多角数を用いた DA 変換器アーキテクチャの設計

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Digital-to-Analog Converter Architectures Based on Polygonal Numbers

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Abstract - This paper describes new configurations of digital-to-analog converters (DACs) based on number theory, and shows their architecture design in 6-bit case. They consist of N current sources, N-angle number weighted resistor networks, switch arrays and decoders (N= 3, 4, 5,..). Their principles, configurations and operations as well as simulation results are presented.

キーワード：デジタルアナログ変換器，多角数，整数論
(Digital-to-Analog Converter, Polygonal Number, Integer Theory)

1. INTRODUCTION

The DAC is widely used in communication equipment, electronic measuring instrument, audio systems and so on [1-6]. It consists of digital circuit and analog circuit as its name. On the other hand, there are many interesting characteristics in integers [7], but conventional DAC architectures have not been fully exploited them. This paper investigates the possibility of several new structures of the DAC based on integer characteristics.

2. POLYGONAL NUMBER

In mathematics, a polygonal number is a number represented as dots or pebbles arranged in the shape of a regular polygon. Polygonal numbers can be divided into triangular numbers, square numbers, pentagonal numbers, hexagonal numbers, heptagonal numbers, octagonal numbers, and so on.

This paper investigates the possibility of several new structures of the DAC based on triangular numbers, square numbers, hexagonal numbers and octagonal numbers.

A triangular number counts objects arranged in an equilateral triangle. The n-th triangular number is the number of dots in the triangular arrangement with n dots on a side, and is equal to the sum of the n natural numbers from 1 to n. The sequence of triangular numbers, starting at the 0-th triangular number, is 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276...

A square number or perfect square is an integer that is the square of an integer; in other words, it is the product of some integer with itself. So we obtain the square numbers as follows:

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256…

A hexagonal number is a figurate number. The n-th hexagonal number h_n is the number of distinct dots in a pattern of dots consisting of the outlines of regular hexagons with sides up to n dots, when the hexagons are overlaid so that they share one vertex. So we obtain the hexagonal numbers as follows:

1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276…

An octagonal number is a figurate number that represents an octagon. The octagonal number for n is given by the formula $3n^2 - 2n$, with n > 0. The first few octagonal numbers are:

1, 8, 21, 40, 65, 96, 133, 176, 225, 280…

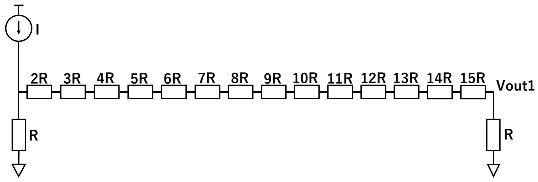
3. Digital-to-Analog Converter

A digital-to-analog converter (DAC) is an electronic circuit that converts a digital signal into an analog signal. An analog-to-digital converter (ADC) performs its reverse function. The DAC converts an abstract finite-precision number into a physical electronic quantity (e.g., a voltage or a current). In particular, DACs are often used to convert finite-precision time series data to a continually varying electronic signal.

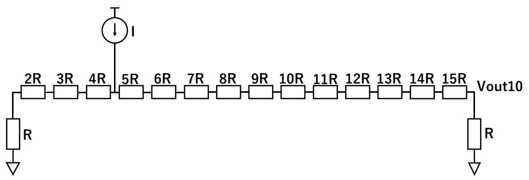
〈３・１〉 Proposed DAC using Triangular Numbers

We propose a DAC based on the triangular number theorem. It consists of 3 current sources, a triangular number weighted resistor network, a switch array and a decoder.

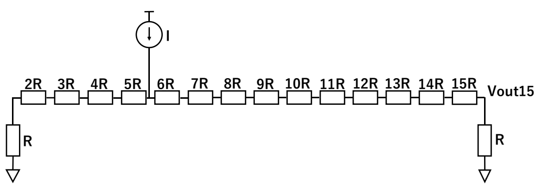
Fig. 1 explains the operation of the proposed triangular weighted resistor network. It generates a triangular number weighted voltage at Vout when a current is injected to one node. According to the superposition principle and the trigonometric principle, any integer weighted voltage proportional to the digital input can be outputted as Vout.



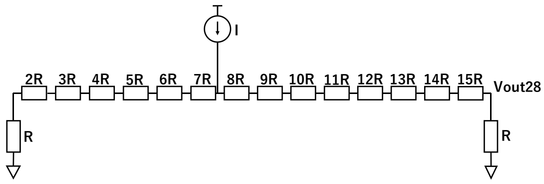
(a) Vout = (1/121) RI.



(b) Vout = (10/121) RI.



(c) Vout = (15/121) RI.



(d) Vout = (28/121) RI.

Fig. 1. Operation of the triangular number weighted resistor network, which generate a triangular number weighted voltage.

Fig. 2 explains the operation of the proposed DAC using the triangular numbers.

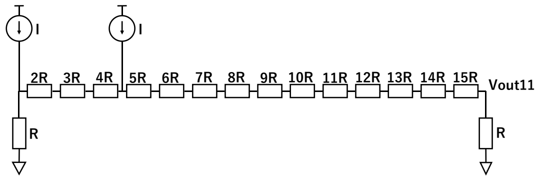
(i)Fig. 2 (a) shows the case that the digital input is 11, and there the analog output voltage is given by Vout11＝ Vout 1 ＋ Vout10 = I*R/121 ＋ 10I*R/121＝11I*R/121.

(ii) Fig. 2 (b) shows the case that the digital input is 16, and there the analog output voltage is Vout16＝Vout1＋Vout15＝I*R/121 ＋15I*R/121 ＝16I*R/121.

(iii) Fig. 2 (c) shows the case that the digital input is 26, and the analog output voltage is Vout26＝ Vout 1 ＋Vout10＋Vout15＝I*R/121＋10I*R/121 ＋15I*R/121＝26I*R/121.

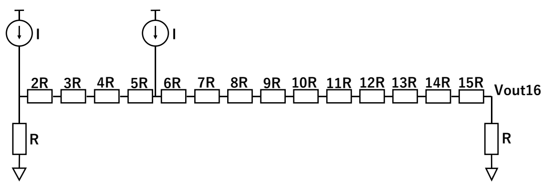
(iv) Fig. 2 (d) shows the case that the digital input is 26, and the analog output voltage is Vout53＝ Vout10 ＋ Vout15 ＋ Vout28 = 10I*R/121 ＋ 15I*R/121＋28I*R/121＝53I*R/121.

These output voltages can be obtained by applying the superposition principle to the circuits in Figs. 2 (a)-(d).



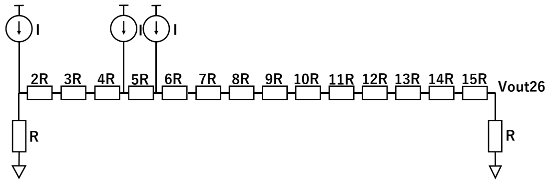
(a) In case that the digital input is 11.

Vout = (11/121) RI.



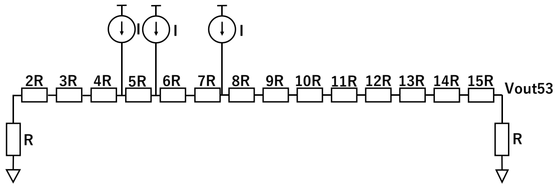
(b) In case that the digital input is 16.

Vout = (16/121) RI.



(c) In case that the digital input is 26.

Vout = (26/121) RI.



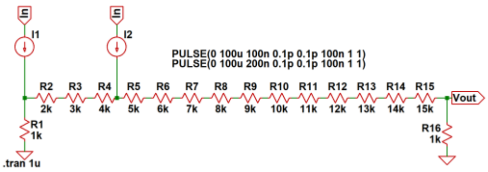
(d) In case that the digital input is 53.

Vout = (53/121) RI.

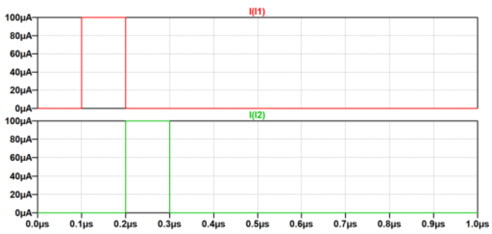
Fig. 2. Operation of the triangular number DAC.

4. SIMULATION VERIFICATION

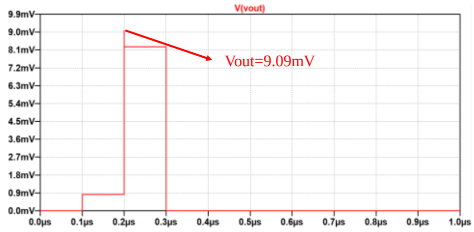
We have performed SPICE simulation for the circuits in Fig. 2 and its simulation circuit is shown in Fig. 3 (a), where R=1kΩ. The waveforms of the input current sources I1~I2 are shown in Fig. 3 (b) and the waveform of Vout is shown in Fig. 3 (c); we see that this simulation result confirms the operations of the circuits in Fig. 2, with R=1kΩ and I=100μA. We see that they confirm the expected operation.



(a) Simulation circuit.



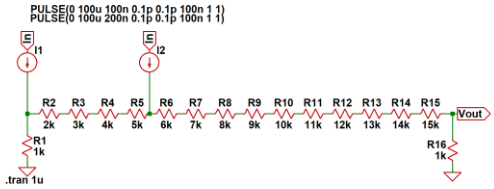
(b) Input current source waveforms.



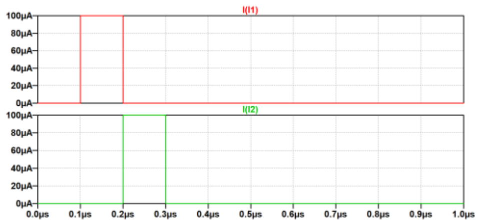
(c) Output voltage (Vout) waveform.

Fig. 3. SPICE simulation result of the proposed triangular number weighted resistor network.

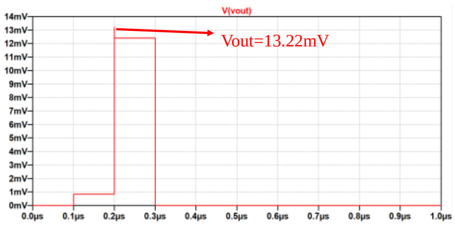
We have also simulated for the input of 16, 26, 53, 83. Their waveforms are shown in Figs. 4, 5, and 6. We see that they are as expected.



(a) Simulation circuit.



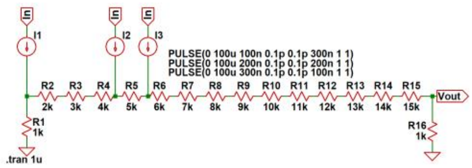
(b) Input current source waveforms.



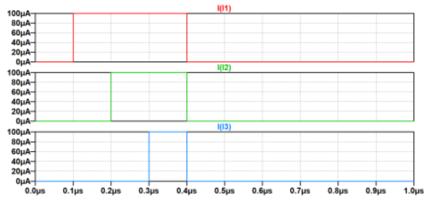
(c) Output voltage (Vout) waveform.

Fig. 4 In case that the digital input is 16.

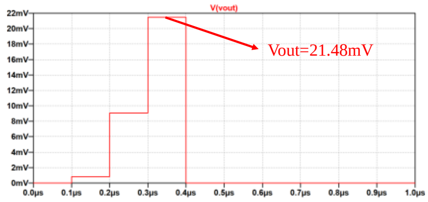
Vout = (16/121) RI.



(a) Simulation circuit.



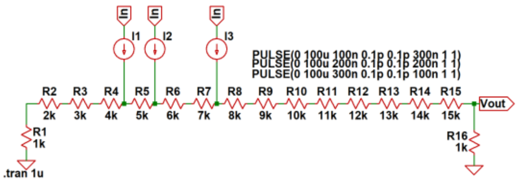
(b) Input current source waveforms.



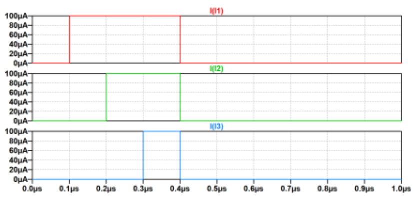
(c) Output voltage (Vout) waveform.

Fig. 5 In case that the digital input is 26.

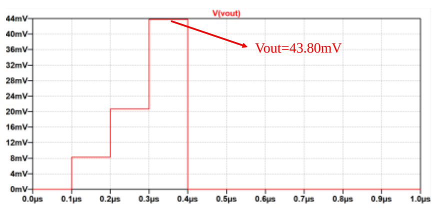
Vout = (26/121) RI.



(a) Simulation circuit.



(b) Input current source waveforms.



(c) Output voltage (Vout) waveform.

Fig. 6 In case that the digital input is 53.

Vout = (53/121) RI.

5. SUMMARY

We have studied the possibility of new DAC architectures using polygonal numbers. By using the polygonal number theorem, a DAC can be composed of N current sources, N switch arrays, an N-polygonal number weighted resistor network and a decoder for N=3, 4, 5, …. Similar to the principle of triangular numbers, we can also obtain the calculation results and the simulation results of square numbers, pentagonal numbers, hexagonal numbers, heptagonal numbers, octagonal numbers... Discussion on their advantages and disadvantages are left for the future work.

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