Study of Behaviors of Motion Models in High-Order Systems

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Outline

1. Research Background
   • Motivation, objectives and achievements

2. Proposed Superposition Formulas
   • Time, frequency responses, and superposition theorems

3. Motion Models for Large-Scale Systems
   • Behaviors of the Earth's Motions

4. Motion Models for Regular-Scale Systems
   • Behaviors of mechanical systems

5. Motion Models for Small-Scale Systems
   • Behaviors of electronic systems

6. Conclusions
1. Research Background

Motivation on High-Order Physical Systems

• Behaviours of complex functions in time and frequency domains are not analysed in detail.
• Limitations of loop gain, differential equations, and heat equations are not pointed out.
• Superposition theorems are not widely used in large-, medium-, and small-scale physical systems.
• Properties of positive and negative impedances, resistance in mechanical systems are not introduced.
• Relationship between periodic motion systems and positive feedback systems is not well investigated.
1. Research Background
Objectives of This Study

• Investigation of some limitations of differential equations and loop gain in motion models
• Study of behaviors of various different scale systems: planets, mechanical systems, and electronic systems
• Models of periodic motion systems using complex functions → Positive feedback systems
→ Ringing test for high-order electronic systems such as transmission lines, passive and active filters.
→ Observation of phase margin at unity gain determines operating regions of high-order systems
1. Research Background

Contributions of This Work

- Three superposition formulas for physical systems.
  - Mechanical superposition formula
  - Electrical superposition formula
  - Multi-source superposition formula
- Proposed motion models for various different scale physical systems:
  - Earth’s motions (large-scale),
  - Simple pendulum systems (regular-scale), and
  - Electronic systems (small-scale)
- Investigation of positive feedback systems
- Ringing test for mechatronic systems such as transmission lines, passive and active low-pass filters
1. Research Background

Limitations of Differential Equations and Loop Gain

Fourth-order differential equation
\[ a_0 y^{\prime\prime\prime} + a_1 y^{\prime\prime} + a_2 y^{\prime} + a_3 y + a_4 y = 0; \]

- Numerical methods don’t solve the high-order differential equations.
- They only approximate the solutions to them.

- Loop gain cannot be used to do the ringing test for mechatronic systems.

Gain reduction in an inverting amplifier

Transfer function
\[ H = \frac{A}{1 + A\beta} \approx \frac{1}{\beta} \]

\( A\beta \): loop gain

Nyquist plot of loop gain

Ringing in mechatronic systems

- Overshoot
- Undershoot

BW = 100 Hz
GBW = 10 MHz

(Unclear operating region)
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2. Proposed Superposition Formulas

Superposition Formulas for Mechatronic Systems

**Force conservation law**

\[
\sum_{l=1}^{m} F_{in}^l(\omega) = \sum_{p=1}^{q} F_{out}^p(\omega) \rightarrow X_{in}(\omega) \sum_{l=1}^{m} \left( \frac{k_l}{j\omega} + c_l + j\omega m_l \right) = X_{out}(\omega) \sum_{p=1}^{q} \left( \frac{k_p}{j\omega} + c_p + j\omega m_p \right)
\]

**Current conservation law**

\[
\sum_{k=1}^{m} I_{in}^k(\omega) = \sum_{p=1}^{q} I_{out}^p(\omega) \rightarrow V_{in}(\omega) \sum_{k=1}^{m} \left( \frac{1}{R_k} + \frac{1}{j\omega L_k} + j\omega C_k \right) = V_{out}(\omega) \sum_{p=1}^{q} \left( \frac{1}{R_p} + \frac{1}{j\omega L_p} + j\omega C_p \right)
\]

**Electrical superposition formula**

**Multi-source superposition formula**

\[
V_O(\omega) \left( \sum_{i=1}^{n} \frac{l}{Z_i} + \sum_{i=1}^{n} \frac{1}{Z_{si}} + \frac{1}{l} \sum_{k=1}^{n} \frac{l}{Z_{pik}} \right) = \sum_{i=1}^{n} \left( \frac{V_i(\omega)}{Z_i} + I_{ai}(\omega) - I_{gi}(\omega) \right)
\]
2. Proposed Superposition Formulas

Time and Frequency Responses of Systems

<table>
<thead>
<tr>
<th>Single-harmonic input signal</th>
<th>High-order mechatronic system</th>
<th>Single-harmonic output signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{in}(t) = A\cos(\omega_0 t + \Theta_0) )</td>
<td>( H(\omega) \rightarrow V_{out}(\omega) )</td>
<td>( V_{out}(t) = A\cos(\omega_0 t + \Theta_0)H(\omega_0) )</td>
</tr>
</tbody>
</table>

(time domain) (all frequency domains) (time domain)

Frequency response of high-order system (all frequency domains)

\[
H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{b_0(j\omega)^n + b_1(j\omega)^{n-1} + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + a_1(j\omega)^{n-1} + \ldots + a_{n-1}(j\omega) + a_n};
\]

Time response of high-order system (single harmonic input wave)

\[
V_{out}(t) = V_{in}(t) \frac{b_0(j\omega_0)^n + b_1(j\omega_0)^{n-1} + \ldots + b_{n-1}(j\omega_0) + b_n}{a_0(j\omega_0)^n + a_1(j\omega_0)^{n-1} + \ldots + a_{n-1}(j\omega_0) + a_n};
\]

\[
\Rightarrow V_{out}(t) = |H(\omega_0)|A\cos(\omega_0 t + \Theta_0 + \angle H(\omega_0));
\]
2. Proposed Superposition Formulas

Self-loop Function in A Transfer Function

Transfer function of high-order system

\[ H(\omega) = \frac{b_0 (j\omega)^n + \ldots + b_{n-1} (j\omega) + b_n}{a_0 (j\omega)^n + \ldots + a_{n-1} (j\omega) + a_n} \]

Simplified transfer function

\[ H(\omega) = \frac{V_{\text{out}}(\omega)}{V_{\text{in}}(\omega)} = \frac{A(\omega)}{1 + L(\omega)} \]

Relationship between output and input

\[ V_{\text{out}}(\omega) = A(\omega) \left[ V_{\text{in}}(\omega) - \frac{L(\omega)}{A(\omega)} V_{\text{out}}(\omega) \right] \]

- Polar chart → Nyquist chart
- Magnitude-frequency plot
- Angular-frequency plot
- Magnitude-angular diagram → Nichols diagram

Variable: angular frequency (\( \omega \))

Graph signal of negative feedback system

\[ A(\omega) : \text{Numerator function} \]
\[ H(\omega) : \text{Transfer function} \]
\[ L(\omega) : \text{Self-loop function} \]
2. Proposed Superposition Formulas
Periodic Motions and Helix Waves

- The Earth’s rotation
- Motion of electronic particles
- Double helix waves in DNA
- Motion of electrons in the crystal structure of silicon atoms

<table>
<thead>
<tr>
<th>Breaking forces in chemical bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Types of breaking forces</td>
</tr>
<tr>
<td>A covalent bond</td>
</tr>
<tr>
<td>A noncovalent bond</td>
</tr>
<tr>
<td>A weak bond</td>
</tr>
</tbody>
</table>
2. Proposed Superposition Formulas

Characteristics of Helix Functions

Positive helix function

\[ S_p(t) = Ahe(\omega_0 t + \theta_0) \]

Negative helix function

\[ S_N(t) = Ahe(-\omega_0 t - \theta_0) \]
# 2. Proposed Superposition Formulas

## Spectra of Common Analog Signals

<table>
<thead>
<tr>
<th>Signal type</th>
<th>Time domain</th>
<th>Half-side spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive helix</td>
<td>$Ahe(\omega_0 t + \theta_0)$</td>
<td>$A\sqrt{2}e^{j(\omega_0 T_0 + \theta_0)}$</td>
</tr>
<tr>
<td>Negative helix</td>
<td>$Ahe(-\omega_0 t - \theta_0)$</td>
<td>$A\sqrt{2}e^{j(-\omega_0 T_0 - \theta_0)}$</td>
</tr>
<tr>
<td>Cosine</td>
<td>$A \cos(\omega_0 t + \theta_0)$</td>
<td>$\frac{A\sqrt{2}}{2}e^{j(\omega_0 T_0 + \theta_0)}$</td>
</tr>
<tr>
<td>Sine</td>
<td>$A \sin(\omega_0 t + \theta_0)$</td>
<td>$\frac{A\sqrt{2}}{2}e^{j(\omega_0 T_0 + \theta_0 + \frac{\pi}{2})}$</td>
</tr>
<tr>
<td>Square</td>
<td>$A_{sq}(\omega_0 t + \theta_0) = A \left(\frac{1}{2} + \sum_{n=1}^{\infty} \frac{\sin[(2n+1)\omega_0 t + \theta_0]}{(2n+1)\pi}\right)$</td>
<td>$</td>
</tr>
</tbody>
</table>
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3. Motion Models for Large-Scale Systems
Behaviors of the Earth’s Motions

Motion of the Earth on its axis

Motion wave

\[ V_{he}(t) = Ahe(\omega_0 t + \theta_0) \]

A is radius of the Earth

Frequency \( f_0 \) is 11.5 \( \mu \)Hz, (or a period of 86400 s)
3. Motion Models for Large-Scale Systems

Motion Model of the Earth on Its Orbit

Loop gain cannot be applied for a large-scale physical system.

Model of the Earth and the Sun

Apply superposition at the node $X_{out}$,

$X_{out}(\omega) = \left[ j\omega m + c + \frac{k}{j\omega} \right] X_{out}(\omega) = \left[ c + \frac{k}{j\omega} \right] X_{in}(\omega)$;

Transfer function

$H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{b_0 j\omega + 1}{1 + a_0 (j\omega)^2 + a_1 j\omega}$;

Self-loop function

$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega$;

Where,

$b_0 = \frac{c}{k}; a_0 = \frac{m}{k}; a_1 = \frac{c}{k}$.
3. Motion Models for Large-Scale Systems
Behaviors of a 2nd-Order Mechanical System

When, \[ a_0 = \frac{m}{k}, \]
\[ b_0 = \frac{c}{k} = a_1 = \frac{c}{k} = 0; \]

Transfer function

\[ H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{1}{1 - \frac{m}{k} \omega^2} \]

Self-loop function

\[ L(\omega) = -\frac{m}{k} \omega^2; \]

Relationship between output and input

\[ X_{out}(\omega) = X_{in}(\omega) + \frac{m}{k} \omega^2 X_{out}(\omega); \]

Graph signal of positive feedback system

Positive feedback system

Motion wave \[ V_{he}(t) = A \text{he}(\omega_0 t + \theta_0) \]

A is radius of the Earth’s orbit, and frequency \( f_0 \) is 31.5 nHz
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4. Motion Models for Regular-Scale Systems

Analysis of a 2\textsuperscript{nd} -Order Pendulum System

Loop gain cannot be applied for a high-order mechanical system.

Model of pendulum system

Apply superposition at the node $X_{\text{out}}$.

Transfer function

$$H(\omega) = \frac{X_{\text{out}}(\omega)}{X_{\text{in}}(\omega)} = \frac{b_0 j\omega + 1}{1 + L(\omega)};$$

Self-loop function

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Where,

$$b_0 = \frac{c}{k}; a_0 = \frac{m}{k}; a_1 = \frac{c}{k}$$

Mechanical superposition formula

$$X_{\text{in}}(\omega) \sum_{l=1}^{m} \left( k_l \right) \left( \frac{1}{j\omega} + c_l + j\omega m_l \right) = X_{\text{out}}(\omega) \sum_{p=1}^{q} \left( k_p \right) \left( \frac{1}{j\omega} + c_p + j\omega m_p \right)$$
4. Motion Models for Regular-Scale Systems
Behaviors of a 2nd-Order Pendulum System

When,
\[ a_0 = \frac{m}{k} \]
\[ b_0 = \frac{c}{k} = a_1 = \frac{c}{k} = 0; \]

The simplified transfer function
\[ H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{1}{1 - a_0 \omega^2}; \]

Relationship between output and input
\[ X_{out}(\omega) = X_{in}(\omega) + \frac{m}{k} \omega^2 X_{out}(\omega); \]

Here, \( l = 1\text{m}, m = 1 \text{ kg}, g = 9.8 \text{ m/s}^2 \)
4. Motion Models for Regular-Scale Systems

Analysis of a 4th-Order Pendulum System

Model of double-pendulum system

Apply superposition at each node, we get

\[
\left( \frac{k_1}{j\omega} + m_1 j\omega + c_1 + \frac{k_2}{j\omega} \right) X_{out1}(\omega) = \frac{k_2}{j\omega} X_{out2}(\omega); \\
\left( \frac{k_2}{j\omega} + m_2 j\omega + c + \frac{k_3}{j\omega} \right) X_{out2}(\omega) = \frac{k_2}{j\omega} X_{out1}(\omega) \\
+ \left( c + \frac{k_1 + k_2 + k_3}{j\omega} \right) X_{in}(\omega);
\]

Transfer functions and self-loop function

\[
H_1(\omega) = \frac{X_{out1}(\omega)}{X_{in}(\omega)} = \frac{b_0 j\omega + b_1}{1 + a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega};
\]

\[
H_2(\omega) = \frac{X_{out2}(\omega)}{X_{in}(\omega)} = \frac{b_2 (j\omega)^3 + b_3 (j\omega)^2 + b_4 j\omega + b_5}{1 + a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega};
\]

\[
L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega;
\]

Pascal’s Triangle

<table>
<thead>
<tr>
<th></th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
4. Motion Models for Regular-Scale Systems

Behaviors of a 4\textsuperscript{th} -Order Pendulum System

**Transfer function of 1\textsuperscript{st} pendulum**

\[ H_1(\omega) = \frac{b_0 j\omega + b_1}{1 + a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega}; \]

As \( c = 0 \), then \( b_0, a_1, a_3 \) are neglected.

\[ H_1(\omega) = \frac{X_{\text{out1}}(\omega)}{X_{\text{in}}(\omega)} = \frac{b_1}{1 + a_0 \omega^4}; \]

**Relationship between output and input**

\[ X_{\text{out1}}(\omega) = \frac{b_1}{1 + a_0 \omega^4} \left( X_{\text{in}}(\omega) + \frac{a_2 \omega^2}{b_1} X_{\text{out}}(\omega) \right); \]

**Graph signal of positive feedback system**

\[ X_{\text{in}}(\omega) \]

\[ + \]

\[ \frac{b_1}{1 + a_0 \omega^4} \]

\[ + \]

\[ \frac{a_2 \omega^2}{b_1} \]

\[ X_{\text{out1}}(\omega) \]

**Variables of double-pendulum system**

<table>
<thead>
<tr>
<th>Var</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_0 )</td>
<td>( \frac{k_c}{k_1 k_2 + k_2 k_3 + k_3 k_1} )</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>( \frac{k_c}{k_1 k_2 + k_2 k_3 + k_3 k_1} )</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>( \frac{k_c}{k_1 k_2 + k_2 k_3 + k_3 k_1} )</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>( \frac{k_c}{k_1 k_2 + k_2 k_3 + k_3 k_1} )</td>
</tr>
<tr>
<td>( b_4 )</td>
<td>( \frac{k_c}{k_1 k_2 + k_2 k_3 + k_3 k_1} )</td>
</tr>
</tbody>
</table>

**Harmonic motion of pendulum system**

Here,

\[ l_1 = l_2 = 1m \]

\[ m_1 = m_2 = 1 \text{ kg} \]

\[ g = 9.8 \text{ m/s}^2 \]
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Analysis of a 2\textsuperscript{nd} -Order Passive Low-Pass Filter

Loop gain cannot be applied for a passive filter.

Electrical superposition formula

\[ \sum_{k=1}^{m} I_{in}^{(\omega)} = \sum_{p=1}^{q} I_{out}^{(\omega)} \]

\[ V_{in}^{(\omega)} \sum_{k=1}^{m} \left( \frac{1}{R_k} + \frac{1}{j\omega L_k} + j\omega C_k \right) = V_{out}^{(\omega)} \sum_{p=1}^{q} \left( \frac{1}{R_p} + \frac{1}{j\omega L_p} + j\omega C_p \right) \]

Apply superposition at the node \( V_{out}^{\omega} \)

\[ V_{out}^{(\omega)} \left( \frac{1}{R + j\omega L} + j\omega C \right) = V_{in}^{(\omega)} \frac{1}{R + j\omega L}; \]

Transfer function and self-loop function

\[ H^{(\omega)} = \frac{V_{out}^{(\omega)}}{V_{in}^{(\omega)}} = \frac{1}{1 + L^{(\omega)}}; \]

\[ L^{(\omega)} = a_0 \left( j\omega \right)^2 + a_1 j\omega; \]

\[ a_0 = LC; a_1 = RC; \]
5. Motion Models for Small-Scale Systems

Measurement Results of a 2\textsuperscript{nd} -Order Passive LPF

**Simulated transient response**

- Over-damping
- Critical damping
- Under-damping

**Bode plot of transfer function**

- Over-damping
- Critical damping
- Under-damping

**Nichols plot of self-loop function**

<table>
<thead>
<tr>
<th>Case</th>
<th>Over-damping</th>
<th>Critical damping</th>
<th>Under-damping</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnitude (transfer function)</strong></td>
<td>-12 dB</td>
<td>-6 dB</td>
<td>1 dB</td>
</tr>
<tr>
<td><strong>Phase margin (self-loop function)</strong></td>
<td>80° (observed at 100°)</td>
<td>60° (observed at 120°)</td>
<td>31° (observed at 149°)</td>
</tr>
</tbody>
</table>
5. Motion Models for Small-Scale Systems

Analysis of Schematic Model of Transmission Line

**Simplified model of coaxial line**

**Parameters of the schematic model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_a )</td>
<td>0.25 nH</td>
<td>( Z_0 )</td>
<td>50 Ω</td>
</tr>
<tr>
<td>( R_a )</td>
<td>1.82 mΩ</td>
<td>Loss</td>
<td>1.6 mdB</td>
</tr>
<tr>
<td>( C_a )</td>
<td>0.1 pF</td>
<td>Skin depth</td>
<td>1.7 um</td>
</tr>
<tr>
<td>( G_a )</td>
<td>6.81 uM/Ω</td>
<td>Delay</td>
<td>0.2 ns</td>
</tr>
</tbody>
</table>

**Parameters of the physical model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal width (W)</td>
<td>0.71 mm</td>
<td>Substrate thickness (H)</td>
<td>2</td>
</tr>
<tr>
<td>Trace length (L)</td>
<td>40 mm</td>
<td>Dielectric constant (( \varepsilon_r ))</td>
<td>4.6</td>
</tr>
<tr>
<td>Metal thickness (( T_m ))</td>
<td>35 mm</td>
<td>Loss tangent (Tan)</td>
<td>0.01</td>
</tr>
<tr>
<td>Metal resistivity</td>
<td>17.2 nΩ</td>
<td>Frequency</td>
<td>1.5 GHz</td>
</tr>
<tr>
<td>Surface roughness</td>
<td>0.1 um</td>
<td>Characteristic Impedance</td>
<td>50 Ω</td>
</tr>
</tbody>
</table>
5. Motion Models for Small-Scale Systems

Ringing Test for Coaxial Line

Physical model of transmission line

General characteristic impedance

\[
Z_0 = \sqrt{\frac{R_a + j\omega L_a}{G_a + j\omega C_a}} \approx \sqrt{\frac{L_a}{C_a}};
\]

Apply superposition at the node \(X_{\text{out}}\)

\[
V_{\text{out}}(\omega) \left(\frac{1}{R_S + Z_0} + \frac{1}{R_L}\right) = V_{\text{in}}(\omega) \frac{1}{R_S + Z_0};
\]

Ringing test for the coaxial line

Transfer function and self-loop function

\[
H(\omega) = \frac{V_{\text{out}}(\omega)}{V_{\text{in}}(\omega)} = \frac{1}{1 + L(\omega)};
\]

\[
L(\omega) = \frac{R_a + j\omega L_a}{G_a + j\omega C_a} \cdot \frac{R_S + \sqrt{R_a + j\omega L_a}}{R_L}.
\]
5. Motion Models for Small-Scale Systems

Simulation Results of Coaxial Line

Simulated transient response

<table>
<thead>
<tr>
<th>Case</th>
<th>Over-damping</th>
<th>Critical damping</th>
<th>Under-damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude (transfer function)</td>
<td>-6 dB</td>
<td>-1 dB</td>
<td>12 dB</td>
</tr>
<tr>
<td>Phase margin (self-loop function)</td>
<td>80° (observed at 100°)</td>
<td>60° (observed at 120°)</td>
<td>11° (observed at 169°)</td>
</tr>
</tbody>
</table>

Bode plot of transfer function

Nichols plot of self-loop function

- Overshoot
- PM 100°
- PM 120°
- PM 169°
5. Motion Models for Small-Scale Systems
Analysis of Fourth-Order Active Low-Pass Filter

Schematic of Akerberg-Mossberg LPF

Apply superposition at each node,

\[
V_A \left( \frac{1}{R_1} + \frac{1}{R_2} + j\omega C_1 + \frac{1}{R_6} \right) = V_{in+}(\omega) - V_B \left( \frac{1}{R_2} + j\omega C_1 \right) - \frac{V_G}{R_6};
\]

\[
V_C \left( \frac{1}{R_3} + j\omega C_2 \right) = \frac{V_B}{R_3} - V_D j\omega C_2; \quad V_B = 2V_A A(\omega); \quad V_G = 2V_C A(\omega);
\]

\[
V_F \left( \frac{1}{R_4} + \frac{1}{R_5} \right) = \frac{V_G}{R_4} - \frac{V_D}{R_4}; \quad V_D = 2V_F A(\omega); \quad V_{out+}(\omega) = 2V_J A(\omega);
\]

\[
V_H \left( \frac{1}{R_7} + \frac{1}{R_8} + j\omega C_3 + \frac{1}{R_{12}} \right) = \frac{V_G}{R_7} - \frac{V_I}{R_8} \left( \frac{1}{R_8} + j\omega C_3 \right) - \frac{V_{out+}(\omega)}{R_{12}};
\]

\[
V_J \left( \frac{1}{R_9} + j\omega C_4 \right) = \frac{V_L}{R_9} - V_K j\omega C_4; \quad V_I = 2V_H A(\omega);
\]

\[
V_L \left( \frac{1}{R_{10}} + \frac{1}{R_{11}} \right) = \frac{V_{out+}(\omega)}{R_{11}} - \frac{V_K}{R_{10}}; \quad V_k = 2V_L A(\omega);
\]

Transfer function and self-loop function

\[
H(\omega) = \frac{b_0}{1 + L(\omega)}; \quad L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega;
\]
5. Motion Models for Small-Scale Systems

Simulation Results of 4th-Order Active LPF

**Simulated transient response**

- **Over-damping**
- **Critical damping**
- **Under-damping**

**Overshoot**

- **Bode plot of transfer function**
  - **Over-damping**
  - **Critical damping**
  - **Under-damping**

- **Magnitude (transfer function)**
  - Over-damping: 12 dB
  - Critical damping: 20 dB
  - Under-damping: 27 dB

- **Phase margin (self-loop function)**
  - Over-damping: 72° (observed at 108°)
  - Critical damping: 67° (observed at 113°)
  - Under-damping: 57° (observed at 123°)

**Nichols plot of self-loop function**

- **PM**
  - Over-damping: 72°
  - Critical damping: 67°
  - Under-damping: 57°
Outline

1. Research Background
   • Motivation, objectives and achievements

2. Proposed Superposition Formulas
   • Time, frequency responses, and superposition theorems

3. Motion Models for Large-Scale Systems
   • Behaviors of the Earth's Motions

4. Motion Models for Regular-Scale Systems
   • Behaviors of mechanical systems

5. Motion Models for Small-Scale Systems
   • Behaviors of electronic systems

6. Conclusions
6. Conclusions

This work:

• Study of limitations of differential equations and loop gain.
• Three superposition formulas are also introduced for deriving the transfer functions in physical systems.
• Investigation of behaviors of physical systems such as the Earth’s motions, pendulum systems, transmission lines, passive and active low-pass filters.

遄 Periodic motion networks are positive feedback systems.
遄 Observation of self-loop function can help us optimize the behaviors of high-order mechatronic systems easily.

遄 Future work:
• Stability test for dynamic load and other mechatronic systems.
References


Thank you very much!