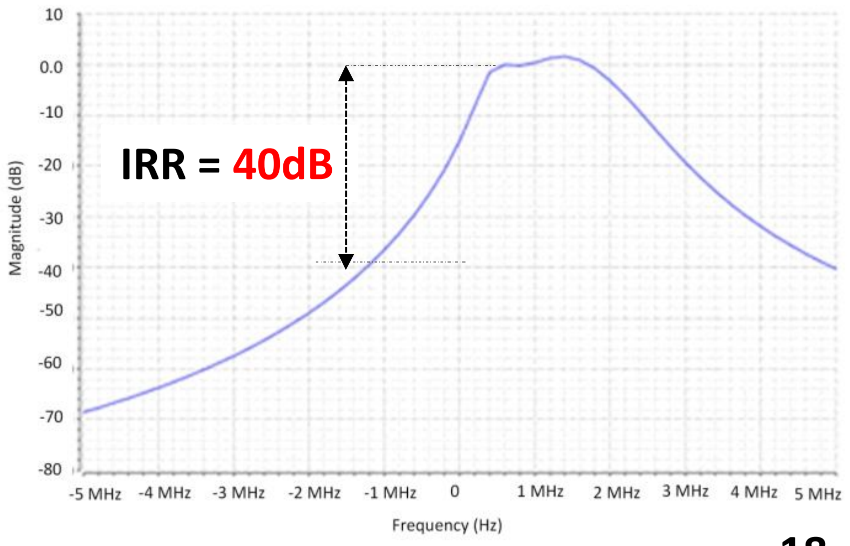

IRR = 40dB

Image rejection ratio (IRR)

$$IRR(\omega)=\frac{H_{Pos}(\omega)}{H_{Neg}(\omega)}=\frac{\left[j\left(\frac{\omega}{\omega_{cut1}}+\frac{R_{21}}{R_{31}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut2}}+\frac{R_{22}}{R_{32}}\right)+1\right]}{\left[j\left(\frac{\omega}{\omega_{cut1}}-\frac{R_{21}}{R_{31}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut2}}-\frac{R_{22}}{R_{32}}\right)+1\right]}$$

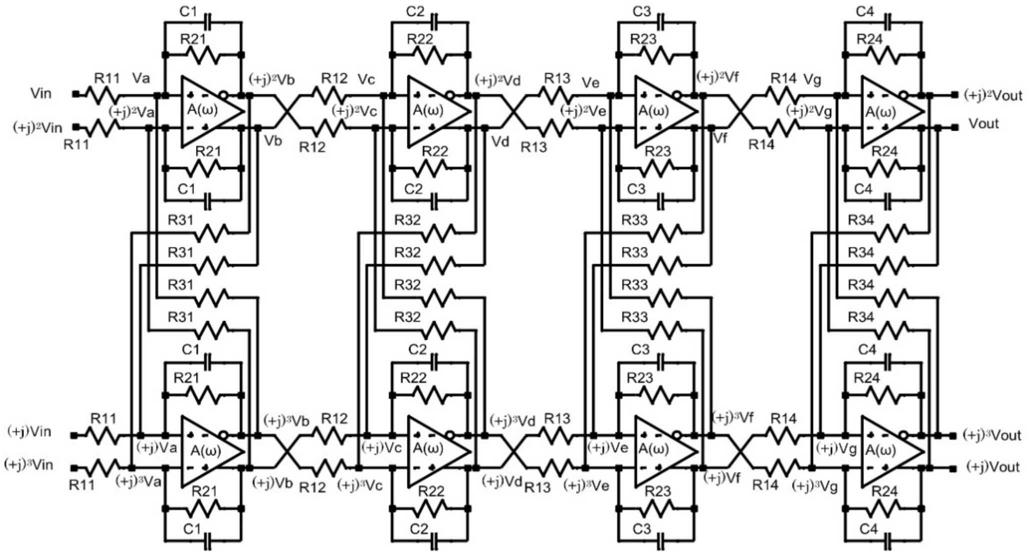
3. Behaviors of High-Order Complex Filters

Behavior of 4th-order Complex Filter

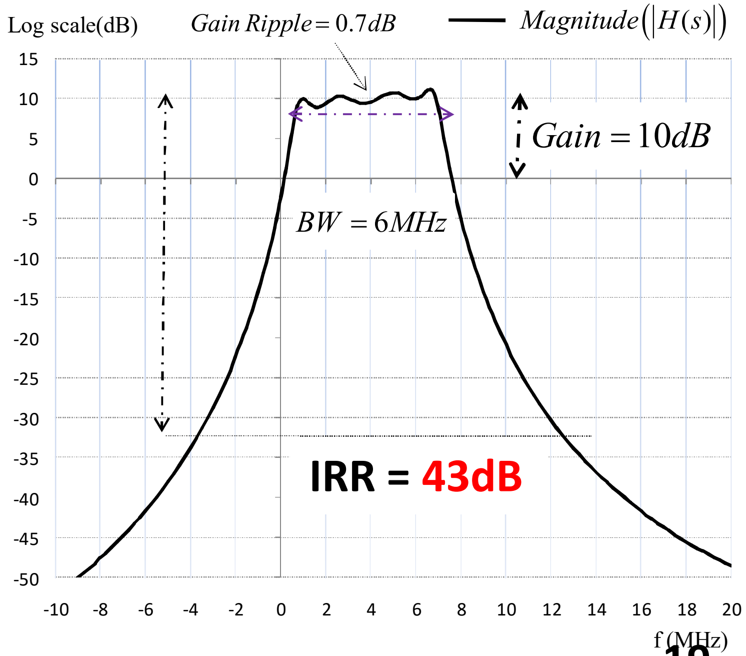
Image rejection ratio (IRR)

$$IRR(\omega)=\frac{\left[j\left(\frac{\omega}{\omega_{cut1}}+\frac{R_{21}}{R_{31}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut2}}+\frac{R_{22}}{R_{32}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut3}}+\frac{R_{23}}{R_{33}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut4}}+\frac{R_{24}}{R_{34}}\right)+1\right]}{\left[j\left(\frac{\omega}{\omega_{cut1}}-\frac{R_{21}}{R_{31}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut2}}-\frac{R_{22}}{R_{32}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut3}}-\frac{R_{23}}{R_{33}}\right)+1\right]\left[j\left(\frac{\omega}{\omega_{cut4}}-\frac{R_{24}}{R_{34}}\right)+1\right]}$$

4th-order complex filter



Bode plot of transfer function



IRR = 43dB

Outline

1. Research Background
   - Motivation, objectives and achievements
   - Superposition formula for multi-source networks
2. Behaviors of High-Order Polyphase Filters
   - Derivation of image rejection ratio for polyphase filters
3. Behaviors of High-Order Complex Filters
   - Derivation of image rejection ratio for complex filters
4. **Conclusions**

4. Comparison (Superposition formula)

Features	Superposition formula	Conventional Superposition	Millan's theorem
Effects of all actuating sources	At one time	Several times	At one time
Transfer function accuracy	Yes	No	No
Single-input network analysis	Yes	Yes	Yes
Polyphase network analysis	Yes	No	No
Complex network analysis	Yes	No	No
Image rejection ratio accuracy	Yes	No	No

4. Discussions (Superposition formula)

Transfer function and image rejection ratio give useful information about the behaviors of polyphase filters and complex filters.

Fundamental network analysis theory for multi-source systems:

- Compute the effects of all sources at one time,
- Reduce the wasteful time,
- Decrease the hand calculation times,
- Get the transfer function faster, and
- Reduce the network complexity.

4. Conclusions

This work:

- **Proposal of superposition formula for multi-source network analysis**
- **Analysis of high-order passive RC poly-phase filters in all frequency domain**
- **Analysis of high-order active complex filters in all frequency domain**
- **Derivation of image rejection ratio in low-IF receivers**

Future of work:

- **Analysis of I/Q mismatches, DC offsets, and parasitic components in polyphase and complex filters**

References

[1] Y. Yin, S. Wang, Y. Ma, S. Kang, H. Guan, G. Jin, X. Qi, "*The design of large image rejection and wideband CMOS active polyphase filter for BeiDou RF receiver*", IEICE Electronics Express, vol. 17, no. 12, pp. 20200110-20200110, 2020.

[3] M. Tran, A.Kuwana, H. Kobayashi,"*Design of Sixth-Order Passive Quadrature Signal Generation Network Based on Polyphase Filter*", IEEE 15th Int. Conf. on Solid-State and Integrated Circuit Technology, Kunming, China, Nov. 2020.

[4] L. Severo, W. Noije, "*A 0.4-V 10.9-µW/Pole third-order complex BPF for low energy RF receivers,*" IEEE Trans. Circuits and Systems I, vol. 66, no. 6, pp. 2017–2026, 2019.

[5] M. Tran, N. Kushita, A. Kuwana, H. Kobayashi, "*Mathematical Analysis and Design of 4-Stage Passive RC Network in RF Front-End System*", 3rd Int. Conf. ICTSS2019, May, 2019.

[6] Z. Xie, J. Wu, C. Chen, "*A compact low-power biquad for active-RC complex filter*", IEEE Trans. Circuits and Systems II, vol. 65, no. 6, pp. 709–713, June 2018.

[7] M. Tran, N. Kushita, A. Kuwana, H. Kobayashi, "*Mathematical Model and Analysis of 4-Stage Passive RC Polyphase Filter for Low-IF Receiver*", J. Mech. Elect. Intel. Syst. Vol. 3, no. 2, May 2020.

[8] H. Kobayashi, N. Kushita, M. Tran, K. Asami, H. San, A. Kuwana, "*Analog - Mixed-Signal - RF Circuits for Complex Signal Processing*", 13th IEEE Int. Conf. ASICON, Oct. 2019.

[9] M. Tran, N. Kushita, A. Kuwana, H. Kobayashi, "*Pass-band Gain Improvement Technique for Passive RC Polyphase Filter in Bluetooth Low-IF Receiver using Two RC Band-stop Filters*", 5th Int. Symp. GUMI & 9th Int. Conf. AMDE AMM, 2020.

[10] H. Tanimoto, "*Exact Design of RC Polyphase Filters and Related Issues*", IEICE Trans. Fund., vol. E96-A, no.2, 2013.

[11] K. Asami, N. Kushita, A. Hatta, M. Tran, Y. Tamura, A. Kuwana, H. Kobayashi, "*Analysis and Evaluation Method of RC Polyphase Filter*", 13th IEEE Int. Conf. ASICON, Oct. 2019.

[12] M. Tran, N. Kushita, A. Kuwana, H. Kobayashi, "*Flat Pass-Band Method with Two RC Band-Stop Filters for 4-Stage Passive RC Polyphase Filter in Low-IF Receiver Systems*", 13th IEEE Int. Conf. ASICON, Oct. 2019.

[13] M. Tran, N. Kushita, A. Kuwana, H. Kobayashi, "*Minimum Gain Ripple Technique for Pass-Band of 4-Stage Passive RC Polyphase Filter in Low IF Receivers*", 5th Taiwan and Japan Conf. CAS, Aug. 2019.

[14] W. Luo, N. Yan, Y. Chen, Y. Lin, H. Xu, "*Design of An Efficient Wideband Quadrature Generator for Multiple Receivers Based on Poly-Phase Filter*", IEEE 15th Int. Conf. on Solid-State and Integrated Circuit Technology, China, Nov. 2020.

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Thank you very much!

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