Input Signal and Sampling Frequencies Requirements for Efficient ADC Testing with Histogram Method

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Outline

- Objective
- ADC Test with Histogram Method
- Input Sine Wave and Sampling Frequencies Relationship in ADC Histogram Test Method
  - Sine Wave Histogram and Waveform Missing
  - Golden Ratio Sampling
  - Metallic Ratio Sampling
  - Prime Number Ratio Sampling
- Conclusion
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• Conclusion
Background

IoT era is coming!

ADC is a key component

◆ Analog signal

Sampling

◆ Digital signal

A/D Conversion

High quality & Low cost ADC test is required
SAR ADC linearity test takes a long time
- low-speed sampling
- high-resolution

Test cost is proportional to test time

This Work

ADC linearity test with histogram method:
Investigation of “high efficiency relationship” between input and sampling frequencies
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Histogram method (Ramp wave input)

- ADC output histograms for all bins are equal if ADC is perfectly linear
- Highly linear ramp signal generation is difficult (limitation up to 14-bit ADC)
Conventional Linearity Testing 2

Histogram method (Single sine wave input)

- Low distortion sine using an analog filter
- Number of samples is small around the middle of output range ➔ Many samples required (long test time)
DNL & INL

- Important ADC testing items

DNL : Difference between actual step width and ideal value

INL : Deviation from ideal conversion line

\[ INL(k) = \sum_{i=1}^{k} DNL(i) \]
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Sine Wave Histogram

- Repetitive waveform sampled asynchronously
- Reconstruct a 1-period waveform
- Sampled histogram is compared with PDF.
- Histogram is obtained.
- DNL, INL are calculated.

PDF:
Probability Distribution Function

\[
p(v) = \frac{1}{\pi \sqrt{A^2 - v^2}}
\]

Number of Samples

Output Code
Waveform Missing

A large amount of data is required to reconstruct the waveform

Test time: long

Repetitive waveform sampled asynchronously

Reconstruct a 1-period waveform

Measured waveform

Sampling CLK

Waveform missing occurs at special ratio \( T_{CLK} / T_{SIG} \)
\[ f_{CLK} \gg f_{sig} \]
\[ f_{CLK} \approx \frac{1}{\alpha} f_{sig} \]
\[ \alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \ldots, \frac{1}{6}, \ldots \]

Special ratio \( T_{CLK} \) and \( T_{sig} \), \( f_{CLK} \) and \( f_{sig} \)

\[ f_{CLK} \approx f_{sig} \]

Yuto Sasaki, Yujie Zhao, Anna Kuwana and Haruo Kobayashi, "Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System", 27th IEEE Asian Test Symposium, Hefei, Anhui, China (Oct. 2018)
Waveform Missing for Saw Signal

Normal situation

Waveform Missing
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Golden Ratio: \[ \lim_{n \to \infty} \frac{F_n}{F_{n-1}} = 1.618033988749895 = \varphi \]

The most beautiful ratio
Golden Ratio $\phi$

$$f_{CLK} = \phi \times f_{sig}$$

$\phi = 1.6180339887…$

Proposal of sampling conditions for the highest waveform acquisition efficiency

Yuto Sasaki, Yujie Zhao, Anna Kuwana and Haruo Kobayashi, "Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System", 27th IEEE Asian Test Symposium, Hefei, Anhui, China (Oct. 2018)
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## Metallic Ratio

**Golden Ratio:** \(\lim_{n \to \infty} \frac{F_n}{F_{n-1}} = 1.61803398874989.. = \phi\)

<table>
<thead>
<tr>
<th>n</th>
<th>Decimal</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>Golden Ratio</td>
</tr>
<tr>
<td>1</td>
<td>(\frac{1 + \sqrt{5}}{2}) = 1.6180339887…</td>
<td>Golden Ratio</td>
</tr>
<tr>
<td>2</td>
<td>1 + \sqrt{2} = 2.4142135623…</td>
<td>Silver Ratio</td>
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<tr>
<td>3</td>
<td>(\frac{3 + \sqrt{13}}{2}) = 3.3027756377…</td>
<td>Bronze Ratio</td>
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<tr>
<td>4</td>
<td>2 + \sqrt{5} = 4.2360679774…</td>
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<tr>
<td>(\ldots)</td>
<td>\ldots</td>
<td></td>
</tr>
<tr>
<td>(n)</td>
<td>(\frac{n + \sqrt{n^2 + 4}}{2})</td>
<td></td>
</tr>
</tbody>
</table>

**Generalization of Golden Ratio**
Histogram of Saw Signal

Total number of samples: $M$
ADC resolution: $N$.

ideal value $h_i(k) = \frac{M}{N}$, $k = 1, 2, 3, \ldots, N$

error $e(k) = \frac{N \cdot h(k)}{M} - 1$
RMS Error Calculation

Root mean square error between actual and ideal histograms

\[ RMS = \sqrt{\frac{\sum(e(k))^2}{N}} \]
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RMS of Prime Number Sampling

\[ f_{CLK} = f_{sig} \times RATIO \]

In contrast, Golden Ratio has a smaller RMS.

RATIO with two large prime numbers ⇒ Small RMS

Total number of samples: \( M = 65536 \)
RMS of Prime Number Sampling: **Big Number Case**

- **$f_{CLK}$**: Big prime number 997
- **$f_{SIG}$**: Some prime numbers 389, 599, 751, 853, 907, 953, 991

$\rightarrow$ almost the same

Total number of samples: $M=65536$
Increasing N to 16384,
Bronze ratio result is better (RMS range is smaller)
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Golden Ratio sampling
Efficiency: high
Sampling frequency: low

Metallic ratio sampling
Efficiency: high
Sampling frequency: high

Prime number ratio sampling
Efficiency: Not good
Sampling frequency: low

Next work

- Like the golden ratio
  Find conditions for efficient sampling
  at a specific location
  (ADC resolution N=256, 512, 1024, 2048, 4096)
Thanks for your attention.