

Segmented-type DAC Unit Cell Selection Algorithm Based on Euler's Knight Tour & Magic Square

オイラーのナイトツアー & 魔方陣に基づく セグメント型DAC単位セル選択アルゴリズム

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Contents

- Research Objective
- Segment-type DAC
- Characteristic of Variation in Circuit Element
- Proposed Layout Method
 - Magic Square
 - Euler's Knight Tour
- Conclusion

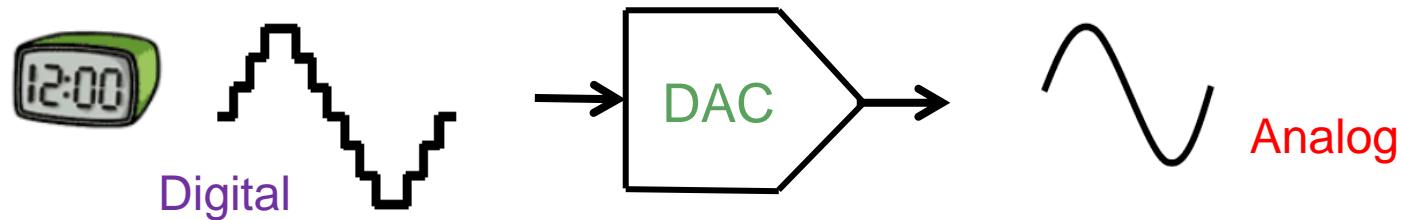
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Research Objective

Objective

- Development of a **highly linear** digital-to-analog converter (DAC)



Our Approach

- DAC layout technique to cancel systematic mismatch effects among unit current cells.

-Layout based on

Euler's Knight Tour and Magic Squares

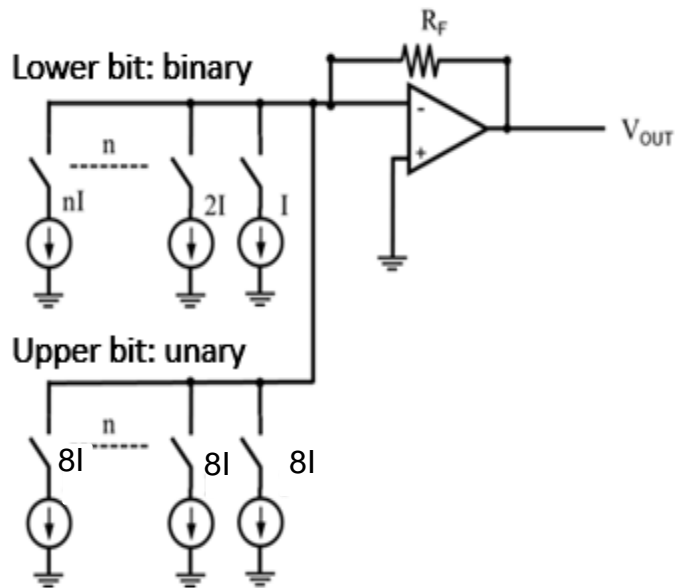
➡ Better linearity

New!!

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Segment-type DAC Configuration



✓ Binary (Lower bits)

- Small circuit
- Large glitch
- Large mismatch effect & Large nonlinearity

✓ Unary (Upper bits)

- Large circuit
- Small glitch
- Small mismatch effect & modest linearity

Segmented DAC

Focus !!

Segment-type DAC (7-bit case)

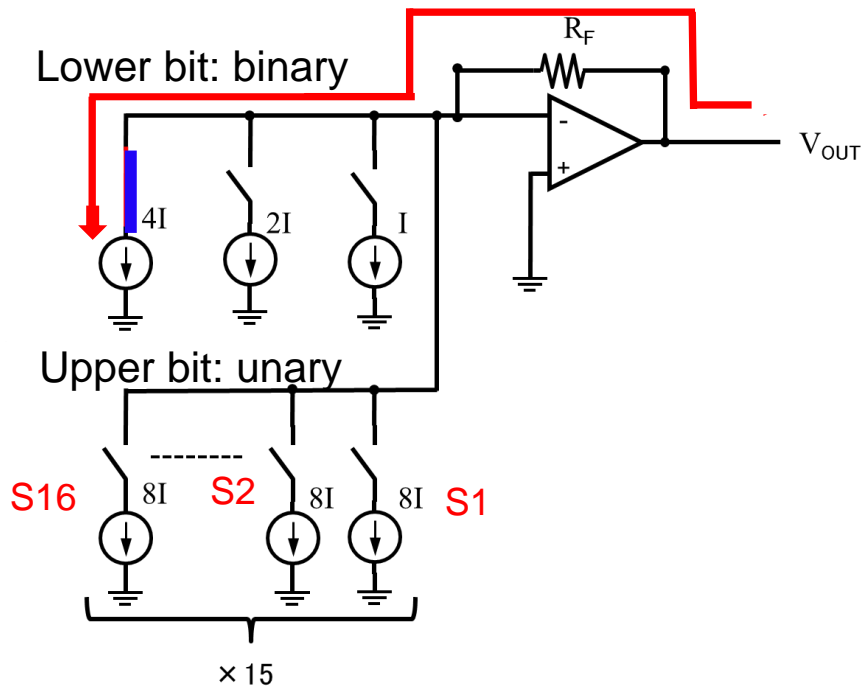
ex.1

In case digital input = 4

(0000100)



$$V_{out} = 4IR_F$$



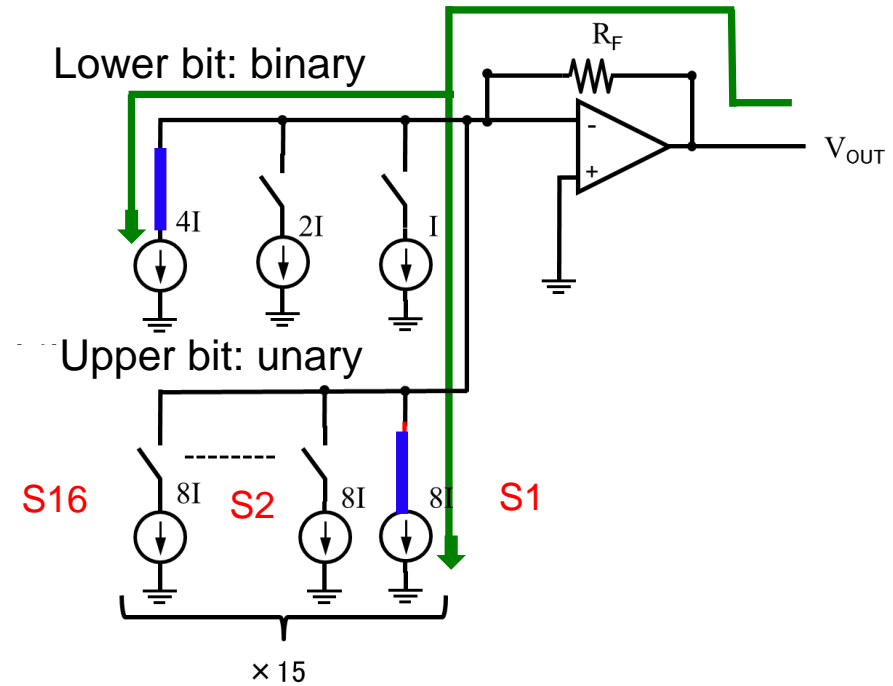
ex.2

In case digital input = 12

(0001100)



$$V_{out} = 12IR_F$$



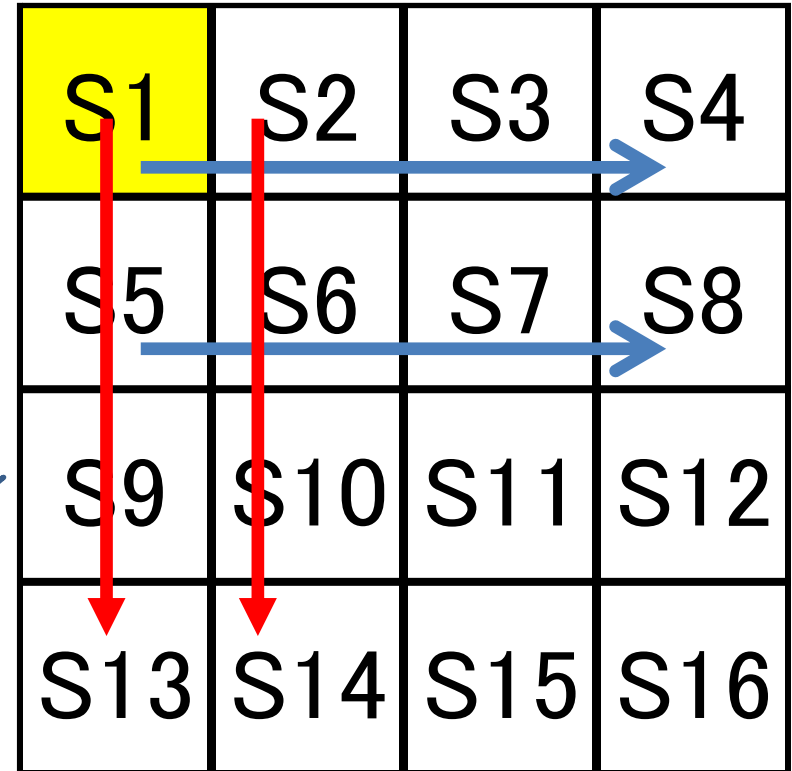
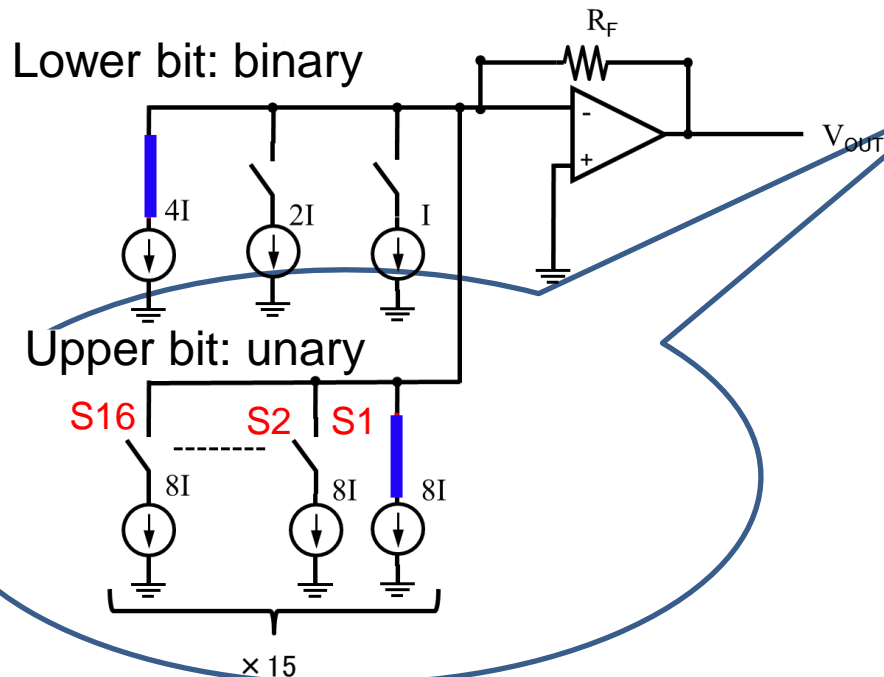
Unary DAC Current Cells Layout

✓ 7bit DAC

(0001100)



$$V_{out} = 12IR_F$$



2D array of
unit current cells (8I)

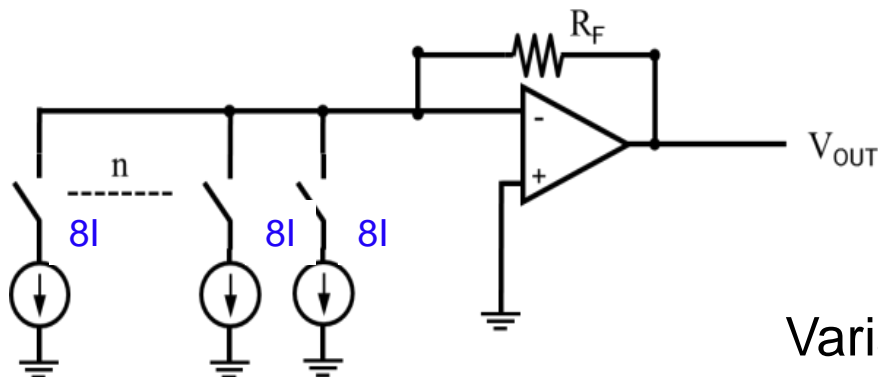
Regular layout

Unary DAC Features

- Identical current sources
- Small glitch
- Inherent monotonicity



- Large circuits
 - Decoder
 - Many switches and current sources



Variations (e_{16}, \dots, e_2, e_1)
among current sources

 Real chip

8I	8I	8I
+	+	+
e_{16}	e_2	e_1



DAC nonlinearity

Problem of Regular Layout

Error e depends on place

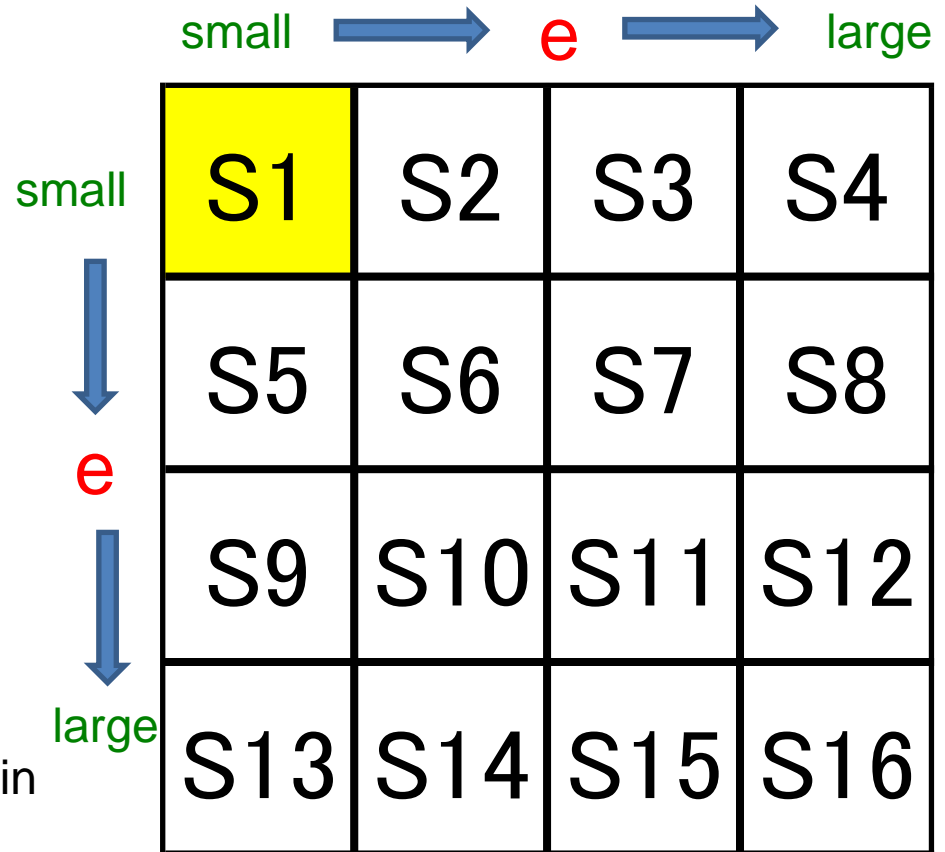
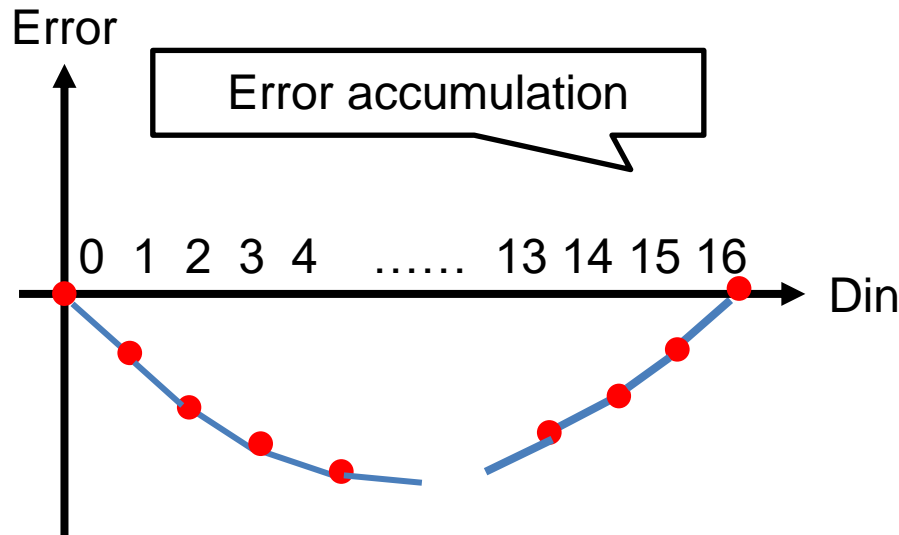
$$V(0) = 0$$

$$V(1) = 8l + e_1$$

$$V(2) = 16l + e_1 + e_2$$

$$V(3) = 32l + e_1 + e_2 + e_3$$

:

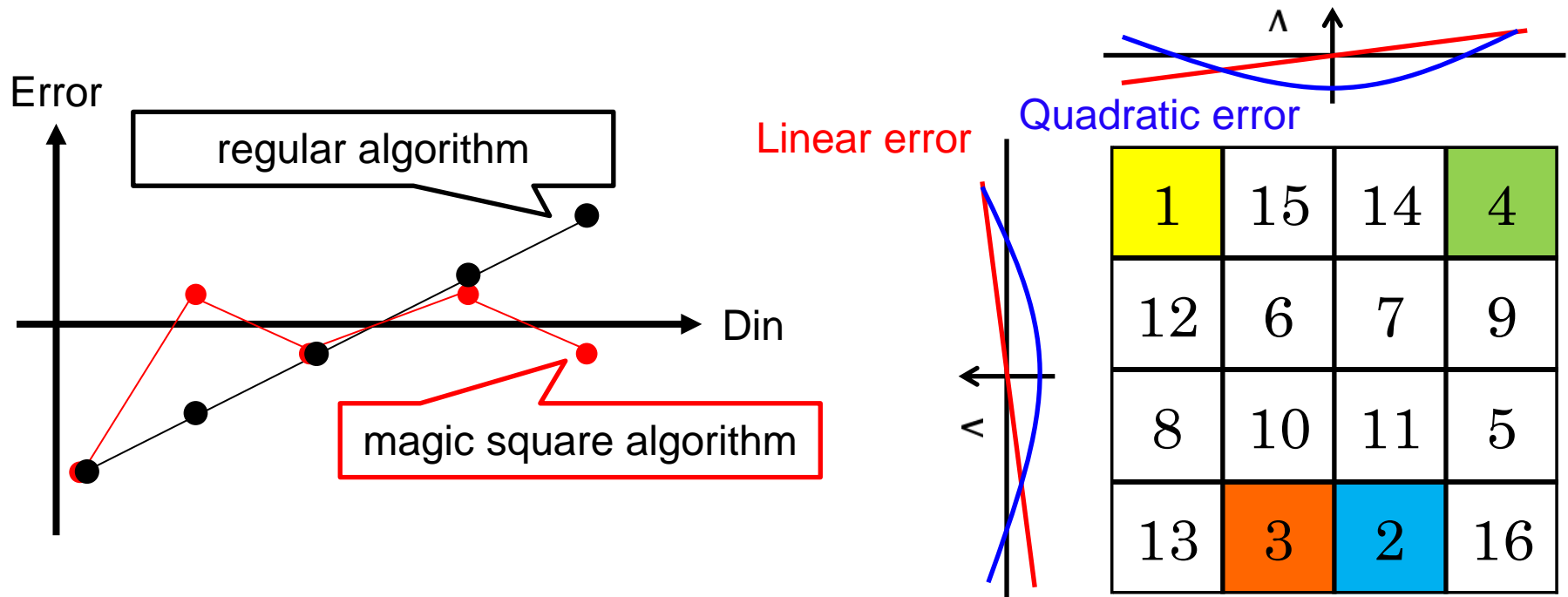


2D array of
unit current cells (8l)

Regular layout

Cell Layout and Systematic Mismatch

- Semiconductor devices have systematic mismatches
 - Changing the unit cell layout order
 - We propose magic squares and Euler's Knight Tour algorithms
- Cancellation of **systematic mismatch effects**



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Variation in Circuit Element Characteristics

◆ Systematic variations

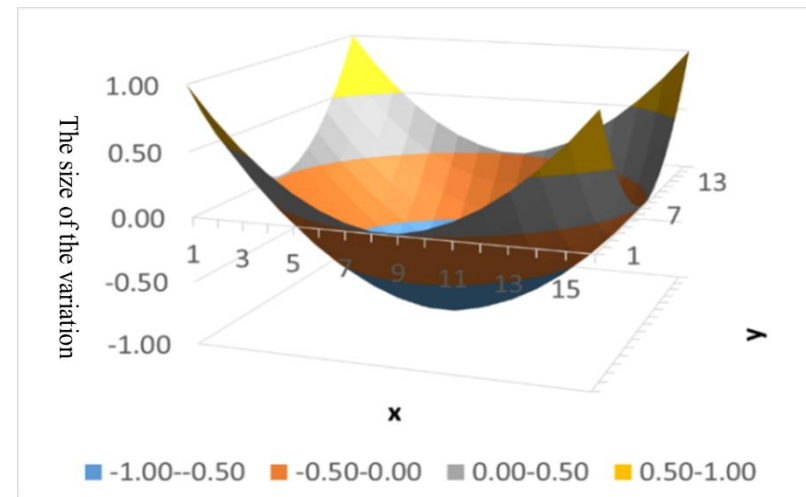
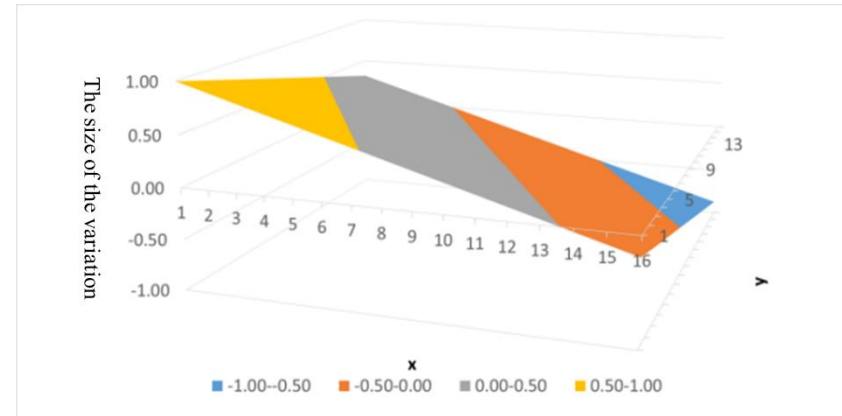
- ✓ Voltage drop
- ✓ Thickness of oxide film
- ✓ Doping
- ✓ Mechanical stress
- ✓ Temperature distribution
- ✓ In wafer plane

Linear
error

Quadratic
error



Joint Error (Sum of both)



Systematic Variation Model

Linear Error

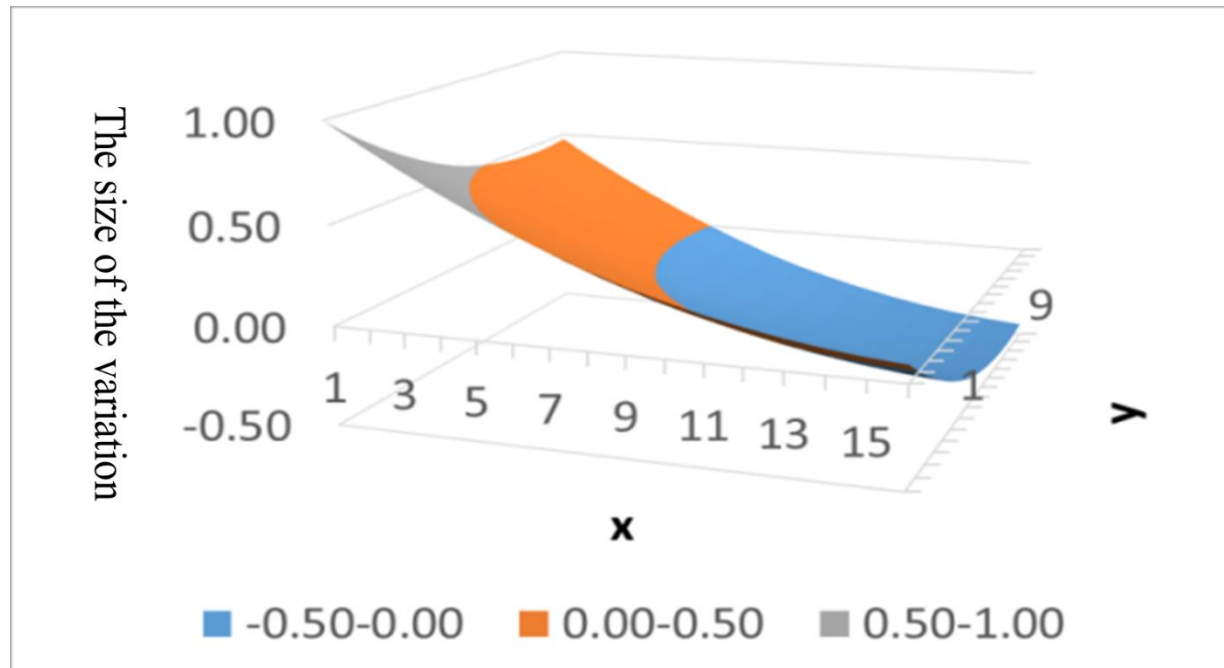
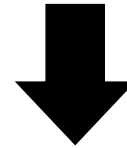
$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

Quadratic Error

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

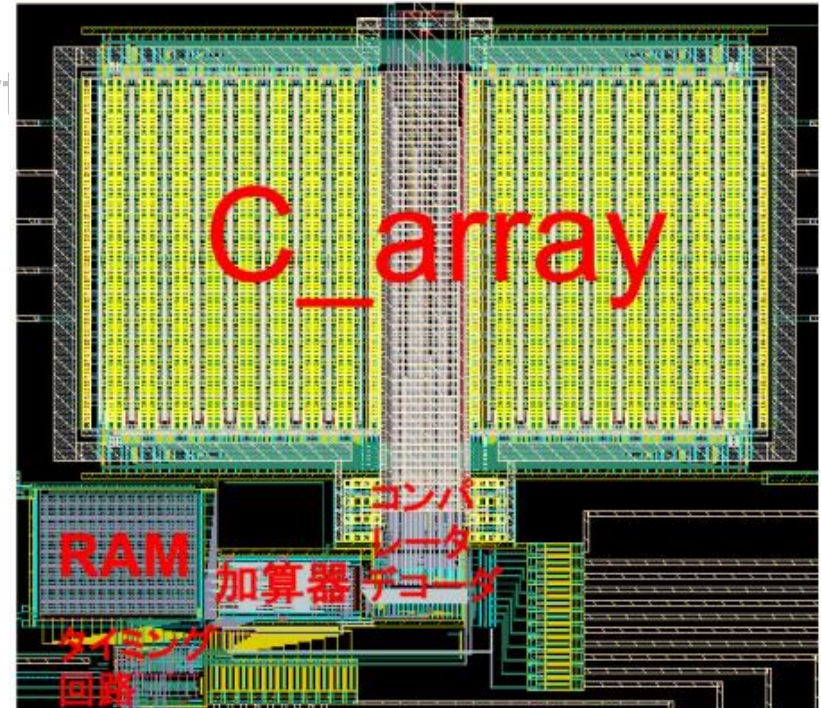
Joint Errors

$$\varepsilon_j(x, y) = \varepsilon_l(x, y) + \varepsilon_q(x, y)$$



Contents

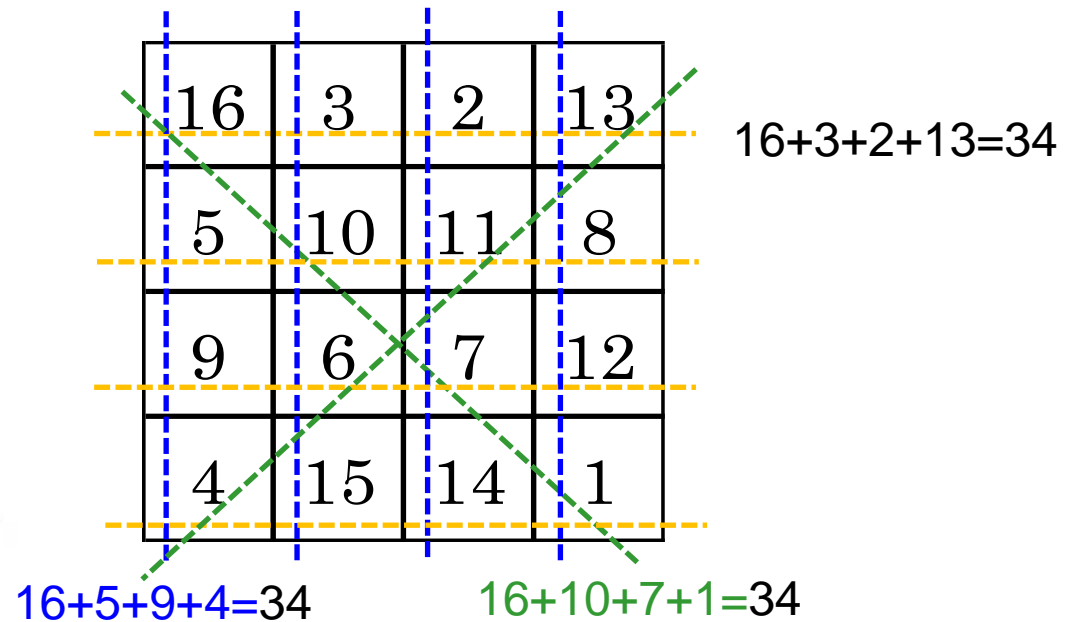
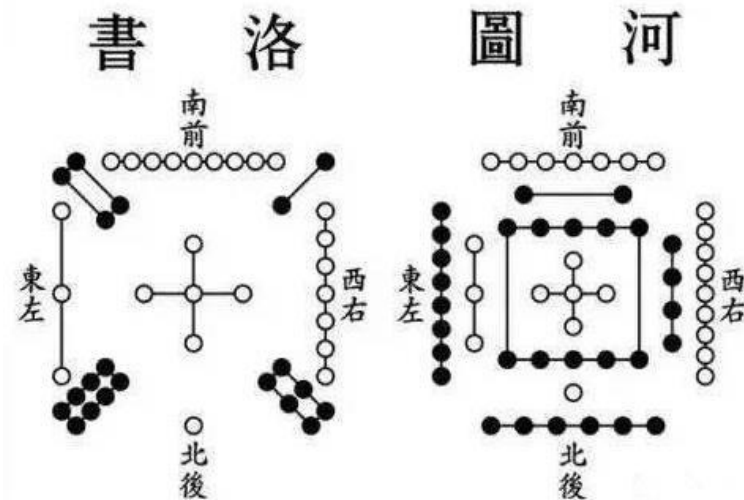
- Research Objective
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IC Layout

What is Magic Square ?

- Origin from Chinese academia
- “Constant sum” characteristics
- Varieties of magic squares



Constant Sum

Row, Column, Diagonal



Magic square has
good balance



16*16 Magic Square

255	2	3	253	252	6	7	249	248	10	11	245	244	14	15	241
17	239	238	20	21	235	234	24	25	231	230	28	29	227	226	32
33	223	222	36	37	219	218	40	41	215	214	44	45	211	210	48
208	50	51	205	204	54	55	201	200	58	59	197	196	62	63	193
192	66	67	189	188	70	71	185	184	74	75	181	180	78	79	177
81	175	174	84	85	171	170	88	89	167	166	92	93	163	162	96
97	159	158	100	101	155	154	104	105	151	150	108	109	147	146	112
144	114	115	141	140	118	119	137	136	122	123	133	132	126	127	129
128	130	131	125	124	134	135	121	120	138	139	117	116	142	143	113
145	111	110	148	149	107	106	152	153	103	102	156	157	99	98	160
161	95	94	164	165	91	90	168	169	87	86	172	173	83	82	176
80	178	179	77	76	182	183	73	72	186	187	69	68	190	191	65
64	194	195	61	60	198	199	57	56	202	203	53	52	206	207	49
209	47	46	212	213	43	42	216	217	39	38	220	221	35	34	224
225	31	30	228	229	27	26	232	233	23	22	236	237	19	18	240
6	242	243	13	12	246	247	9	8	250	251	5	4	254	255	1

2056

2056

2056

Constant Sum: Row, Column, Diagonal = 2056

Contents

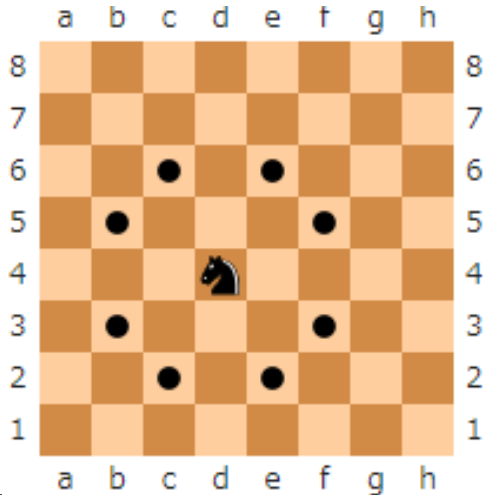
- Research Objective
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 - **Euler's Knight Tour**
- Conclusion



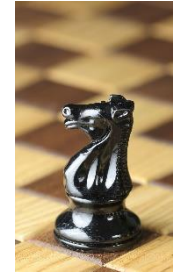
Harry Potter

What is Knight Tour ?

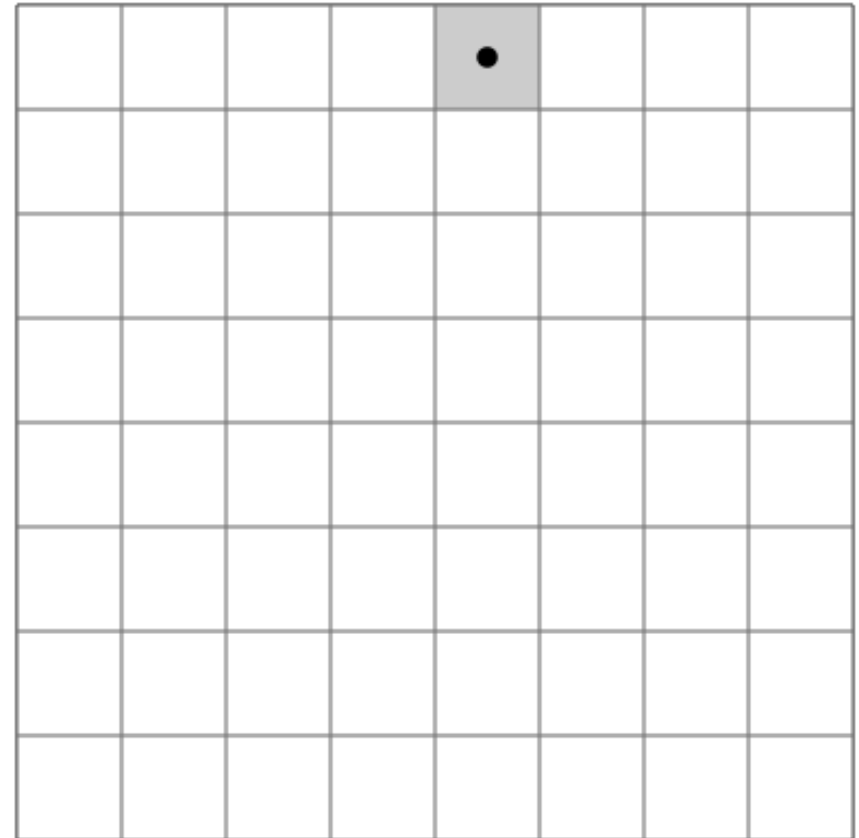
Chess



Knight



15	62	19	34	1	50	31	46
18	35	16	63	32	47	2	49
61	14	33	20	51	4	45	30
36	17	60	13	64	29	48	3
11	58	21	40	5	54	27	44
22	37	12	59	28	41	6	53
57	10	39	24	55	8	43	26
38	23	56	9	42	25	54	7



What is Euler's Knight Tour ?



- Found out
By Leonhard Euler
- Magic square
+
Knight tour

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

260
260

8x8 Euler's Knight Tour

16x16 Euler's Knight Tour

84	217	170	75	188	219	172	77	228	37	86	21	230	39	88	25
169	74	185	218	171	76	189	220	85	20	229	38	87	24	231	40
216	183	68	167	222	187	78	173	36	227	22	83	42	237	26	89
73	168	215	186	67	174	221	190	19	84	35	238	23	90	41	232
182	213	166	69	178	223	176	79	226	33	82	31	236	43	92	27
165	72	179	214	175	66	191	224	81	18	239	34	91	30	233	44
212	181	70	163	210	177	80	161	48	225	32	95	46	235	28	93
71	164	211	180	65	162	209	192	17	96	47	240	29	94	45	234
202	13	126	61	208	15	128	49	160	241	130	97	148	243	132	103
125	60	203	14	127	64	193	16	129	112	145	242	131	102	149	244
12	201	62	123	2	207	5	113	256	159	98	143	246	147	104	133
59	124	11	204	63	114	1	194	111	144	255	146	101	134	245	150
200	9	122	55	206	3	116	51	158	253	142	99	154	247	136	105
121	58	205	10	115	54	195	4	141	110	155	254	135	100	151	248
8	199	56	119	6	197	52	117	252	157	108	139	250	153	106	137
57	120	7	198	53	118	5	106	109	140	251	156	107	138	249	152

2056

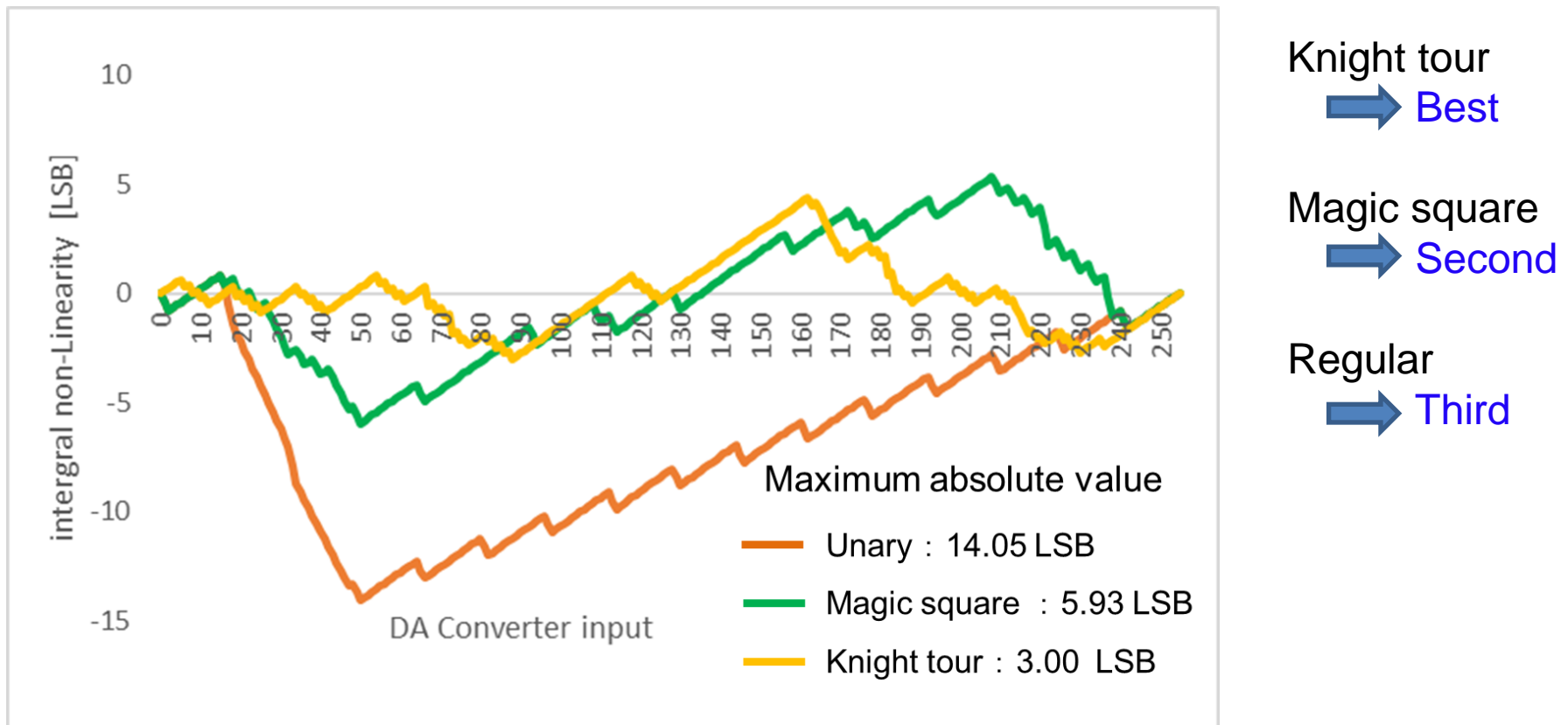
2056

Constant Sum: Row, Column, Diagonal = 2056

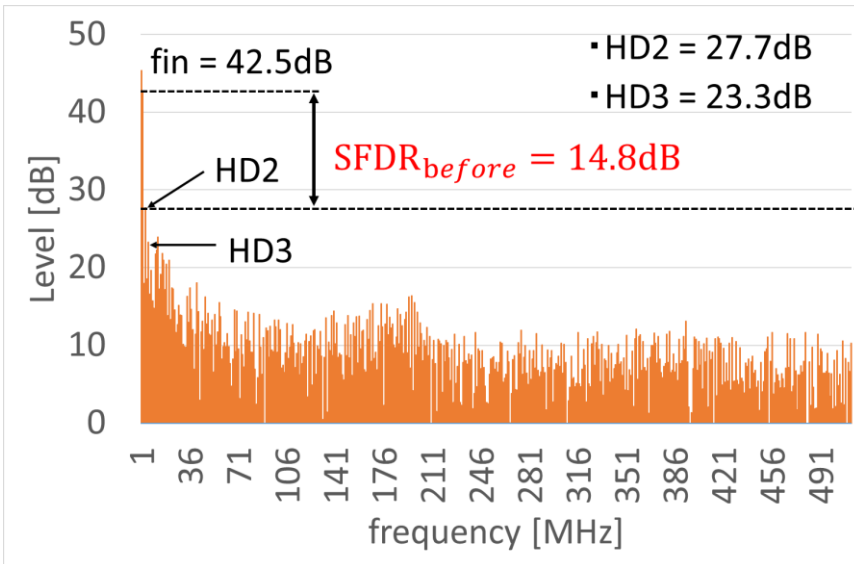
2056

DAC Simulation Results (1)

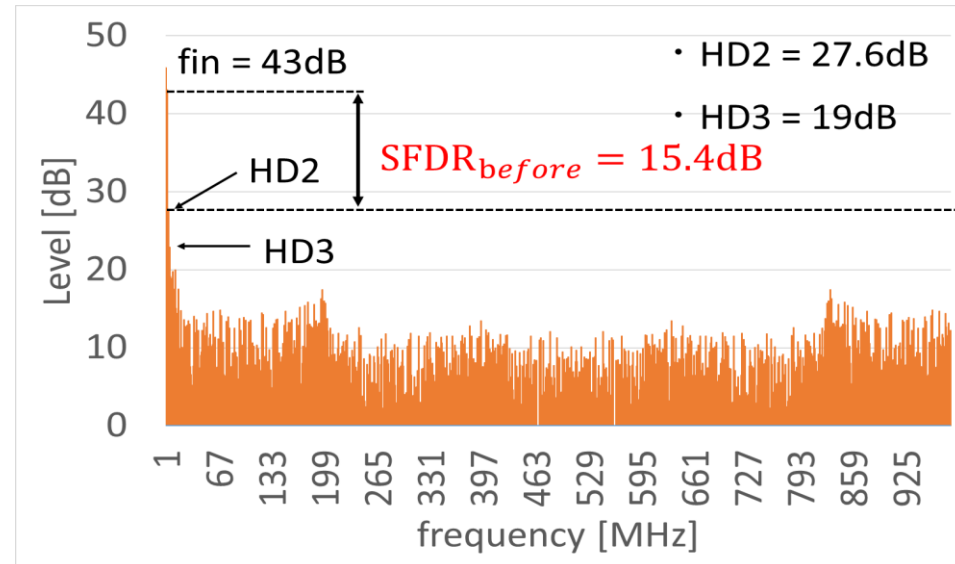
- Randomize error e by magic square / knight tour layout.
- Linear gradient e case
- DAC integral non-linearity (INL)



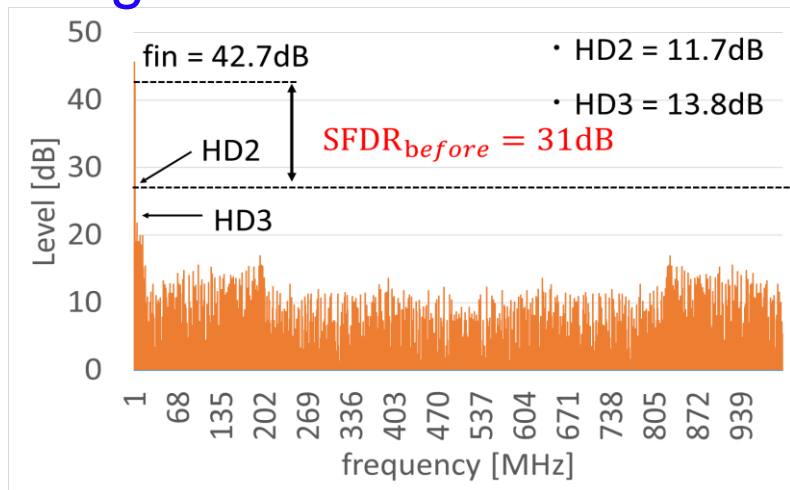
DAC Simulation Results (2)



Regular



Magic square



Euler's Knight Tour

SFDR: Spurious Free Dynamic Range

Regular: 14.8 dB

⇒ Third

Magic square: 15.4 dB

⇒ Second

Knight tour : 31.0 dB

⇒ Best

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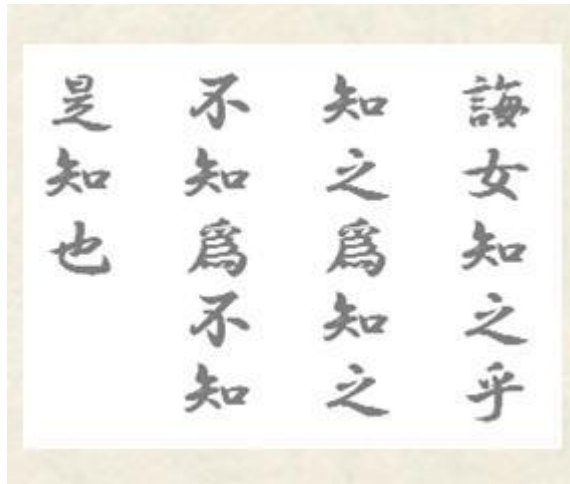
Conclusion

- Unary DAC linearity improvement
 - Unit current cell systematic mismatch effects cancellation
 - Unit current cell layout algorithm based on Magic Square and Euler's Knight Tour
- DAC Simulation validation
 - INL improvement
 - SFDR improvement



ご清聴ありがとうございました

Thank you for listening



謝謝

Q&A

1. From the result, Euler's Knight Tour is better than magic square.
What caused this result ?

Euler's Knight Tour has both its own properties and magic square mathematical properties that the sum of the row column and diagonal is the same
This is good balance, so it is better than magic square

2. It can be seen from the above that Euler's Knight Tour is not only a path.
How did you choose ?

If it is just Knight Tour , there are actually many paths,
but the path with the characteristics of Euler's Knight Tour and
magic square is 8bit, and I found this one path.