DERIVATION OF IMAGE REJECTION RATIO FOR HIGH-ORDER COMPLEX FILTERS

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1. Research Background
   - Motivation, objectives and achievements
   - Superposition formula for multi-source networks

2. Behaviors of High-Order Polyphase Filters
   - Derivation of image rejection ratio for polyphase filters

3. Behaviors of High-Order Complex Filters
   - Derivation of image rejection ratio for complex filters

4. Conclusions
1. Research Background
Motivation of Study

Common types of noise:
- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Device noise:
- Flicker noise,
- Thermal noise,
- White noise.

Multi-phase networks
- Image noise,
- I/Q mismatches
- DC offsets

Signal to Noise Ratio: \[ SNR = \frac{\text{Signal power}}{\text{Noise power}} \]

Figure of Merit: \[ F = \frac{\text{Output SNR}}{\text{Input SNR}} \]
1. Research Background

Objectives of Study

- Derivation of transfer function in multi-source systems using superposition theorem
- Investigation of behaviors of high-order passive RC polyphase filter networks
- Investigation of behaviors of high-order complex filter networks
- Derivation of image rejection ratio in low-IF receivers
1. Research Background

Achievements of Study

Superposition formula for multi-source networks

\[ V_o(t) \sum_{i=1}^{n} \frac{1}{Z_i} + V_o(t) \sum_{i=1}^{n} \frac{1}{Z_{si}} + \sum_{k=1}^{n} \frac{1}{Z_{pik}} = \sum_{i=1}^{n} \left( \frac{V_i(t)}{Z_i} + I_{ai}(t) - I_{gi}(t) \right) \]

2\textsuperscript{nd}-order polyphase filter

Image rejection ratio

\[ IRR(\omega) = \frac{(1+b_1\omega)(1+b_2\omega)}{(1-b_1\omega)(1-b_2\omega)} \]

Image rejection ratio

\[ IRR(\omega) = \left[ j \left( \frac{\omega}{\omega_{cut1}} + \frac{R_{21}}{R_{31}} \right) + 1 \right] \left[ j \left( \frac{\omega}{\omega_{cut2}} + \frac{R_{22}}{R_{32}} \right) + 1 \right] \left[ j \left( \frac{\omega}{\omega_{cut3}} + \frac{R_{23}}{R_{33}} \right) + 1 \right] \left[ j \left( \frac{\omega}{\omega_{cut4}} + \frac{R_{24}}{R_{34}} \right) + 1 \right] \]
1. Research Background
Low-IF Receiver System Architecture

Block diagram of low-IF receiver

This Work

Applications: Wi-Fi, WiMax, UWB, GSM, WCDMA, LTE, 4G, Cordless Phones, RFID, ZigBee, Bluetooth, TV Set Top Box, Sensing, Radar...

Merits
- Low-cost
- Small-size
- High-integration

Demerits
- Image Noises
- Power Loss
- Noise Figure
1. Research Background

Characteristics of Low-IF Receiver Signals
1. Research Background

Positive Polyphase Signals on Frequency Domain

Positive polyphase signals

\[ S_{\text{Pos poly}} \{ V_1(t); V_2(t); V_3(t); V_4(t) \} = \{ 1; +j; (+j)^2; (+j)^3 \} V_{\text{pos}}(t) \]
1. Research Background

Negative Polyphase Signals on Frequency Domain

**Negative polyphase signals**

\[ V_{n4} = +jV_n \]
\[ V_{n3} = -V_n \]
\[ V_{n1} = V_n \]
\[ V_{neg2} = -jV_{neg} \]

Clockwise

\[ S_{Neg\_poly}\{V_1(t); V_2(t); V_3(t); V_4(t)\} = \{1; -j; (-j)^2; (-j)^3\}V_{neg}(t) \]

- (a) Cosine wave on real plane
- (b) Spectrum of cosine wave
- (c) Minus sine wave on imaginary plane
- (d) Spectrum of minus sine wave
- (e) Negative angular frequency wave
- (f) Spectrum of negative angular frequency wave

(c) Angular frequency plane for polyphase signals

Amplitude [V]

\[ \begin{align*}
\angle V &= S_n(\omega_0) \\
\omega &= -\omega_0 \quad \omega_0 \\
\end{align*} \]
1. Research Background

Polyphase Signals on Frequency Domain

**Negative polyphase signals**

\[ V_{n3} = -V_n \]
\[ V_{n1} = V_n \]
\[ V_{n4} = +jV_n \]
\[ V_{neg2} = -jV_{neg} \]

Clockwise

**Positive polyphase signals**

\[ V_{p3} = -V_p \]
\[ V_{p1} = V_p \]
\[ V_{p2} = jV_p \]
\[ V_{p4} = -jV_p \]

Anti-clockwise

**Image Signals**

\[ S_{Neg\_poly} \{V_1(t); V_2(t); V_3(t); V_4(t)\} = \{1; -j; (-j)^2; (-j)^3\}V_{neg}(t) \]

**Wanted Signals**

\[ S_{Pos\_poly} \{V_1(t); V_2(t); V_3(t); V_4(t)\} = \{1; +j; (j)^2; (j)^3\}V_{pos}(t) \]
1. Research Background

Superposition Theorem for Multi-Source Systems

Superposition formula:

\[
V_O(t)\sum_{i=1}^{n} \frac{1}{Z_i} + V_O(t)\sum_{i=1}^{n} \frac{1}{Z_{si}} + \sum_{k=1}^{n} \frac{1}{Z_{pik}} = \sum_{i=1}^{n} \left( \frac{V_i(t)}{Z_i} + I_{ai}(t) - I_{gi}(t) \right)
\]

- \(V_O(t)\): Voltage at one node
- \(V_i(t)\): Input voltage sources
- \(I_{ai}(t)\): Ahead-toward current sources
- \(I_{gi}(t)\): Ground-toward current sources
- \(Z_{i, si, pi, t}\): Impedances at each branch

- Multi-source systems, feedback networks (op amps, amplifiers), polyphase filters, complex filters...
Outline

1. Research Background
   • Motivation, objectives and achievements
   • Superposition formula for multi-source networks

2. Behaviors of High-Order Polyphase Filters
   • Derivation of image rejection ratio for polyphase filters

3. Behaviors of High-Order Complex Filters
   • Derivation of image rejection ratio for complex filters

4. Conclusions
2. Investigation of Multi-Phase Networks
   Design Principle for Polyphase Filter Networks

Complementation between low-pass and high-pass circuits → a passive polyphase filter

**Wanted Signals**

- Real part
- Imaginary part (jX)

**Image Signals**

- Real part
- Imaginary part (jX)

(c) Pass-band filter (wanted signals)
(d) Notch-band filter (image signal)
2. Investigation of Multi-Phase Networks

Analysis of 2nd–Order Polyphase Filter

Transfer function for positive polyphase signal

\[ H_P(\omega) = \frac{V_{out}}{V_{in}} = \frac{1 + (j)^3 b_1 j\omega}{a_0 (j\omega)^2 + a_1 j\omega + 1} \left[ 1 + (j)^3 b_2 j\omega \right] \]

Transfer function for negative polyphase signal

\[ H_N(\omega) = \frac{V_{out}}{V_{in}} = \frac{1 + (j)^3 b_1 j\omega}{a_0 (j\omega)^2 + a_1 j\omega + 1} \left[ 1 + (j)^3 b_2 j\omega \right] \]

Here:

\[ b_0 = R_1 C_1; b_1 = R_2 C_2; a_0 = b_0 b_1; a_1 = b_0 + b_1 + 2 R_2 C_1; \]

Image rejection ratio (IRR)

\[ IRR(\omega) = \left| \frac{H_P(\omega)}{H_N(\omega)} \right| = \frac{|(1 + b_1 \omega)(1 + b_2 \omega)|}{|(1 - b_1 \omega)(1 - b_2 \omega)|} \]
2. Investigation of Multi-Phase Networks Behaviors of 2\textsuperscript{nd}–Order Polyphase Filter

Transfer function in all frequency domain

\[
|H(\omega)| = \frac{(1 + b_1 \omega)(1 + b_2 \omega)}{\sqrt{(1 - a_0 \omega^2)^2 + (a_1 \omega)^2}}; \quad \omega \in R
\]

Here, \(R_1 = 1\) k\(\Omega\), \(C_1 = 227\) pF, \(R_2 = 1\) k\(\Omega\), \(C_2 = 114\) pF, at \(f_1 = 700\) kHz, \(f_2 = 1.4\) MHz,

Bode plot of transfer function in all frequency domain
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4. Conclusions
3. Behaviors of High-Order Complex Filters

Design Principle for Complex Filter Networks

Frequency shifting of real low-pass filter in all frequency domains → an active complex filter

Low-pass filter (LPF) | Complex filter (CF)

$H_{\text{LPF}}(\omega) = -\frac{A_{21}}{j\frac{\omega}{\omega_{\text{cut}}} + 1}$

$\omega - \omega_{\text{cr}} \rightarrow H_{\text{CF}}(\omega) = -\frac{A_{21}}{j\frac{\omega - \omega_{\text{cr}}}{\omega_{\text{cut}}} + 1}$

$\omega_{\text{cr}}$: cross angular frequency

$\omega_{\text{cr}} = \frac{1}{R_3C_1}$
3. Behaviors of High-Order Complex Filters

Behavior of 2\textsuperscript{nd}-order Complex Filter

Apply superposition at each node

\begin{align*}
V_a \left( \frac{1}{Z_{C1} R_{21}} + \frac{1}{R_{21}} \right) &= V_{in} + \left( +j \right)^3 V_b + V_b \left( \frac{1}{Z_{C1}} + \frac{1}{R_{21}} \right); \\
V_c \left( \frac{1}{Z_{C2} R_{22}} + \frac{1}{R_{22}} \right) &= V_{b} + \left( +j \right)^3 V_{out} + V_{out} \left( \frac{1}{Z_{C2}} + \frac{1}{R_{22}} \right); \\
V_b &= \left[ V_a - \left( +j \right)^2 V_a \right] A(\omega); \\
V_{out} &= \left[ V_c - \left( +j \right)^2 V_c \right] A(\omega); \\

\text{Transfer function for positive polyphase signals}

H_P(\omega) &= \frac{V_{out}}{V_{in}} = \frac{R_{21}}{R_{11}} \frac{R_{22}}{R_{12}} \left[ 1 + j \left( \frac{\omega}{\omega_{cut1} R_{31}} - \frac{R_{21}}{R_{31}} \right) \right] \left[ 1 + j \left( \frac{\omega}{\omega_{cut2} R_{32}} - \frac{R_{22}}{R_{32}} \right) \right];

\text{Transfer function for negative polyphase signals}

H_N(\omega) &= \frac{V_{out}}{V_{in}} = \frac{R_{21}}{R_{11}} \frac{R_{22}}{R_{12}} \left[ j \left( \frac{\omega}{\omega_{cut1} R_{31}} + 1 \right) \right] \left[ j \left( \frac{\omega}{\omega_{cut2} R_{32}} + 1 \right) \right].
\end{align*}

Here, cut-off angular frequencies:

\begin{align*}
\omega_{cut1} &= \frac{1}{R_{21} C_1}; \\
\omega_{cut2} &= \frac{1}{R_{22} C_2};
\end{align*}
3. Behaviors of High-Order Complex Filters

Behavior of 2\textsuperscript{nd}-order Complex Filter

Component parameters

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R11</td>
<td>2kΩ</td>
<td>R12</td>
<td>1kΩ</td>
</tr>
<tr>
<td>R21</td>
<td>7kΩ</td>
<td>R22</td>
<td>3.5kΩ</td>
</tr>
<tr>
<td>R31</td>
<td>2kΩ</td>
<td>R32</td>
<td>1kΩ</td>
</tr>
<tr>
<td>C1</td>
<td>86pF</td>
<td>C2</td>
<td>52pF</td>
</tr>
</tbody>
</table>

Image rejection ratio (IRR)

\[ IRR(\omega) = \frac{H_{pos}(\omega)}{H_{neg}(\omega)} = \frac{j \left( \frac{\omega}{\omega_{cut1}} + \frac{R_{21}}{R_{31}} \right) + 1}{j \left( \frac{\omega}{\omega_{cut1}} - \frac{R_{21}}{R_{31}} \right) + 1} \times \frac{j \left( \frac{\omega}{\omega_{cut2}} + \frac{R_{22}}{R_{32}} \right) + 1}{j \left( \frac{\omega}{\omega_{cut2}} - \frac{R_{22}}{R_{32}} \right) + 1} \]

Bode plot of transfer function

IRR = 40dB
3. **Behaviors of High-Order Complex Filters**

**Behavior of 4th-order Complex Filter**

**Image rejection ratio (IRR)**

\[
IRR(\omega) = \left[ j\left( \frac{\omega}{\omega_{\text{cut1}}} + \frac{R_{21}}{R_{31}} \right) + 1 \right] \left[ j\left( \frac{\omega}{\omega_{\text{cut2}}} - \frac{R_{22}}{R_{32}} \right) + 1 \right] \left[ j\left( \frac{\omega}{\omega_{\text{cut3}}} + \frac{R_{23}}{R_{33}} \right) + 1 \right] \left[ j\left( \frac{\omega}{\omega_{\text{cut4}}} - \frac{R_{24}}{R_{34}} \right) + 1 \right]
\]

**4th-order complex filter**

![Diagram of 4th-order complex filter]

**Bode plot of transfer function**

- **Gain Ripple** = 0.7 dB
- **Magnitude** \(|H(s)|
- **Gain** = 10 dB
- **BW** = 6 MHz
- **IRR** = 43 dB
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4. Conclusions
## 4. Comparison (Superposition formula)

<table>
<thead>
<tr>
<th>Features</th>
<th>Superposition formula</th>
<th>Conventional Superposition</th>
<th>Millan’s theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effects of all actuating sources</td>
<td>At one time</td>
<td>Several times</td>
<td>At one time</td>
</tr>
<tr>
<td>Transfer function accuracy</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Single-input network analysis</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Polyphase network analysis</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Complex network analysis</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Image rejection ratio accuracy</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
4. Discussions (Superposition formula)

Transfer function and image rejection ratio give useful information about the behaviors of polyphase filters and complex filters.

Fundamental network analysis theory for multi-source systems:

• **Compute** the effects of all sources at one time,
• **Reduce** the wasteful time,
• **Decrease** the hand calculation times,
• **Get** the transfer function faster, and
• **Reduce** the network complexity.
4. Conclusions

This work:

• **Proposal of superposition formula** for multi-source network analysis

• **Analysis of high-order passive RC poly-phase filters** in all frequency domain

• **Analysis of high-order active complex filters** in all frequency domain

• **Derivation of image rejection ratio** in low-IF receivers

Future of work:

• **Analysis of I/Q mismatches, DC offsets, and parasitic components** in polyphase and complex filters
References


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Thank you very much!

Q&A