Application of Residue Sampling to RF/AMS Device Testing

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Outline

1. Research Objective
2. Residue Sampling
   - Chinese Remainder Theorem
   - Residue Sampling Principle
3. Application to RF/AMS Device Testing
   - Two-Tone Signal Testing
   - High Frequency Narrow-Band Signal Testing
4. Conclusion

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Research Objective

Next Generation Communication System "6G"

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High frequencies in communication systems

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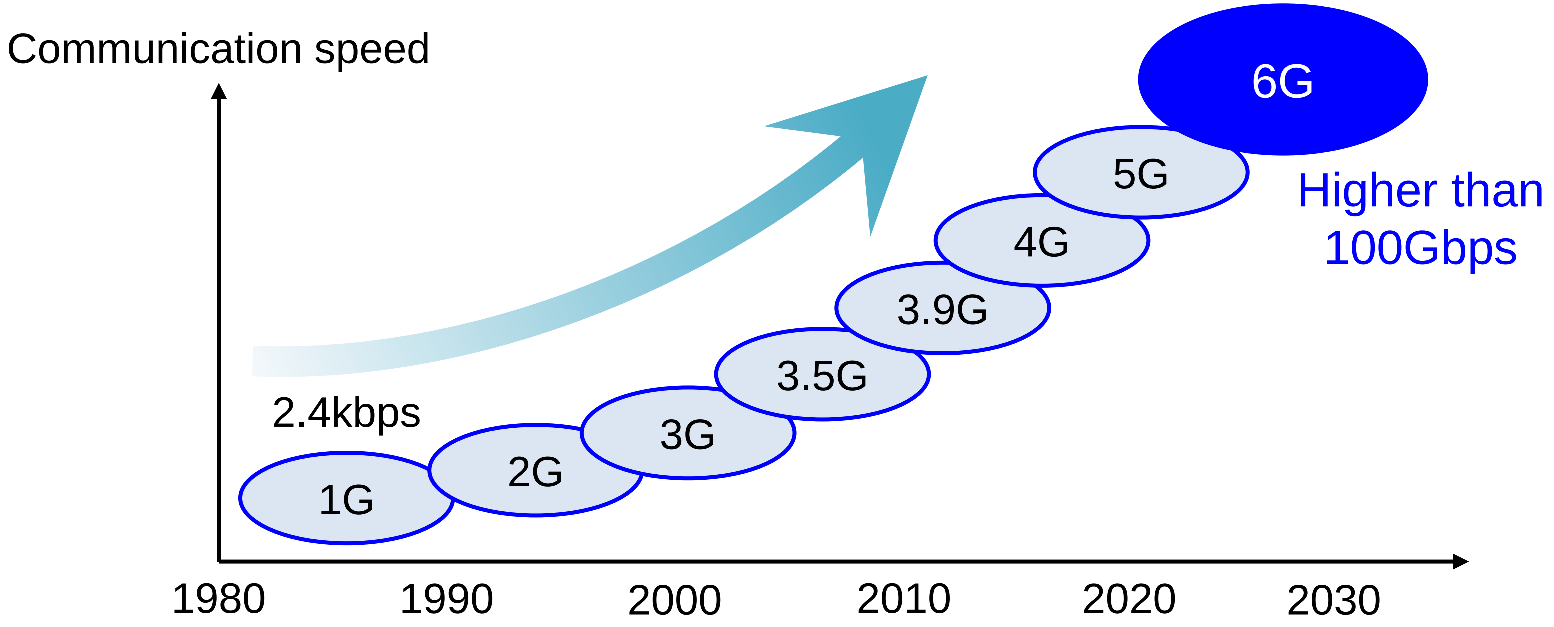
Development of their efficient testing method



Research Goal

Low cost testing for high-frequency signals

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Application of residue sampling
to RF/AMS device testing

RF: Radio Frequency
AMS: Analog/Mixed-Signal

Residue sampling:
Measure high-frequency signal
with multiple low-frequency sampling clocks
utilizing spectrum folding

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Chinese Remainder Theorem

Sun Tzu

Chinese arithmetic book
'Sun Tzu calculation'

孫子算経

"When dividing by 3, its residue is 2,
dividing by 5, its residue is 3,
dividing by 7,its residue is 2.
What is the original number ?"

孫子算經卷上

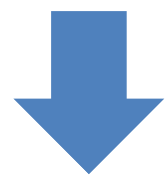
唐朝議大夫行太史令上輕車都尉臣李淳風等奉 勅注釋

度之所起起於忽欲知其忽蠶吐絲爲忽十忽
爲一絲十絲爲一毫十毫爲一氂十氂爲一分
十分爲一寸十寸爲一尺十尺爲一丈十丈爲
一引五十尺爲一端四十尺爲一疋六尺爲一
步二百四十步爲一畝三百步爲一里
稱之所起起於黍十黍爲一絫十絫爲一銖二
十四銖爲一兩十六兩爲一斤三十斤爲一鈞

Sun Tzu calculation

Answer 23

Generalization



Chinese Remainder Theorem

How to use Chinese remainder theorem

He quickly found out how many soldiers were.

Sun Tzu

“Divide by 3.”

Remainder: 2

...

How to use Chinese remainder theorem

He quickly found out how many soldiers were.

Sun Tzu

“Divide by 5.”

Remainder: 3

How to use Chinese remainder theorem

He quickly found out how many soldiers were.

“Divide by 7.”

“There are 23 soldiers in all.”

Sun Tzu

Remainder: 2

Example of Residue Number System

$\mathrm{mod}_3 23 = 2,\ \mathrm{mod}_5 23 = 3,\ \mathrm{mod}_7 23 = 2$

- Natural numbers
  3, 5, 7 (relatively prime)
  $N = 3 \times 5 \times 7 = 105$
- k $(\, 0 \leq k \leq N-1\ (= 104)\,)$

a : Remainder of k divided by 3 $a = \mathrm{mod}_3(k)$
b : divided by 5 $b = \mathrm{mod}_5(k)$
c : divided by 7 $c = \mathrm{mod}_7(k)$

$k \Longleftrightarrow (a, b, c)$

one to one

Chinese remainder theorem

a	b	c	k
0	0	1	15
1	1	2	16
2	2	3	17
0	3	4	18
1	4	5	19
2	0	6	20
0	1	0	21
1	2	1	22
2	**3**	**2**	**23**
0	4	3	24
1	0	4	25
2	1	5	26
0	2	6	27
1	3	0	28
2	4	1	29

Residue number system

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   - Residue Sampling Principle

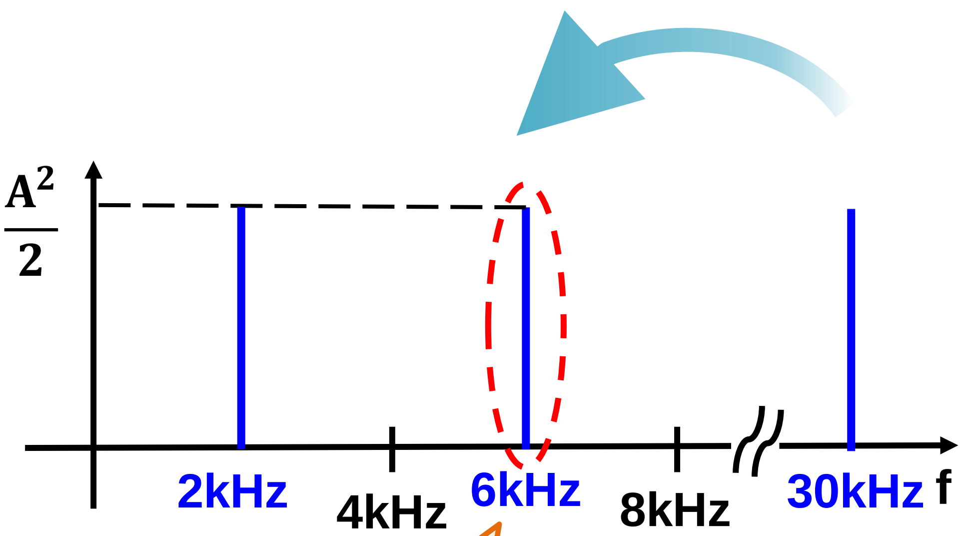
[1] Y. Abe, S. Katayama, C. Li, A. Kuwana, H. Kobayashi, "Frequency Estimation Sampling Circuit Using Analog Hilbert Filter and Residue Number System", IEEE ASICON (Oct. 2019).

Spectrum Folding by Sampling

Sampling frequency : 8 kHz

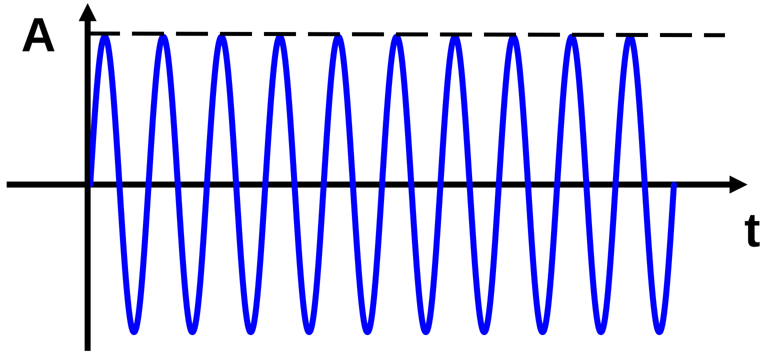
FFT

Spectrum is folded
within sampling frequency band



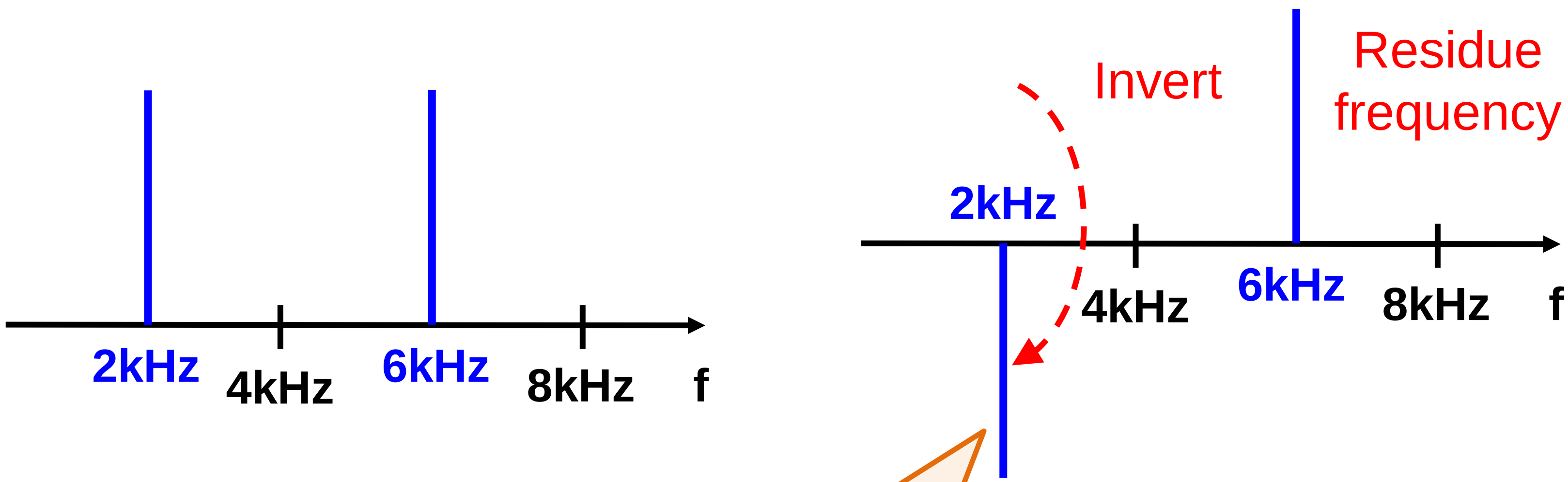
Residue frequency
$\mathrm{mod}_8 30 = 6$

Waveform frequency : 30kHz



Complex FFT of $j \times \sin(2\pi f_{in} t)$

Complex FFT
Input frequency : 30 kHz
Sampling frequency : 8 kHz

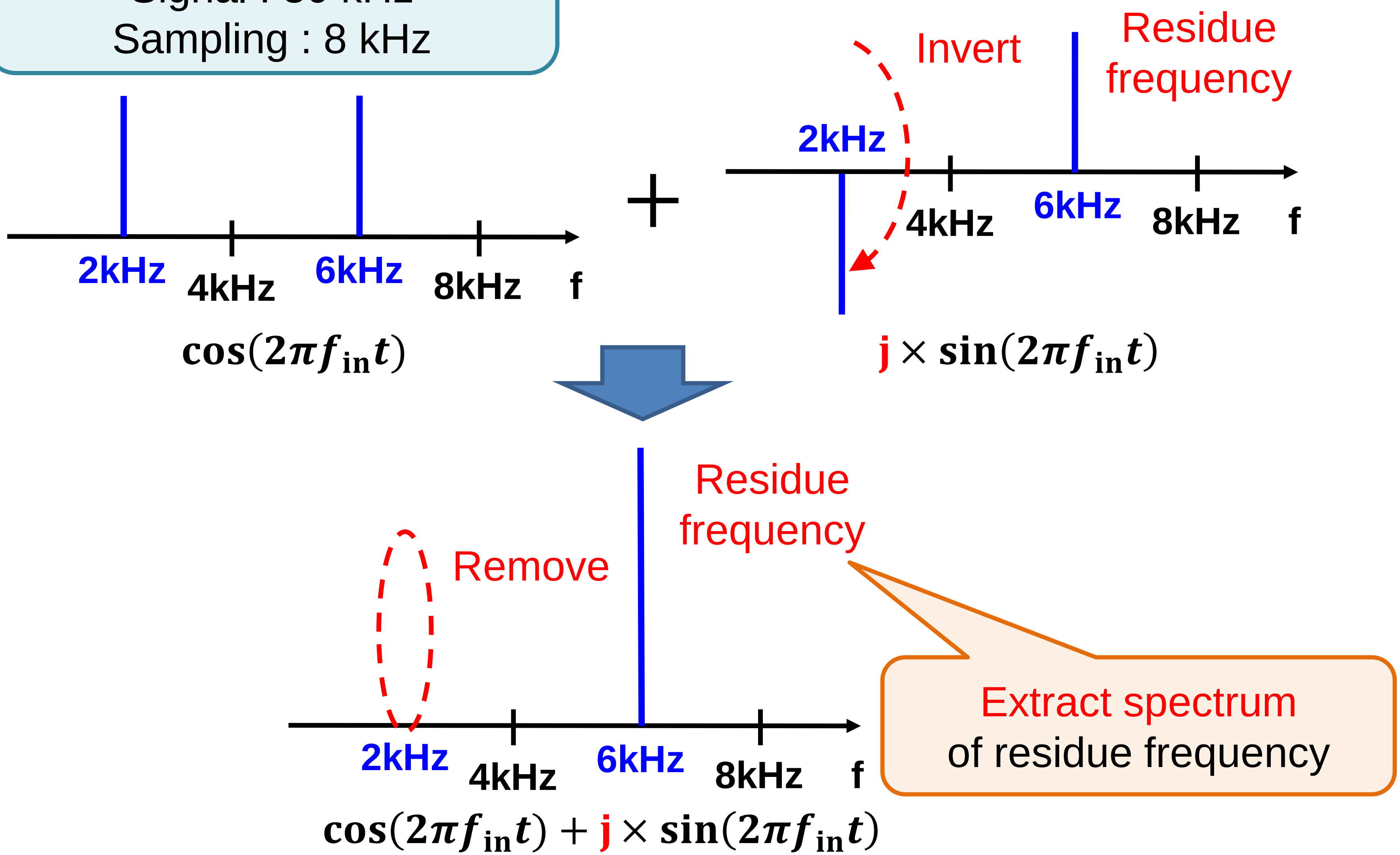


$\cos(2\pi f_{in} t)$ $j \times \sin(2\pi f_{in} t)$

Inverted spectrum
anti-symmetric at Nyquist frequency

Complex FFT of $\cos(2\pi f_{\text{in}} t) + \text{j} \times \sin(2\pi f_{\text{in}} t)$

Complex FFT
Signal : 30 kHz
Sampling : 8 kHz



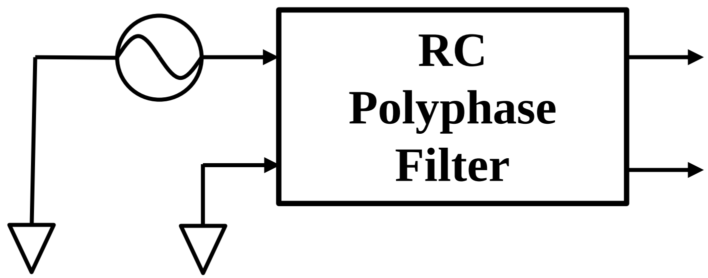
How to Generate $j \times \sin(2\pi f_{in} t)$

Use analog Hilbert filter

↓

RC polyphase filter

$\mathbf{I_{in} = \cos(\omega t)}$



$\mathbf{I_{out}} = A\cos(\omega t + \theta)$

$\mathbf{Q_{out}} = A\sin(\omega t + \theta)$

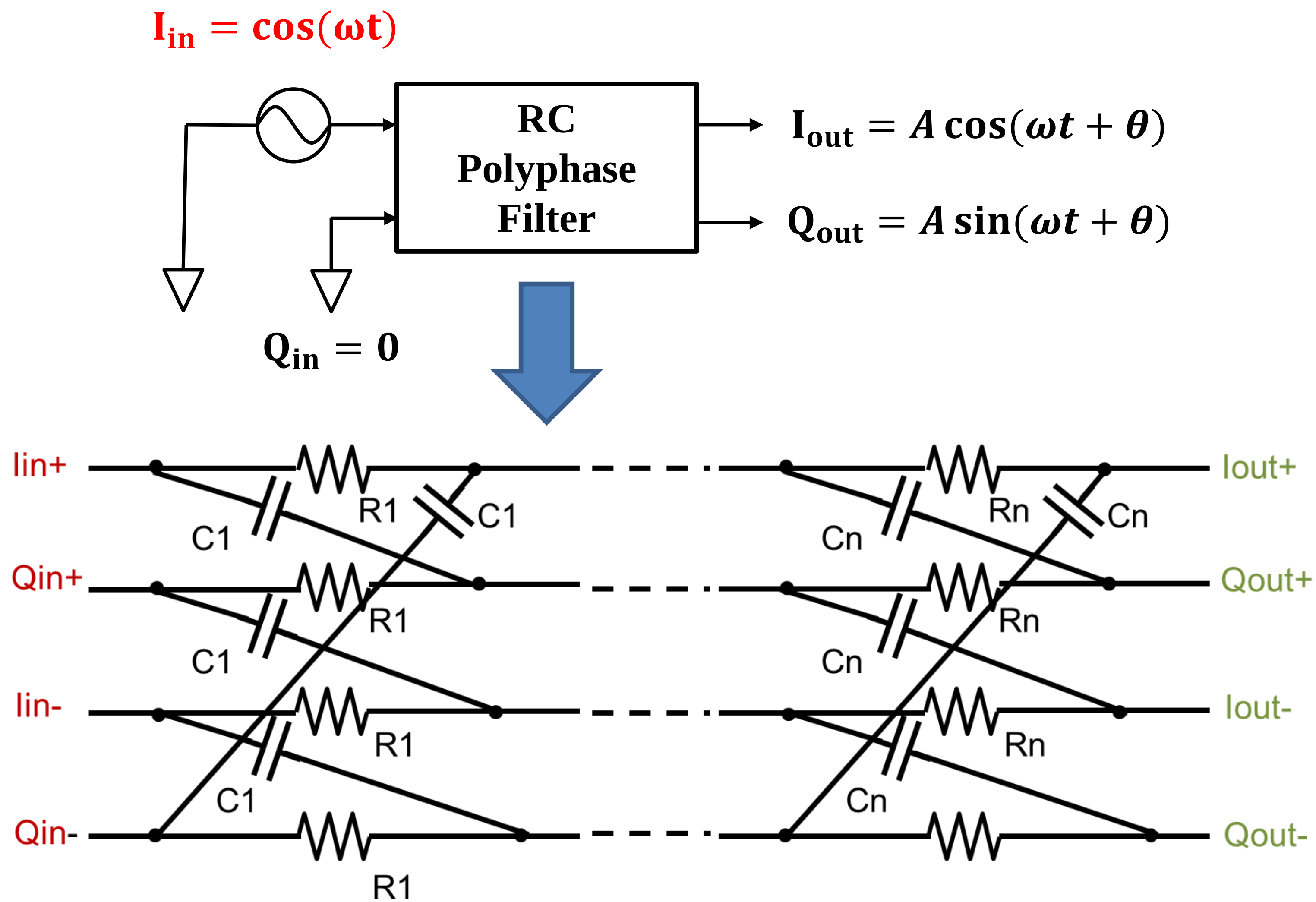
$\mathbf{Q_{in} = 0}$

David Hilbert
1862 - 1943

> Generate in-phase and quadrature waves from a cosine wave

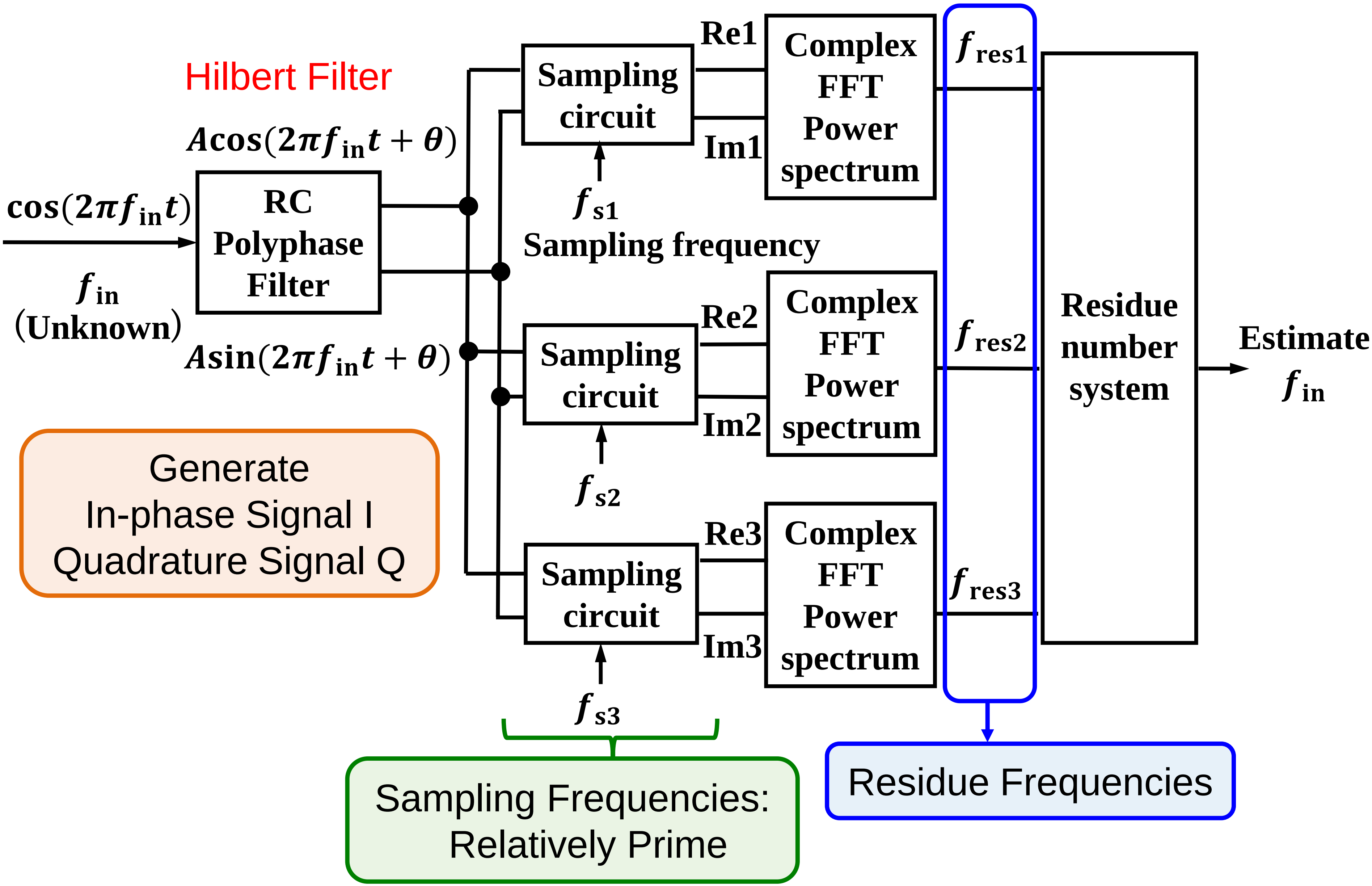
[2] Y. Tamura, R. Sekiyama, K. Asami, H. Kobayashi, "RC Polyphase Filter As Complex Analog Hilbert Filter", IEEE ICSICT (Oct. 2016).

RC Polyphase Filter



Passive analog bandstop filter

Residue Sampling Circuit



Frequency Estimation by Residue Number System

Residue frequencies

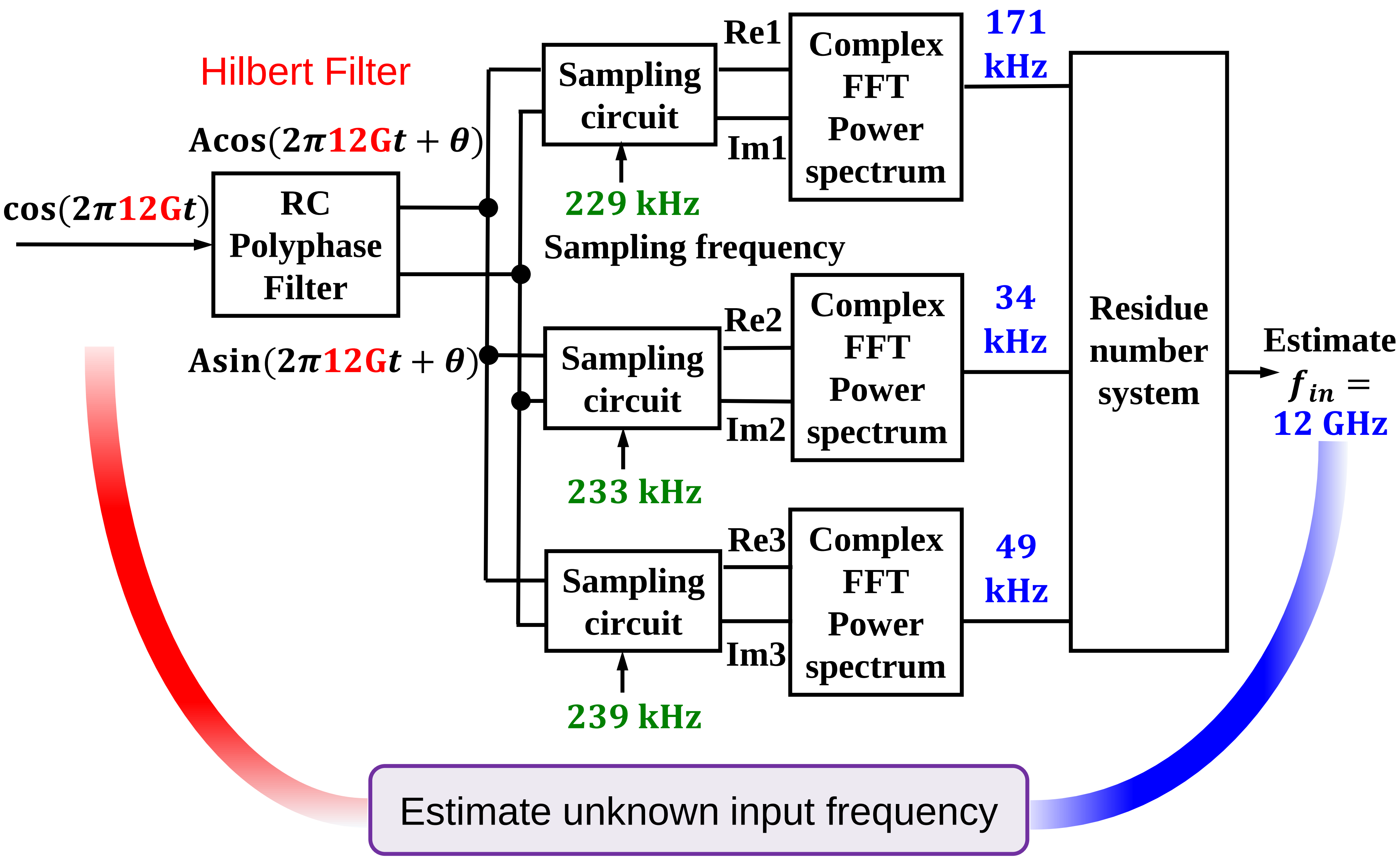
171 kHz, 34 kHz, 49 kHz

Input frequency estimation using residue frequencies and residue number system

Estimate input frequency 12 GHz

a [kHz]	b [kHz]	c [kHz]	k [kHz]
0	0	0	0
1	1	1	1
2	2	2	2
⋮	⋮	⋮	⋮
169	[illegible]	47	11999998
170	33	48	11999999
171	**34**	**49**	**12000000**
172	35	50	12000001
173	36	51	12000002
⋮	⋮	⋮	⋮
226	230	[illegible]	12752320
227	231	237	12752321
228	232	238	12752322

Simulation Result Overview



Frequency Resolution of Residue Sampling

Frequency resolution: $\frac{f_s}{N} = \frac{1}{t_{max}}$

N: Sampling points

t_{max} : Measurement time

N: large ➡ Frequency resolution $\frac{f_s}{N}$: fine

Fine resolution frequency measurement can be achieved by taking a large number of sampled data

Example : Input signal frequency $f_{in} = 19.386$ [kHz]

f_s [kHz]	Theorical residue frequency [kHz]	Residue frequency [kHz] (bin/number of points)			
		$t_{max} = 1$ [ms]	$t_{max} = 10$ [ms]	$t_{max} = 100$ [ms]	$t_{max} = 1000$ [ms]
3	1.386	1 (2/3)	1.4 (15/30)	1.39 (140/300)	1.386 (1387/3000)
5	4.386	4 (5/5)	4.4 (45/50)	4.39 (440/500)	4.386 (4387/5000)
7	5.386	5 (6/7)	5.4 (55/70)	5.39 (540/700)	5.386 (5387/7000)
11	8.386	8 (9/11)	8.4 (85/110)	8.39 (840/1100)	8.386 (8387/11000)
13	6.386	6 (7/13)	6.4 (65/130)	6.39 (640/1300)	6.386 (6387/13000)

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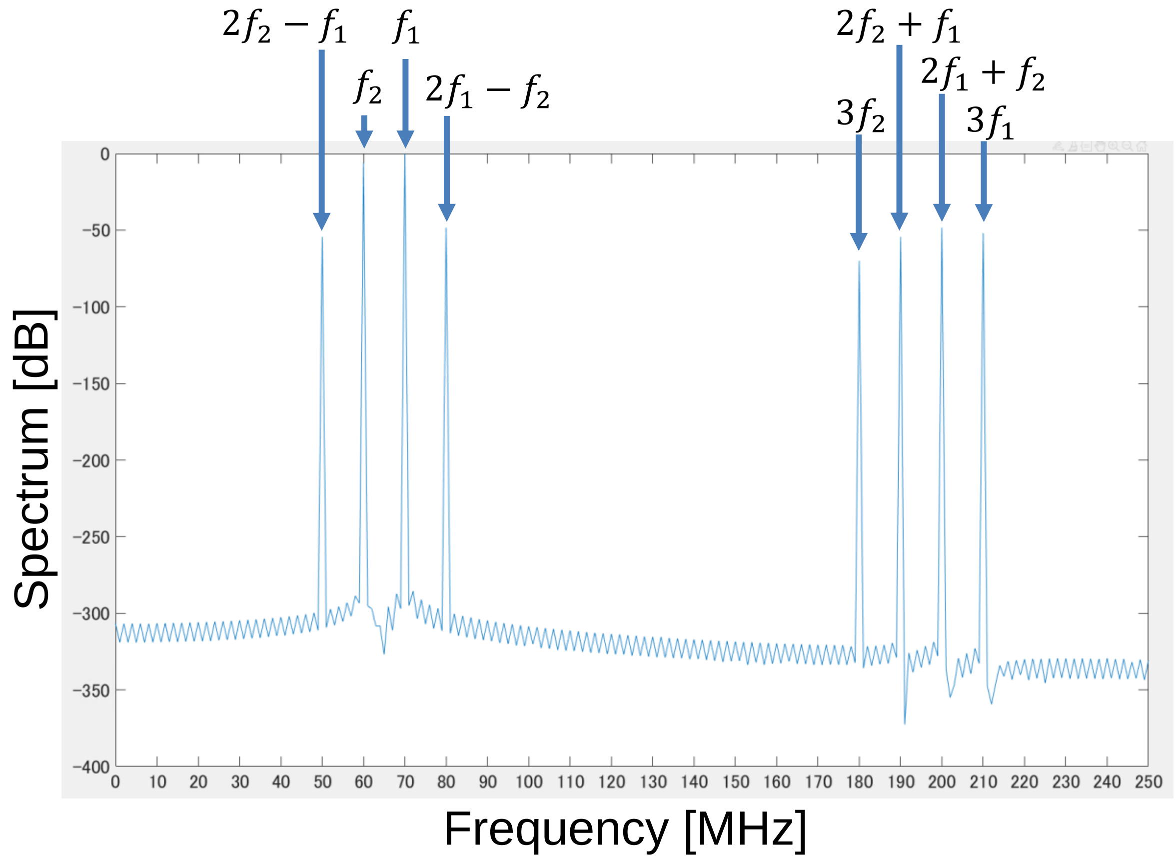
Two-Tone Test by Residue Sampling

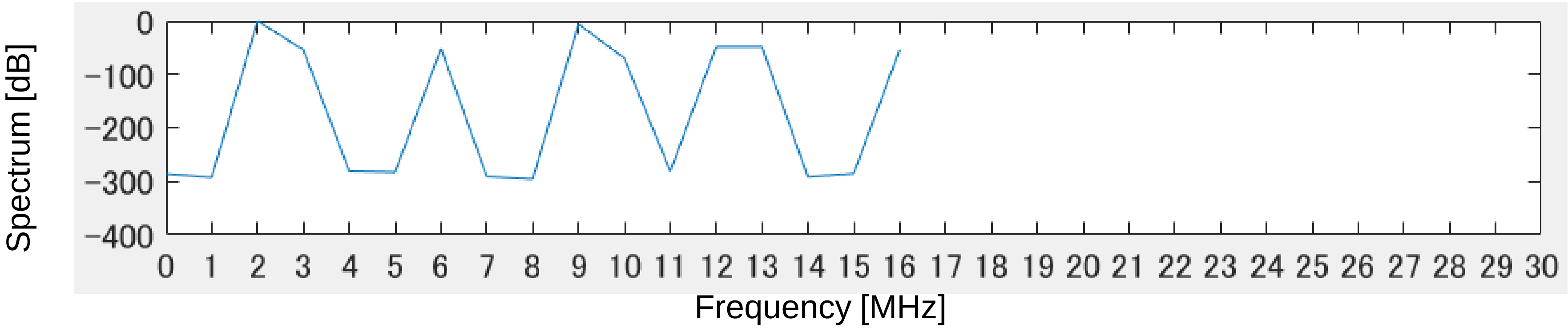
Consider multi-tone testing for residue sampling.
First, simulate two-tone test.

Input: $x(t) = \cos(2\pi f_1 t) + 0.5\cos(2\pi f_2 t), f_1 = 70$ MHz, $f_2 = 60$ MHz
Output: $y(t) = x(t) - 0.01\, x(t)^3$



Two-Tone Test Simulation ($f_{s1} = 17$ MHz)

Sampling Frequency: $f_{s1} = 17$ MHz

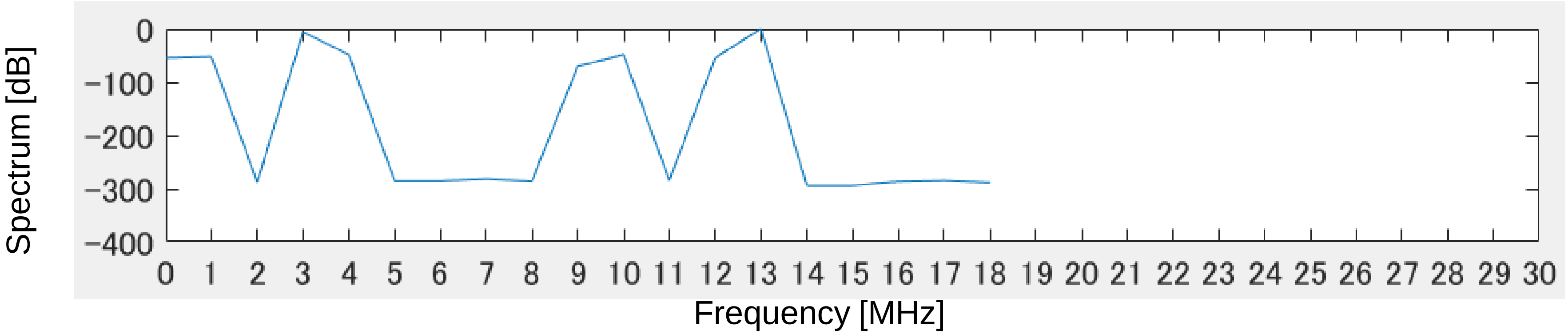


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	2	0.00
f_2	60	-6.07	9	-6.07
$3f_1$	210	-51.9	6	-51.9
$3f_2$	180	-70.0	10	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	12	-48.4
$2f_2 - f_1$	50	-54.4	16	-54.4
$2f_1 + f_2$	200	-48.4	13	-48.4
$2f_2 + f_1$	190	-54.4	3	-54.4

Two-Tone Test Simulation $(f_{s2} = 19\ \mathrm{MHz})$

Sampling Frequency: $f_{s2} = 19\ \mathrm{MHz}$

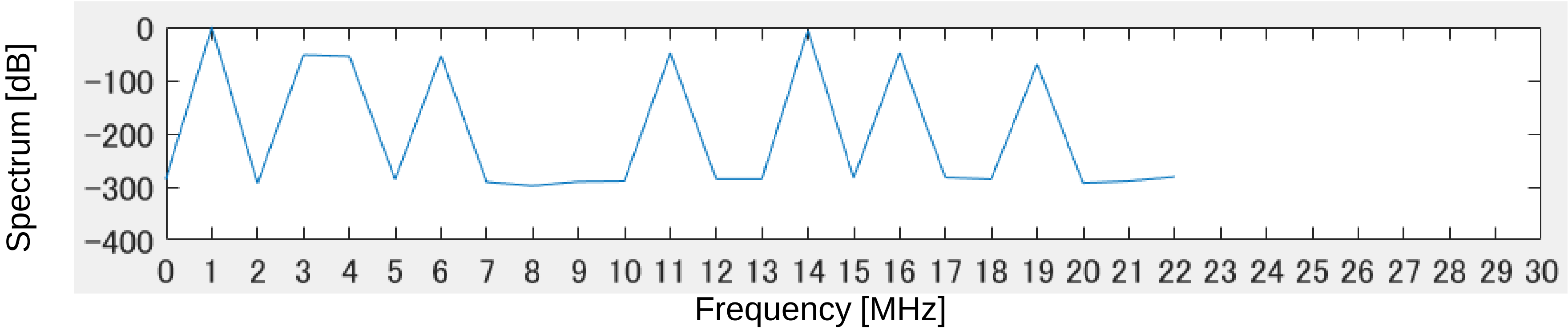


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	13	0.00
f_2	60	-6.07	3	-6.07
$3f_1$	210	-51.9	1	-51.9
$3f_2$	180	-70.0	9	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	4	-48.4
$2f_2 - f_1$	50	-54.4	12	-54.4
$2f_1 + f_2$	200	-48.4	10	-48.4
$2f_2 + f_1$	190	-54.4	0	-54.4

Two-Tone Test Simulation $(f_{s3} = 23\ \mathrm{MHz})$

Sampling Frequency: $f_{s3} = 23\ \mathrm{MHz}$

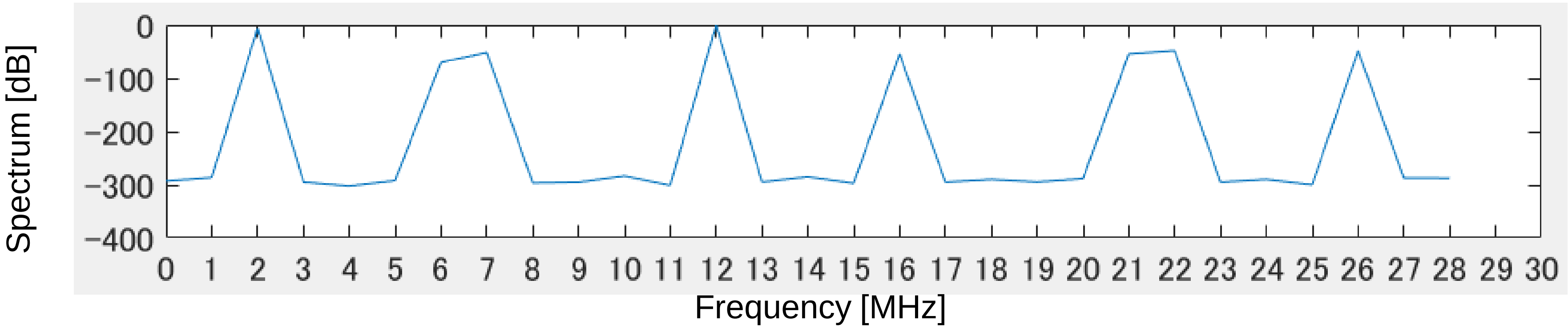


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	1	0.00
f_2	60	-6.07	14	-6.07
$3f_1$	210	-51.9	3	-51.9
$3f_2$	180	-70.0	19	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	11	-48.4
$2f_2 - f_1$	50	-54.4	4	-54.4
$2f_1 + f_2$	200	-48.4	16	-48.4
$2f_2 + f_1$	190	-54.4	6	-54.4

Two-Tone Test Simulation ($f_{s4} = 29$ MHz)

Sampling Frequency: $f_{s4} = 29$ MHz

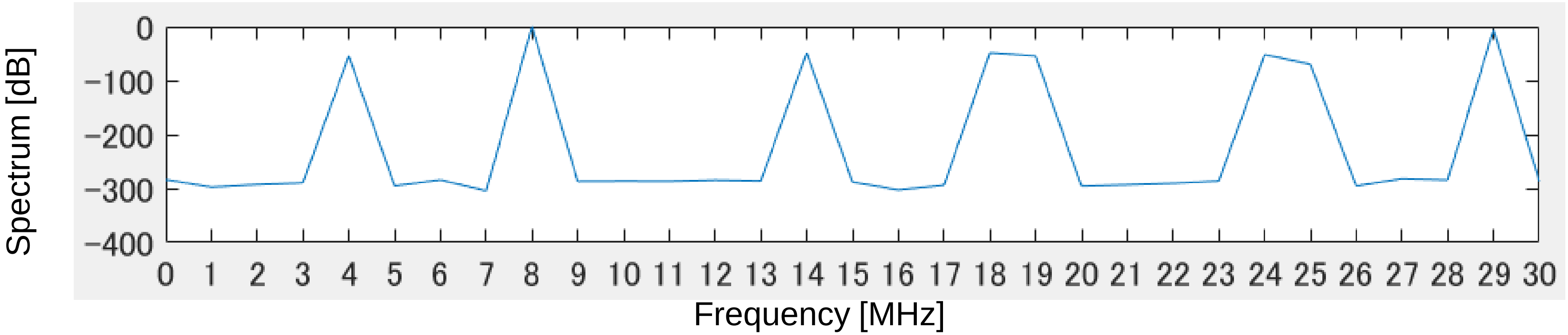


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	12	0.00
f_2	60	-6.07	2	-6.07
$3f_1$	210	-51.9	7	-51.9
$3f_2$	180	-70.0	6	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	22	-48.4
$2f_2 - f_1$	50	-54.4	21	-54.4
$2f_1 + f_2$	200	-48.4	26	-48.4
$2f_2 + f_1$	190	-54.4	16	-54.4

Two-Tone Test Simulation ($f_{s5} = 31$ MHz)

Sampling Frequency: $f_{s5} = 31$ MHz



Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	8	0.00
f_2	60	-6.07	29	-6.07
$3f_1$	210	-51.9	24	-51.9
$3f_2$	180	-70.0	25	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	18	-48.4
$2f_2 - f_1$	50	-54.4	19	-54.4
$2f_1 + f_2$	200	-48.4	14	-48.4
$2f_2 + f_1$	190	-54.4	4	-54.4

Residue HD, IMD power are the same as theorical HD, IMD power
Residue sampling is applicable to multi-tone test

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Bluetooth BR Frequency Measurement

Bluetooth basic rate (BR)

Carrier frequency: $2402 + k$ [MHz], $k = 0, 1, 2, \cdots, 78$
Channel space: 1 MHz
Signal: 1 Msps Gaussian Frequency Shift Keying (GFSK)
Bandwidth-Time product (BT): 0.5
Modulation index: $0.28 \sim 0.35$

GFSK signal:
Hopped between 2.402 GHz and 2.480 GHz

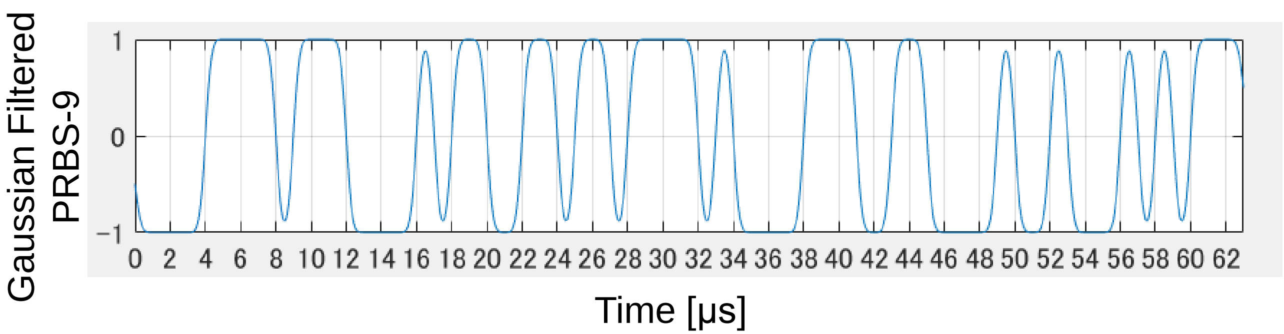
Consider frequency hopping signal testing
by residue sampling

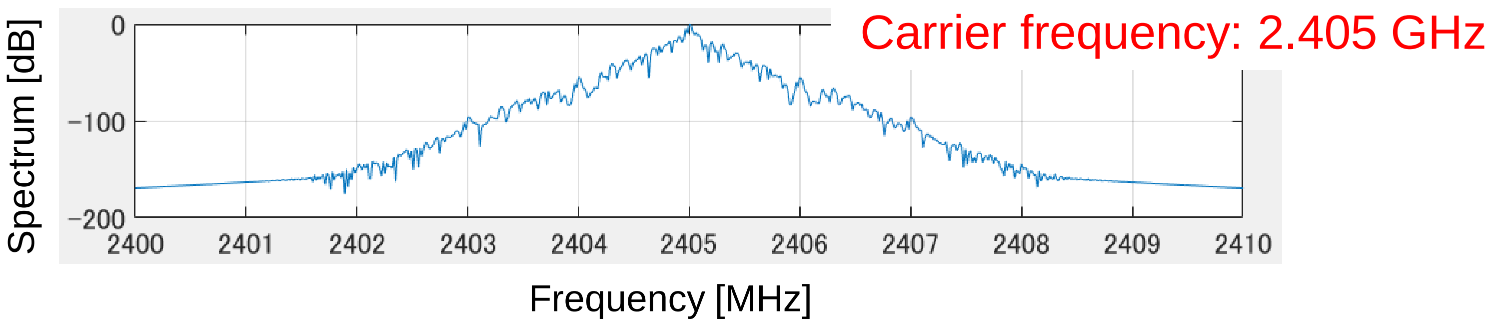
Residue sampling:
Lower frequency sampling clocks (7, 11, 13, 17 MHz) than bandwidth (78 MHz)

Simulation Conditions

Carrier frequency: 2405 MHz
Signal: Gaussian filtered PRBS-9 (BT = 0.5)
Modulation Index: 0.3
Sampling frequencies:

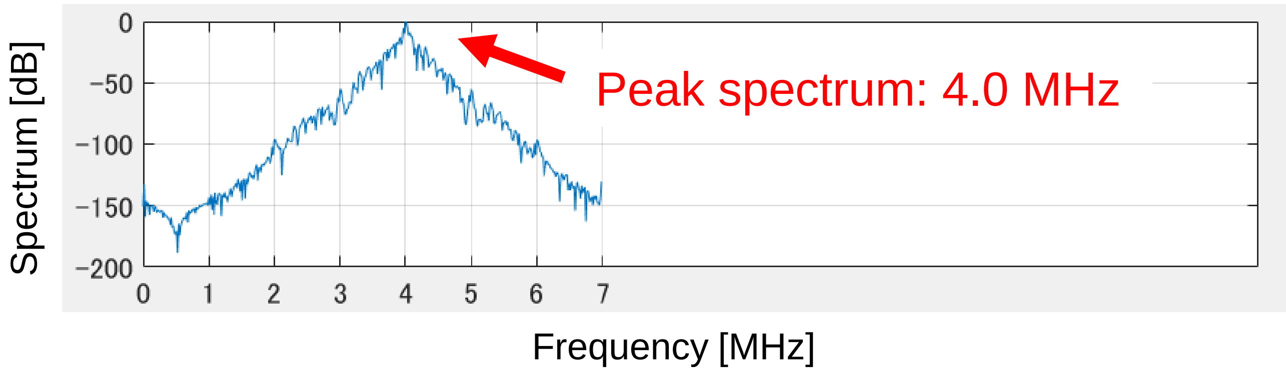
$$f_{s1} = 7\ \text{MHz},\ f_{s2} = 11\ \text{MHz},\ f_{s3} = 13\ \text{MHz},\ f_{s4} = 17\ \text{MHz}$$





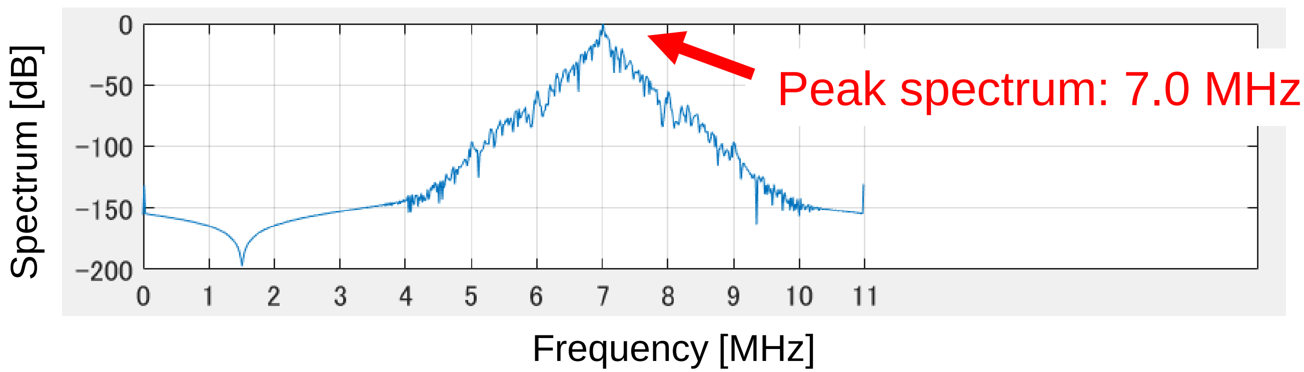
Bluetooth BR Simulation (f_s = 7MHz, 11 MHz)

Sampling frequency: $f_{s1} = 7$ MHz



$$\mathrm{mod}_7(2405) = 4$$

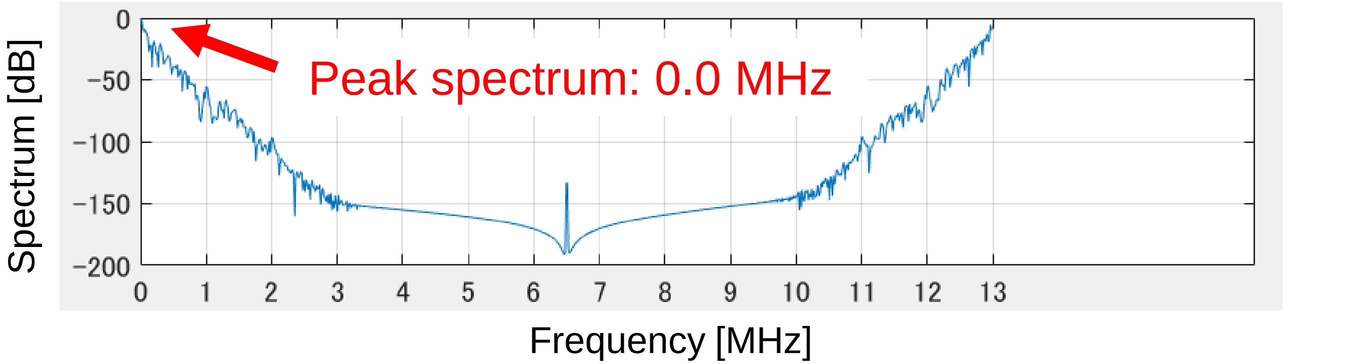
Sampling frequency: $f_{s2} = 11$ MHz



$$\mathrm{mod}_{11}(2405) = 7$$

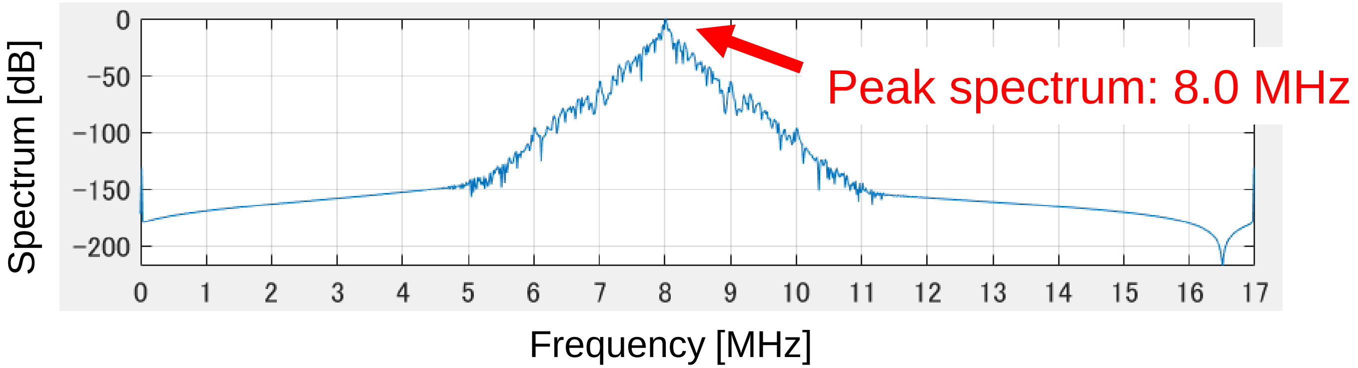
Bluetooth BR Simulation ($f_s = 13\text{MHz}, 17\ \text{MHz}$)

Sampling frequency: $f_{s3} = 13\ \text{MHz}$



$$\text{mod}_{13}(2405) = 0$$

Sampling frequency: $f_{s4} = 17\ \text{MHz}$



$$\text{mod}_{17}(2405) = 8$$

Frequency hopping signal testing can be achieved by residue sampling

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Conclusion

- Residue sampling:
  High-frequency signal measurement
  with multiple low-frequency sampling clocks

- Application to RF/AMS device testing
  - Fine frequency resolution measurement
    by large number of sampled data
    (long measurement time)
  - Two-tone signal test:
    Residue HD, IMD power are the same as
    original HD, IMD power
  - High frequency narrow-band signal test:
    Frequency hopping signal testing can be achieved
    with residue sampling.

Number Theory for RF/AMS Testing

Carolus Fridericus Gauss
(1777-1855)

"Number theory is
the queen of mathematics"

Past Number theory
Beautiful and mysterious
NEVER practical

Current Number theory
For information communication processing
➡ good match to digital technology

Number theory application
for RF/AMS device testing is a frontier.
There are great chances for new discovering!