Digital-to-Analog Converter Architectures Based on Polygonal Numbers

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Outline

- Research Objective
- Background
- Polygonal Number
- Triangular Number DAC
  - Calculation results
  - Simulation results
- Summary
- Square Numbers DAC Results
- Conclusion

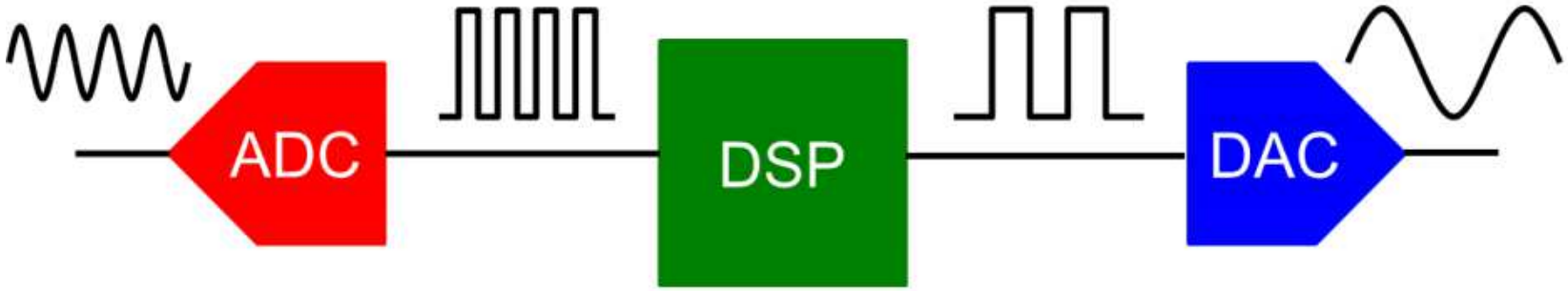
Research Objective

- **Interesting properties of number theory**

- **Possibility of new configurations of DAC（ 6-bit case ）**

Importance of ADC / DAC

- Rapid development of digital electronics technology
- A natural signal is analog



DACs are Everywhere !

Communication equipment

Electronic measuring instrument

Audio systems

Integer Theory and Electronic Circuit Design

Many interesting properties of Integers

Currently
No Link

Electronic circuit designs

Carolus Fridericus Gauss
(1777-1855)

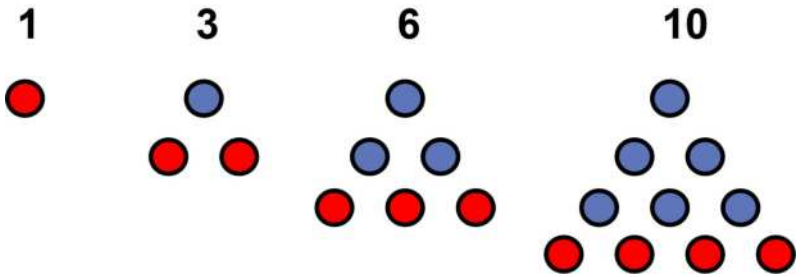
Our research here makes their links !

Integer theory is the queen of Mathematics

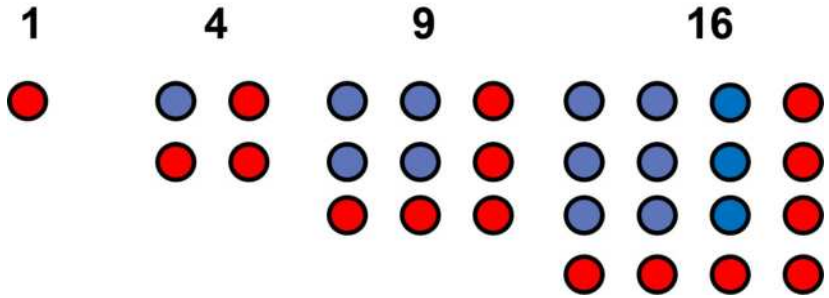
Polygonal Number

Polygonal Number

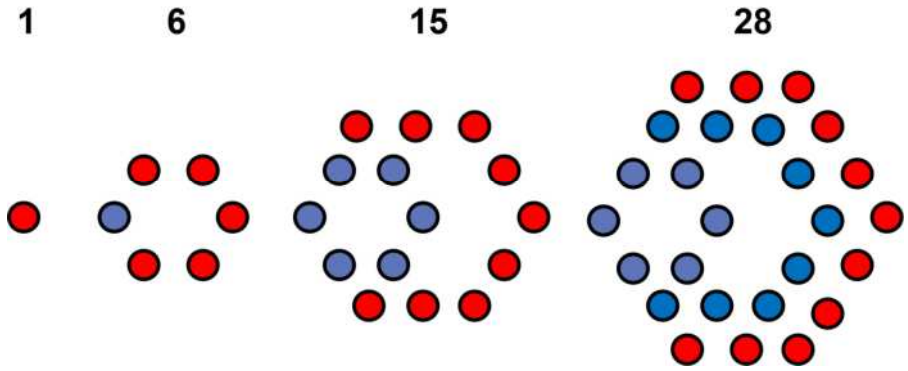
Represented as dots or pebbles arranged in the shape of a regular polygon.



(a)Triangular numbers.



(b) Square numbers.



(c) Hexagonal numbers.

Polygonal Number Theorem

Any natural number

expressed by

Sum of N N-angular numbers

k-th of N-angular number, m(N, k) can be expressed by

$$m(N, k) = (1/2)\, k\, [(N-2)k - (N-4)]$$

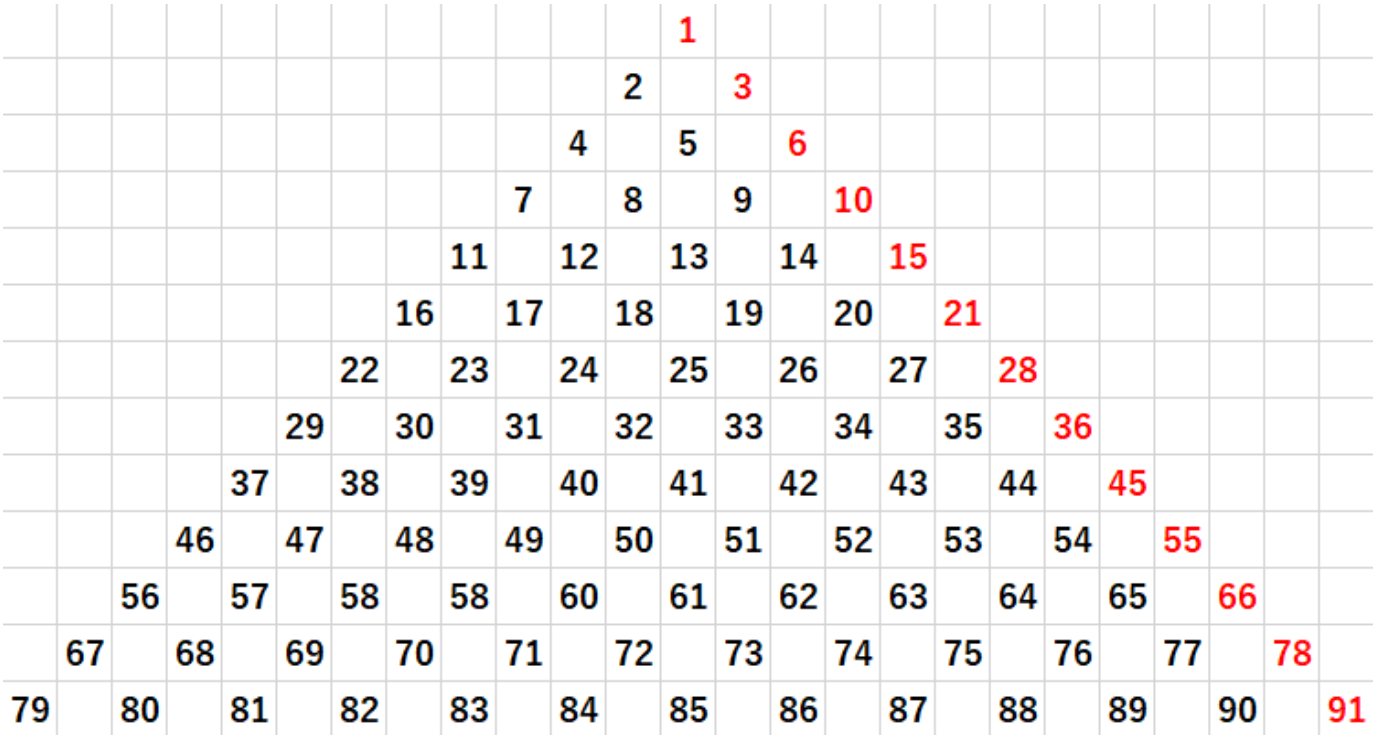
Then N-angular numbers are given by

1, N, 3N-3, 6N-8, 10N-15, …

for k=1, 2, 3, 4, 5, …

What is Triangular Number ?

Triangular Number：1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



Theory of Trigonometric Numbers

Any natural number → Sum of 3 triangular numbers

expressed by

1:	1	16:	1+15	31:	3+28	46:	1+45
2:	1+1	17:	1+1+15	32:	1+3+28	47:	1+1+45
3:	3	18:	3+15	33:	6+6+21	48:	3+45
4:	1+3	19:	1+3+15	34:	6+28	49:	1+3+45
5:	1+1+3	20:	10+10	35:	1+6+28	50:	1+21+28
6:	6	21:	21	36:	36	51:	15+36
7:	1+6	22:	1+21	37:	1+36	52:	1+6+45
8:	1+1+6	23:	1+1+21	38:	1+1+36	53:	10+15+28
9:	3+6	24:	3+21	39:	3+36	54:	3+6+45
10:	10	25:	1+3+21	40:	1+3+36	55:	55
11:	1+10	26:	1+10+15	41:	3+10+28	56:	1+55
12:	1+1+10	27:	6+21	42:	6+36	57:	1+1+55
13:	3+10	28:	28	43:	1+6+36	58:	3+55
14:	1+3+10	29:	1+28	44:	6+10+28	59:	1+3+55
15:	15	30:	1+1+28	45:	45	60:	15+45

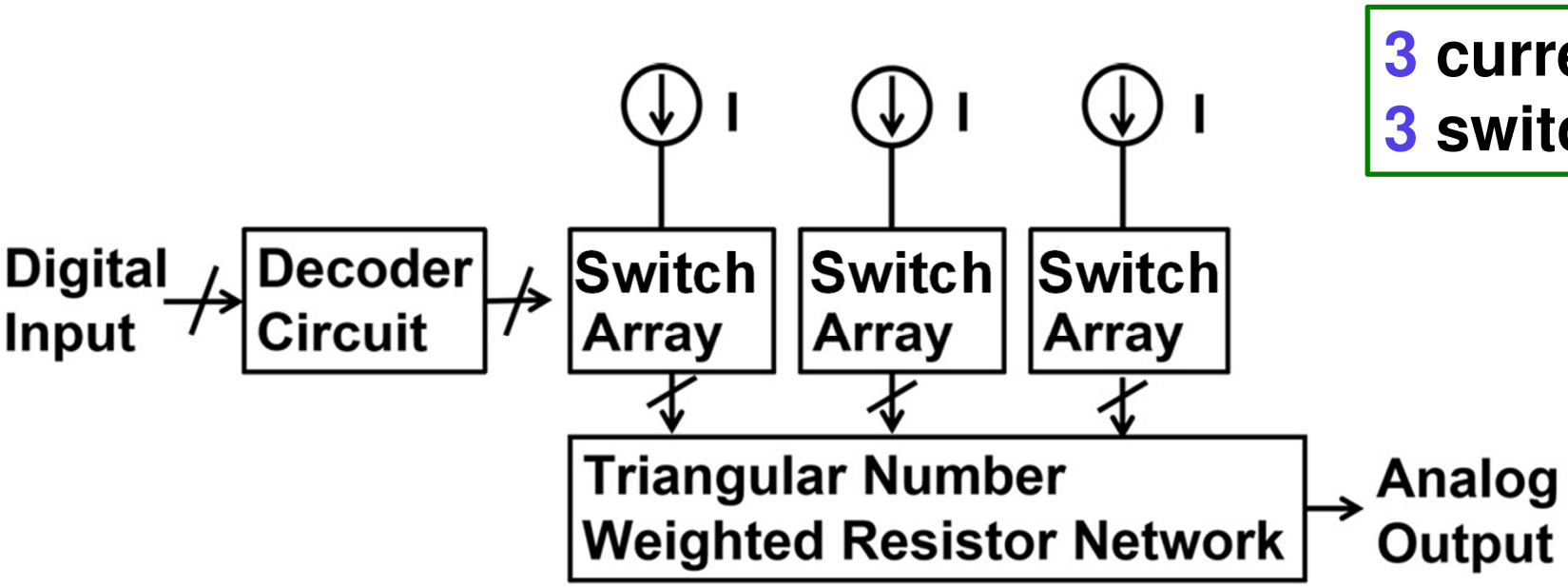
Theory of Trigonometric Numbers

Any natural number → Sum of 3 triangular numbers

expressed by

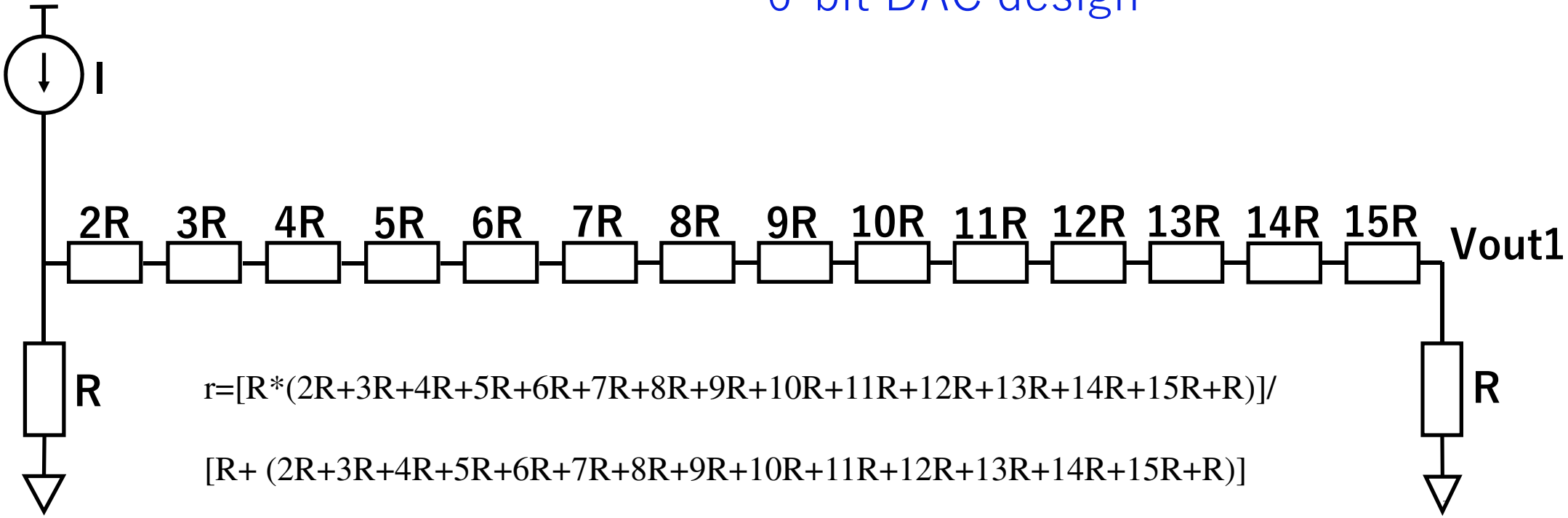
61:	6+55	76:	10+66	91:	91	106:	1+105	121:	1+120
62:	1+6+55	77:	1+10+66	92:	1+91	107:	1+1+105	122:	1+1+120
63:	3+15+45	78:	78	93:	1+1+91	108:	3+105	123:	3+120
64:	3+6+55	79:	1+78	94:	3+91	109:	1+3+105	124:	1+3+120
65:	10+55	80:	1+1+78	95:	1+3+91	110:	55+55	125:	10+10+105
66:	66	81:	3+78	96:	3+15+78	111:	6+105	126:	6+120
67:	1+66	82:	1+3+78	97:	6+91	112:	1+6+105	127:	1+6+120
68:	1+1+66	83:	28+55	98:	1+6+91	113:	1+21+91		
69:	3+66	84:	6+78	99:	21+78	114:	3+6+105		
70:	1+3+66	85:	1+6+78	100:	3+6+91	115:	10+105		
71:	6+10+55	86:	10+10+66	101:	10+91	116:	1+10+105		
72:	6+66	87:	3+6+78	102:	1+10+91	117:	6+6+105		
73:	1+6+66	88:	10+78	103:	6+6+91	118:	3+10+105		
74:	1+28+45	89:	1+10+78	104:	3+10+91	119:	28+91		
75:	3+6+66	90:	6+6+78	105:	105	120:	120		

Proposed Triangular Number DAC



Triangular Number DAC **Input 1**

6-bit DAC design



r=[R*(2R+3R+4R+5R+6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)]/

[R+ (2R+3R+4R+5R+6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)]

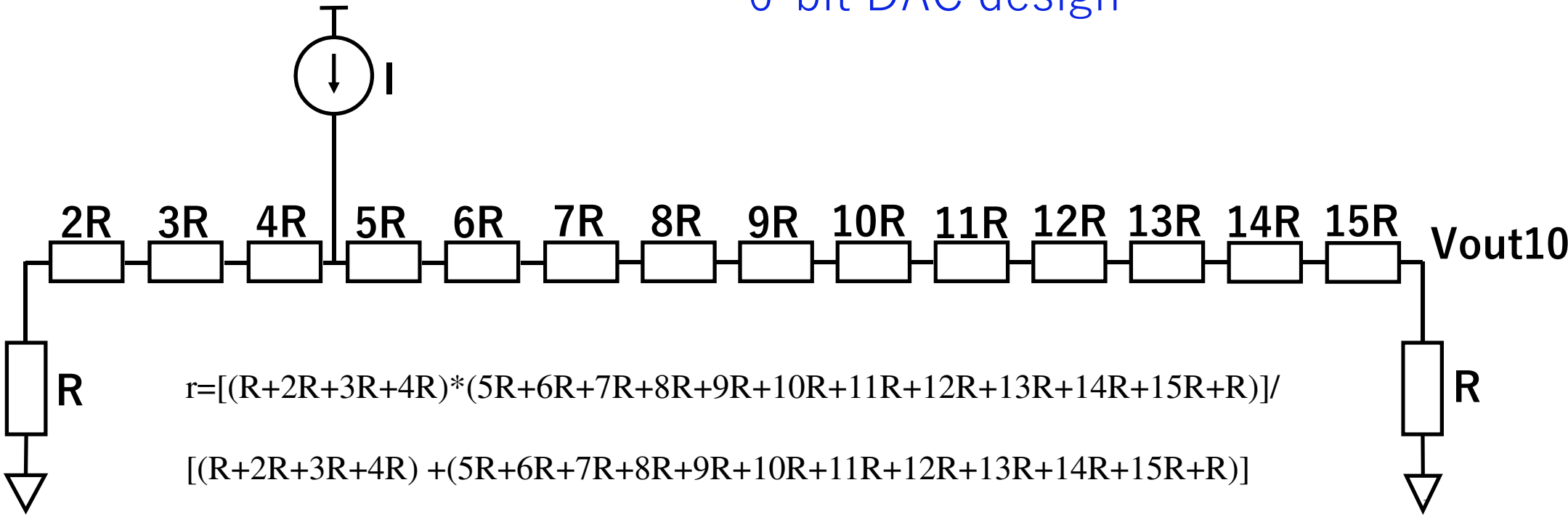
V=I*r

Vout1/V=R/(2R+3R+4R+5R+6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)

Vout1=I*R/121

Triangular Number DAC Input 10

6-bit DAC design



r=[(R+2R+3R+4R)*(5R+6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)]/

[(R+2R+3R+4R) +(5R+6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)]

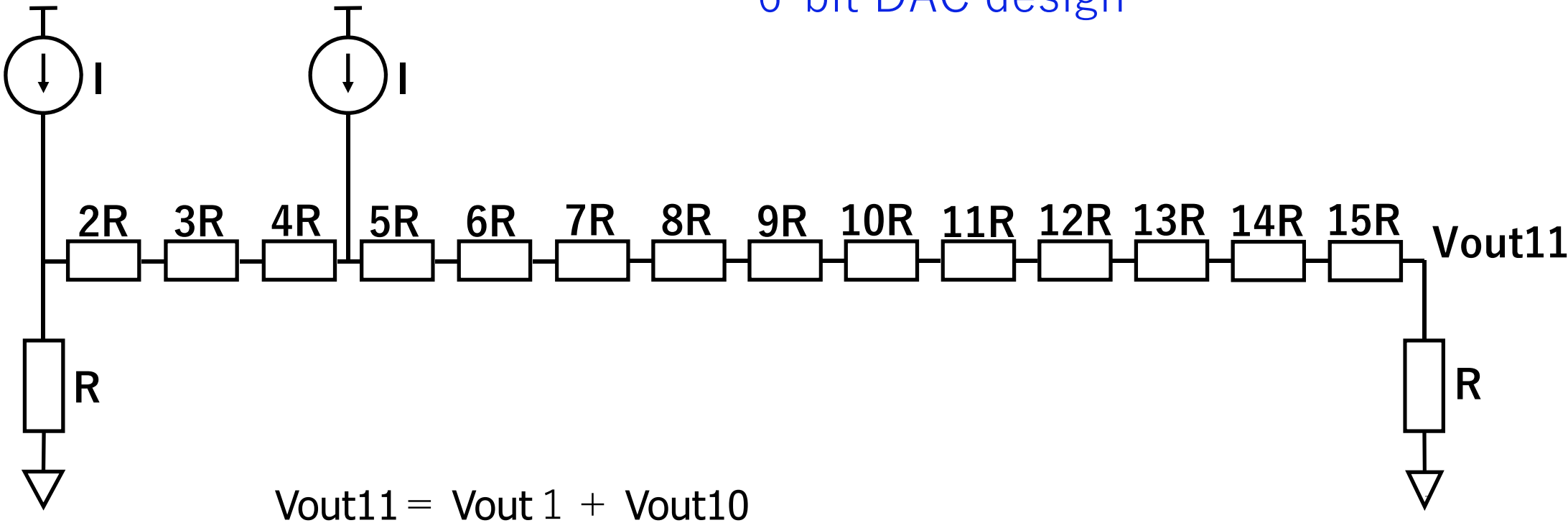
V=I*r

Vout10/V=R/(5R+6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)

Vout10=10I*R/121

Triangular Number DAC **Input 11**

6-bit DAC design



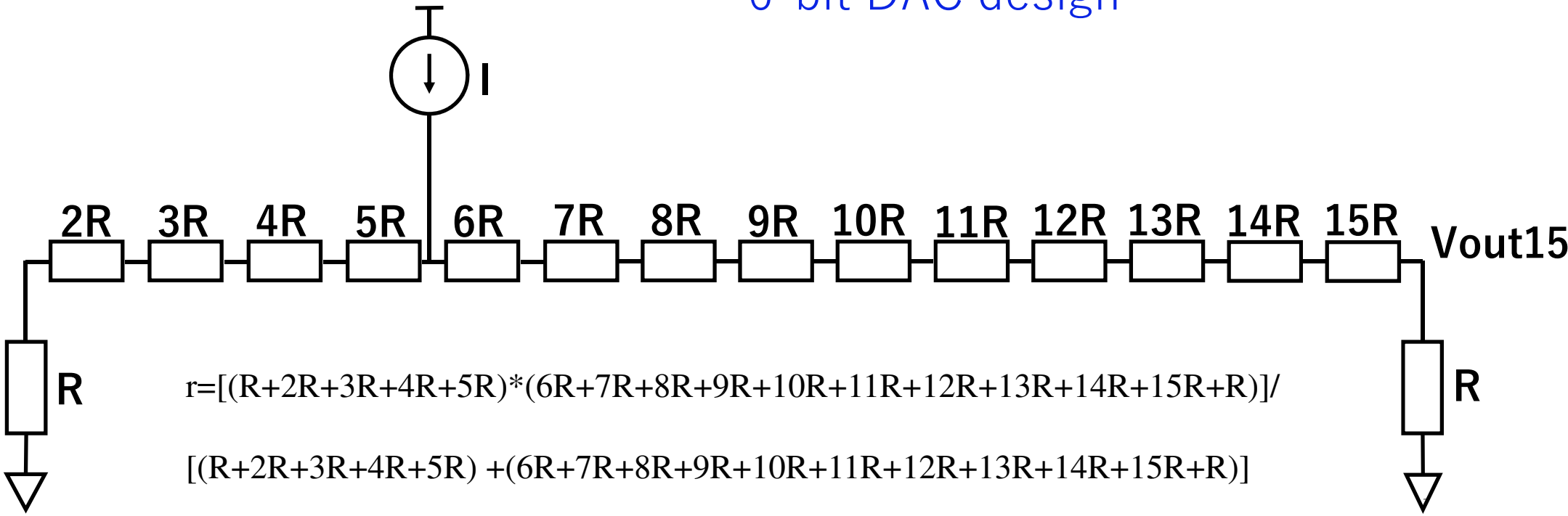
$$Vout11 = Vout1 + Vout10$$

$$= I*R/121 + 10I*R/121$$

$$= 11I*R/121$$

Triangular Number DAC Input 15

6-bit DAC design



r=[(R+2R+3R+4R+5R)*(6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)]/

[(R+2R+3R+4R+5R) +(6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)]

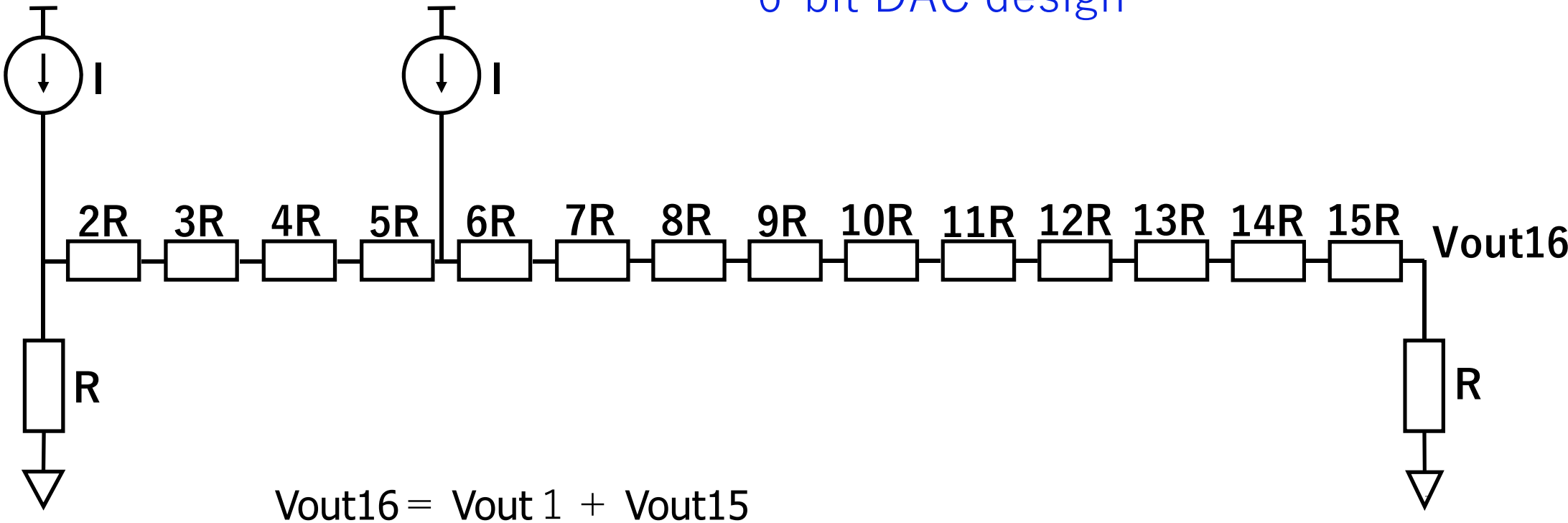
V=I*r

Vout15/V=R/(6R+7R+8R+9R+10R+11R+12R+13R+14R+15R+R)

Vout15=15I*R/121

Triangular Number DAC **Input 16**

6-bit DAC design



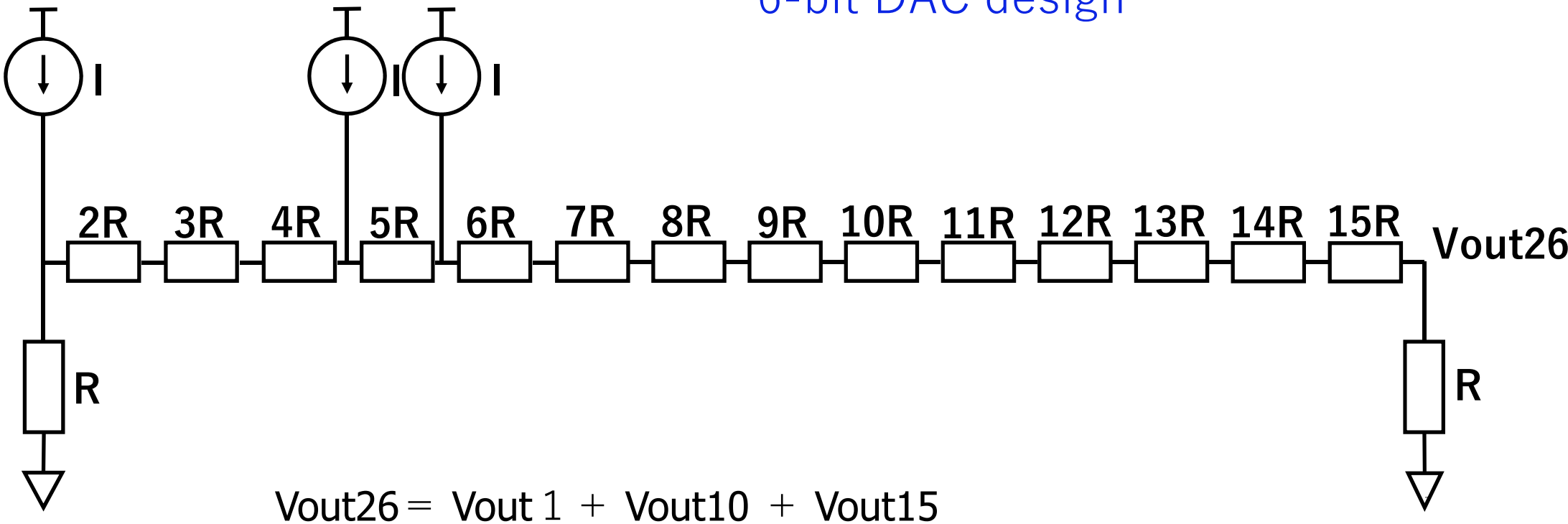
$$Vout16 = Vout1 + Vout15$$

$$= I*R/121 + 15I*R/121$$

$$= 16I*R/121$$

Triangular Number DAC Input 26

6-bit DAC design



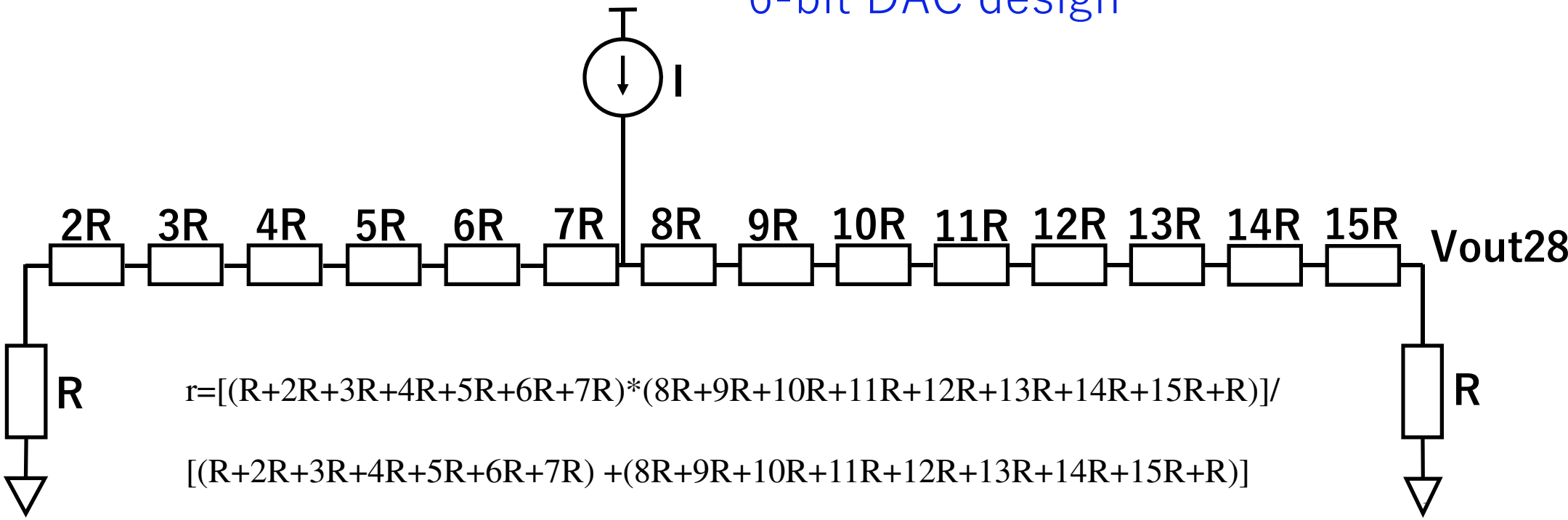
$$Vout26 = Vout1 + Vout10 + Vout15$$

$$= I*R/121 + 10I*R/121 + 15I*R/121$$

$$= 26I*R/121$$

Triangular Number DAC Input 28

6-bit DAC design



r=[(R+2R+3R+4R+5R+6R+7R)*(8R+9R+10R+11R+12R+13R+14R+15R+R)]/

[(R+2R+3R+4R+5R+6R+7R) +(8R+9R+10R+11R+12R+13R+14R+15R+R)]

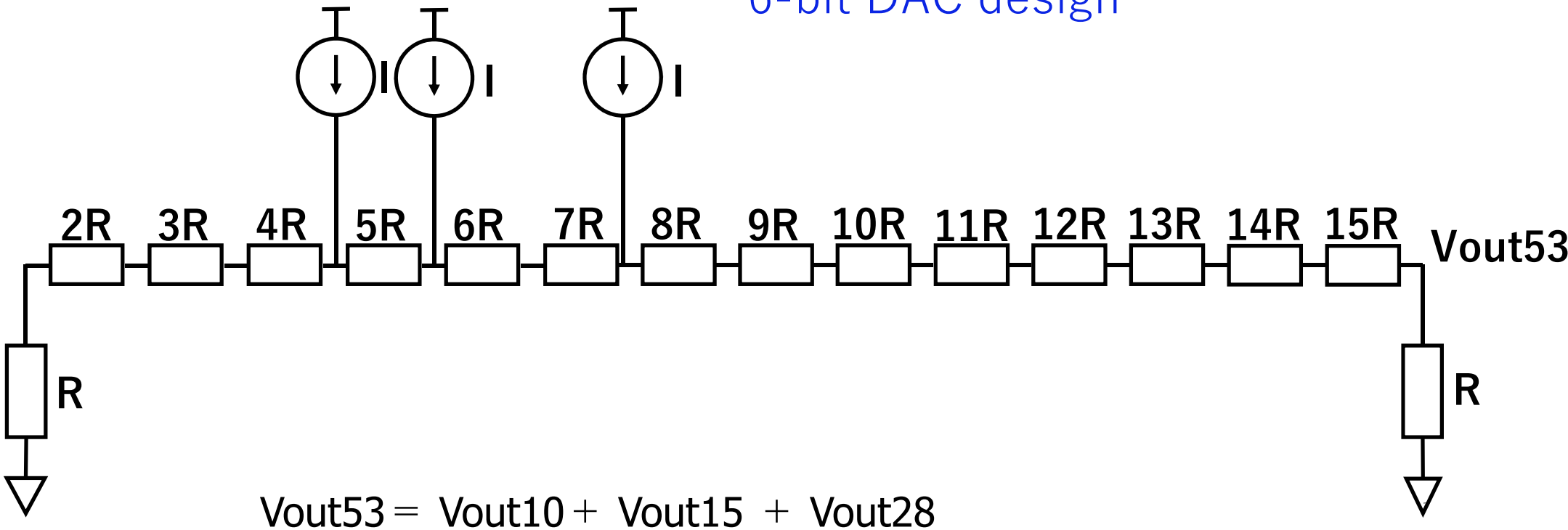
V=I*r

Vout28/V=R/(8R+9R+10R+11R+12R+13R+14R+15R+R)

Vout28=28I*R/121

Triangular Number DAC Input 53

6-bit DAC design



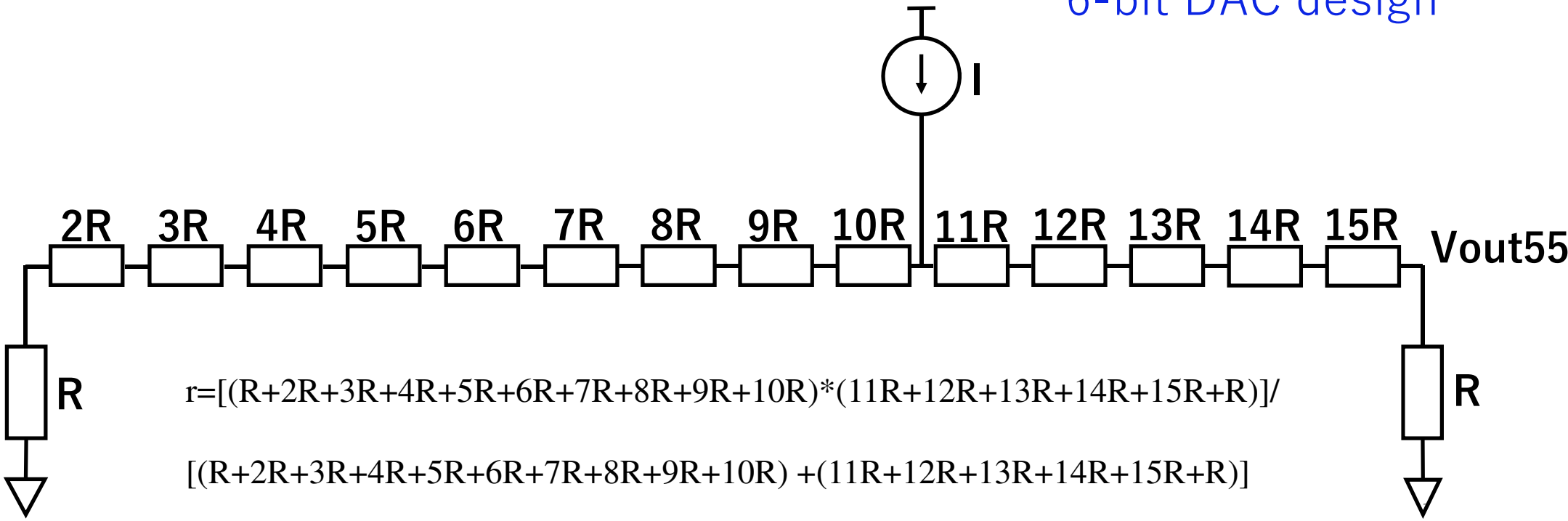
$$Vout53 = Vout10 + Vout15 + Vout28$$

$$= 10I*R/121 + 15I*R/121 + 28I*R/121$$

$$= 53I*R/121$$

Triangular Number DAC Input 55

6-bit DAC design



r=[(R+2R+3R+4R+5R+6R+7R+8R+9R+10R)*(11R+12R+13R+14R+15R+R)]/

[(R+2R+3R+4R+5R+6R+7R+8R+9R+10R) +(11R+12R+13R+14R+15R+R)]

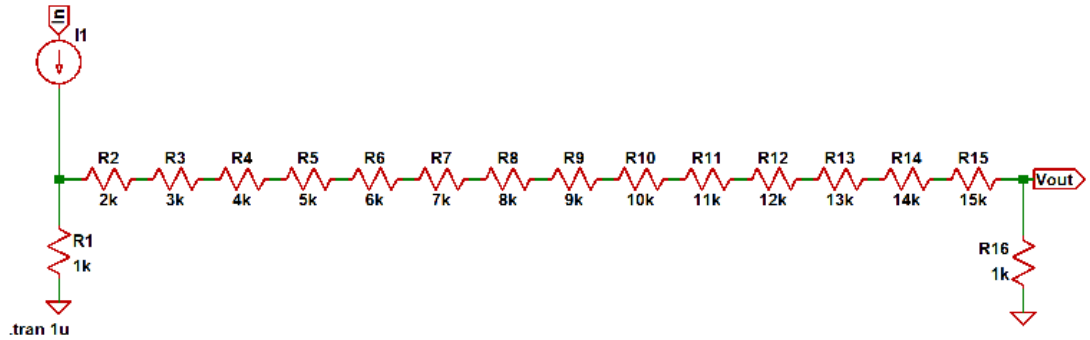
V=I*r

Vout55/V=R/(11R+12R+13R+14R+15R+R)

Vout55=55I*R/121

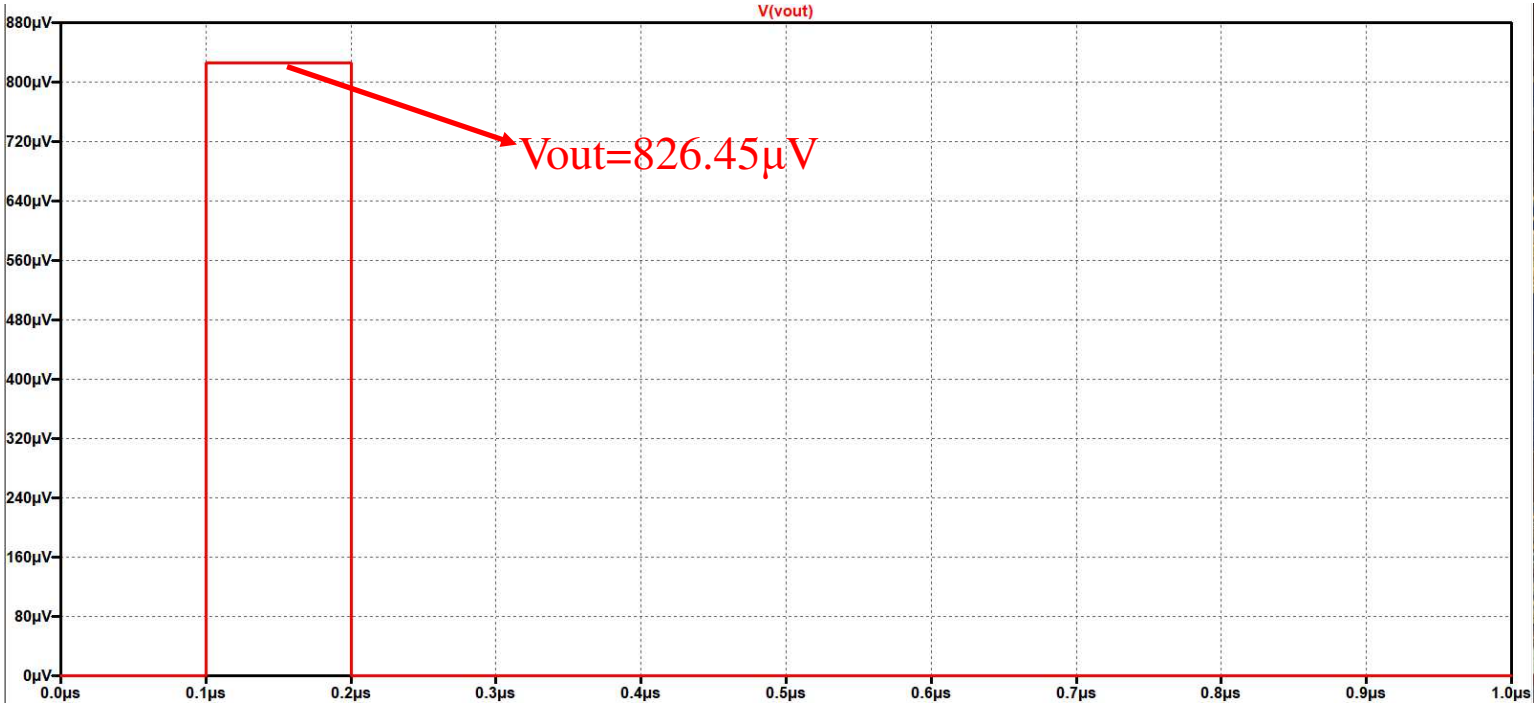
Triangular Number DAC **Input 1**

6-bit DAC simulation



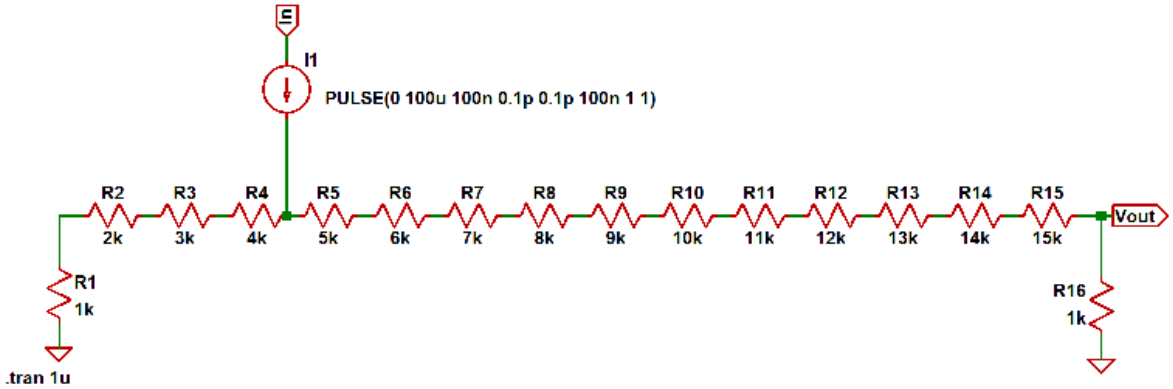
Theoretical Value Vout=I*R/121=826.45μV

Simulation Value Vout=826.45μV



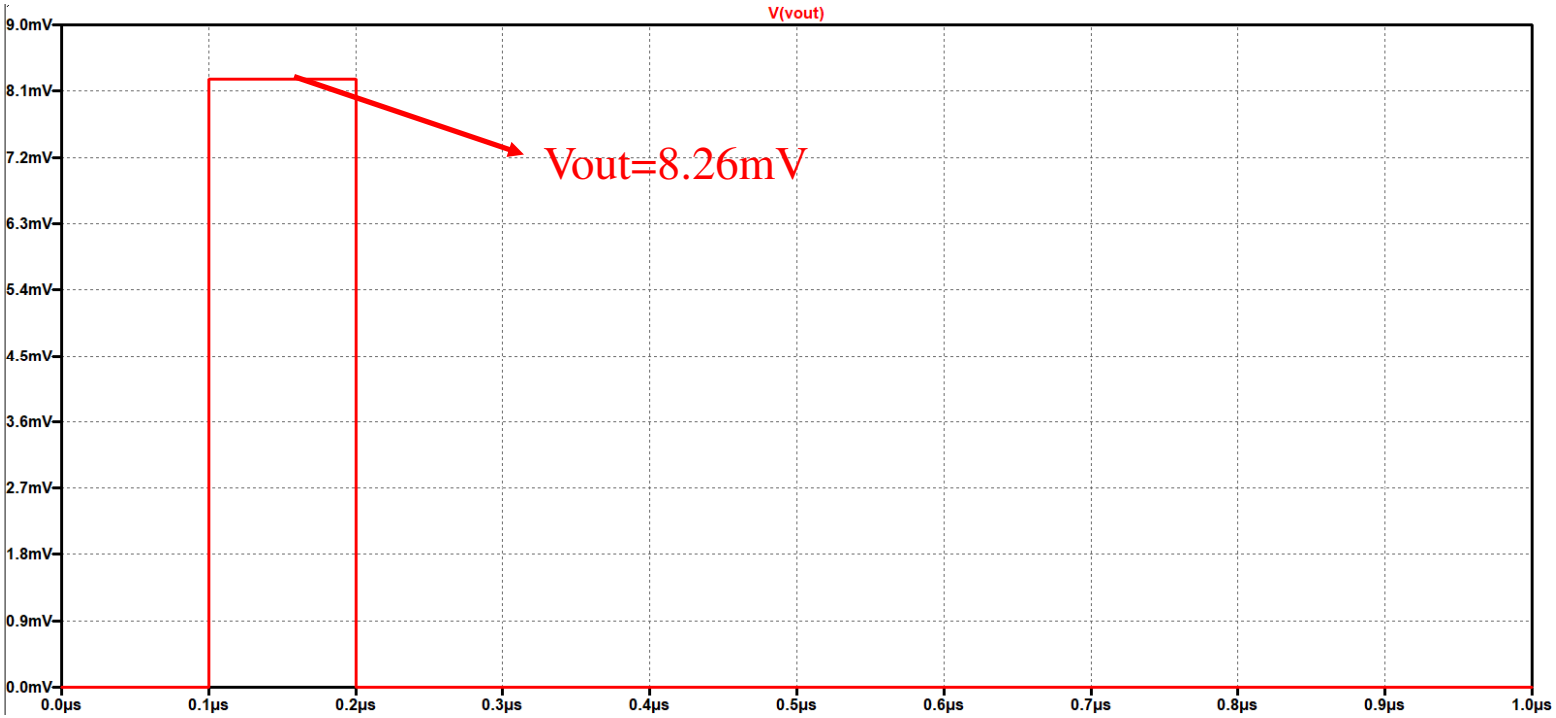
Triangular Number DAC Input 10

6-bit DAC simulation



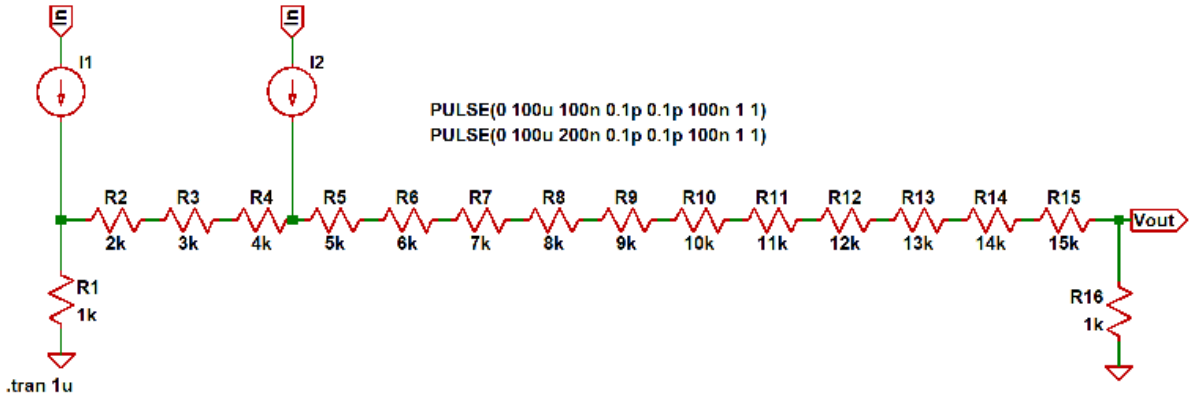
Theoretical Value Vout=10I*R/121=8.26mV

Simulation Value Vout=8.26mV



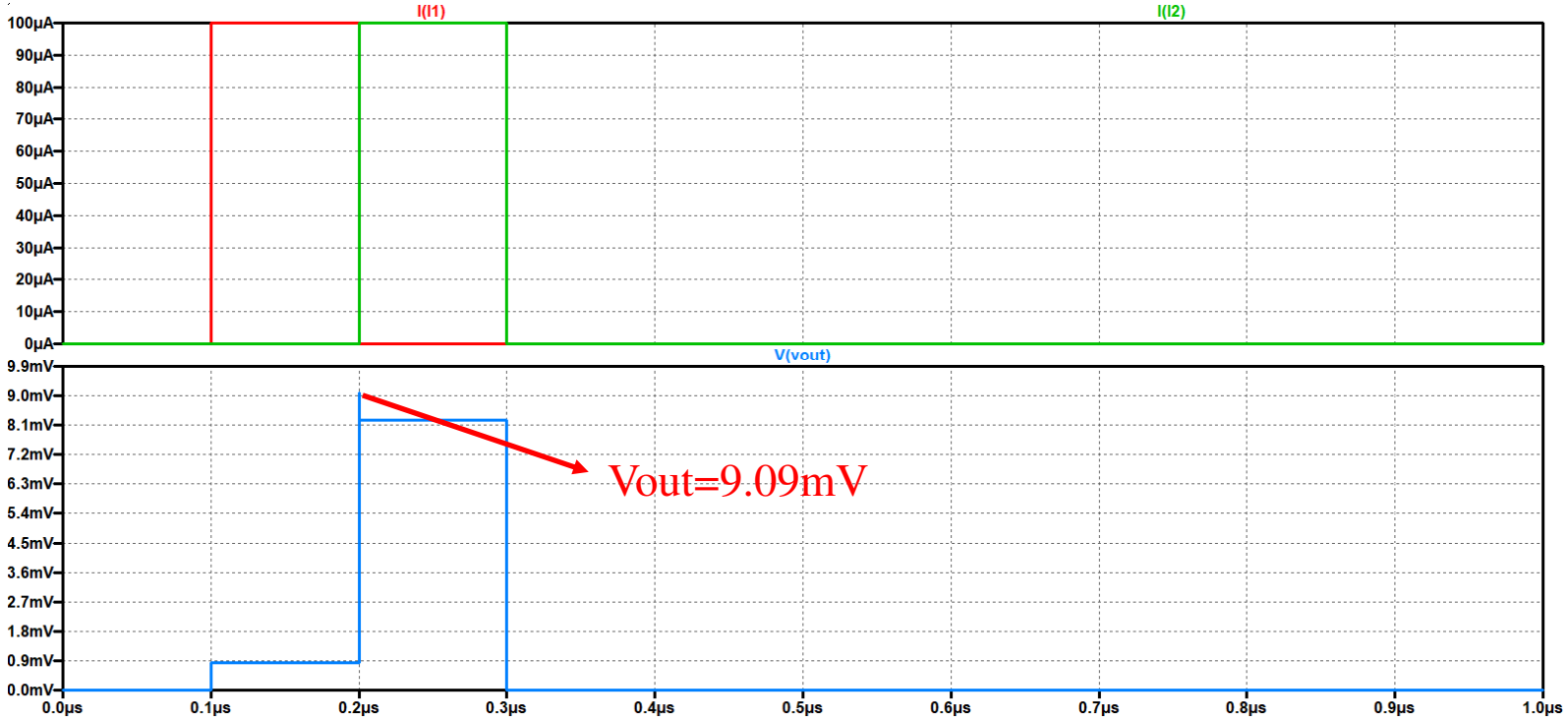
Triangular Number DAC Input 11

6-bit DAC simulation



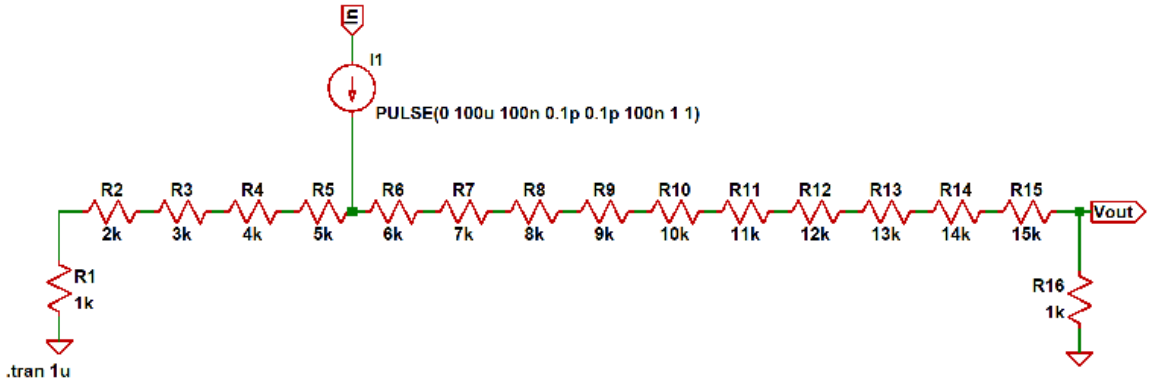
Theoretical Value Vout=11I*R/121=9.09mV

Simulation Value Vout=9.09mV



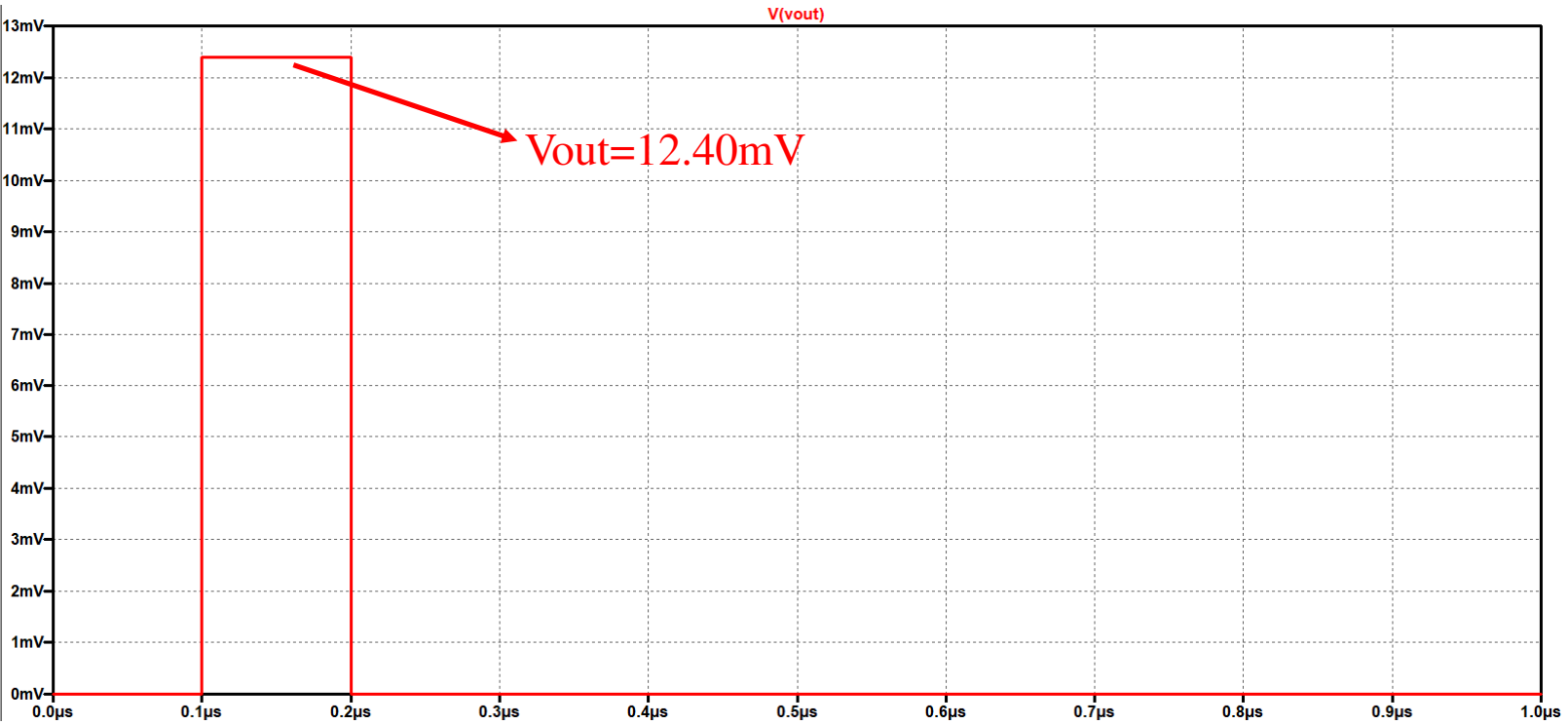
Triangular Number DAC **Input 15**

6-bit DAC simulation



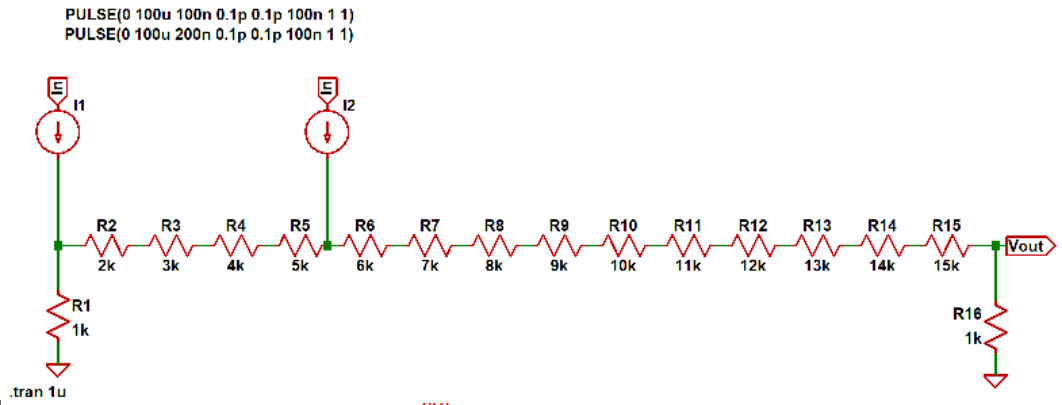
Theoretical Value Vout=15I*R/121=12.40mV

Simulation Value Vout=12.40mV



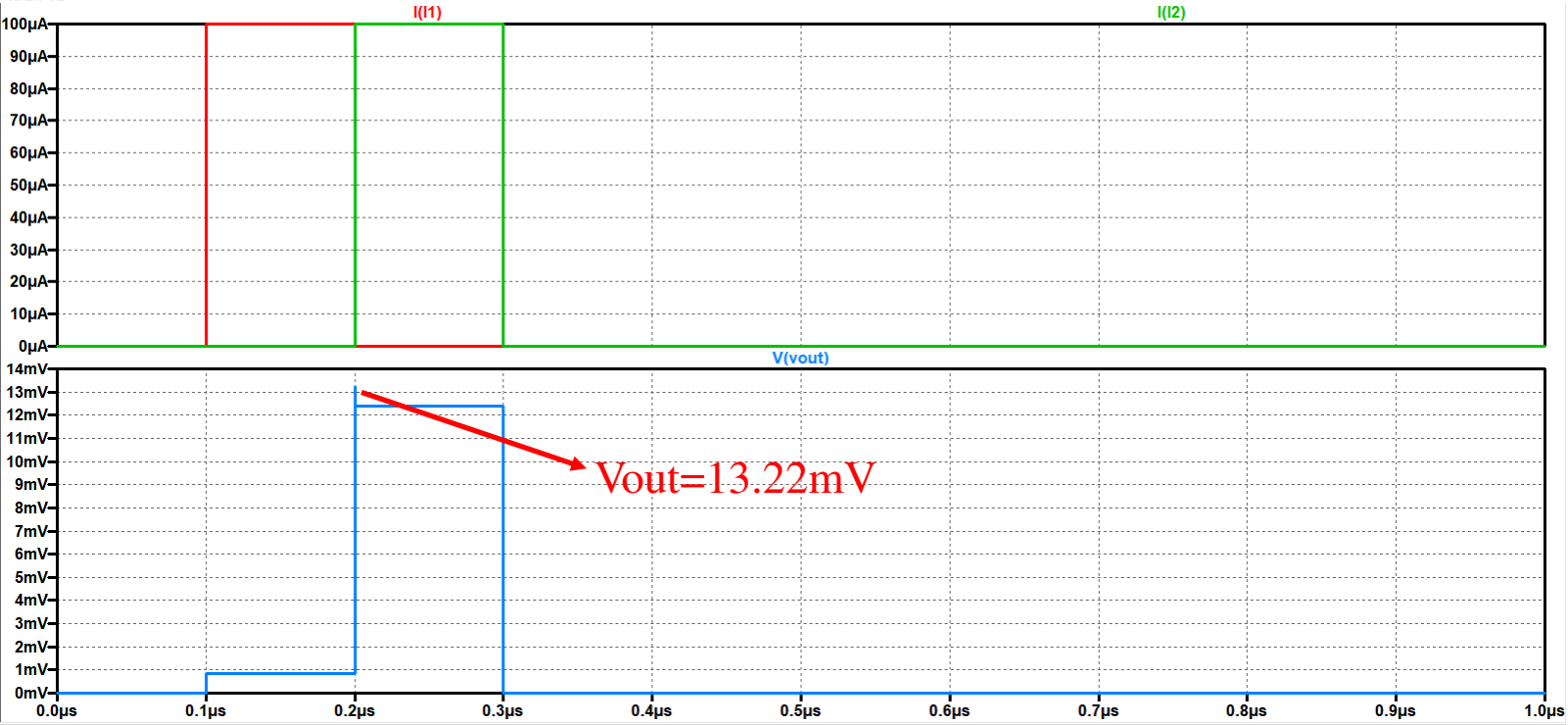
Triangular Number DAC Input 16

6-bit DAC simulation



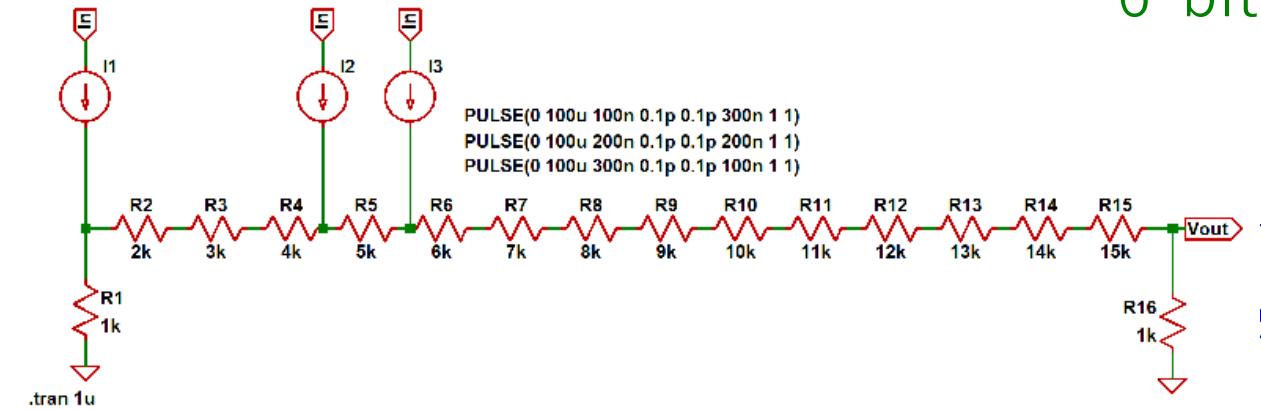
Theoretical Value Vout=16I*R/121=13.22mV

Simulation Value Vout=13.22mV



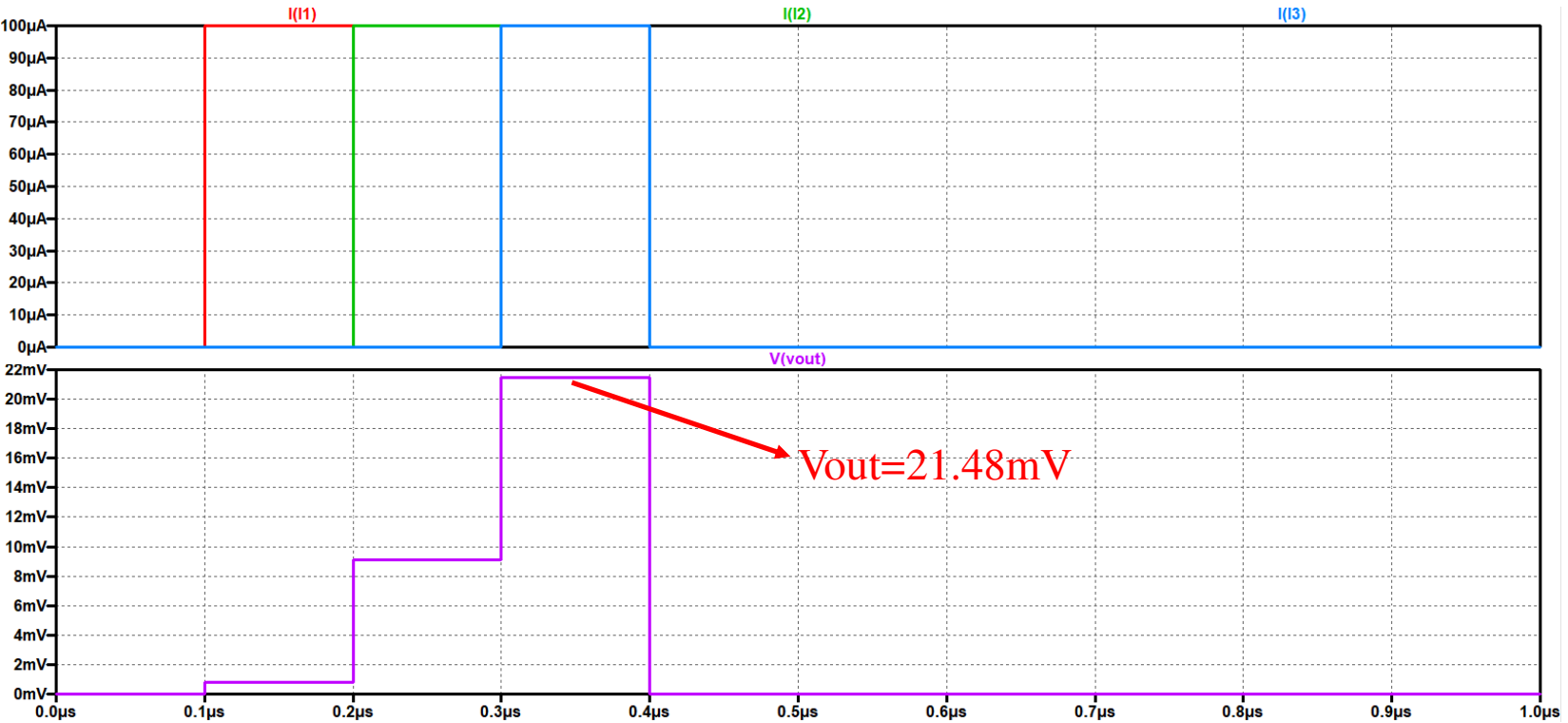
Triangular Number DAC **Input 26**

6-bit DAC simulation



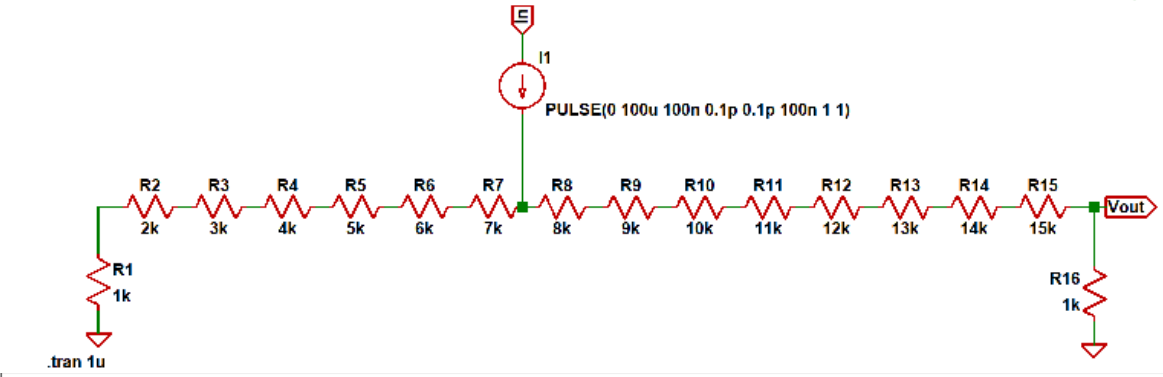
Theoretical Value Vout=26I*R/121=21.48mV

Simulation Value Vout=21.48mV



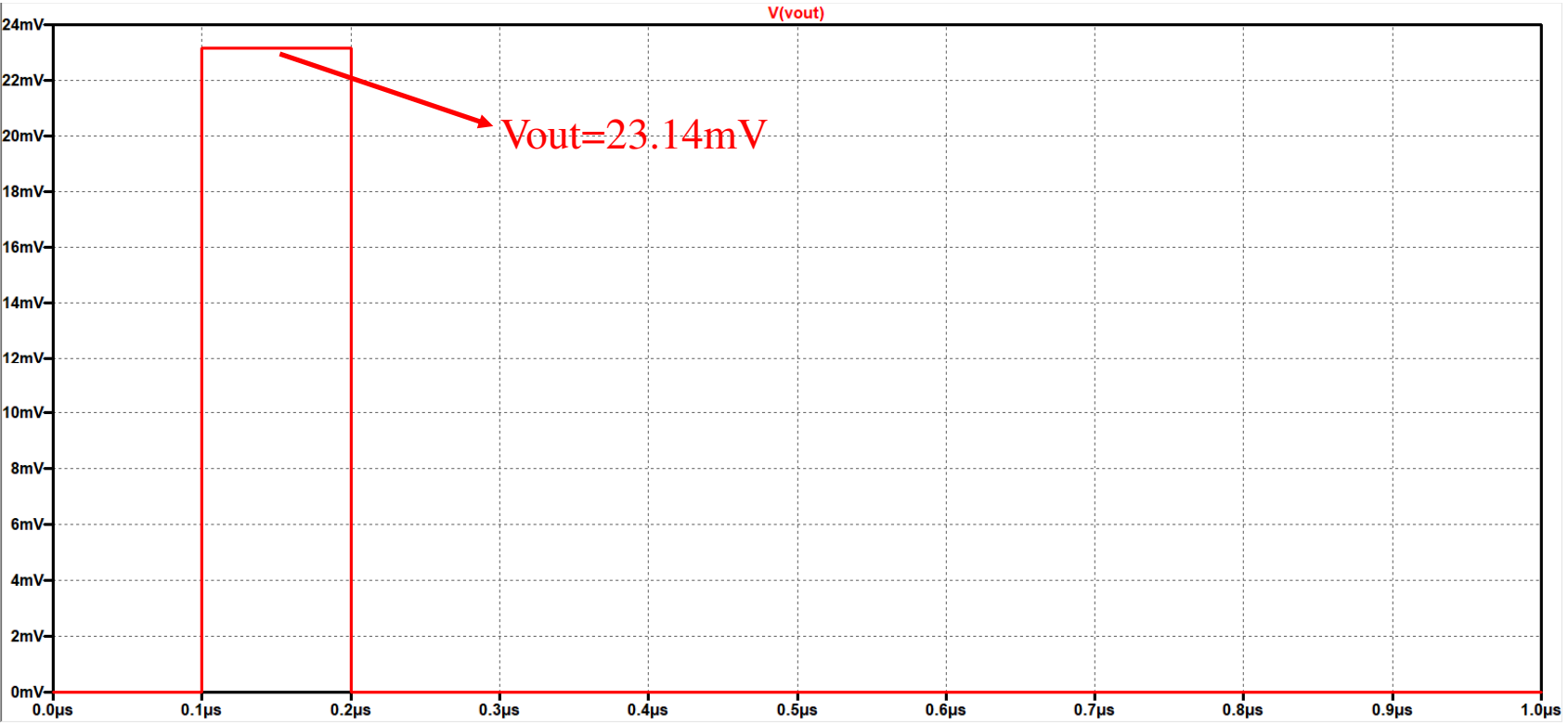
Triangular Number DAC Input 28

6-bit DAC simulation



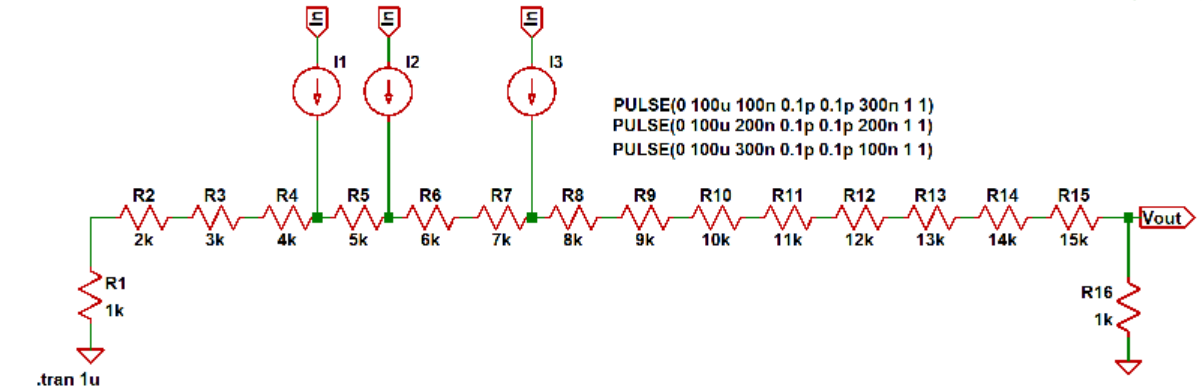
Theoretical Value Vout=28I*R/121=23.14mV

Simulation Value Vout=23.14mV



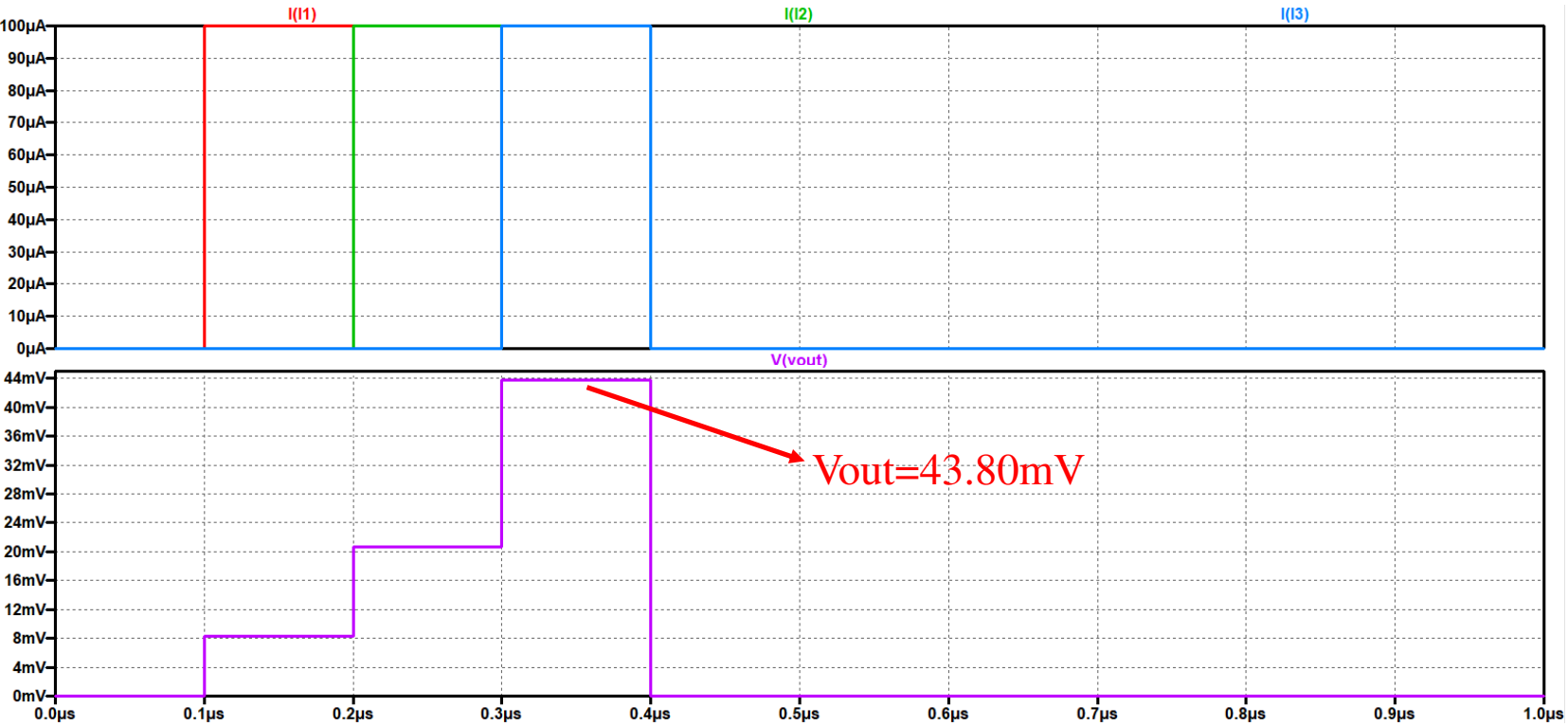
Triangular Number DAC Input 53

6-bit DAC simulation



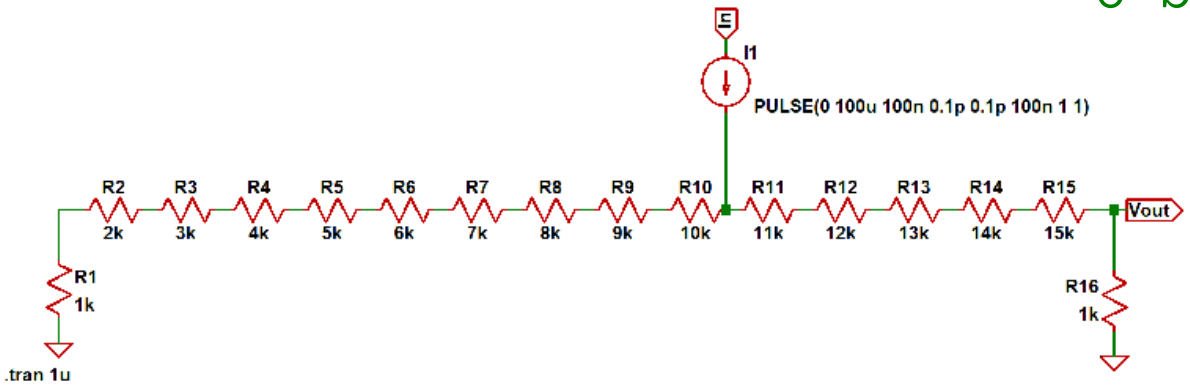
Theoretical Value Vout=53I*R/121=43.80mV

Simulation Value Vout=43.80mV



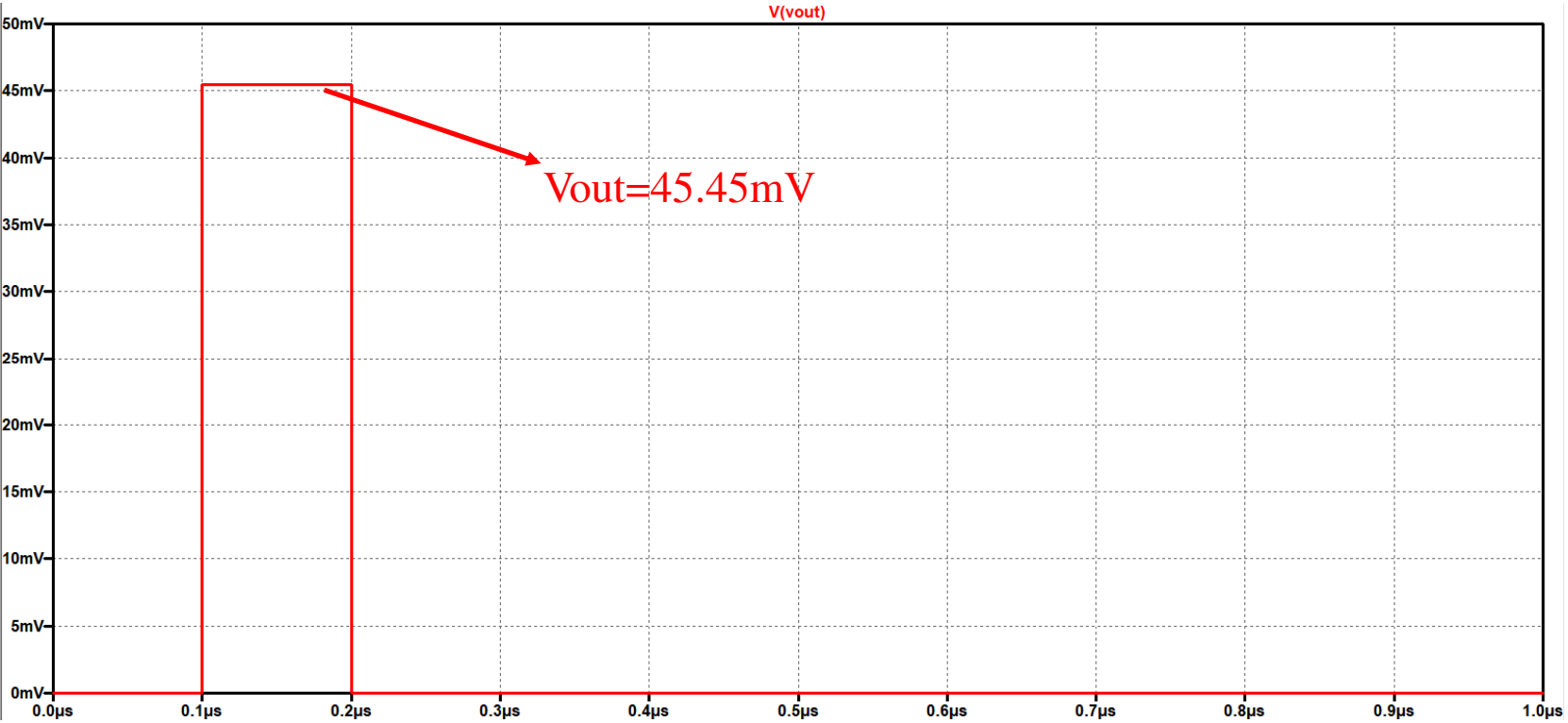
Triangular Number DAC Input 55

6-bit DAC simulation

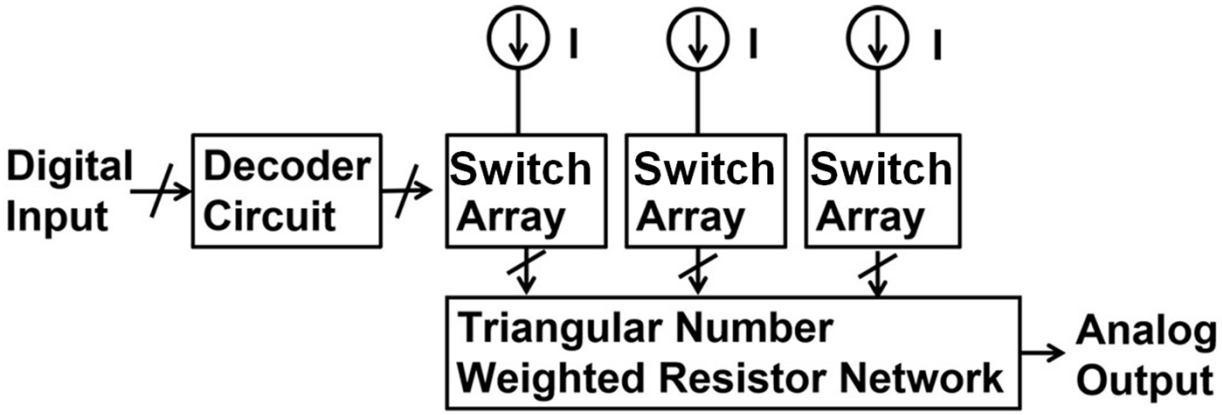


Theoretical Value Vout=55I*R/121=45.45mV

Simulation Value Vout=45.45mV



Summary



Triangular number DAC

3 current sources
3 switch arrays

6-bit DAC design

Square Numbers DAC Results

Input	1	4	9	16
Theoretical Value	Vout=I*R/79 =1.27mV	Vout=6I*R/79 =7.59mV	Vout=10I*R/79 =12.6mV	Vout=15I*R/79 =19.00mV
Simulation Value	Vout=1.27mV	Vout=7.59mV	Vout=12.6mV	Vout=19.00mV

Input	24	81	94	97
Theoretical Value	Vout=I*R/79 =34.18mV	Vout=55I*R/79 =69.62mV	Vout=71I*R/79 =89.87mV	Vout=70I*R/79 =88.61mV
Simulation Value	Vout=34.18mV	Vout=69.62mV	Vout=89.87mV	Vout=88.61mV

Conclusion

- Completely new DAC architectures based on number theory
- Design of 6-bit DAC based on triangular number theory

  ⇒ Verified by SPICE simulation
- Next, decoder design
- Discussions on their pros and cons are left for the future work.

Q&A

Q: Why and how do you combine DAC with the integer theory?

A: Because there is no link between the interesting properties of Integers and the Electronic circuit designs.

We completed new DAC architectures based on number theory. And designed of 6-bit DAC based on triangular number theory. Next, we will complete the decoder design.