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Study of Behaviors of Motion Models in High-Order Systems

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Outline

1. Research Background
- Motivation, objectives and achievements
2. Proposed Superposition Formulas
- Time, frequency responses, and superposition theorems
3. Motion Models for Large-Scale Systems
- Behaviors of the Earth's Motions
4. Motion Models for Regular-Scale Systems
- Behaviors of mechanical systems
5. Motion Models for Small-Scale Systems
- Behaviors of electronic systems
6. Conclusions

1. Research Background

Motivation on High-Order Physical Systems

- Behaviours of complex functions in time and frequency domains are not analysed in detail.
- Limitations of loop gain, differential equations, and heat equations are not pointed out.
- Superposition theorems are not widely used in large-, medium-, and small-scale physical systems.
- Properties of positive and negative impedances, resistance in mechanical systems are not introduced.
- Relationship between periodic motion systems and positive feedback systems is not well investigated.

1. Research Background

Objectives of This Study

- Investigation of some limitations of differential equations and loop gain in motion models
- Study of behaviors of various different scale systems: planets, mechanical systems, and electronic systems
- Models of periodic motion systems using complex functions → Positive feedback systems

→ Ringing test for high-order electronic systems such as transmission lines, passive and active filters.

→ Observation of phase margin at unity gain determines operating regions of high-order systems

1. Research Background

Contributions of This Work

- **Three superposition formulas for physical systems.**
  - **Mechanical superposition formula**
  - **Electrical superposition formula**
  - **Multi-source superposition formula**
- **Proposed motion models for various different scale physical systems:**
  - **Earth's motions (large-scale),**
  - **Simple pendulum systems (regular-scale), and**
  - **Electronic systems (small-scale)**
- **Investigation of positive feedback systems**
- **Ringing test for mechatronic systems such as transmission lines, passive and active low-pass filters**

1. Research Background

Limitations of Differential Equations and Loop Gain

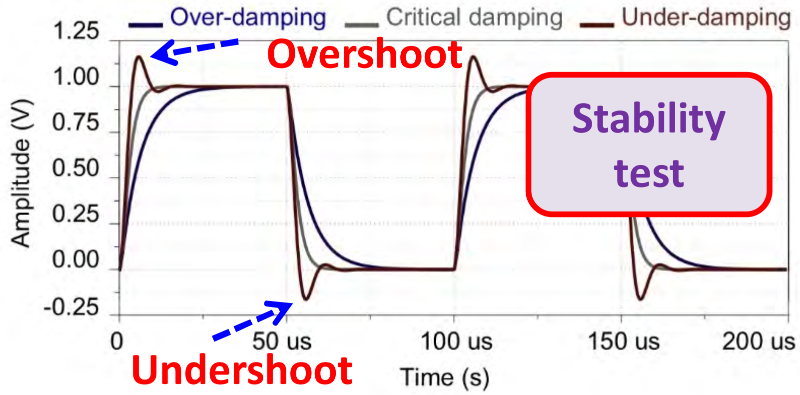
Fourth-order differential equation

$$a_0 y'''' + a_1 y''' + a_2 y'' + a_3 y' + a_4 y = 0;$$

- **Numerical methods don't solve the high-order differential equations.**

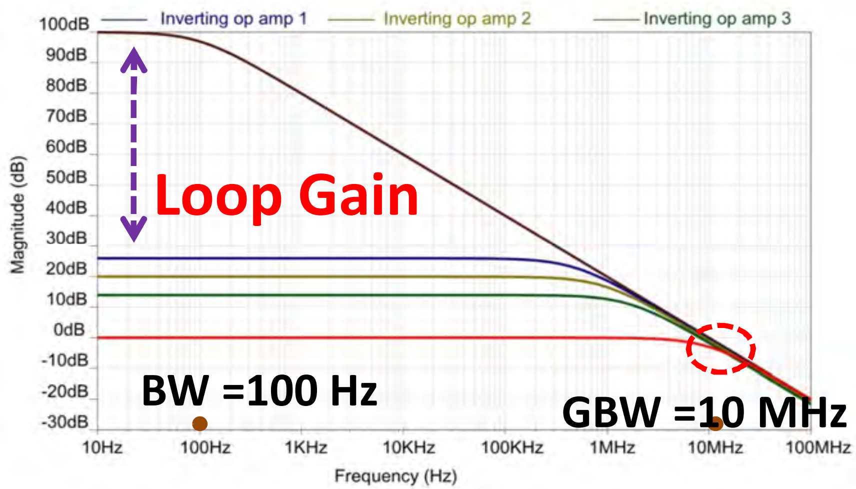
→ **They only approximate the solutions to them.**

Ringing in mechatronic systems



- **Loop gain cannot be used to do the ringing test for mechatronic systems.**

Gain reduction in an inverting amplifier

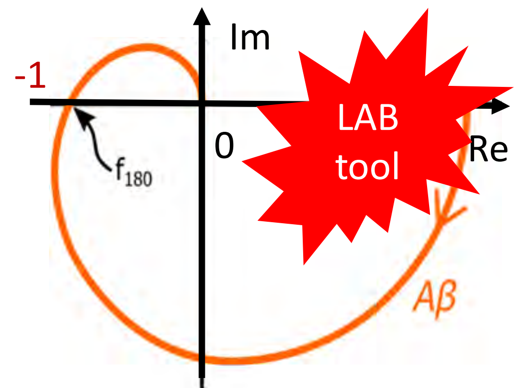


Transfer function

$$H = \frac{A}{1 + A\beta} \approx \frac{1}{\beta}$$

$A\beta$: loop gain

Nyquist plot of loop gain



(Unclear operating region)

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2. Proposed Superposition Formulas

Superposition Formulas for Mechatronic Systems

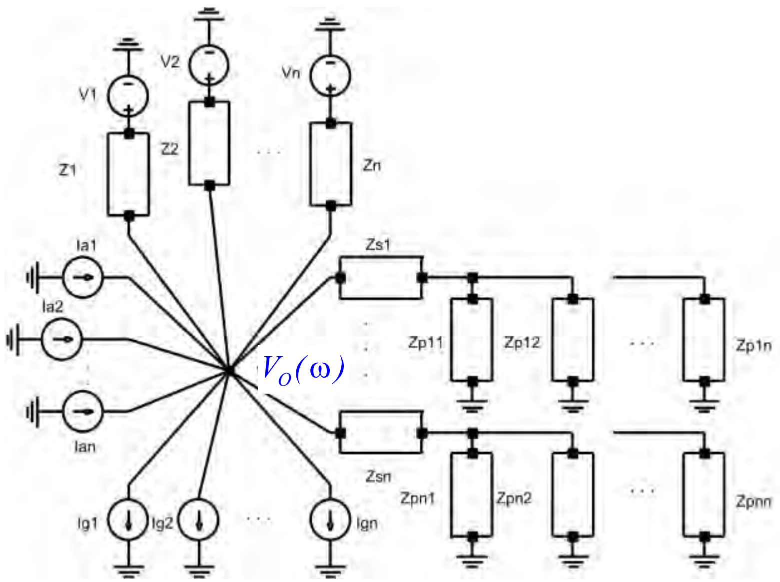
Force conservation law / Mechanical superposition formula

$$\sum_{l=1}^{m} F_{in}(\omega) = \sum_{p=1}^{q} F_{out}(\omega) \rightarrow X_{in}(\omega)\sum_{l=1}^{m}\left(\frac{k_l}{j\omega} + c_l + j\omega m_l\right) = X_{out}(\omega)\sum_{p=1}^{q}\left(\frac{k_p}{j\omega} + c_p + j\omega m_p\right)$$

Current conservation law / Electrical superposition formula

$$\sum_{k=1}^{m} I_{in}(\omega) = \sum_{p=1}^{q} I_{out}(\omega) \rightarrow V_{in}(\quad)\sum_{k=1}^{m}\left(\frac{1}{R_k} + \frac{1}{j\omega L_k} + j\omega C_k\right) = V_{out}(\omega)\sum_{p=1}^{q}\left(\frac{1}{R_p} + \frac{1}{j\omega L_p} + j\omega C_p\right)$$

ω



Multi-source superposition formula

$$V_O(\omega)\left(\sum_{i=1}^{n}\frac{1}{Z_i} + \sum_{i=1}^{n}\frac{1}{Z_{si} + \frac{1}{\sum_{k=1}^{n}\frac{1}{Z_{pik}}}}\right) = \sum_{i=1}^{n}\left(\frac{V_i(\omega)}{Z_i} + I_{ai}(\omega) - I_{gi}(\omega)\right)$$

2. Proposed Superposition Formulas

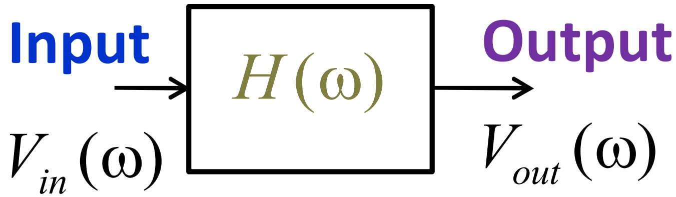
Time and Frequency Responses of Systems

Single-harmonic input signal

$V_{in}(t)=A\cos(\omega_0 t+\Theta_0)$

(time domain)

High-order mechatronic system



(all frequency domains)

Single-harmonic output signal

$V_{out}(t)=A\cos(\omega_0 t+\Theta_0)H(\omega_0)$

(time domain)

Frequency response of high-order system (all frequency domains)

$$H(\omega)=\frac{V_{out}(\omega)}{V_{in}(\omega)}=\frac{b_0(j\omega)^n+b_1(j\omega)^{n-1}+\ldots+b_{n-1}(j\omega)+b_n}{a_0(j\omega)^n+a_1(j\omega)^{n-1}+\ldots+a_{n-1}(j\omega)+a_n};$$

Time response of high-order system (single harmonic input wave)

$$V_{out}(t)=V_{in}(t)\frac{b_0(j\omega_0)^n+b_1(j\omega_0)^{n-1}+\ldots+b_{n-1}(j\omega_0)+b_n}{a_0(j\omega_0)^n+a_1(j\omega_0)^{n-1}+\ldots+a_{n-1}(j\omega_0)+a_n};$$

$$\Rightarrow V_{out}(t)=\left|H(\omega_0)\right|A\cos\left(\omega_0 t+\theta_0+\angle H(\omega_0)\right);$$

2. Proposed Superposition Formulas

Self-loop Function in A Transfer Function

Transfer function of high-order system

$$H(\omega) = \frac{b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n}$$

$A(\omega)$: **Numerator function**

$H(\omega)$: **Transfer function**

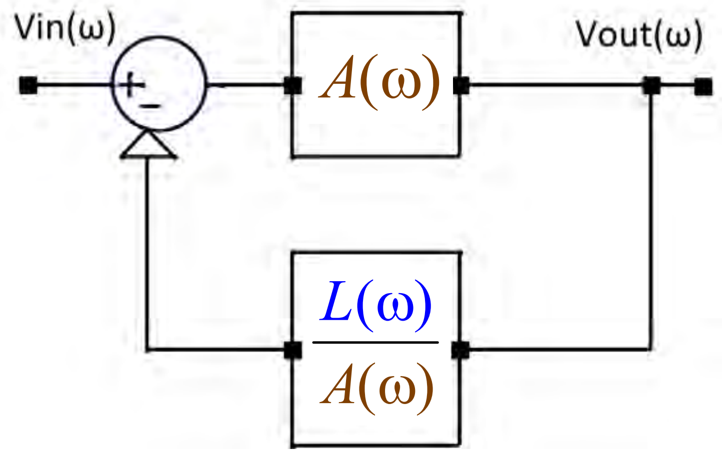
$L(\omega)$: **Self-loop function**

Variable: angular frequency (ω)

Simplified transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)};$$

Graph signal of negative feedback system



Negative feedback system

Relationship between output and input

$$V_{out}(\omega) = A(\omega)\left[V_{in}(\omega) - \frac{L(\omega)}{A(\omega)} V_{out}(\omega)\right]$$

- ○ **Polar chart → Nyquist chart**
- ○ **Magnitude-frequency plot**
- ○ **Angular-frequency plot**

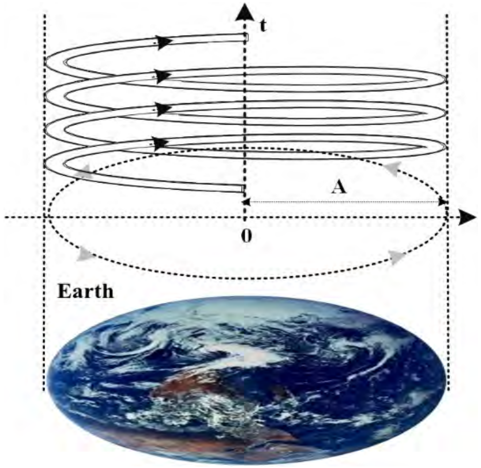
} **Bode plots**

- ○ **Magnitude-angular diagram → Nichols diagram**

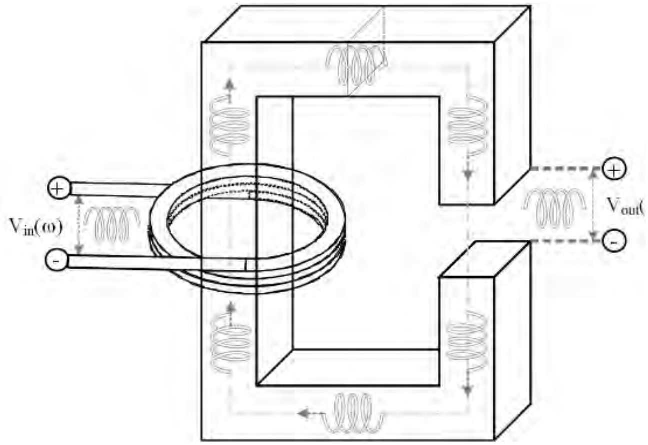
2. Proposed Superposition Formulas

Periodic Motions and Helix Waves

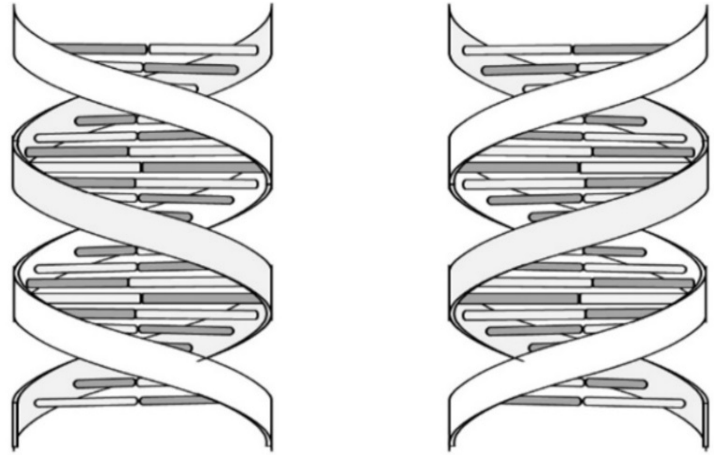
The Earth's rotation



Motion of electronic particles

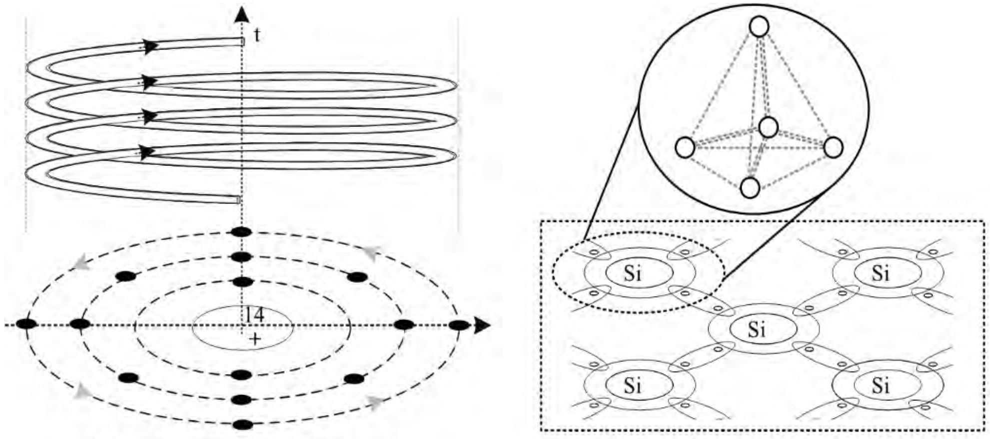


Double helix waves in DNA



(a) Negative helix wave (b) Positive helix wave

Motion of electrons in the crystal structure of silicon atoms



(a) Outermost orbit of silicon atom (b) Crystal structure of silicon atom

Breaking forces in chemical bonds

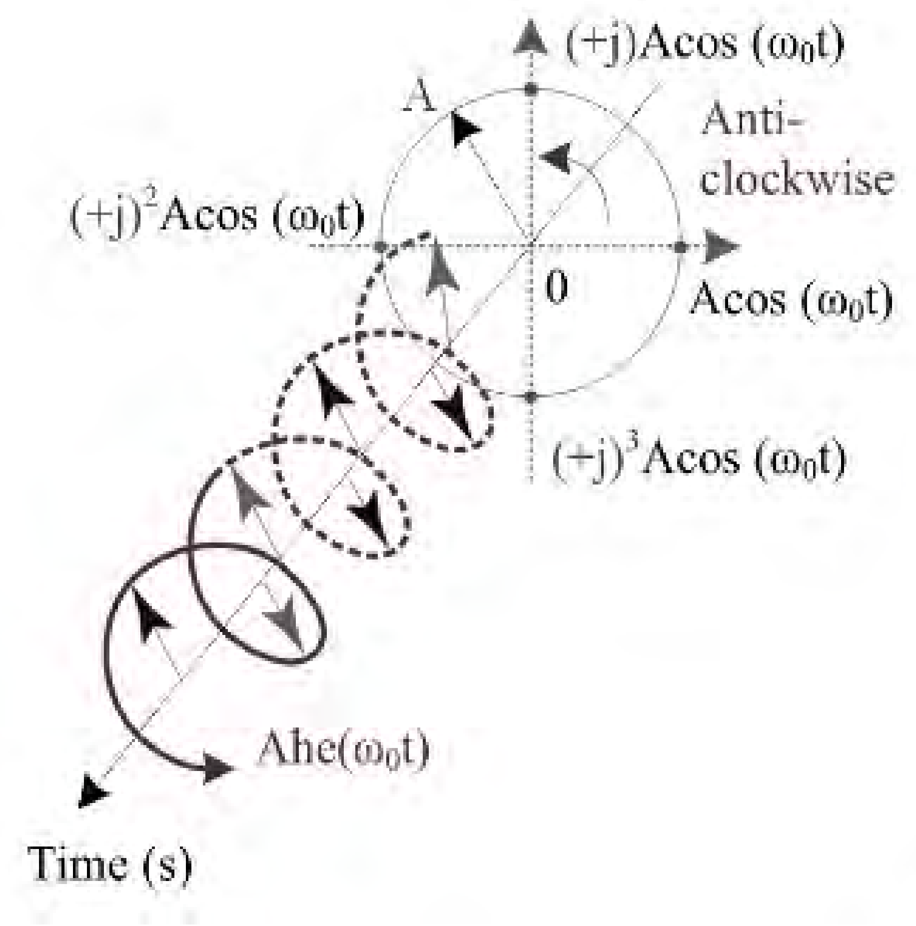
Types of breaking forces	Value
A covalent bond	1600 pN
A noncovalent bond	160 pN
A weak bond	4 pN

2. Proposed Superposition Formulas

Characteristics of Helix Functions

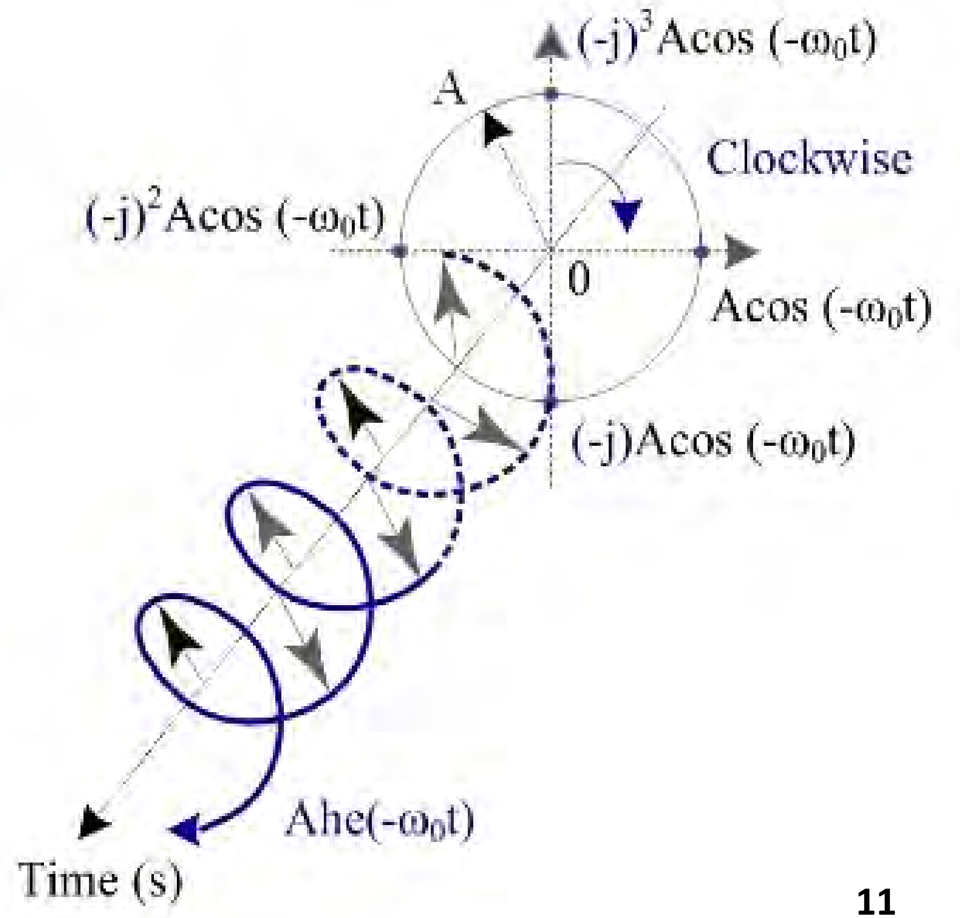
Positive helix function

$$S_p(t) = Ahe(\omega_0 t + \theta_0)$$



Negative helix function

$$S_N(t) = Ahe(-\omega_0 t - \theta_0)$$



2. Proposed Superposition Formulas

Spectra of Common Analog Signals

Signal type	Time domain	Half-side spectrum
Positive helix	$Ahe(\omega_0 t+\theta_0)$	$A\sqrt{2}e^{j(\omega_0 T_0+\theta_0)}$
Negative helix	$Ahe(-\omega_0 t-\theta_0)$	$A\sqrt{2}e^{j(-\omega_0 T_0-\theta_0)}$
Cosine	$A\cos(\omega_0 t+\theta_0)$	$\frac{A\sqrt{2}}{2}e^{j(\omega_0 T_0+\theta_0)}$
Sine	$A\sin(\omega_0 t+\theta_0)$	$\frac{A\sqrt{2}}{2}e^{j\left(\omega_0 T_0+\theta_0+\frac{\pi}{2}\right)}$
Square	$Asq(\omega_0 t+\theta_0)=A\left(\frac{1}{2}+\sum_{n=1}^{\infty}\frac{\sin\left[(2n+1)\omega_0 t+\theta_0\right]}{(2n+1)\pi}\right)$	$\lvert Asq(\omega_0 T_0+\theta_0\rvert=A\left(\frac{1}{2}+\sum_{n=1}^{\infty}\frac{\sqrt{2}e^{j\left((2n+1)\omega_0 T_0+\theta_0+\frac{\pi}{2}\right)}}{2(2n+1)\pi}\right)$

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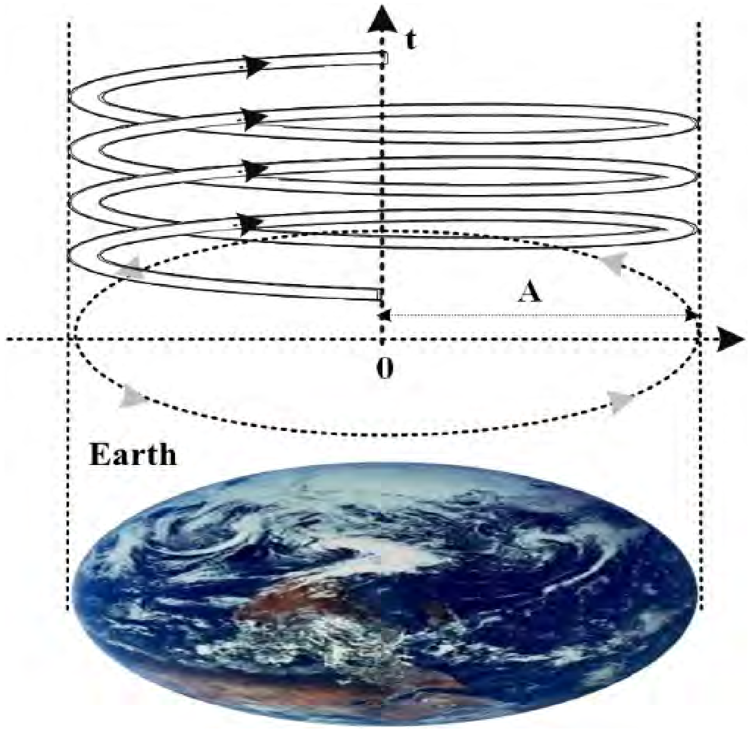
3. Motion Models for Large-Scale Systems

Behaviors of the Earth's Motions

Motion of the Earth on its axis

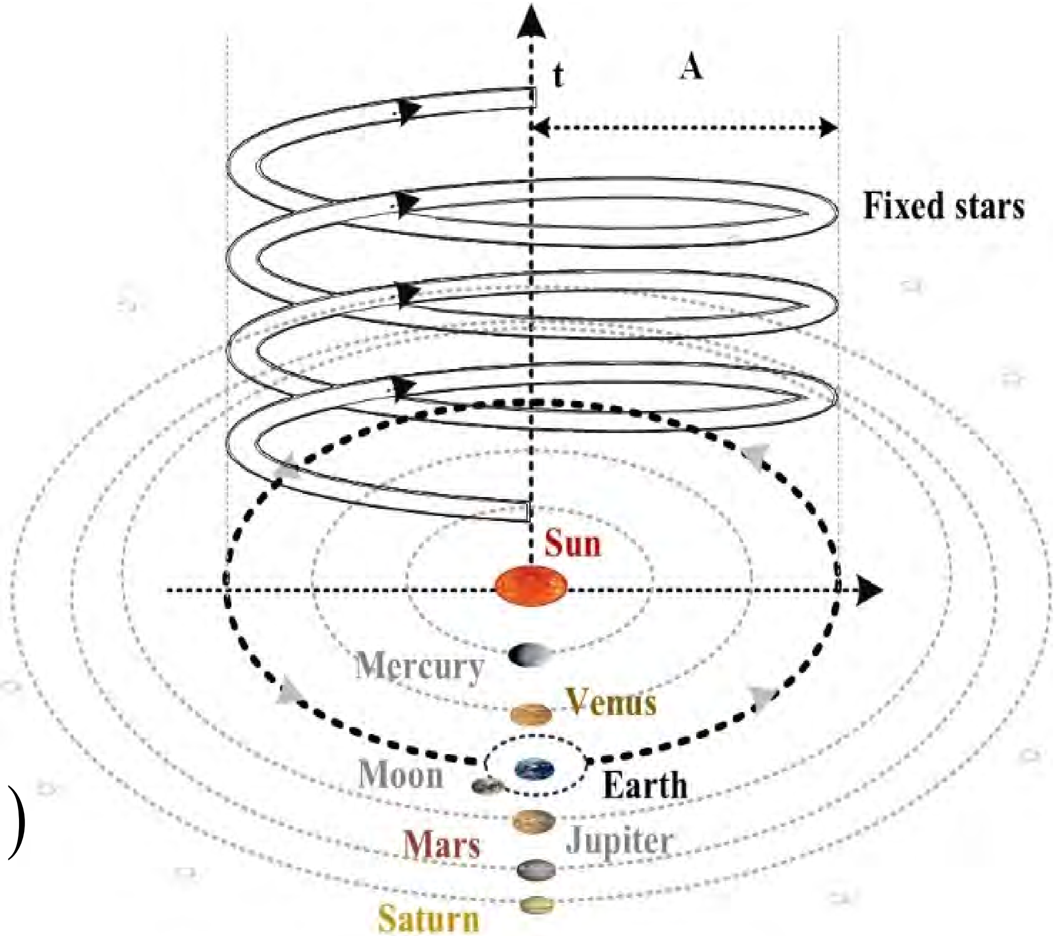


Motion wave $V_{he}(t) = Ahe(\omega_0 t + \theta_0)$

A is radius of the Earth

Frequency f_0 is 11.5 μHz, (or a period of 86400 s)

Motion of the Earth on its orbit

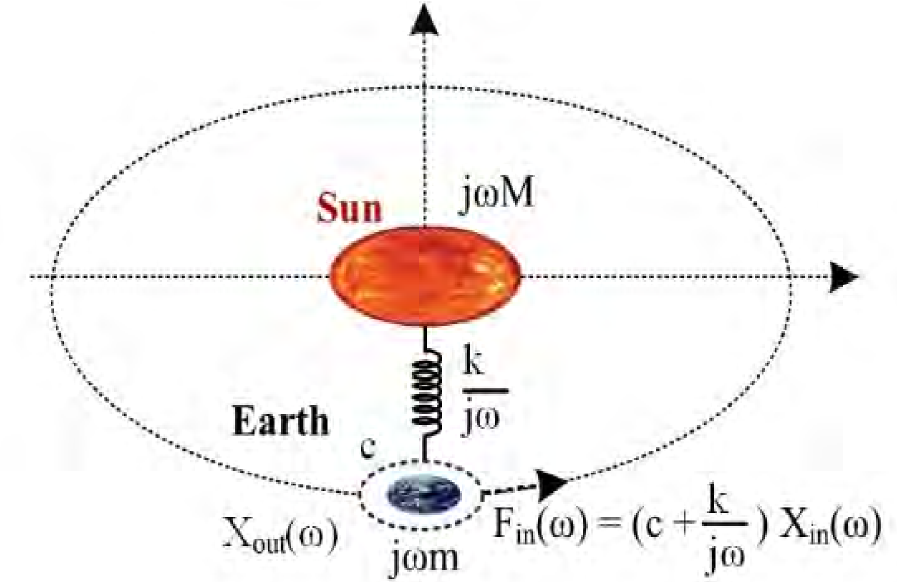


3. Motion Models for Large-Scale Systems

Motion Model of the Earth on Its Orbit

Loop gain cannot be applied for a large-scale physical system.

Model of the Earth and the Sun



Mechanical superposition formula

$$\sum_{l=1}^{m} F_{in}(\omega) = \sum_{p=1}^{q} F_{out}(\omega)$$

$$X_{in}(\omega)\sum_{l=1}^{m}\left(\frac{k_l}{j\omega} + c_l + j\omega m_l\right) = X_{out}(\omega)\sum_{p=1}^{q}\left(\frac{k_p}{j\omega} + c_p + j\omega m_p\right)$$

Apply superposition at the node X_{out},

$$\left[j\omega m + c + \frac{k}{j\omega}\right] X_{out}(\omega) = \left[c + \frac{k}{j\omega}\right] X_{in}(\omega);$$

Transfer function

$$H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{b_0 j\omega + 1}{1 + a_0 (j\omega)^2 + a_1 j\omega};$$

Self-loop function

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Where,

$$b_0 = \frac{c}{k};\ a_0 = \frac{m}{k};\ a_1 = \frac{c}{k}$$

3. Motion Models for Large-Scale Systems

Behaviors of a 2nd -Order Mechanical System

When, $a_0 = \frac{m}{k}$

$$b_0 = \frac{c}{k} = a_1 = \frac{c}{k} = 0;$$

Transfer function

$$H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{1}{1 - \frac{m}{k}\omega^2}$$

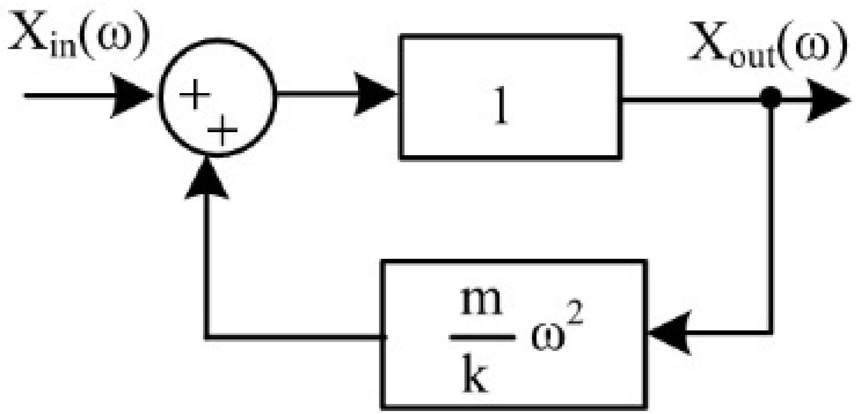
Self-loop function

$$L(\omega) = -\frac{m}{k}\omega^2;$$

Relationship between output and input

$$X_{out}(\omega) = X_{in}(\omega) + \frac{m}{k}\omega^2 X_{out}(\omega);$$

Graph signal of positive feedback system



Positive feedback system

Motion wave $V_{he}(t) = Ahe(\omega_0 t + \theta_0)$

A is radius of the Earth's orbit, and frequency f_0 is 31.5 nHz

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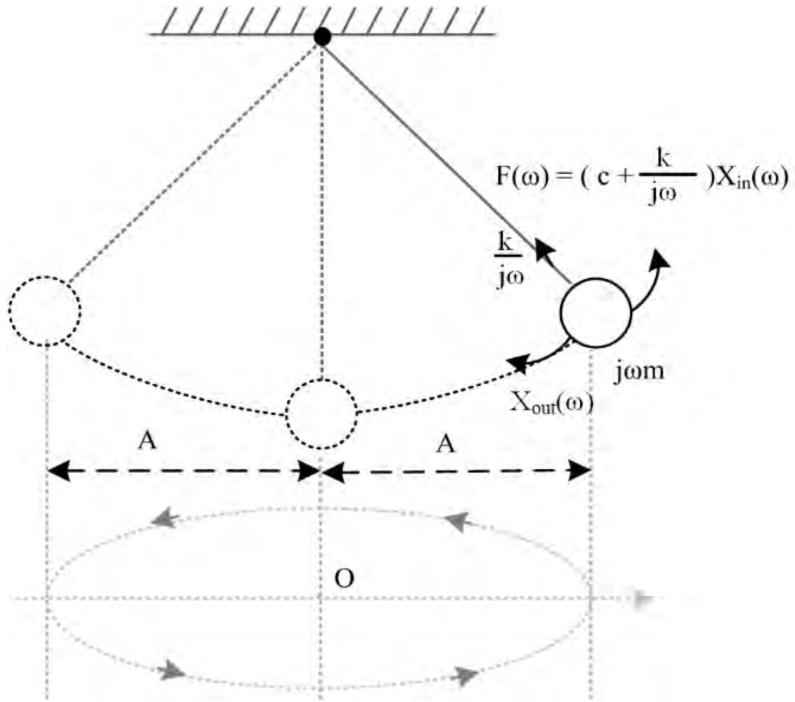
4. Motion Models for Regular-Scale Systems

Analysis of a 2nd -Order Pendulum System

Loop gain cannot be applied for a high-order mechanical system.

Model of pendulum system



Mechanical superposition formula

$$X_{in}(\omega)\sum_{l=1}^{m}\left(\frac{k_l}{j\omega}+c_l+j\omega m_l\right)=X_{out}(\omega)\sum_{p=1}^{q}\left(\frac{k_p}{j\omega}+c_p+j\omega m_p\right)$$

Apply superposition at the node X_{out},

$$\left[j\omega m+c+\frac{k}{j\omega}\right]X_{out}(\omega)=\left[c+\frac{k}{j\omega}\right]X_{in}(\omega);$$

Transfer function

$$H(\omega)=\frac{X_{out}(\omega)}{X_{in}(\omega)}=\frac{b_0 j\omega+1}{1+L(\omega)};$$

Self-loop function

$$L(\omega)=a_0(j\omega)^2+a_1 j\omega;$$

Where,

$$b_0=\frac{c}{k};a_0=\frac{m}{k};a_1=\frac{c}{k}$$

4. Motion Models for Regular-Scale Systems

Behaviors of a 2nd -Order Pendulum System

When, $a_0 = \frac{m}{k}$

$$b_0 = \frac{c}{k} = a_1 = \frac{c}{k} = 0;$$

The simplified transfer function

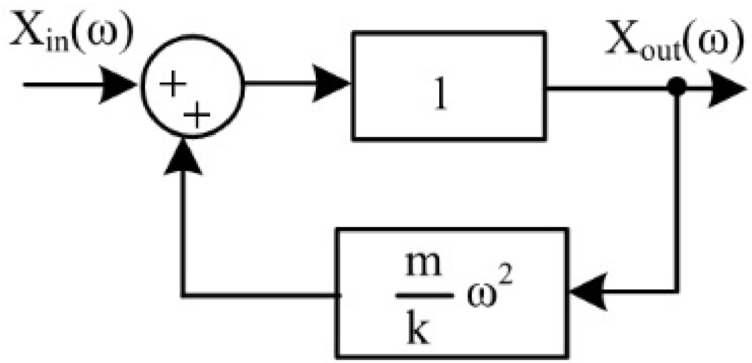
$$H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{1}{1 - a_0\omega^2};$$

Relationship between output and input

$$X_{out}(\omega) = X_{in}(\omega) + \frac{m}{k}\omega^2 X_{out}(\omega);$$

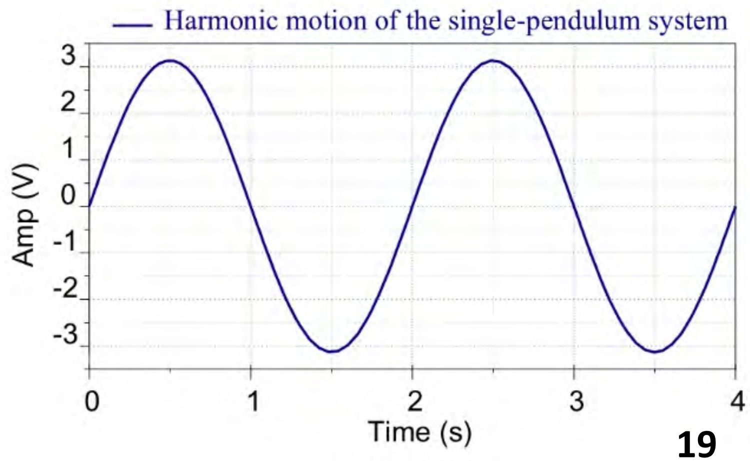
Here, l = 1m, m = 1 kg, g = 9.8 m/s²

Graph signal of positive feedback system



Positive feedback system

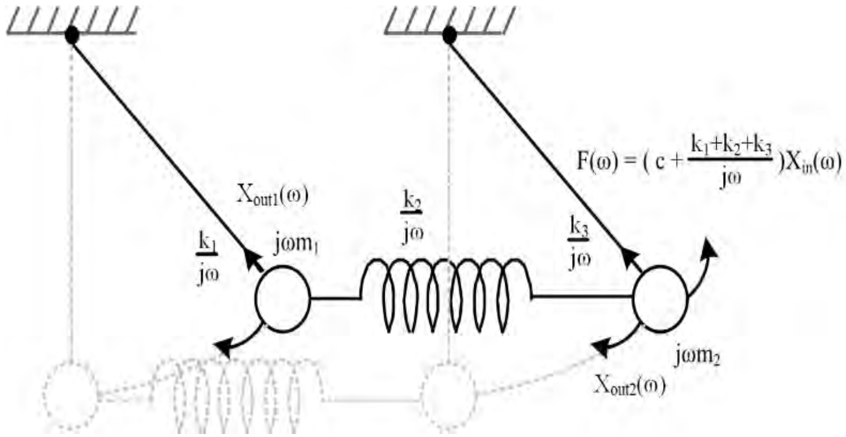
Harmonic motion of pendulum system



4. Motion Models for Regular-Scale Systems

Analysis of a 4th -Order Pendulum System

Model of double-pendulum system



Apply superposition at each node, we get

$$\left(\frac{k_1}{j\omega}+m_1 j\omega+c_1+\frac{k_2}{j\omega}\right)X_{out1}(\omega)=\frac{k_2}{j\omega}X_{out2}(\omega);$$

$$\left(\frac{k_2}{j\omega}+m_2 j\omega+c+\frac{k_3}{j\omega}\right)X_{out2}(\omega)=\frac{k_2}{j\omega}X_{out1}(\omega)$$

$$+\left(c+\frac{k_1+k_2+k_3}{j\omega}\right)X_{in}(\omega);$$

Transfer functions and self-loop function

$$H_1(\omega)=\frac{X_{out1}(\omega)}{X_{in}(\omega)}=\frac{b_0 j\omega+b_1}{1+a_0(j\omega)^4+a_1(j\omega)^3+a_2(j\omega)^2+a_3 j\omega};$$

$$H_2(\omega)=\frac{X_{out2}(\omega)}{X_{in}(\omega)}=\frac{b_2(j\omega)^3+b_3(j\omega)^2+b_4 j\omega+b_5}{1+a_0(j\omega)^4+a_1(j\omega)^3+a_2(j\omega)^2+a_3 j\omega};$$

$$L(\omega)=a_0(j\omega)^4+a_1(j\omega)^3+a_2(j\omega)^2+a_3 j\omega;$$

Pascal's Triangle

2nd		1	2	1	
3rd	1	3	3	1	
4th	1	4	6	4	1

4. Motion Models for Regular-Scale Systems

Behaviors of a 4th -Order Pendulum System

Transfer function of 1st pendulum

$$H_1(\omega)=\frac{b_0 j\omega+b_1}{1+a_0(j\omega)^4+a_1(j\omega)^3+a_2(j\omega)^2+a_3 j\omega};$$

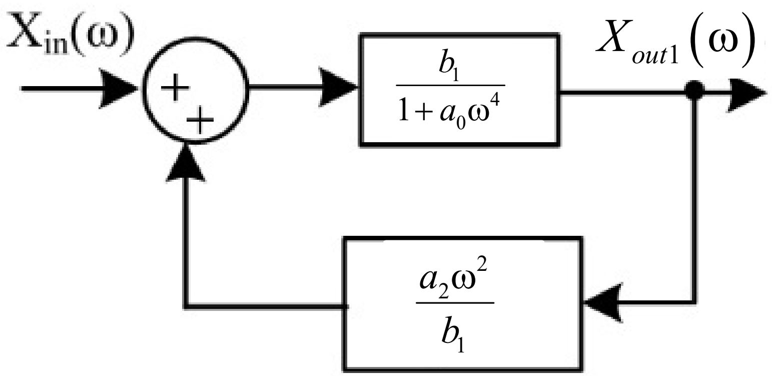
As c = 0, then b_0, a_1, a_3 are neglected.

$$H_1(\omega)=\frac{X_{out1}(\omega)}{X_{in}(\omega)}=\frac{\dfrac{b_1}{1+a_0\omega^4}}{1-\dfrac{a_2\omega^2}{1+a_0\omega^4}};$$

Relationship between output and input

$$X_{out1}(\omega)=\frac{b_1}{1+a_0\omega^4}\left(X_{in}(\omega)+\frac{a_2\omega^2}{b_1}X_{out}(\omega)\right);$$

Graph signal of positive feedback system

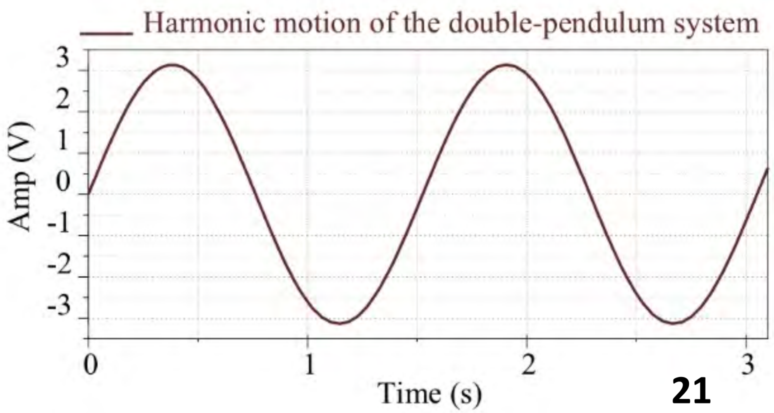


Here,
$l_1 = l_2 = 1m$
$m_1 = m_2 = 1$ kg
$g = 9.8$ m/s^2

Variables of double-pendulum system

Var	Value	Var	Value
b_0	$\frac{k_2 c}{k_1k_2+k_2k_3+k_3k_1}$	b_5	$\frac{(k_1+k_2)(k_1+k_2+k_3)}{k_1k_2+k_2k_3+k_3k_1}$
b_1	$\frac{k_2(k_1+k_2+k_3)}{k_1k_2+k_2k_3+k_3k_1}$	a_0	$\frac{m_1m_2}{k_1k_2+k_2k_3+k_3k_1}$
b_2	$\frac{k_1 c}{k_1k_2+k_2k_3+k_3k_1}$	a_1	$\frac{(m_1+m_2)c}{k_1k_2+k_2k_3+k_3k_1}$
b_3	$\frac{c^2+m_1(k_1+k_2+k_3)}{k_1k_2+k_2k_3+k_3k_1}$	a_2	$\frac{m_1(k_2+k_3)+m_2(k_1+k_2)+c^2}{k_1k_2+k_2k_3+k_3k_1}$
b_4	$\frac{c(2k_1+2k_2+k_3)}{k_1k_2+k_2k_3+k_3k_1}$	a_3	$\frac{(k_1+2k_2+k_3)c}{k_1k_2+k_2k_3+k_3k_1}$

Harmonic motion of pendulum system



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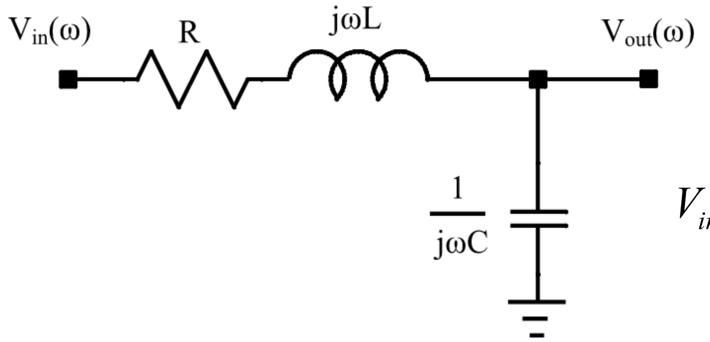
6. Conclusions

5. Motion Models for Small-Scale Systems

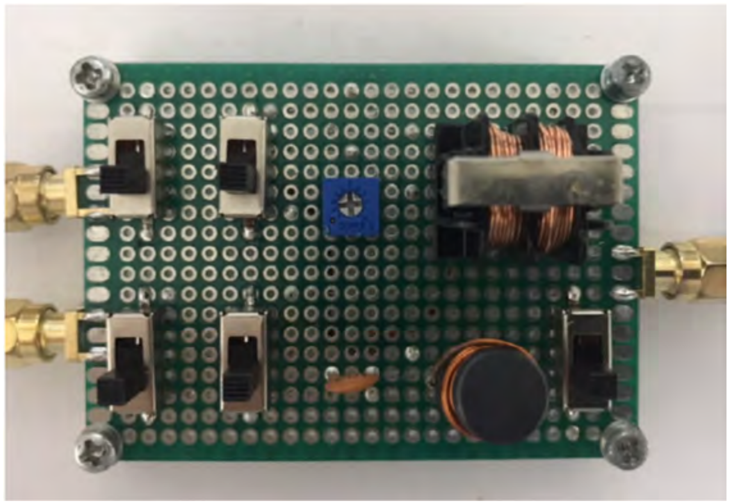
Analysis of a 2nd -Order Passive Low-Pass Filter

Loop gain cannot be applied for a passive filter.

Schematic of RLC low-pass filter



Implemented circuit of RLC LPF



Electrical superposition formula

$$\sum_{k=1}^{m} I_{in}(\omega) = \sum_{p=1}^{q} I_{out}(\omega)$$

$$V_{in}(\omega)\sum_{k=1}^{m}\left(\frac{1}{R_k}+\frac{1}{j\omega L_k}+j\omega C_k\right)=V_{out}(\omega)\sum_{p=1}^{q}\left(\frac{1}{R_p}+\frac{1}{j\omega L_p}+j\omega C_p\right)$$

Apply superposition at the node V_{out},

$$V_{out}(\omega)\left(\frac{1}{R+j\omega L}+j\omega C\right)=V_{in}(\omega)\frac{1}{R+j\omega L};$$

Transfer function and self-loop function

$$H(\omega)=\frac{V_{out}(\omega)}{V_{in}(\omega)}=\frac{1}{1+L(\omega)};$$

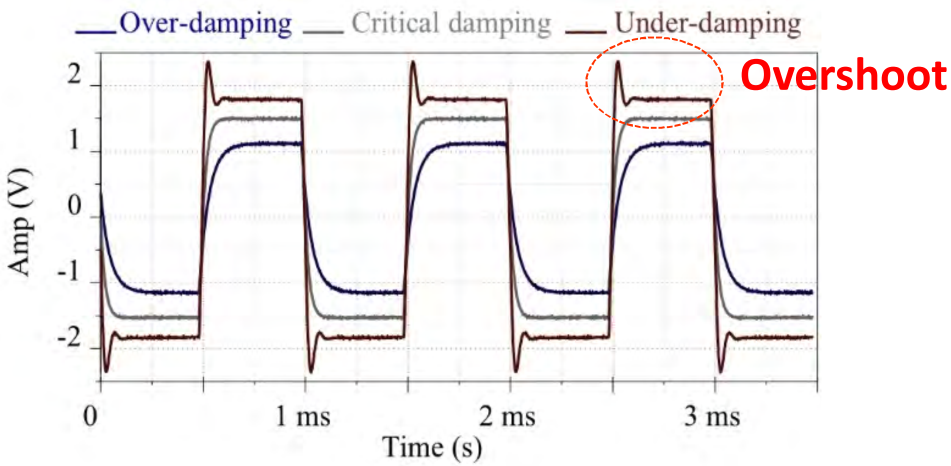
$$L(\omega)=a_0(j\omega)^2+a_1 j\omega;$$

$$a_0=LC;a_1=RC;$$

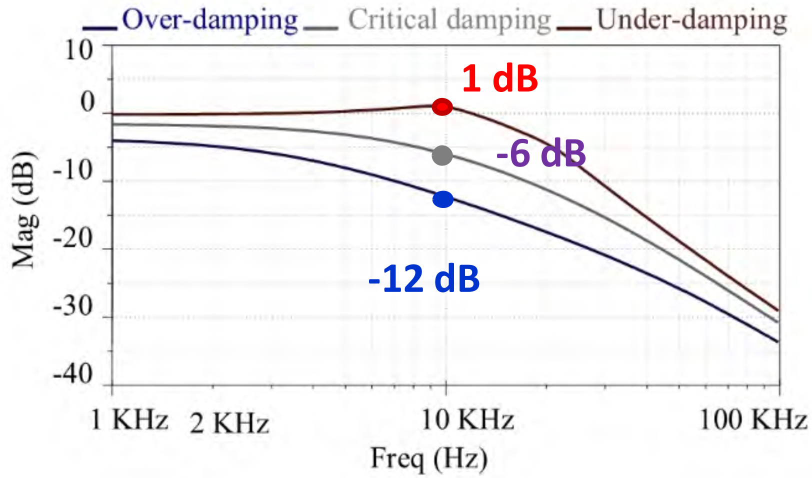
5. Motion Models for Small-Scale Systems

Measurement Results of a 2nd -Order Passive LPF

Simulated transient response

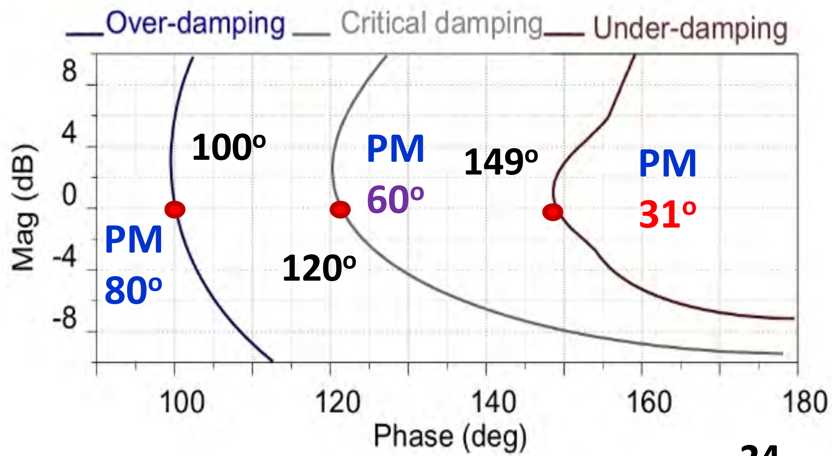


Bode plot of transfer function



Case	Over-damping	Critical damping	Under-damping
Magnitude (transfer function)	-12 dB	-6 dB	1 dB
Phase margin (self-loop function)	80º (observed at 100º)	60º (observed at 120º)	31º (observed at 149º)

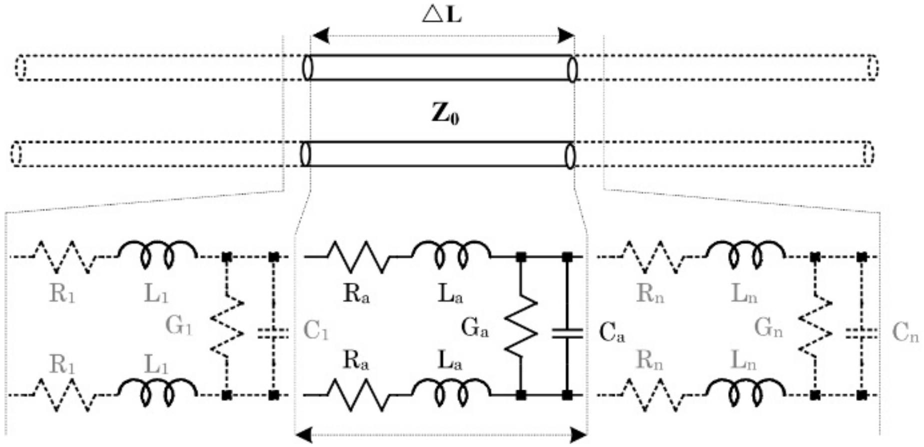
Nichols plot of self-loop function



5. Motion Models for Small-Scale Systems

Analysis of Schematic Model of Transmission Line

Simplified model of coaxial line



Parameters of the schematic model

Variable	Value	Variable	Value
L_a	0.25 nH	Z_0	50 Ω
R_a	1.82 mΩ	Loss	1.6 mdB
C_a	0.1 pF	Skin depth	1.7 um
G_a	6.81 uM/Ω	Delay	0.2 ns

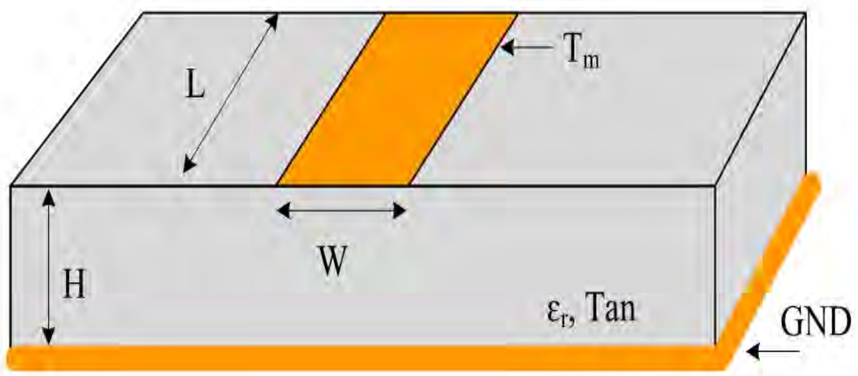
Parameters of the physical model

Parameter	Value	Parameter	Value
Metal width (W)	0.71 mm	Substrate thickness (H)	2
Trace length (L)	40 mm	Dielectric constant (ε_r)	4.6
Metal thickness (T_m)	35 mm	Loss tangent (Tan)	0.01
Metal resistivity	17.2 nΩ	Frequency	1.5 GHz
Surface roughness	0.1 um	Characteristic Impedance	50 Ω

5. Motion Models for Small-Scale Systems

Ringing Test for Coaxial Line

Physical model of transmission line



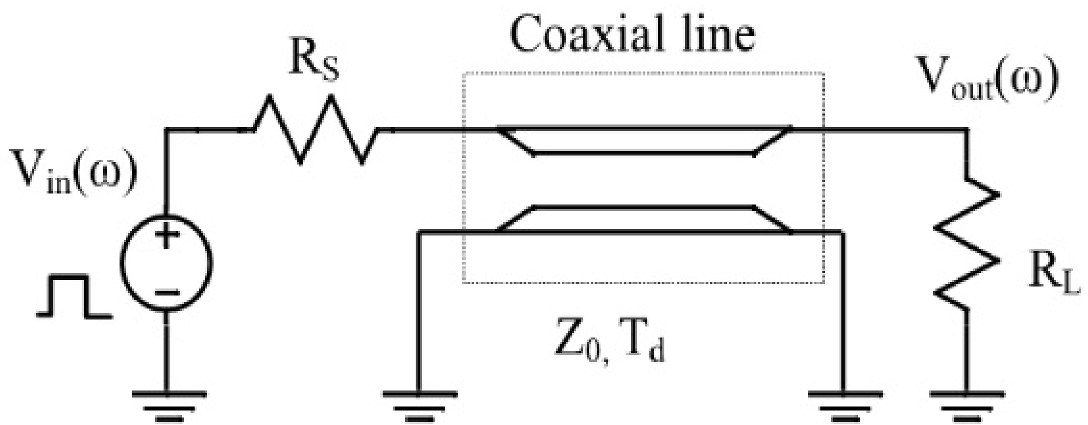
General characteristic impedance

$$Z_0 = \sqrt{\frac{R_a + j\omega L_a}{G_a + j\omega C_a}} \approx \sqrt{\frac{L_a}{C_a}};$$

Apply superposition at the node X_{out},

$$V_{out}(\omega)\left(\frac{1}{R_S + Z_0} + \frac{1}{R_L}\right) = V_{in}(\omega)\frac{1}{R_S + Z_0};$$

Ringing test for the coaxial line



Transfer function and self-loop function

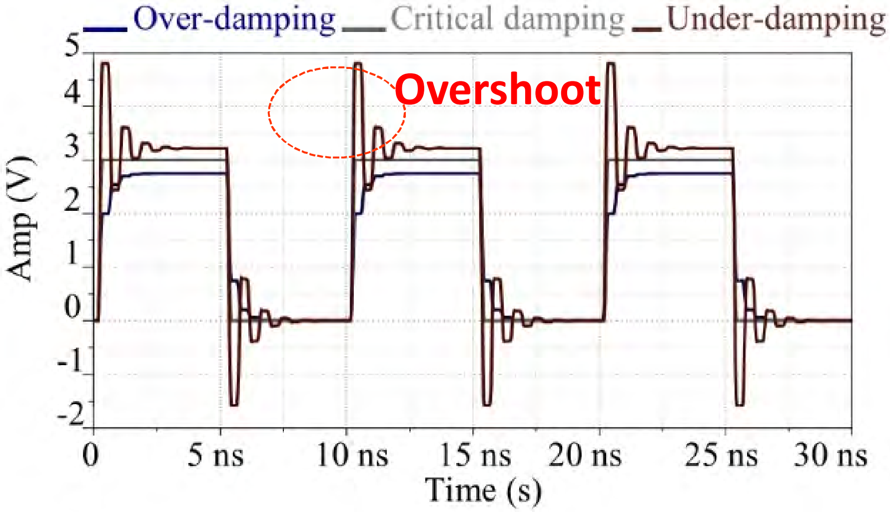
$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + L(\omega)};$$

$$L(\omega) = \frac{R_S + \sqrt{\frac{R_a + j\omega L_a}{G_a + j\omega C_a}}}{R_L};$$

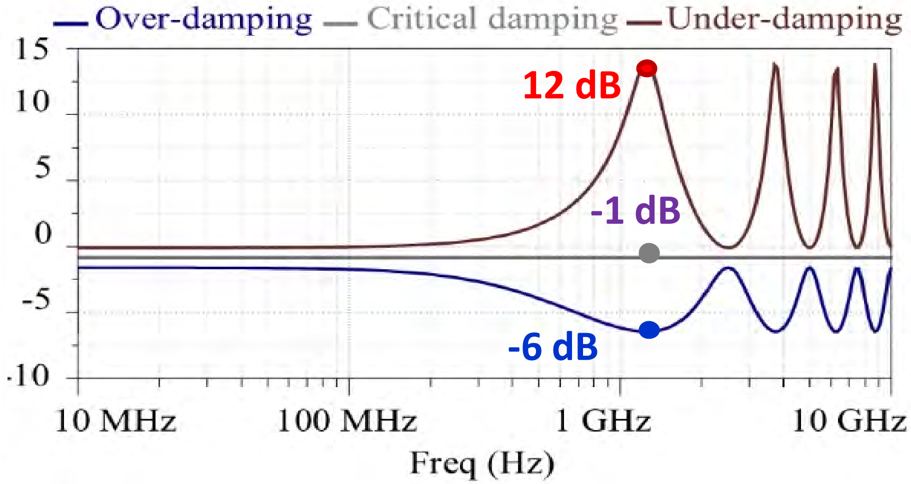
5. Motion Models for Small-Scale Systems

Simulation Results of Coaxial Line

Simulated transient response

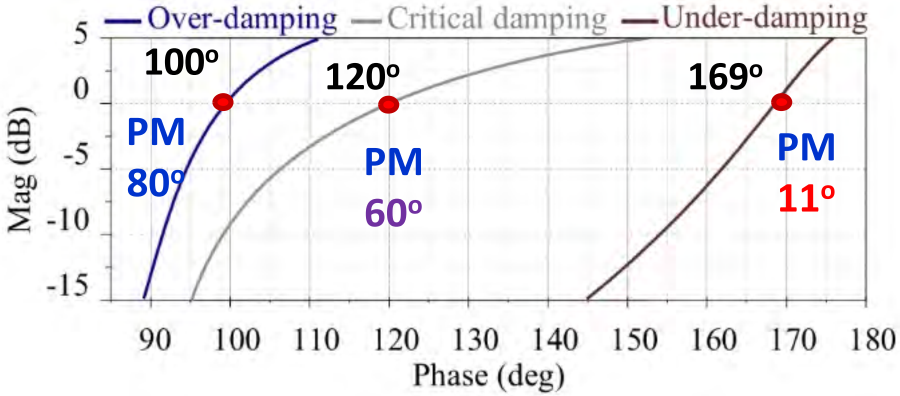


Bode plot of transfer function



Case	Over-damping	Critical damping	Under-damping
Magnitude (transfer function)	-6 dB	-1 dB	12 dB
Phase margin (self-loop function)	80° (observed at 100°)	60° (observed at 120°)	11° (observed at 169°)

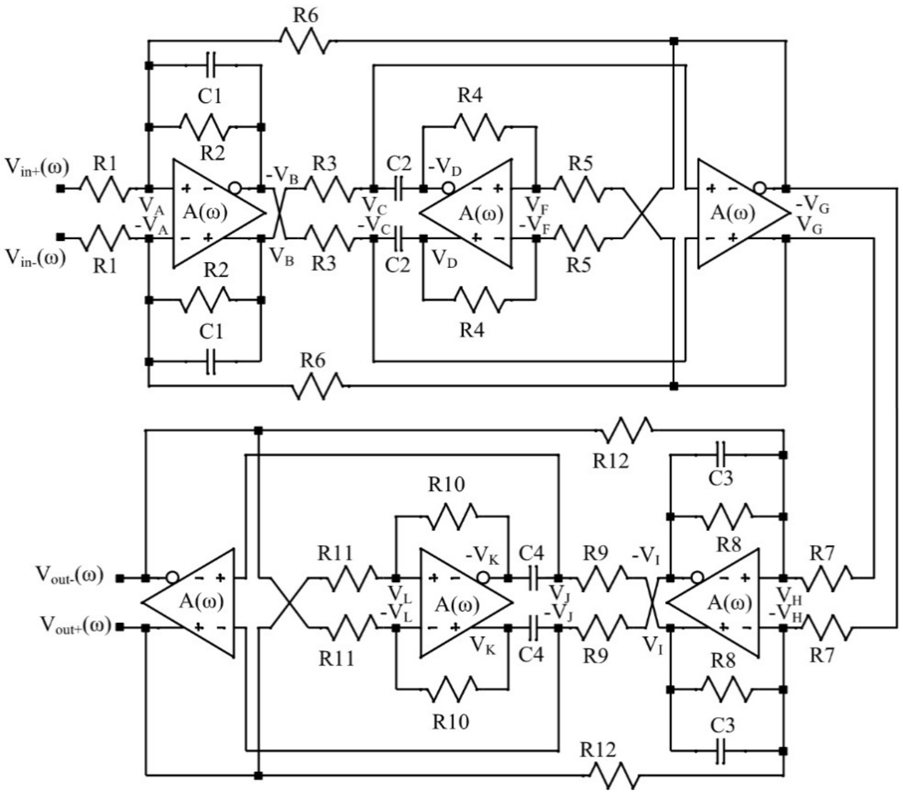
Nichols plot of self-loop function



5. Motion Models for Small-Scale Systems

Analysis of Fourth-Order Active Low-Pass Filter

Schematic of Akerberg-Mossberg LPF



Apply superposition at each node,

$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+j\omega C_1+\frac{1}{R_6}\right)=\frac{V_{in+}(\omega)}{R_1}-V_B\left(\frac{1}{R_2}+j\omega C_1\right)-\frac{V_G}{R_6};$$

$$V_C\left(\frac{1}{R_3}+j\omega C_2\right)=\frac{V_B}{R_3}-V_D j\omega C_2;\ V_B=2V_A A(\omega);\ V_G=2V_C A(\omega);$$

$$V_F\left(\frac{1}{R_4}+\frac{1}{R_5}\right)=\frac{V_G}{R_5}-\frac{V_D}{R_4};V_D=2V_F A(\omega);V_{out+}(\omega)=2V_J A(\omega);$$

$$V_H\left(\frac{1}{R_7}+\frac{1}{R_8}+j\omega C_3+\frac{1}{R_{12}}\right)=\frac{V_G}{R_7}-V_I\left(\frac{1}{R_8}+j\omega C_3\right)-\frac{V_{out+}(\omega)}{R_{12}};$$

$$V_J\left(\frac{1}{R_9}+j\omega C_4\right)=\frac{V_I}{R_9}-V_K j\omega C_4;\ V_I=2V_H A(\omega);$$

$$V_L\left(\frac{1}{R_{10}}+\frac{1}{R_{11}}\right)=\frac{V_{out+}(\omega)}{R_{11}}-\frac{V_K}{R_{10}};V_K=2V_L A(\omega);$$

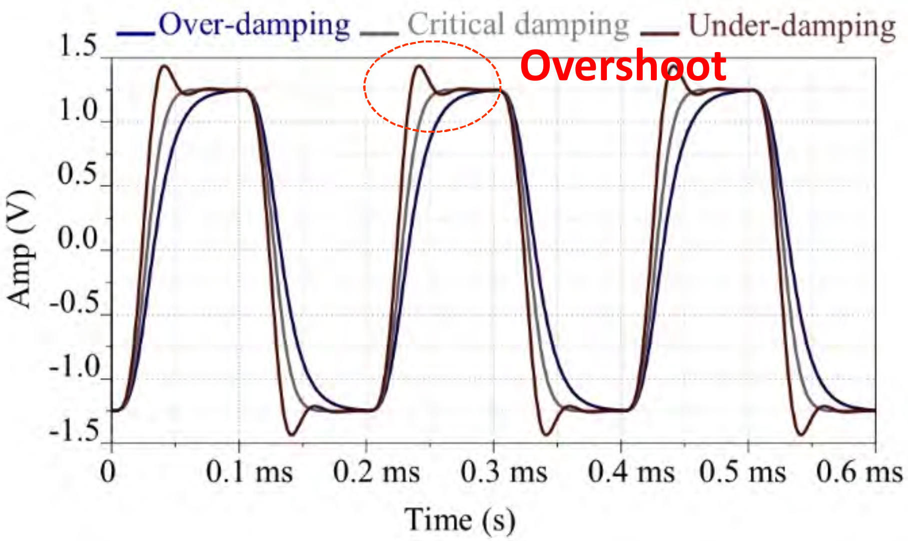
Transfer function and self-loop function

$$H(\omega)=\frac{b_0}{1+L(\omega)};L(\omega)=a_0(j\omega)^4+a_1(j\omega)^3+a_2(j\omega)^2+a_3 j\omega;$$

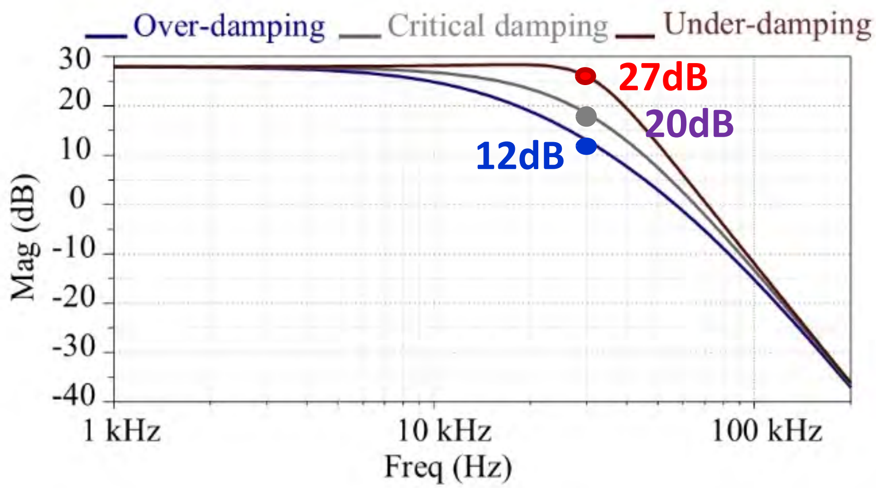
5. Motion Models for Small-Scale Systems

Simulation Results of 4th -Order Active LPF

Simulated transient response

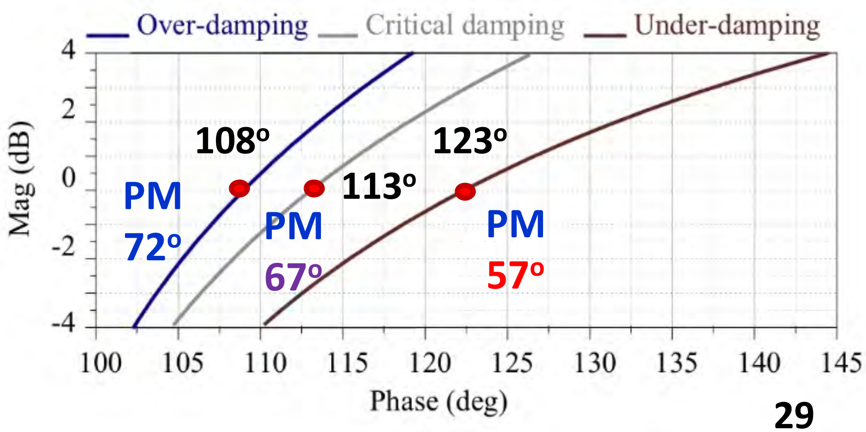


Bode plot of transfer function



Case	Over-damping	Critical damping	Under-damping
Magnitude (transfer function)	12 dB	20 dB	27 dB
Phase margin (self-loop function)	72° (observed at 108°)	67° (observed at 113°)	57° (observed at 123°)

Nichols plot of self-loop function



Outline

1. Research Background
- Motivation, objectives and achievements
2. Proposed Superposition Formulas
- Time, frequency responses, and superposition theorems
3. Motion Models for Large-Scale Systems
- Behaviors of the Earth's Motions
4. Motion Models for Regular-Scale Systems
- Behaviors of mechanical systems
5. Motion Models for Small-Scale Systems
- Behaviors of electronic systems
6. Conclusions

6. Conclusions

This work:

- **Study of limitations of differential equations and loop gain.**
- **Three superposition formulas are also introduced for deriving the transfer functions in physical systems.**
- **Investigation of behaviors of physical systems such as the Earth's motions, pendulum systems, transmission lines, passive and active low-pass filters.**

→ Periodic motion networks are positive feedback systems.

→ Observation of self-loop function can help us optimize the behaviors of high-order mechatronic systems easily.

→Future work:

- **Stability test for dynamic load and other mechatronic systems.**

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Thank you very much!