ATS Doctoral Thesis Award

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Semi-Final of 2022 TTTC’s E. J. McCluskey Doctoral Thesis Award

Histogram Method for Efficient ADC Linearity Test: Input Signal for Code Selection and Ratio of Input and Sampling Frequencies

Student: Yujie Zhao

Supervisor: Prof. Haruo Kobayashi

Division of Electronics and Informatics,
Gunma University, Japan

群馬大学
GUNMA UNIVERSITY

Kobayashi
Laboratory

Outline

- **1. Research Background and Objective**
- **2. ADC Input Signal for Code Selective ADC Histogram Test Method**
  - ➢ **ADC Test with Histogram Method and Waveform Missing**
  - ➢ **Combine Multiple Sine Waves**
  - ➢ **General Two-Tone Input Signal**
- **3. Ratio of Input and Sampling Frequencies for Efficient ADC Histogram Test Method**
  - ➢ **Golden Ratio Sampling**
  - ➢ **Metallic Ratio Sampling**
  - ➢ **Prime Number Ratio Sampling**
- **4. Conclusion**

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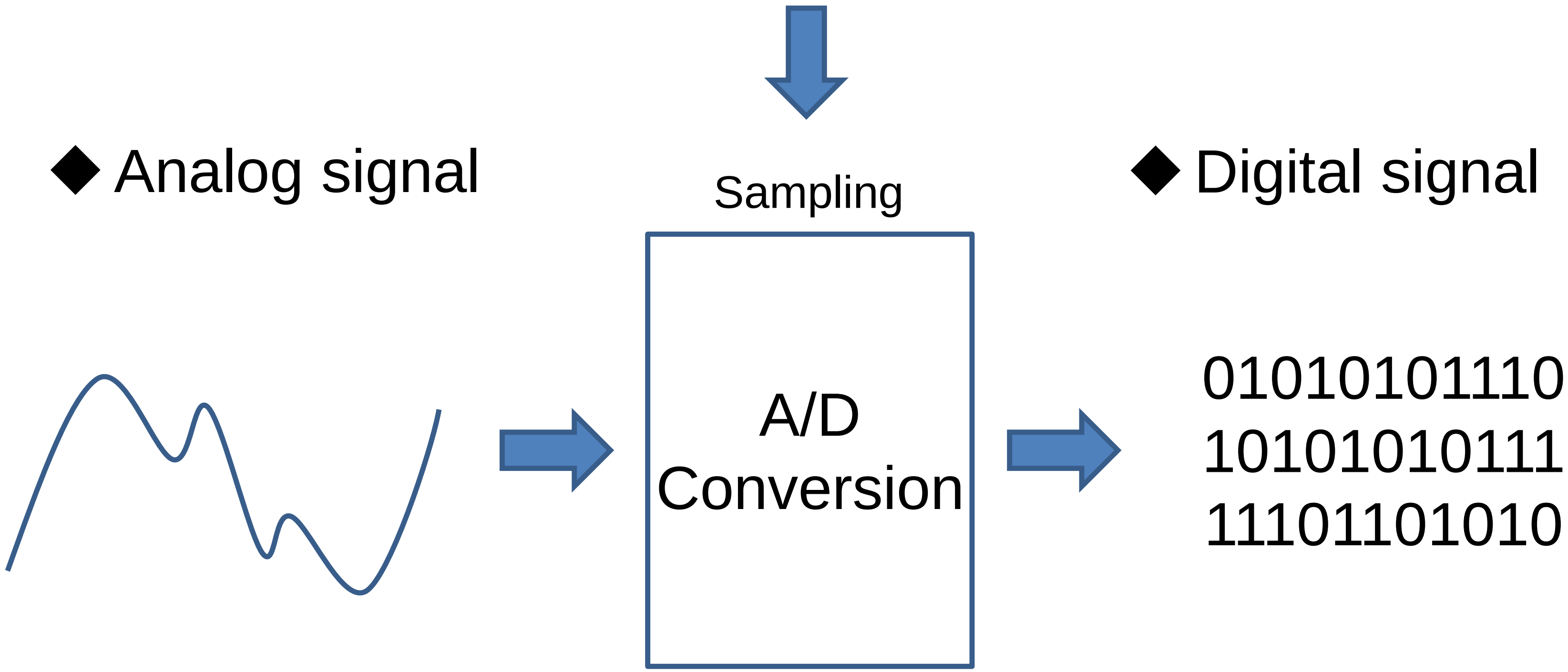
Background

IoT era is coming !

ADC is a key component



High quality & low cost ADC test is required

Research Objective

SAR ADC linearity test takes a long time

- low-speed sampling
- high-resolution

Test cost → Proportional to test time

Development of
efficient ADC testing method with histogram

Our Approach

ADC linearity test with histogram method

➡ Test time reduction

- Code selective histogram method

  Two-tone input signal:

  Output code concentration on codes where ADC nonlinearity likely occur

- Ratio of input and sampling frequencies of ramp input for better test accuracy

  Metallic ratio, prime number ratio

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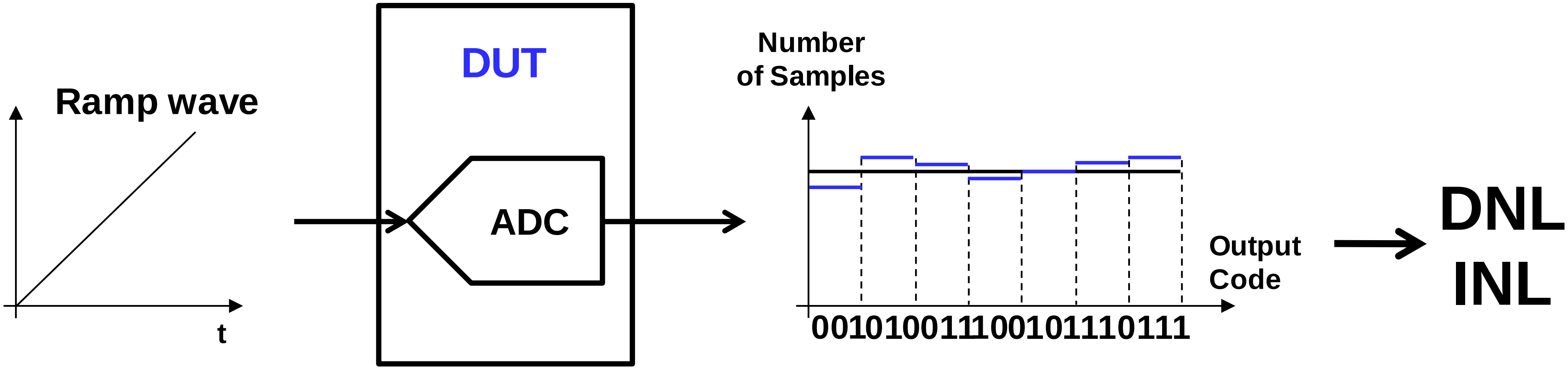
Conventional Linearity Testing 1

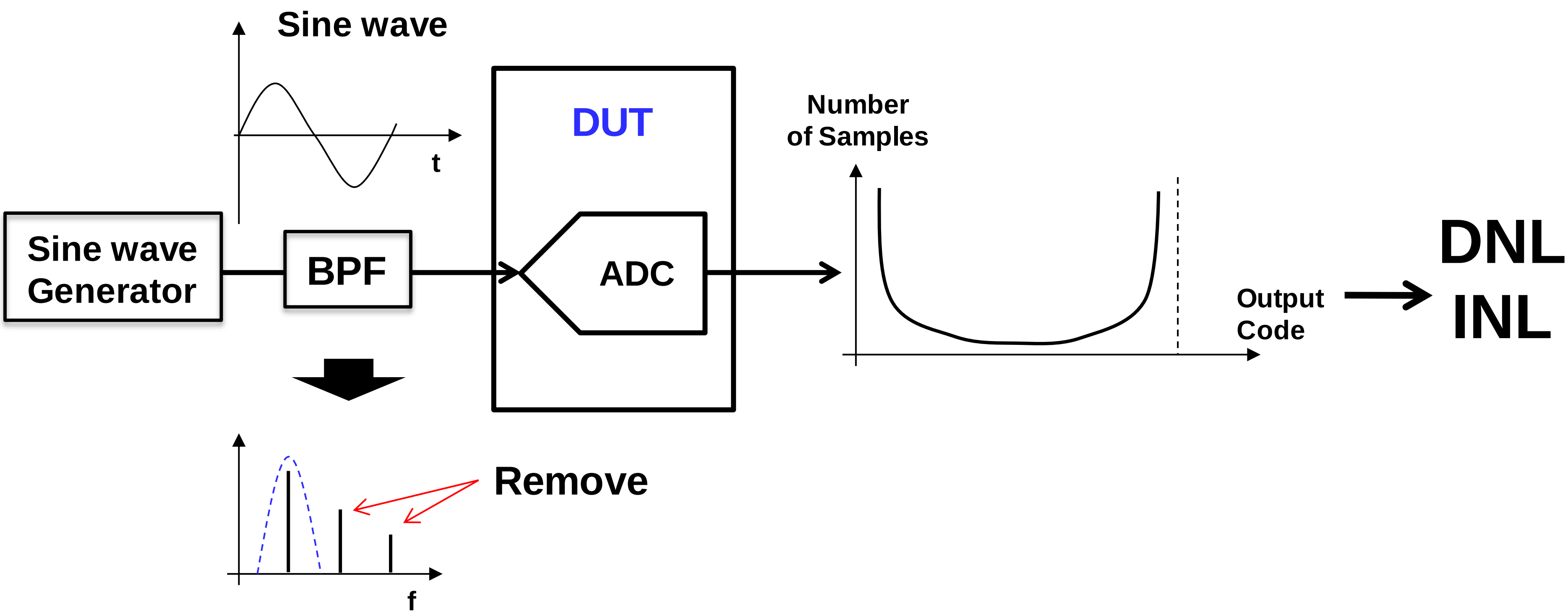
■Histogram method (Ramp wave input)



- if ADC is perfectly linear ➡
  "ADC output histograms for all bins ➡Equal"
- Highly linear ramp signal generation ➡Difficult
  (limitation up to 14-bit ADC)

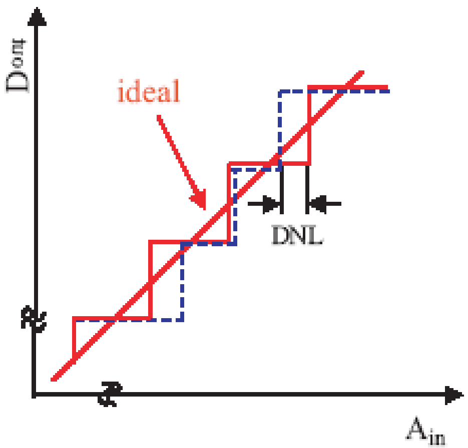
Conventional Linearity Testing 2

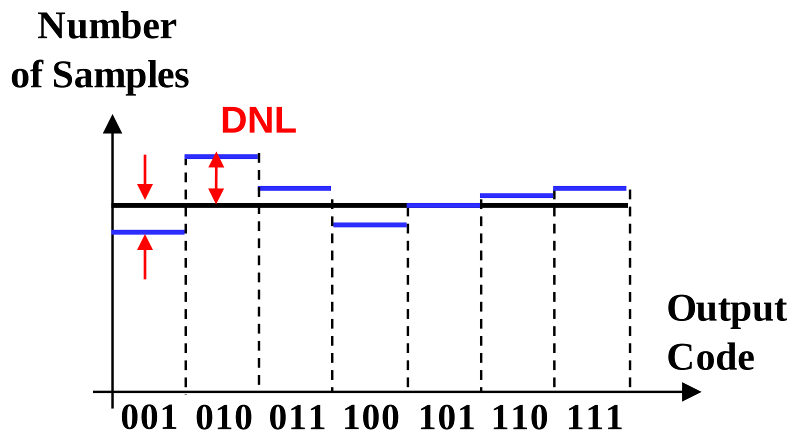
■Histogram method(Single sine wave input)

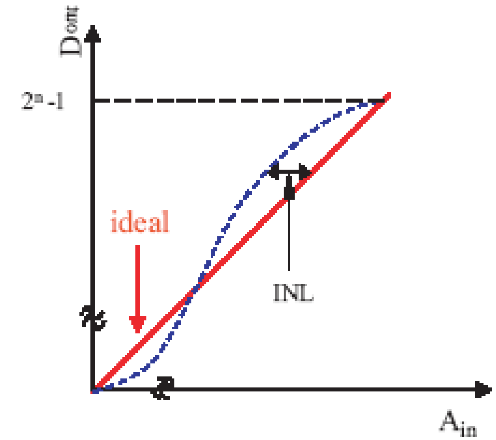


- Low distortion sine using an analog filter
- Number of samples is small around the middle of output range ➡ Many samples required (long test time)

DNL & INL







- Important ADC testing items

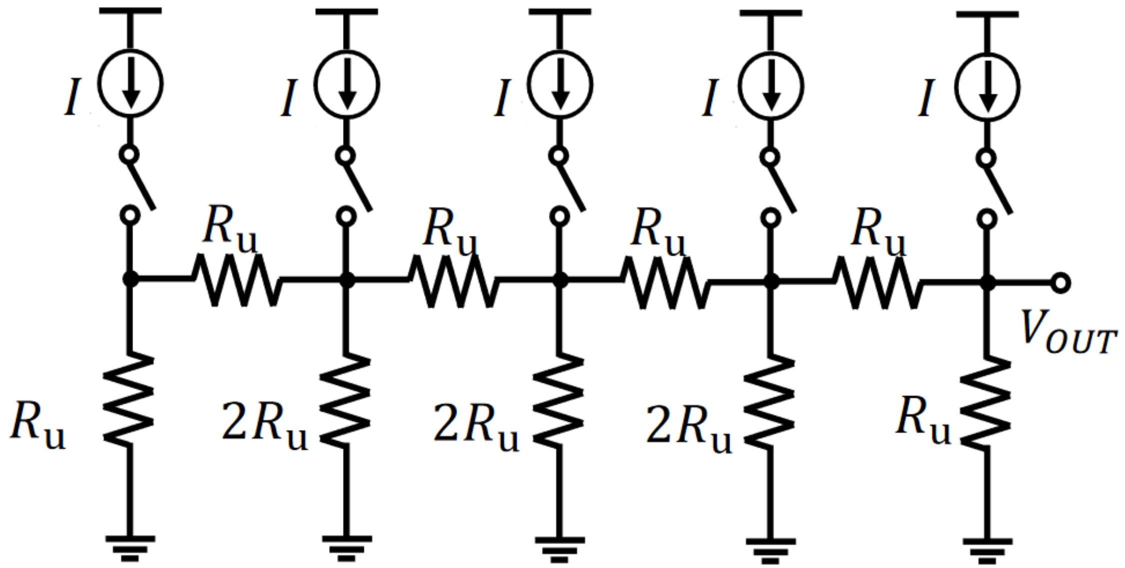
DNL : Difference between actual step width and ideal value

INL : Deviation from ideal conversion line

$$INL(k) = \sum_{i=1}^{k} DNL(i)$$

DAC Inside ADC

DAC linearity inside ADC ➡ Entire ADC linearity



R-2R ladder network DAC.

Target SAR ADC under test ➡ Binary-weighted DAC inside.

In 10-bit case, large DNL ➡
At digital codes of 512, 256, 768, 128, 384, 640, 896, …

ADC Codes Prone to Non-linearity

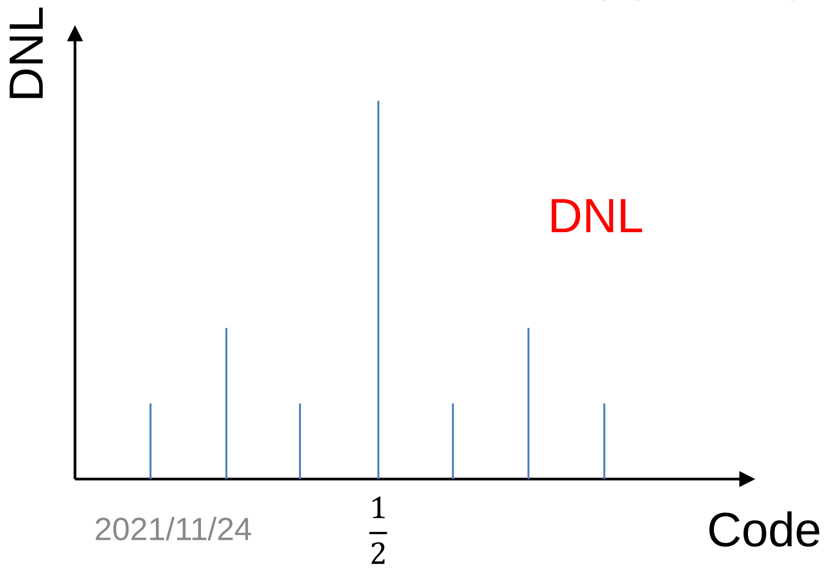
Increase the number of histogram points in these codes.
→ Measure DNL with high accuracy.

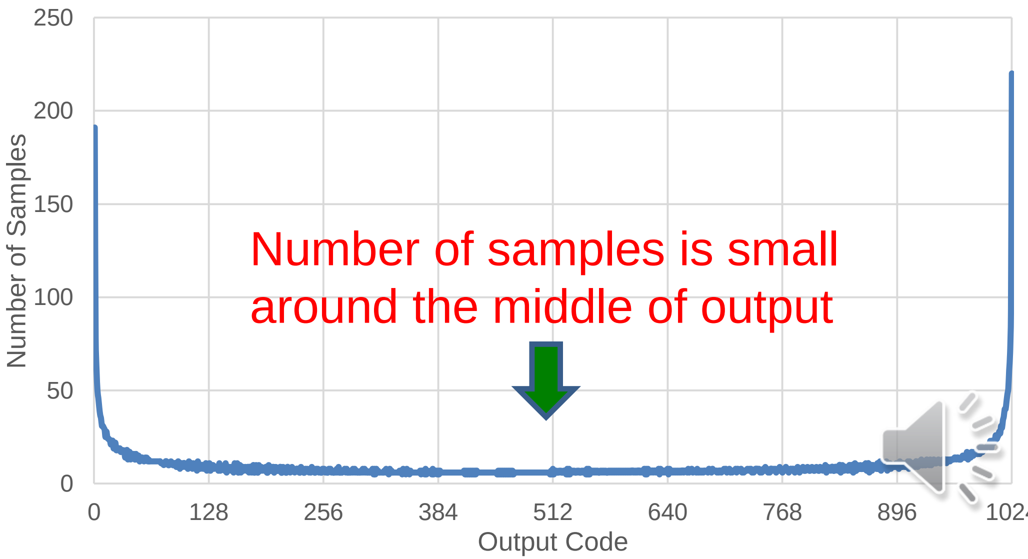
All codes of ADC digital output during testing.

Change

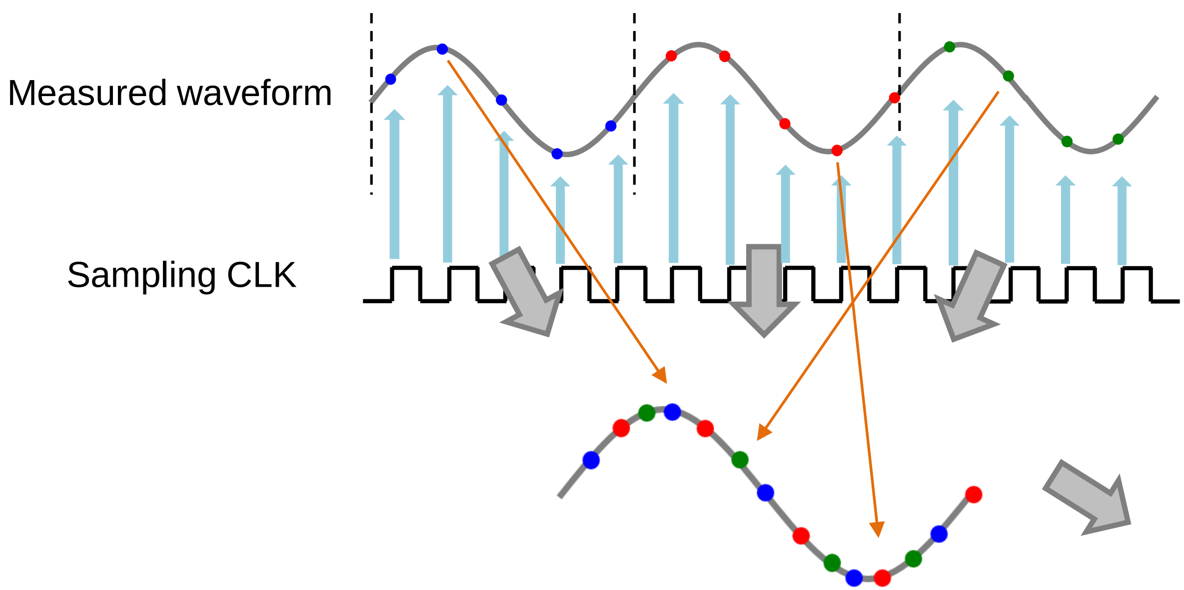
Focus on specific codes prone to non-linearity.

Test time can be shortened.





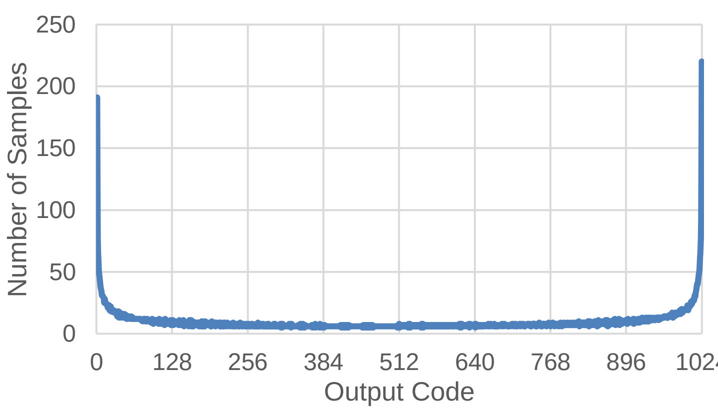
Sine Wave Histogram



- Sampled histogram is compared with PDF.
- Histogram is obtained.
- DNL , INL are calculated.

PDF:
Probability Distribution Function

$$p_{(v)} = \frac{1}{\pi\sqrt{A^2 - v^2}}$$



Repetitive waveform sampled asynchronously

⇩

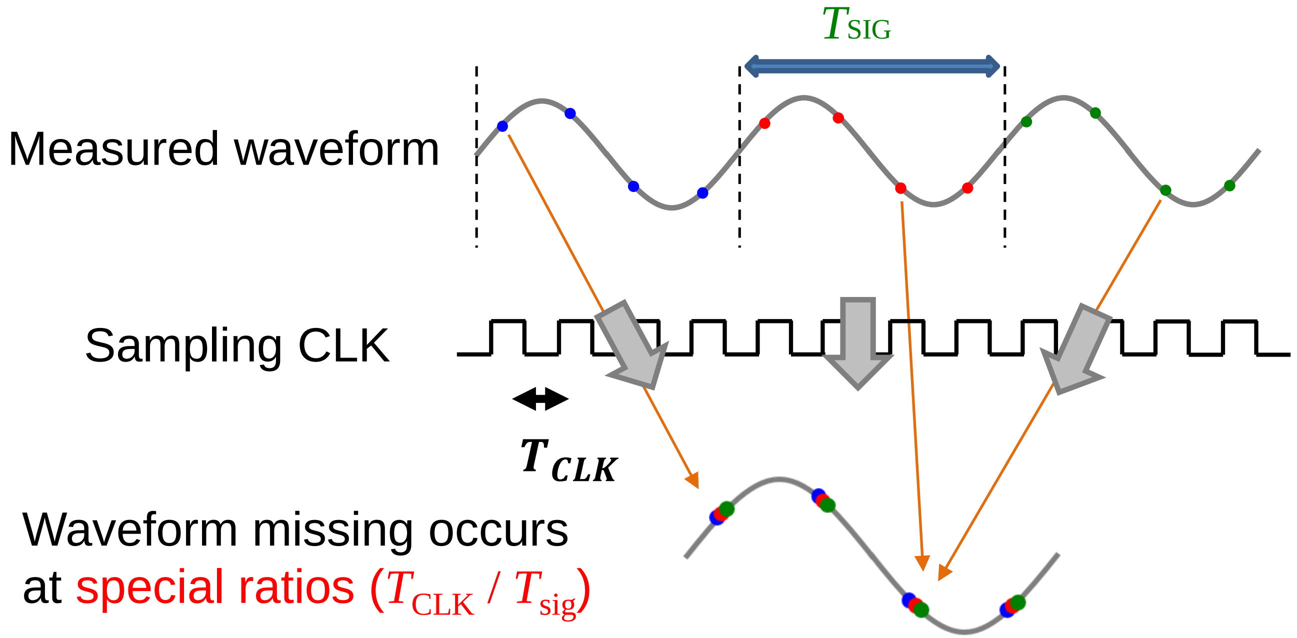
Reconstruct a 1-period waveform

Waveform Missing Phenomena

Repetitive waveform sampled asynchronously

⇩

Reconstruct a 1-period waveform

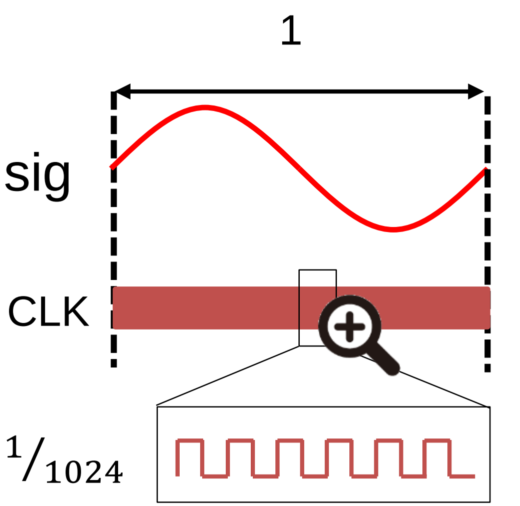


Waveform missing occurs
at special ratios (T_{CLK} / T_{sig})

A large amount of data are required to reconstruct the waveform ➡ Test time: long

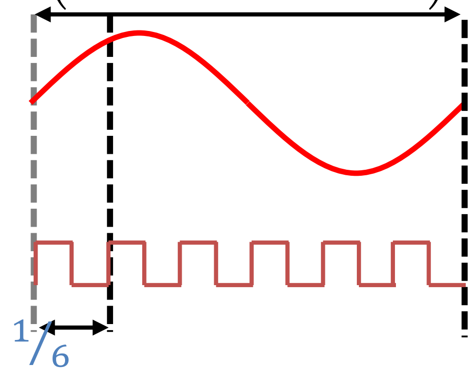
Waveform Missing for Sine Signal

$f_{CLK} \gg f_{sig}$



$f_{CLK} \approx \frac{1}{\alpha} f_{sig}$

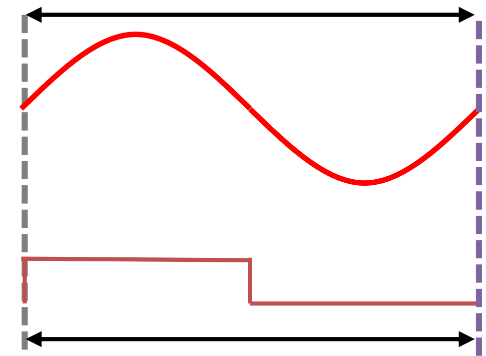
$\left(\alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \cdots, \frac{1}{6}, \cdots\right)$

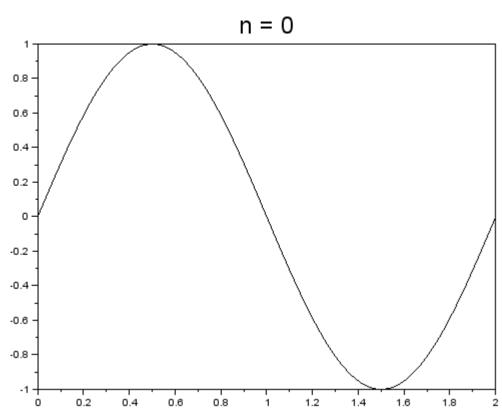


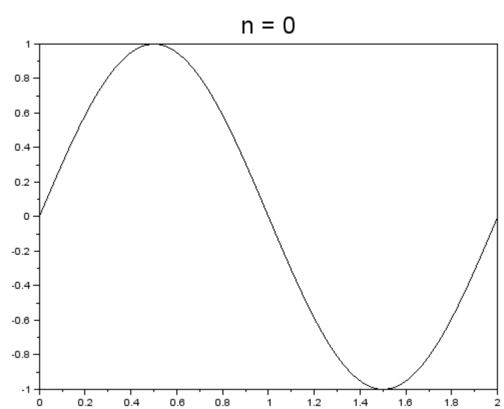
Special ratios

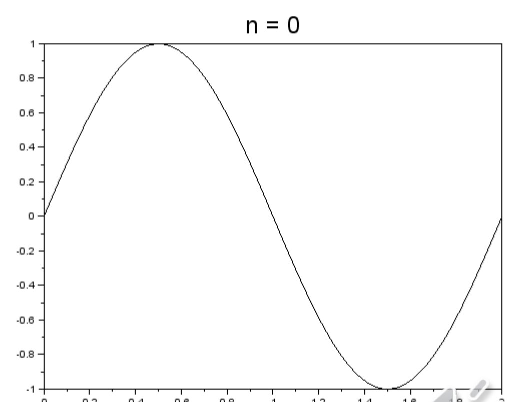
T_{CLK} and T_{sig} , f_{CLK} and f_{sig}

$f_{CLK} \approx f_{sig}$



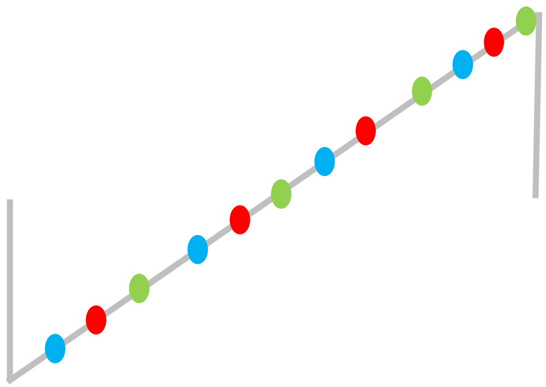


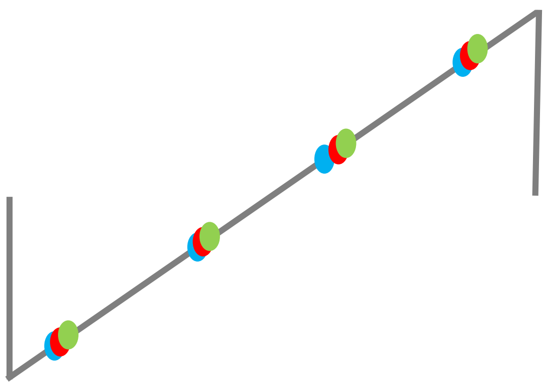




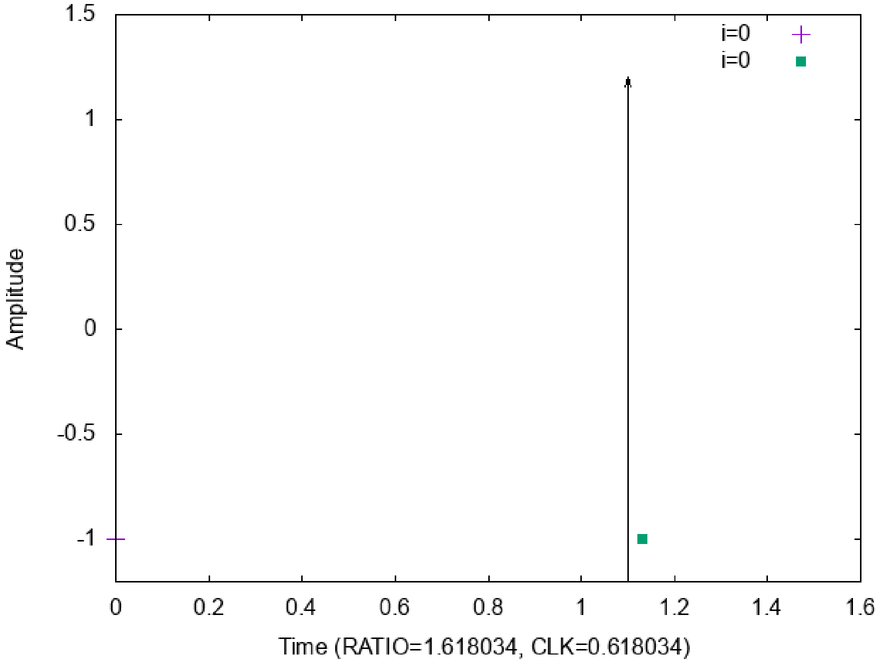
Yuto Sasaki, Yujie Zhao, Anna Kuwana Haruo Kobayashi, "Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System", 27th IEEE Asian Test Symposium, Hefei, Anhui, China (Oct. 2018)

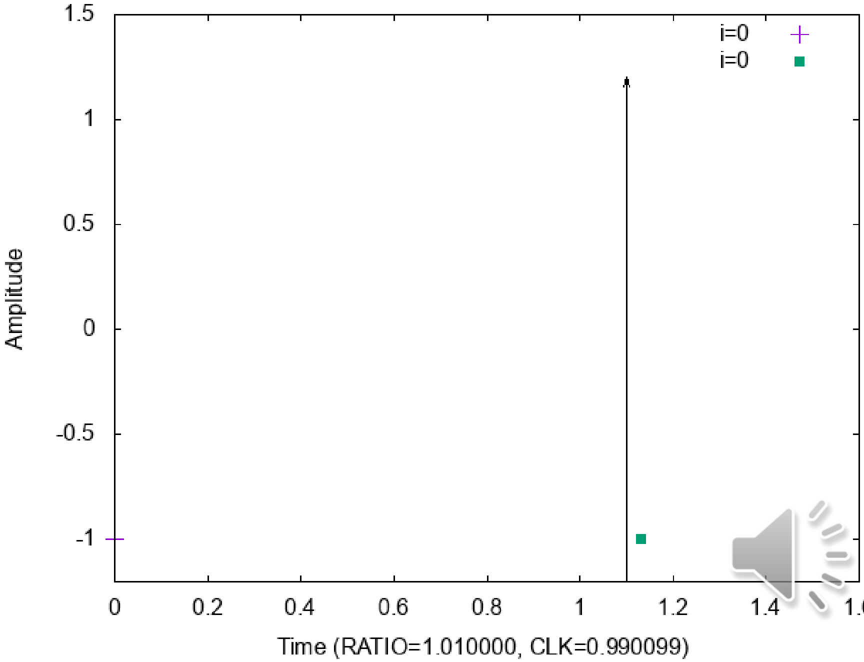
Waveform Missing for Saw Signal





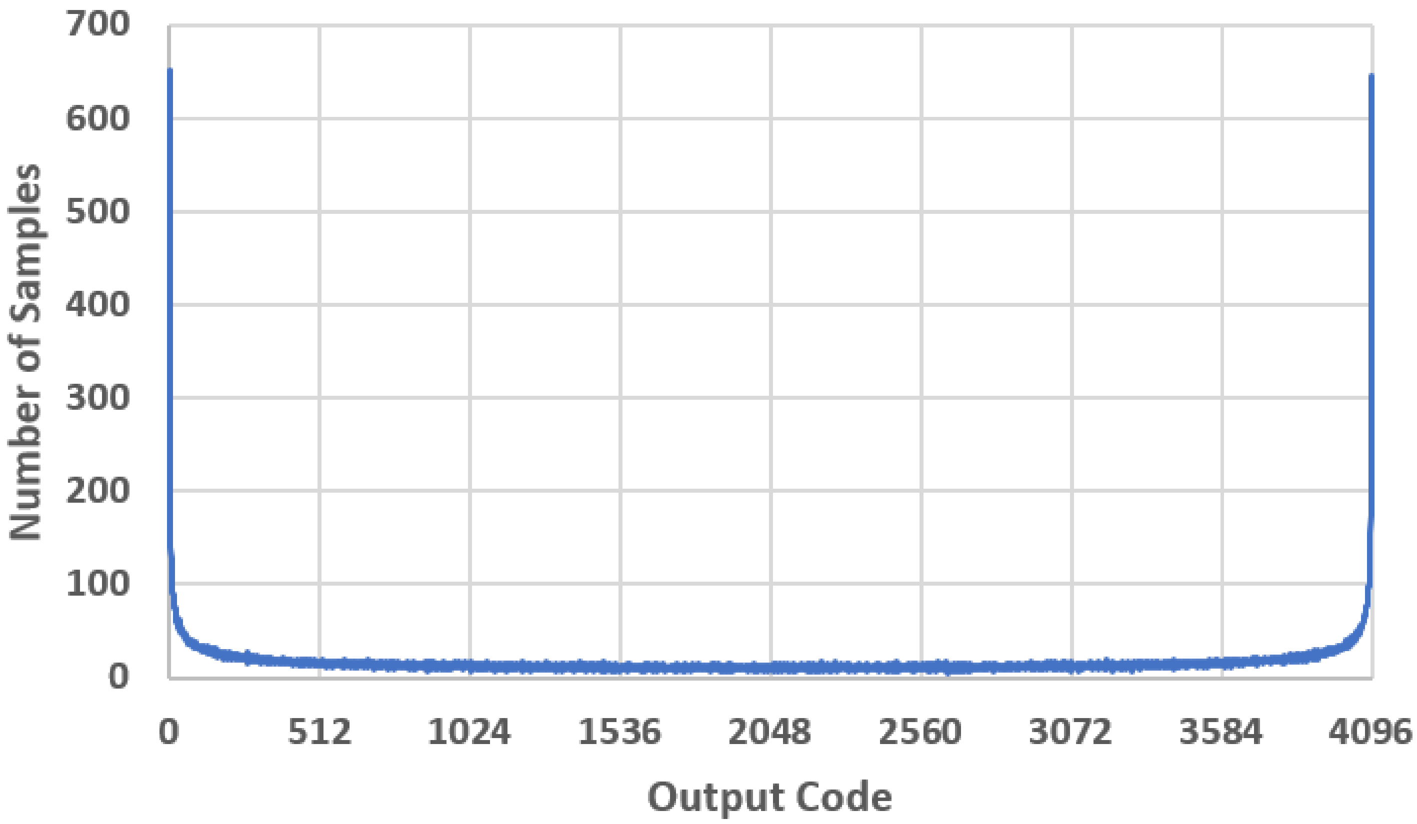
Normal situation	Waveform Missing





Use Waveform Missing

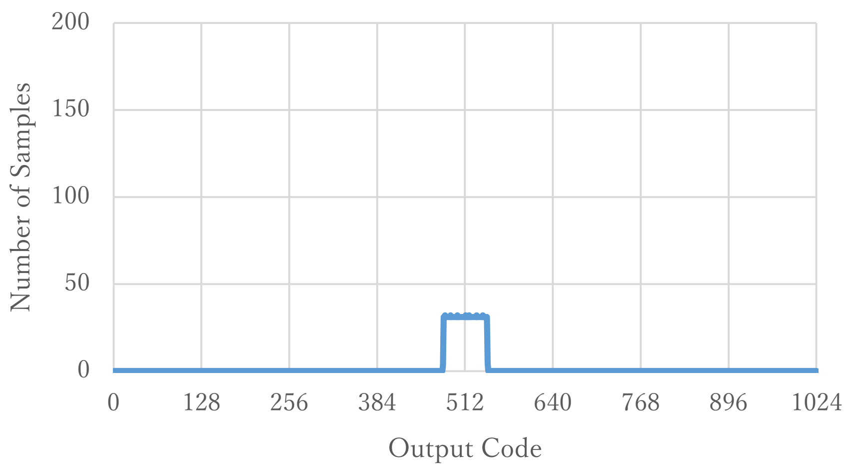
Using waveform missing → Focus on specific codes

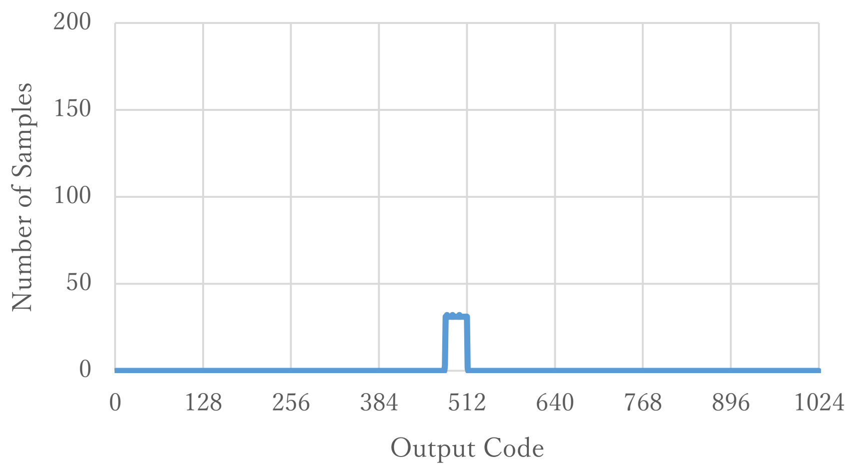


Sine Wave Histogram

Examples of Using Waveform Missing

Histogram can be focused on specific codes:



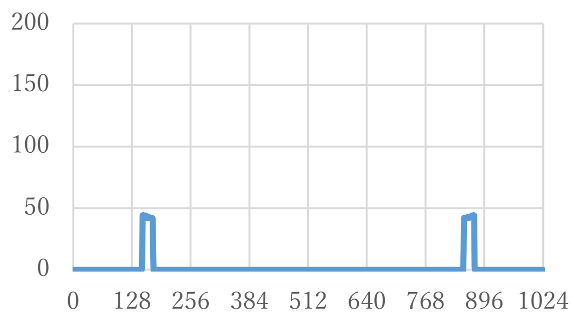
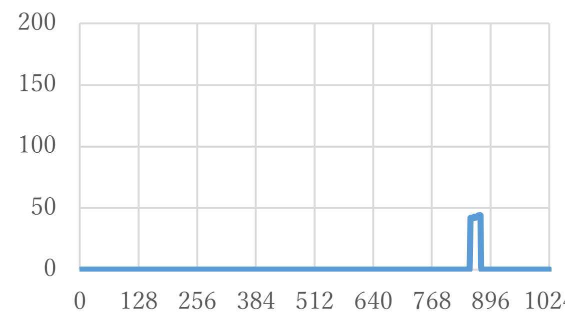
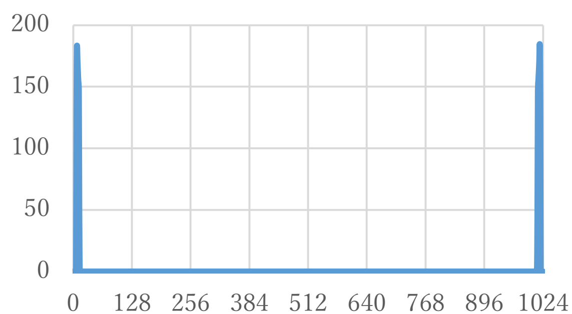
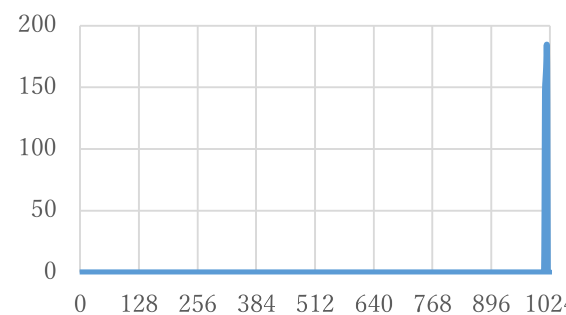


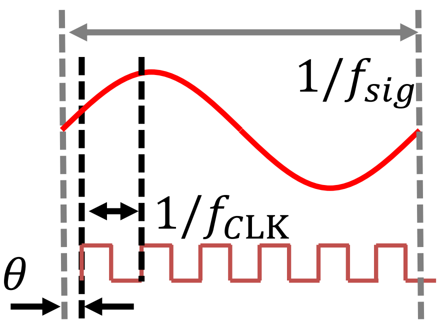
$$f_{\mathrm{CLK}} = f_{sig} \times \frac{2 \times (1 + 100000)}{100000}$$

$$f_{\mathrm{CLK}} = f_{sig} \times \frac{1 + 100000}{100000}$$

 Relationship of f_{CLK}、f_{sig}、θ(Initial phase of input sine wave)

Problem of Using Waveform Missing

	$f_{\mathrm{CLK}} = f_{sig} \times \frac{2 \times (1 + 100000)}{100000}$	$f_{\mathrm{CLK}} = f_{sig} \times \frac{1 + 100000}{100000}$
$\theta = \pi/2 \times 0.5$		
$\theta = \pi/2 \times 0.9$		



$f_{\mathrm{CLK}} = f_{sig} \times$ ● → Concentration
θ → Position to concentrate

→Can be adjusted

For implementation, free adjust θ ➡ difficult

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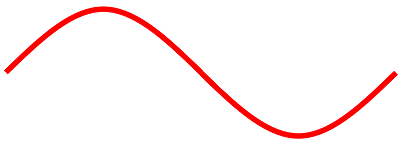
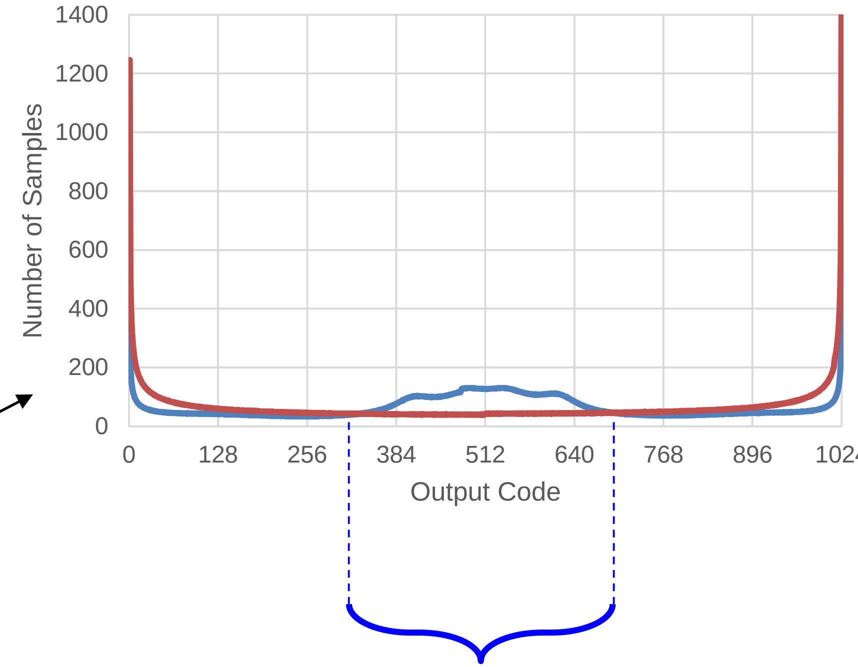
Our Previous research

Single sine wave (red)



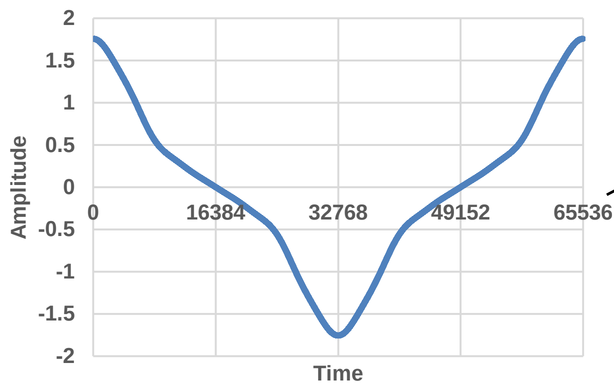
$f(t) = \sin \omega t$

histogram



Histogram focused on a specific code

Two-tone signal (blue)



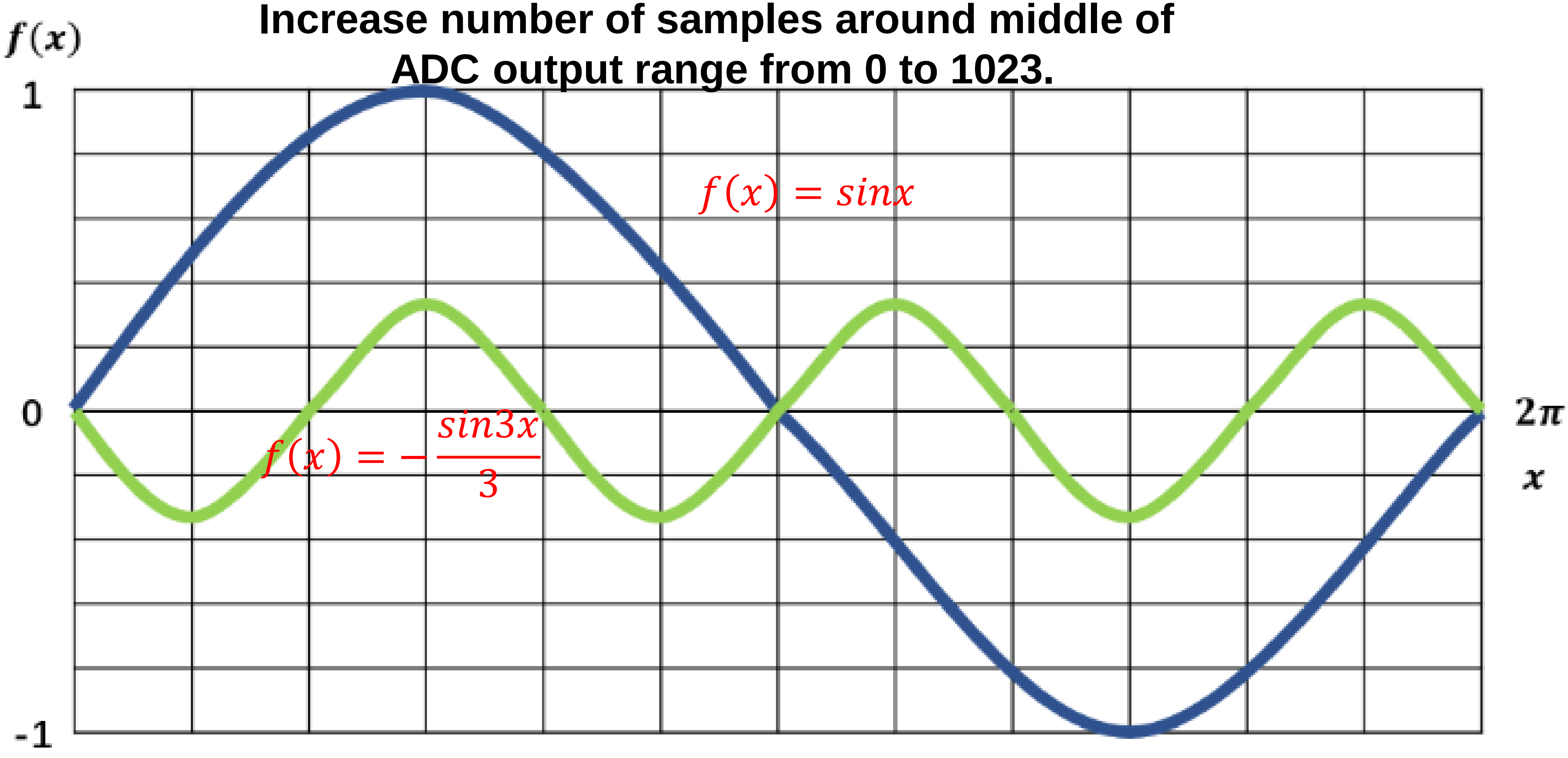
$$f(t) = A(W_1 + 2.6 \cdot W_2 + 1.8 \cdot W_3 + 1.4 \cdot W_6 + 1.2 \cdot W_7) + V_{OS}$$

$$W_m = \frac{\cos((2m-1)\omega t)}{(2m-1)^2} \qquad A = 2.90[V] \qquad V_{OS} = 4.0[V]$$

S. Uemori, T. Yamaguchi, S. Ito, Y. Tan, H. Kobayashi, N. Takai, K. Niitsu, N. Ishikawa,
"ADC linearity test signal generation algorithm", 2010 IEEE Asia Pacific Circuits and Systems Conference (APCCAS).

Multiple Sine Waves Combination

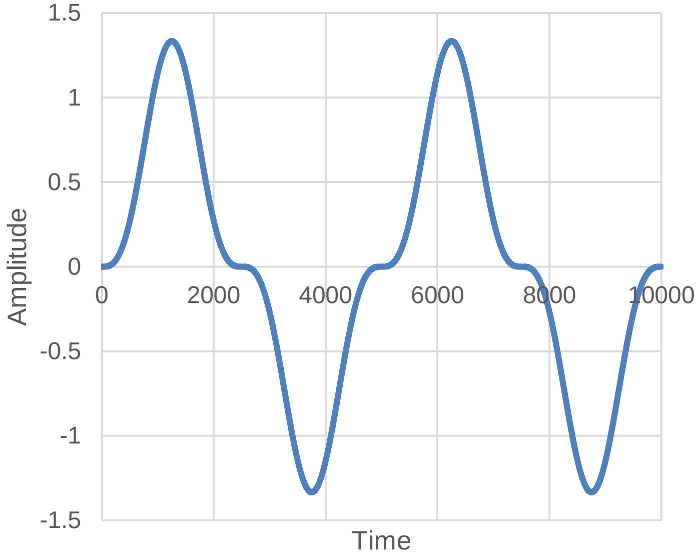
Increase number of samples around middle of ADC output range from 0 to 1023.



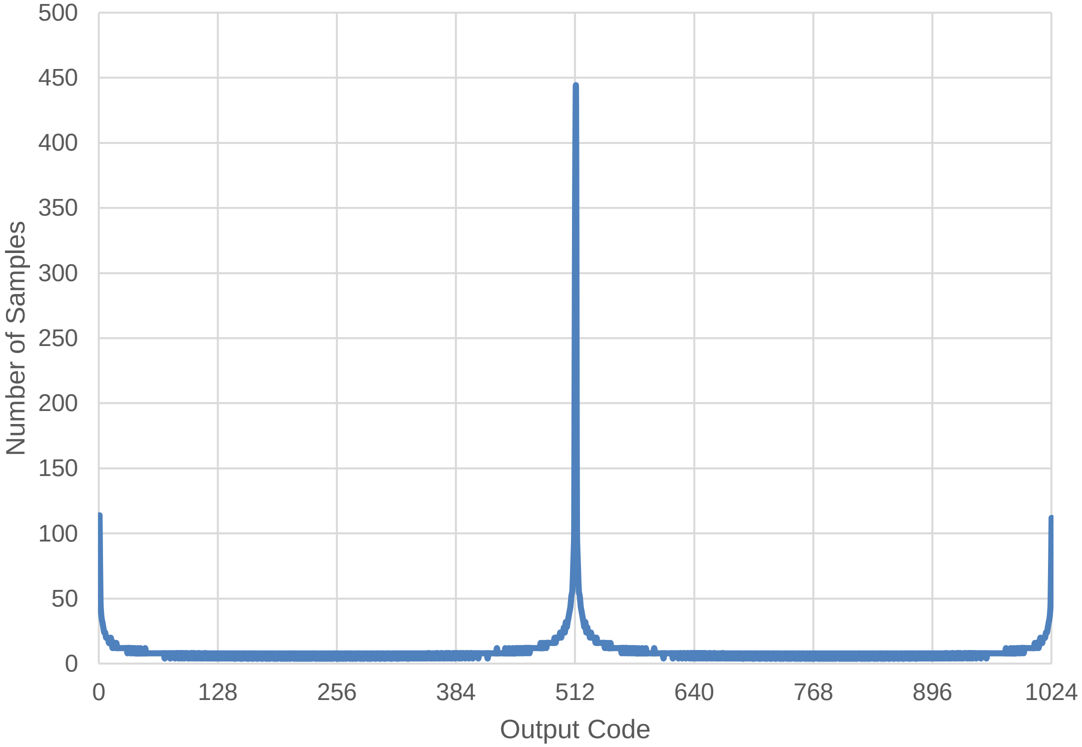
Smaller slope of input signal → More samples in histogram

Input signal slope reductionat target amplitude positions by combining sine waves.

Result of Multiple Sine Waves

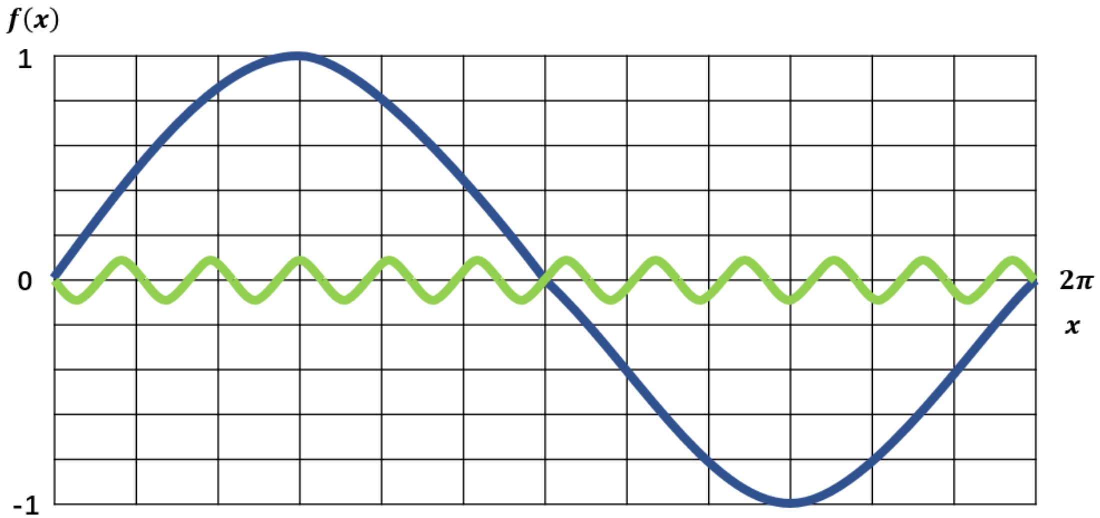


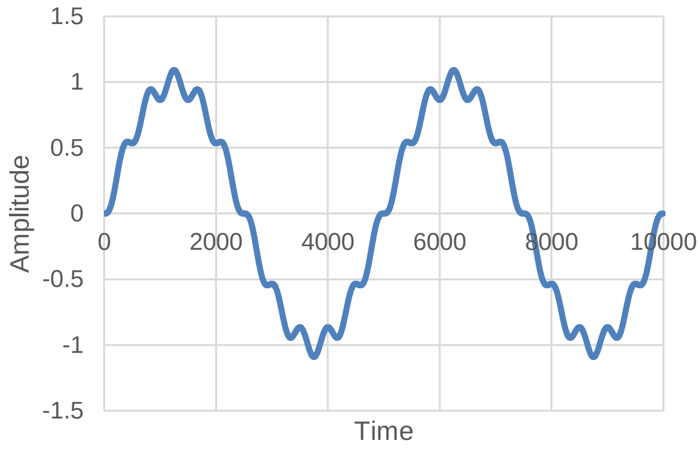
$$f(x) = sinx - \frac{sin3x}{3}$$



Number of samples around the middle (digital output 512) is increased.

Result of Other Codes



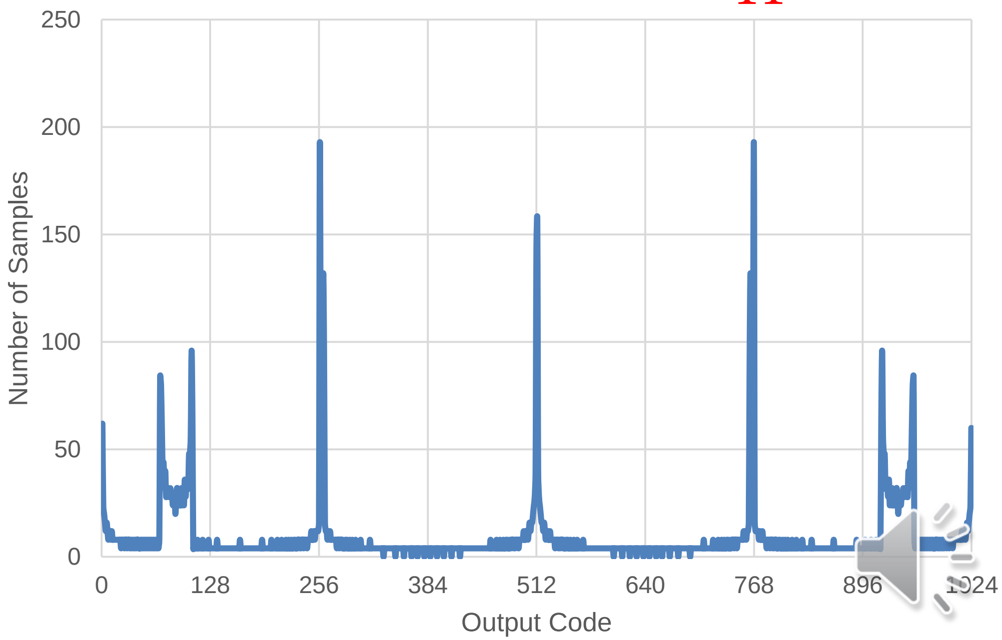


$$f(x) = \sin x - \frac{\sin 11x}{11}$$

Consider to increase 256 and 768 in output range from 0 to 1023.

$\arcsin\frac{1}{2} = \frac{\pi}{6}$ ($\frac{1}{12}$ of period 2π)

Try to use 11.

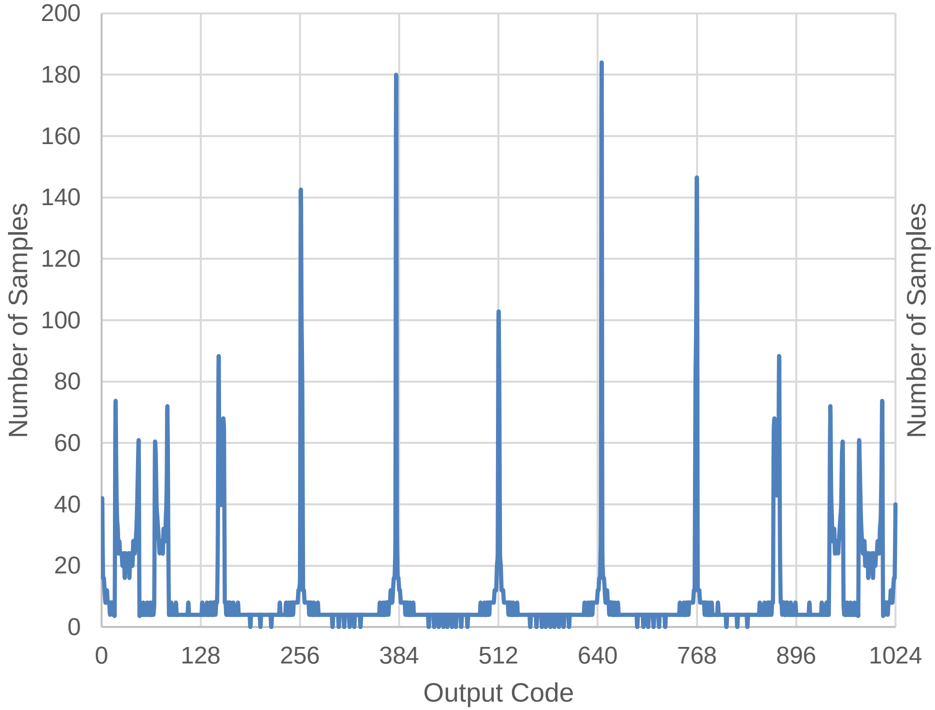


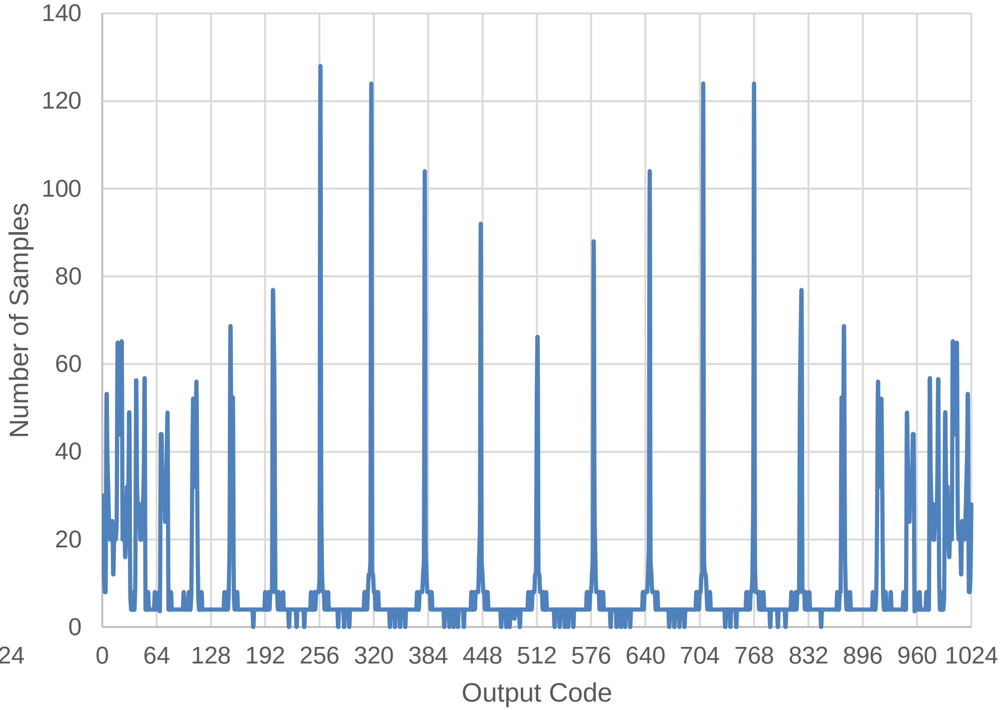
Results of More Codes

In the same way

$$f(x) = \sin x - \frac{\sin 23x}{23}$$

$$f(x) = \sin x - \frac{\sin 49x}{49}$$





Numbers of samples around the target digital output codes ➡ Increase.

$$f(x) = \sin(\omega_1 x) - \frac{\sin(\omega_2 x)}{k}$$

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Two-Tone Input Signal Configuration

$$f(x) = sin(\omega_1 x) - \frac{sin(\omega_2 x)}{k}$$

General two-tone input signal for ADC linearity histogram test

Simulating the following waveforms and obtained ADC histograms.

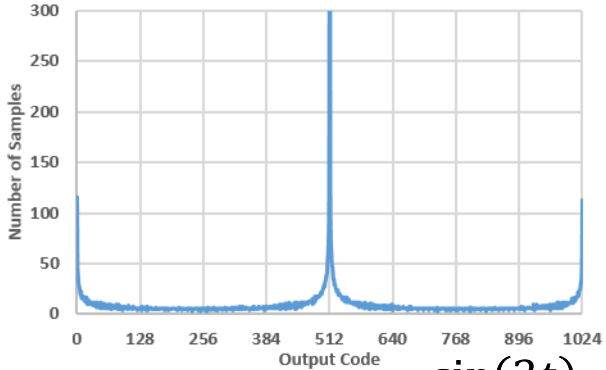
$$g_{3,3}(t) = \sin(t) - \frac{\sin(3t)}{3} \qquad g_{11,11}(t) = \sin(t) - \frac{\sin(11t)}{11}$$

$$g_{3,3.5}(t) = \sin(t) - \frac{\sin(3t)}{3.5} \qquad g_{11,11.5}(t) = \sin(t) - \frac{\sin(11t)}{11.5}$$

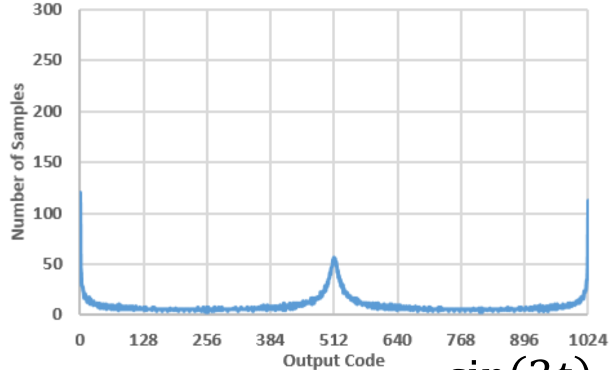
$$g_{3,4}(t) = \sin(t) - \frac{\sin(3t)}{4} \qquad g_{11,12}(t) = \sin(t) - \frac{\sin(11t)}{12}$$

Using AWG (Arbitrary Waveform Generator) for signal generation

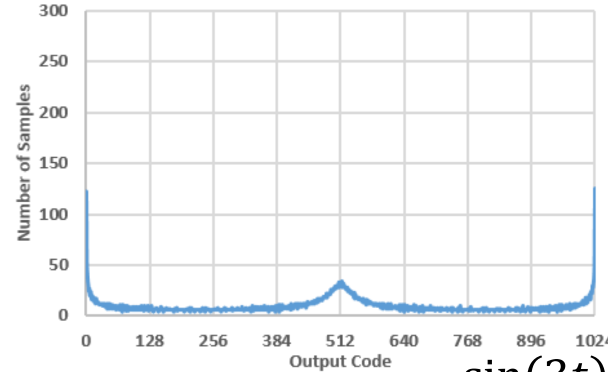
Results of Simple Configuration



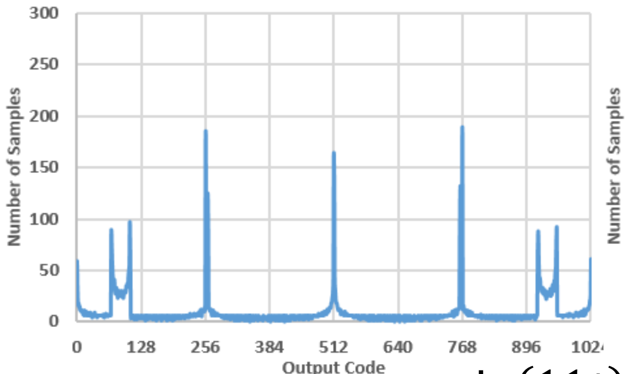
$$g_{3,3}(t) = \sin(t) - \frac{\sin(3t)}{3}$$



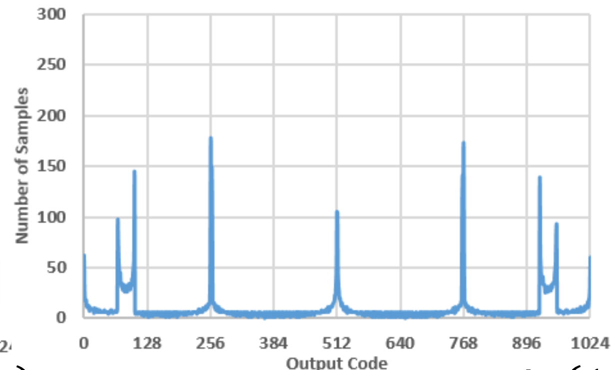
$$g_{3,3.5}(t) = \sin(t) - \frac{\sin(3t)}{3.5}$$



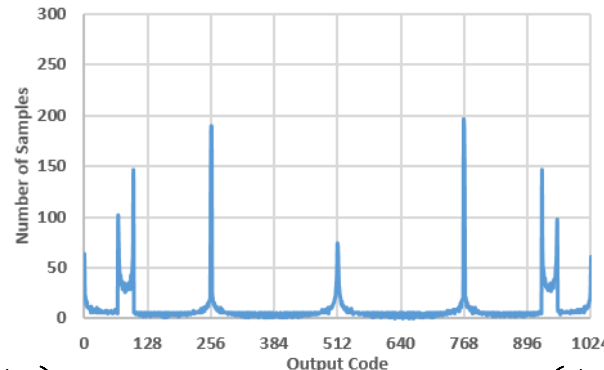
$$g_{3,4}(t) = \sin(t) - \frac{\sin(3t)}{4}$$



$$g_{11,11}(t) = \sin(t) - \frac{\sin(11t)}{11}$$



$$g_{11,11.5}(t) = \sin(t) - \frac{\sin(11t)}{11.5}$$

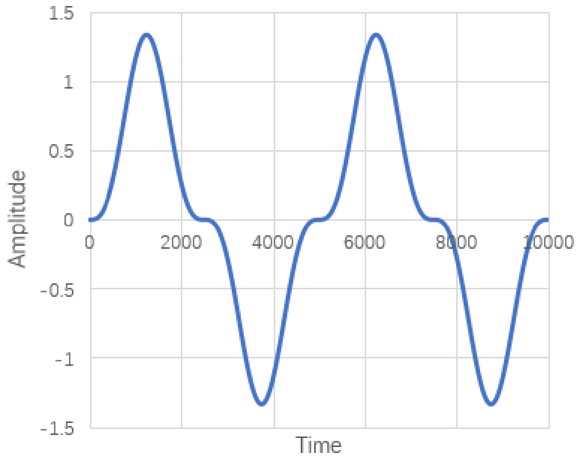


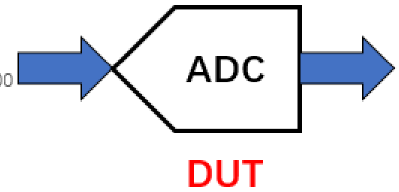
$$g_{11,12}(t) = \sin(t) - \frac{\sin(11t)}{12}$$

$$f(x) = sin(\omega_1 x) - \frac{sin(\omega_2 x)}{k}$$

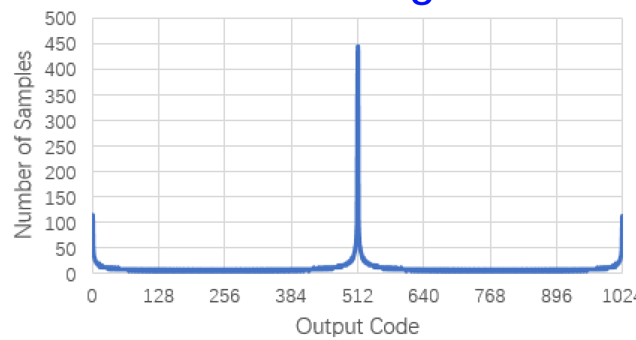
k increases → Histogram peaks spread.

Problem of Two-Tone Input Signal



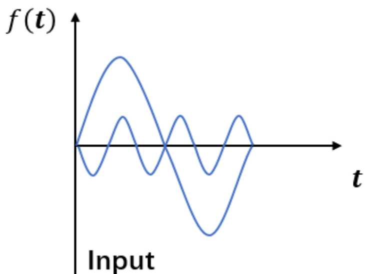


Actual histogram



DNL
INL

PDF is required to calculate the ideal histogram for DNL.



Proposed signal → No inverse function

Theoretical derivation of probability density function (PDF) → difficult

Histogram method requires explicit PDF.

Alternatively, PDF obtained by simulation can be used.

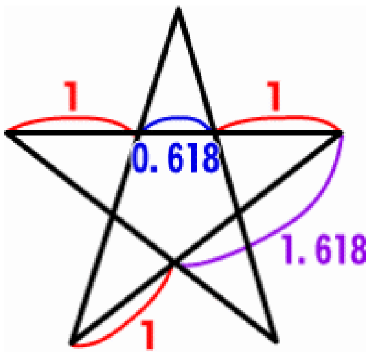
Outline

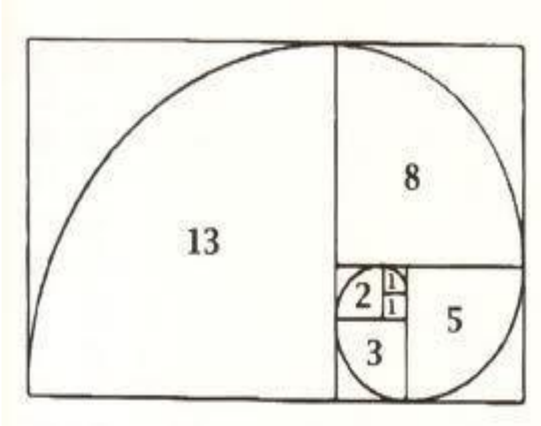
- 1. Research Background and Objective
- 2. ADC Input Signal
  for Code Selective ADC Histogram Test Method
  - ADC Test with Histogram Method and Waveform Missing
  - Combine Multiple Sine Waves
  - General Two-Tone Input Signal
- **3. Ratio of Input and Sampling Frequencies for Efficient ADC Histogram Test Method**
  - **Golden Ratio Sampling**
  - Metallic Ratio Sampling
  - Prime Number Ratio Sampling
- 4. Conclusion

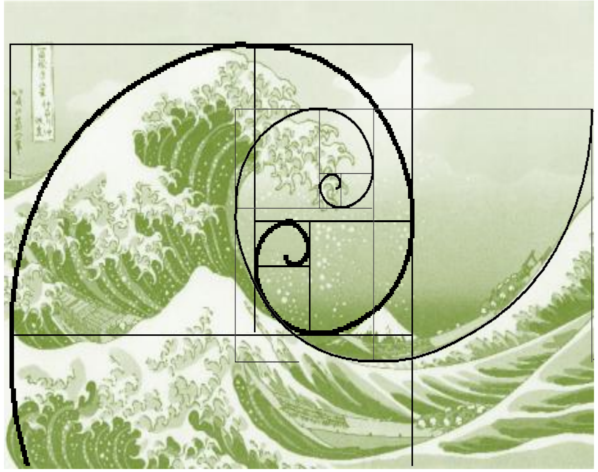
Golden Ratio

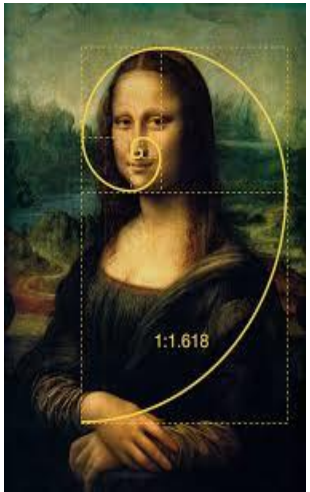
Golden Ratio: $\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = 1.618033988749895 = \varphi$

The most beautiful ratio







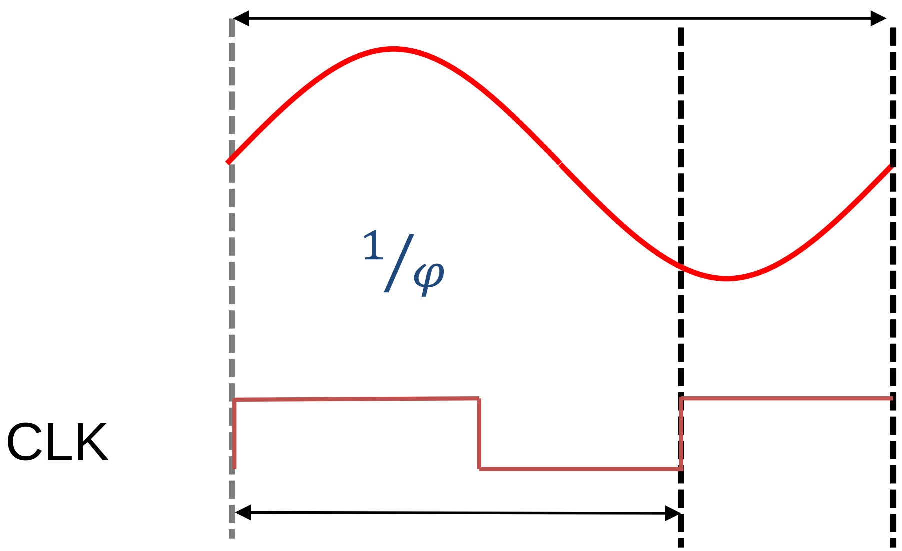


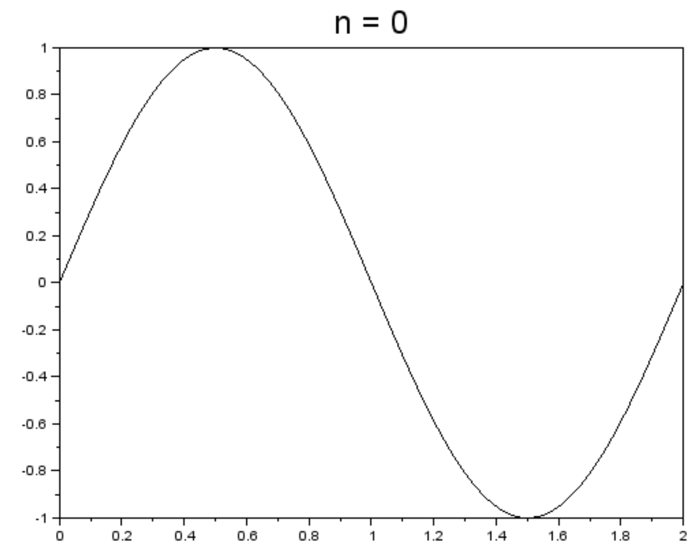
Golden Ratio Sampling

Golden Ratio φ

$$f_{CLK} = \varphi \times f_{sig}$$

φ = 1.6180339887…





Proposal of sampling conditions for the highest waveform acquisition efficiency

Yuto Sasaki, Yujie Zhao, Anna Kuwana and Haruo Kobayashi,
"Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System"
27th IEEE Asian Test Symposium, Hefei, Anhui, China (Oct. 2018)

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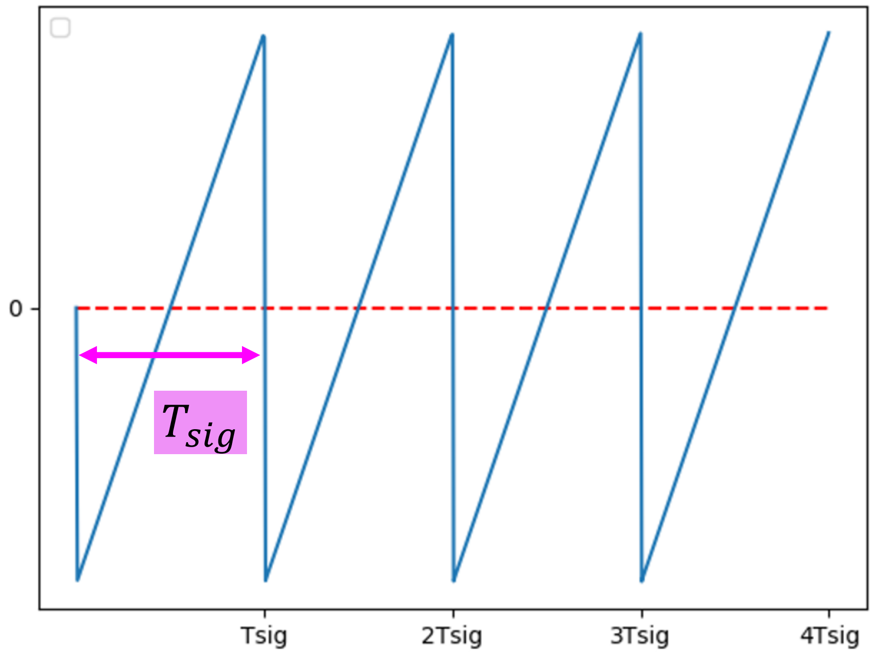
Metallic Ratio

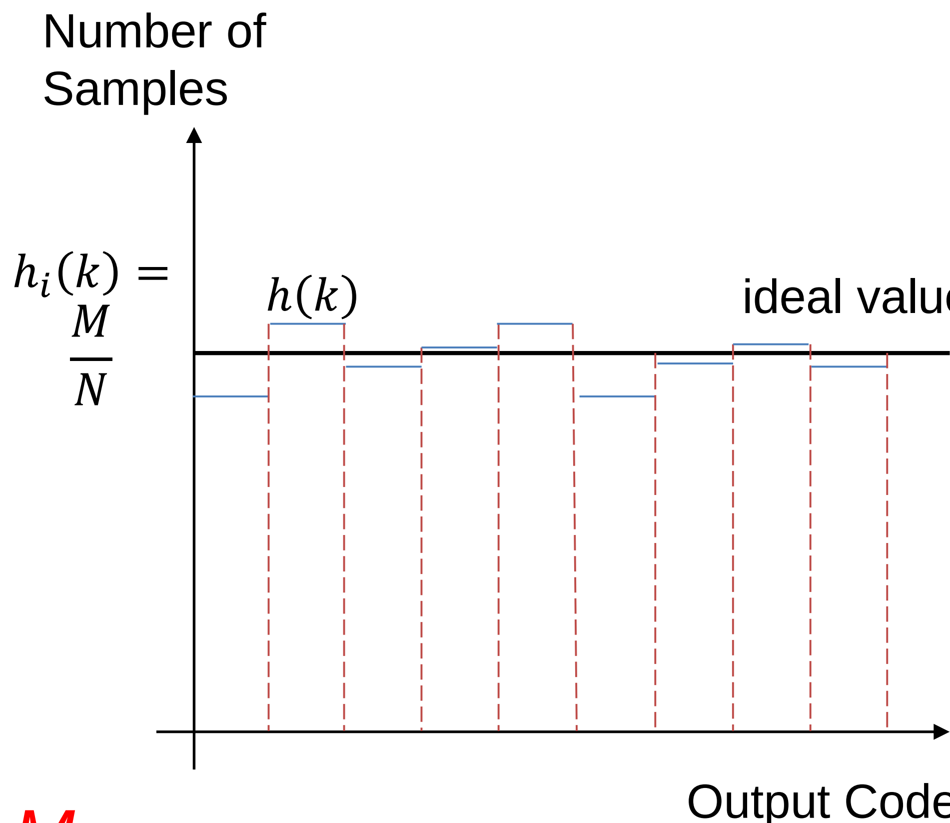
Golden Ratio: $\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = 1.61803398874989.. = \varphi$

n		Decimal	
0	1		
1	$\frac{1+\sqrt{5}}{2}$	1.6180339887…	Golden Ratio
2	$1+\sqrt{2}$	2.4142135623…	Silver Ratio
3	$\frac{3+\sqrt{13}}{2}$	3.3027756377…	Bronze Ratio
4	$2+\sqrt{5}$	4.2360679774…	
…	…		
n	$\frac{n+\sqrt{n^2+4}}{2}$		

Generalization of Golden Ratio

Histogram of Saw Signal





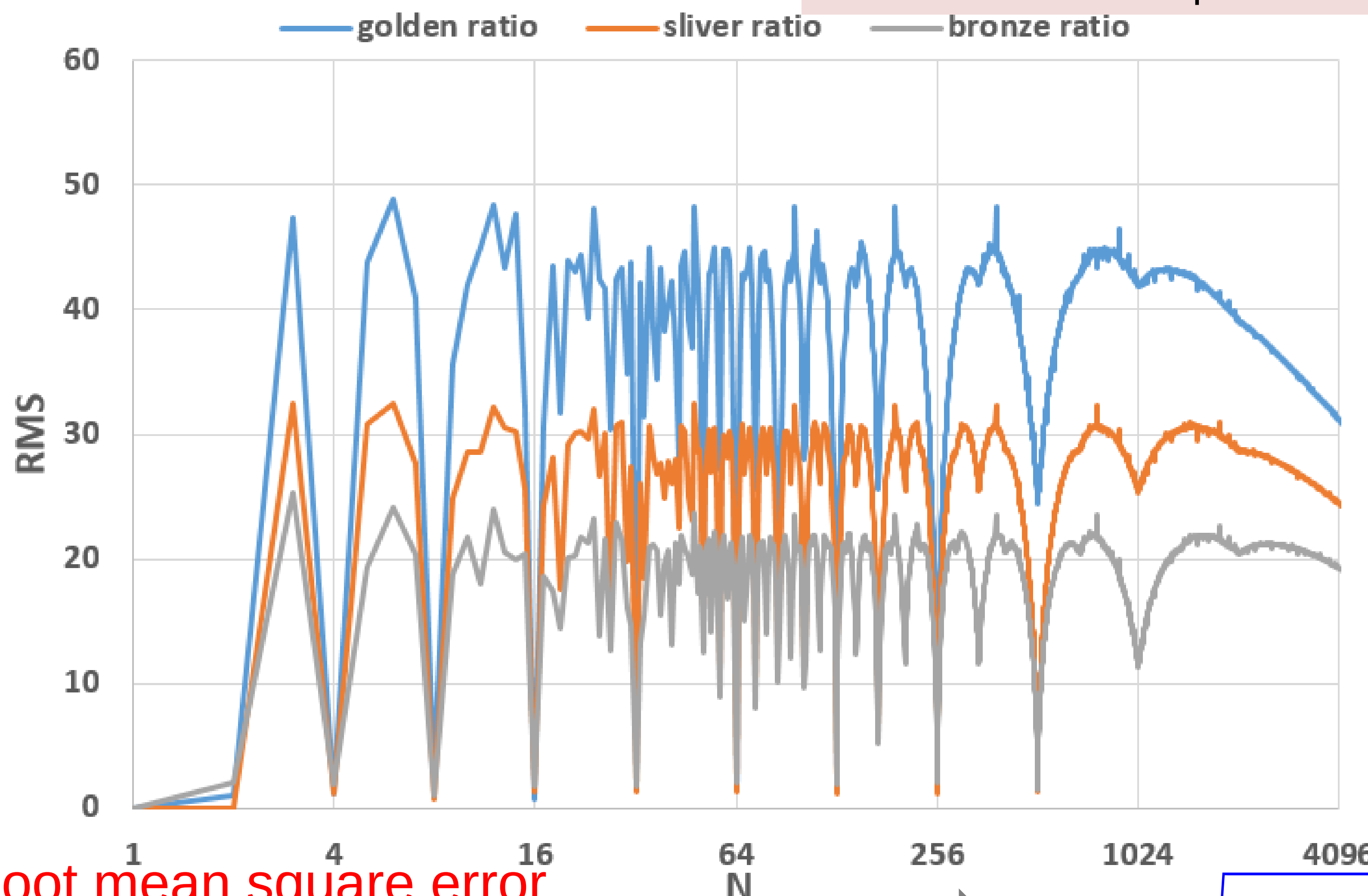
Total number of samples: *M*

ADC resolution :*N*.

ideal value $h_i(k) = \frac{M}{N}, k = 1,2,3,\ldots,N$ error $e(k) = \frac{N \cdot h(k)}{M} - 1$

RMS Error Calculation

Total number of samples: M=65536



Root mean square error
between actual and ideal histograms

$$RMS = \sqrt{\frac{\sum\left(\mathrm{e}(k)\right)^2}{N}}$$

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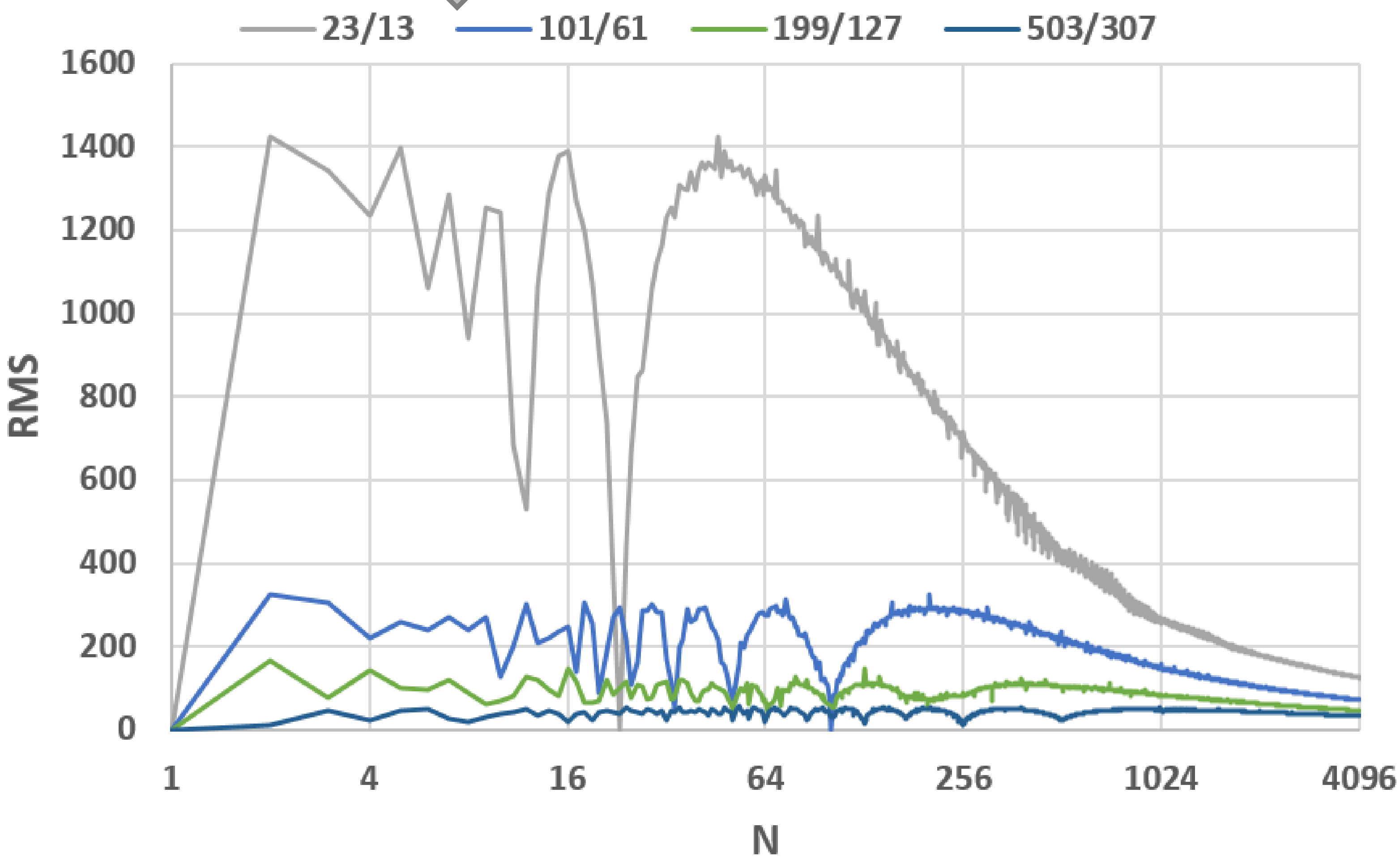
RMS of Prime Number Sampling

$f_{CLK} = f_{sig} \times RATIO$

Total number of samples: M=65536

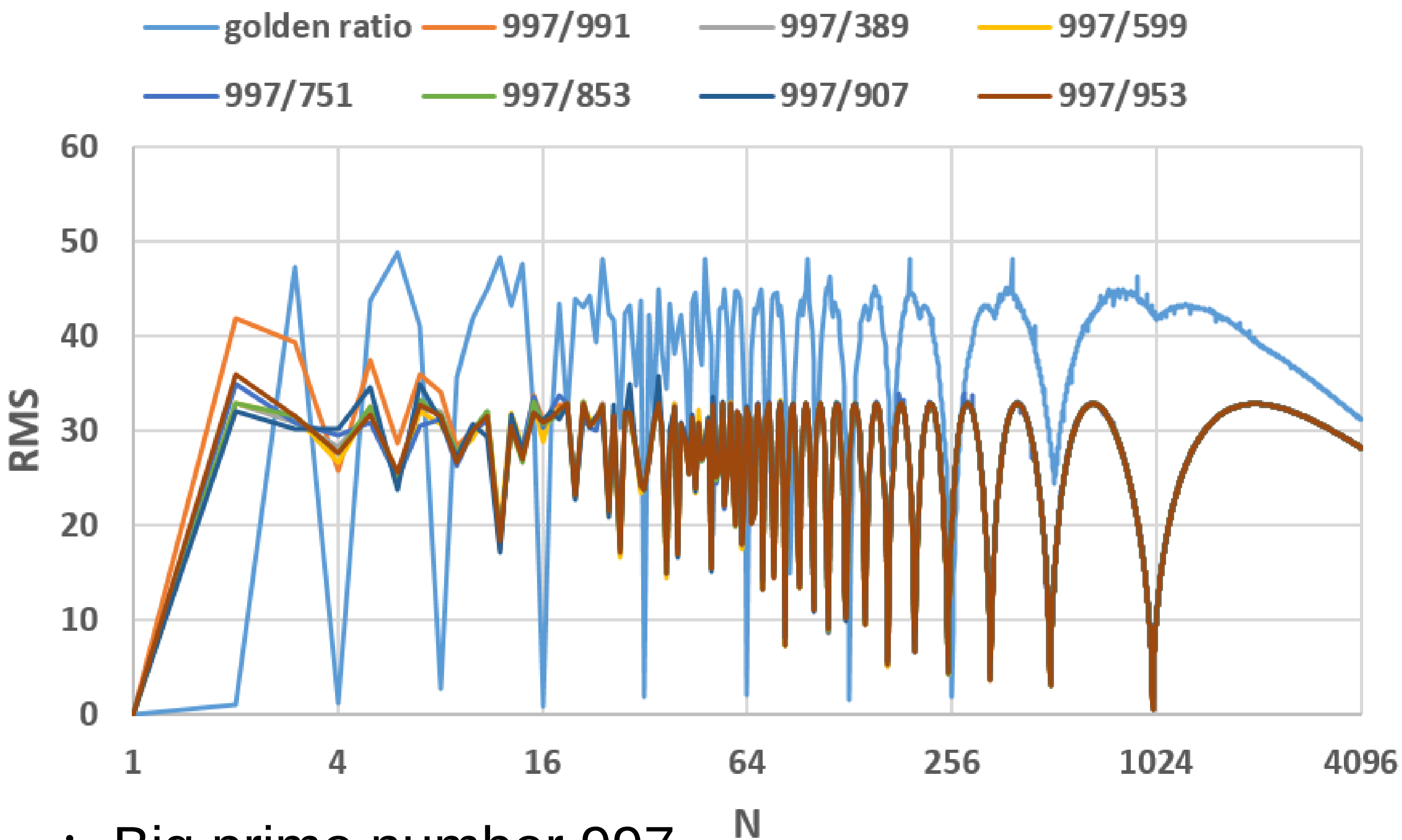


Ratio with two large prime numbers ⇒ Small RMS

Golden ratio has a smaller RMS

RMS of Prime Number Sampling: Big Number Case

Total number of samples: M=65536



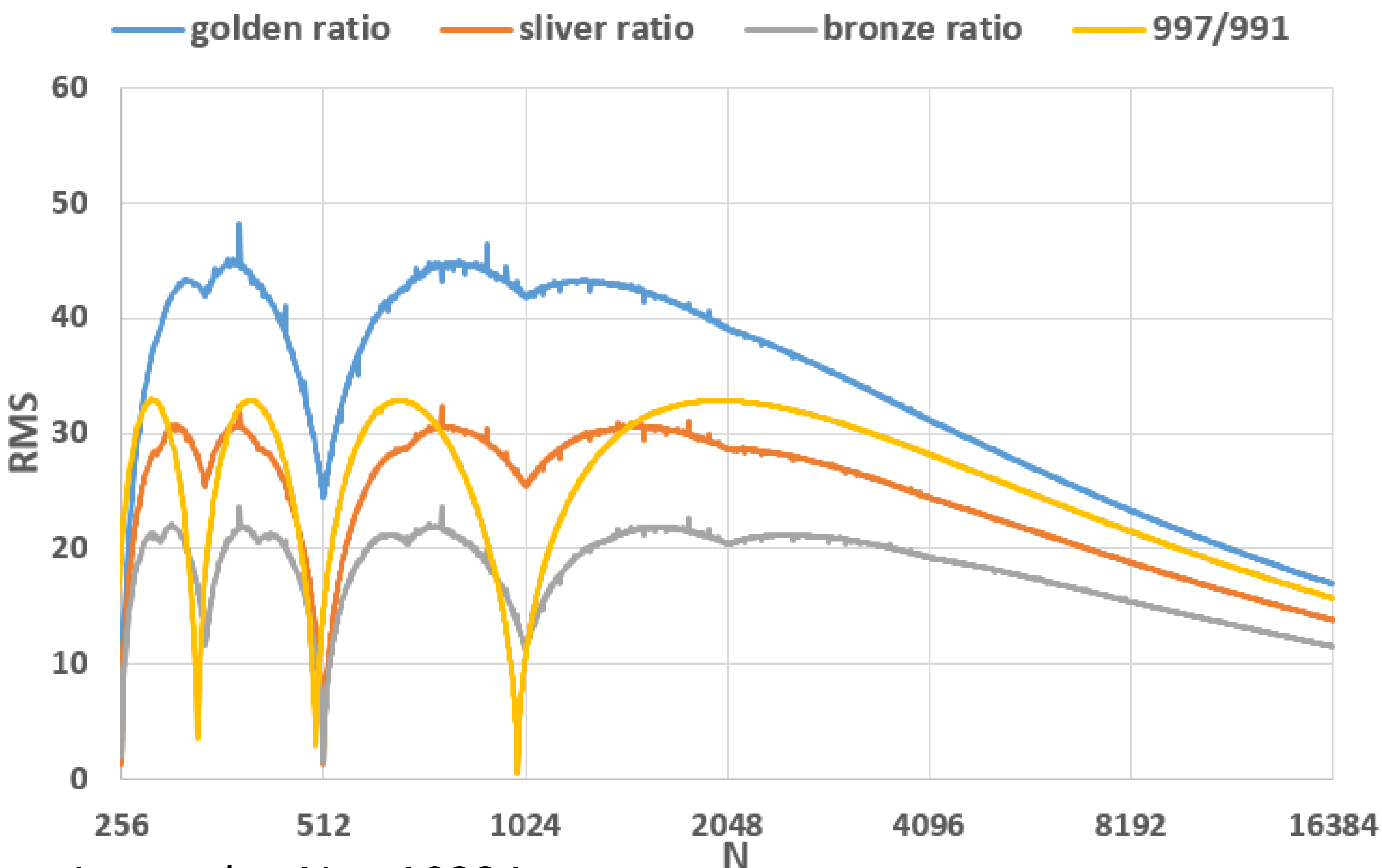
f_{CLK} : Big prime number 997

f_{SIG} : Some prime numbers 389,599,751,853,907,953,991

→ almost the same

RMS Comparison

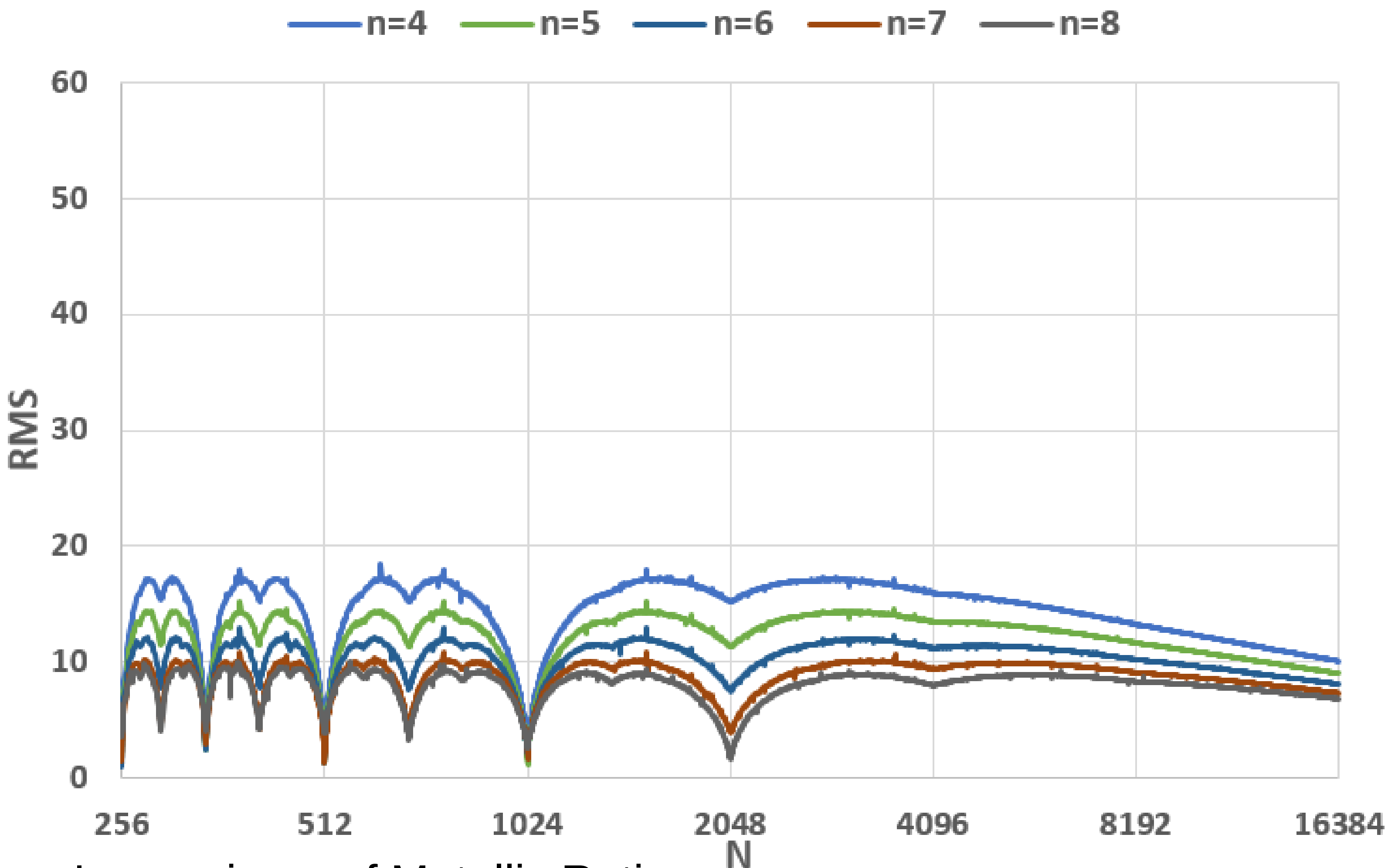
Total number of samples: M=65536



Increasing N to 16384,
Bronze ratio result is better (RMS range is smaller)

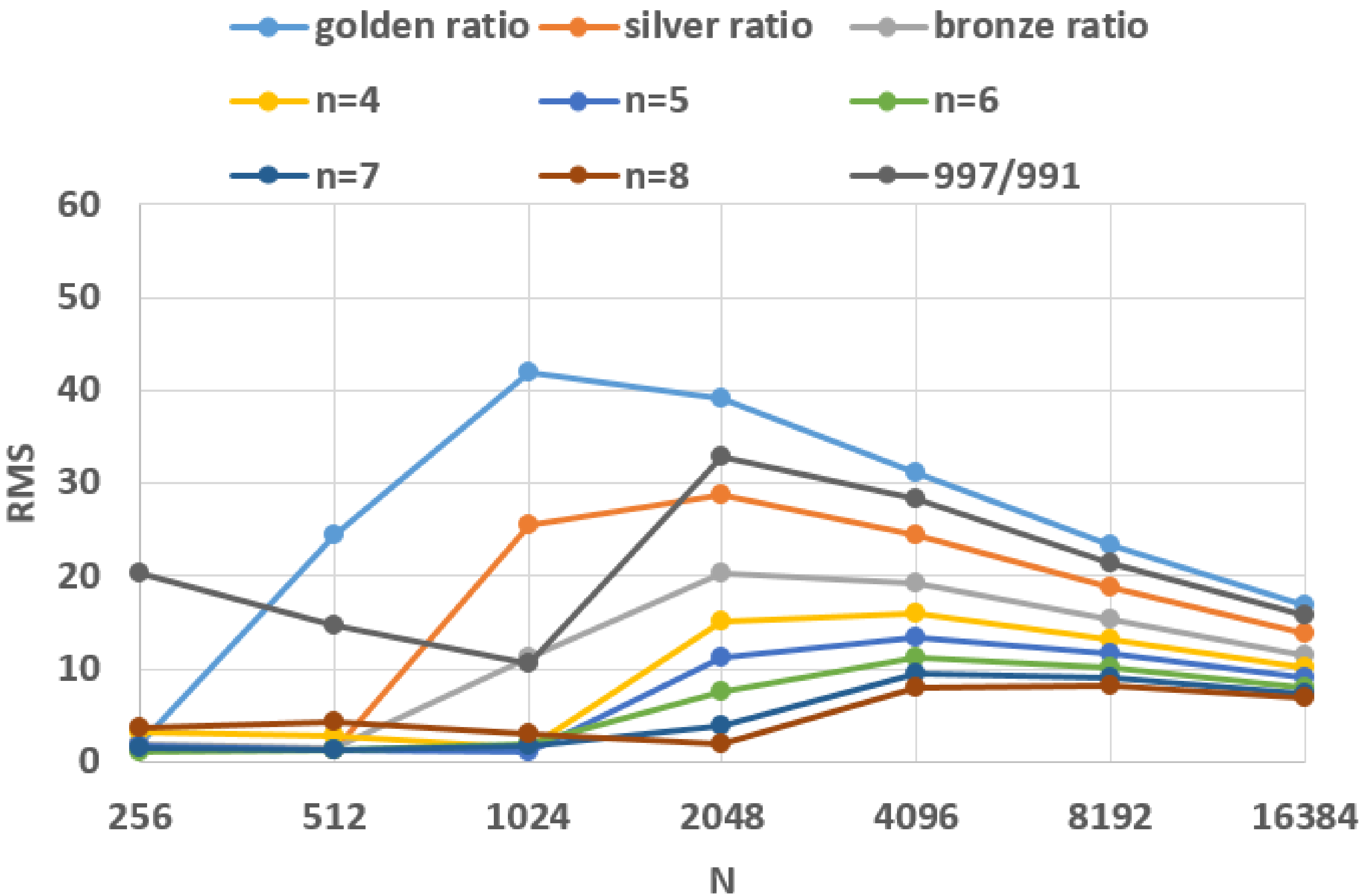
RMS Comparison of Metallic Ratio

Total number of samples: M=65536



Increasing n of Metallic Ratio,
result become better (RMS range is smaller)

Result of RMS: Big N Case



Outline

Conclusion

1. Input Signal for Code Selective ADC Histogram

- ADC output histogram on specific codes by a two-tone wave for short time ADC linearity testing.

2. Ratio of Input and Sampling Frequencies For Efficient ADC Histogram Test

- Proper ratio between input signal and sampling frequencies can attain better testing accuracy with fewer samples.

  → Metallic ratio, Prime number ratio

Conclusion

Golden Ratio sampling
Efficiency: high
Sampling frequency: low

Metallic ratio sampling
Efficiency: high
Sampling frequency: high

Prime number ratio sampling
Efficiency:Not good
Sampling frequency: low

Next work

- Like the golden ratio

  Find conditions for efficient sampling

  at a specific location

  (ADC resolution N=256,512,1024,2048,4096)

Thanks for your attention.

Q&A

Is it possible to combine the two proposed techniques?
We haven‘t discussed it before. I think this can be done.

Can your method DNL/INL for the entire code space?
Because the probability density function cannot be used to calculate the ideal histogram, it cannot be calculated. We are solving this problem.

Why have you used some functions like sin(t) - sin (3t)/3.5? Reference slide 27
The histogram obtained before is too concentrated, we want to disperse the peaks of the histogram.

How much can your methods reduce test time?
In previous studies, the total number of samples can be reduced by half when the number of central samples is close.