

# Segmented DAC Unit Cell Selection Algorithm and Layout/Routing Based on Euler's Knight Tour

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*Gunma University*

*Jedat Inc.*



# Contents

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- Research Objective
- Segment-type DAC
- Circuit Element Characteristic Variation
- Proposed Layout Method
  - Magic Square
  - Euler's Knight Tour
- Trial Layout and Routing
- Conclusion

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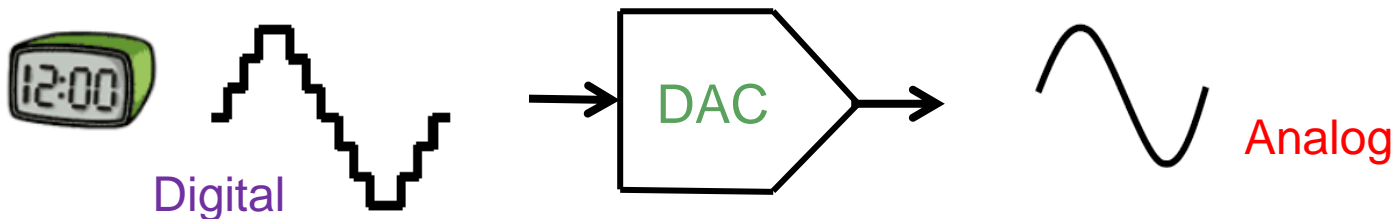
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# Research Objective

## Objective

- Digital-to-analog converter (DAC) is a key component in modern ULSIs
- Development of a **highly linear** DAC



## Our Approach

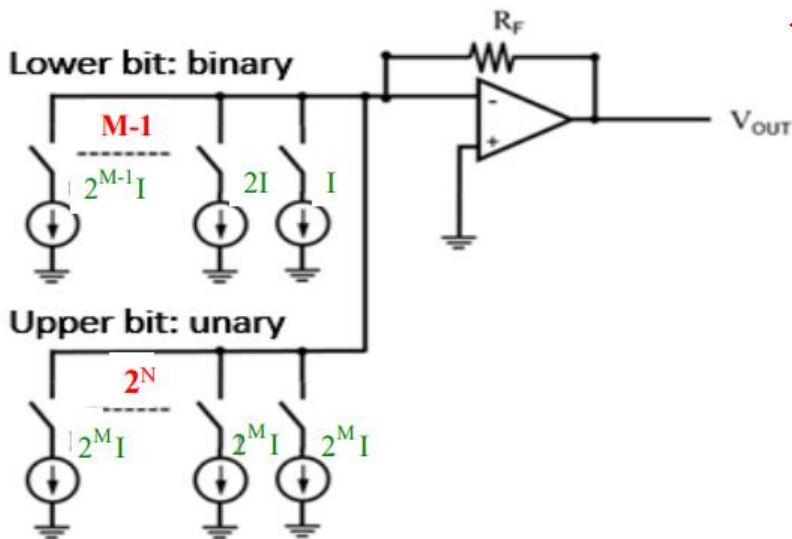
- DAC layout technique to cancel systematic mismatch effects among unit current cells. ➔ Better linearity
- Layout based on **Euler's Knight Tour and Magic Squares** New!!

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# Segment-type DAC Configuration



## ✓ Binary (Lower bits)

- Small circuit
- Large glitch
- Large mismatch effect & nonlinearity

## ✓ Unary (Upper bits)

- Large circuit
- Small glitch
- Small mismatch effect & modest nonlinearity

Segmented DAC

Focus !!

# Segment-type DAC (7-bit case)

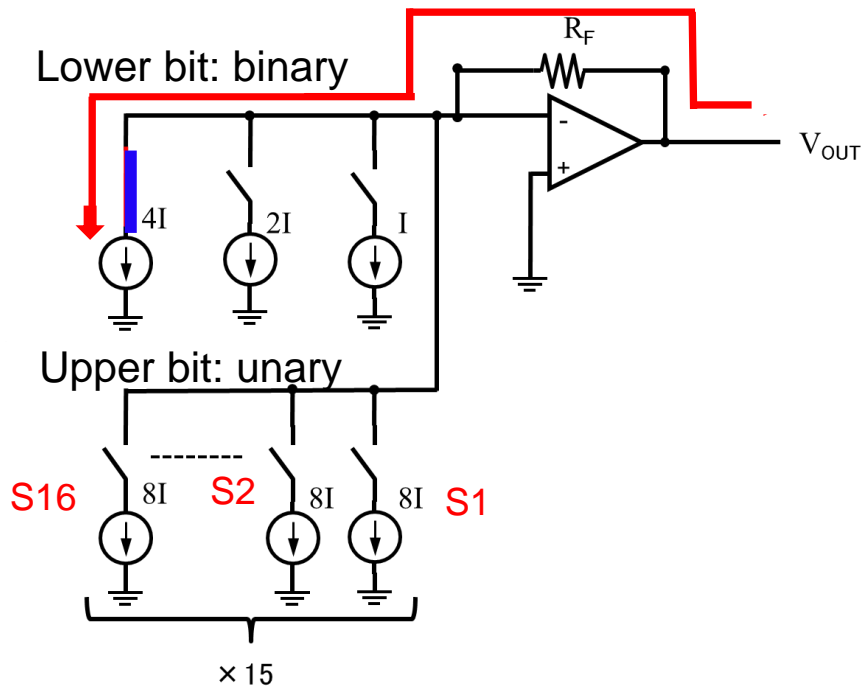
ex.1

digital input = 4

(0000100)



$$V_{out} = 4IR_F$$



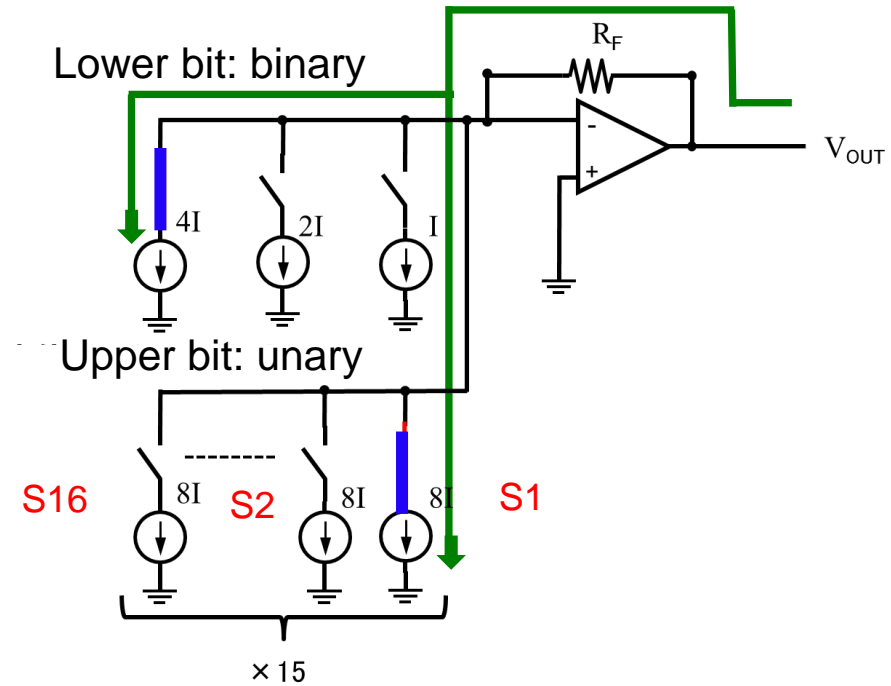
ex.2

digital input = 12

(0001100)



$$V_{out} = 12IR_F$$

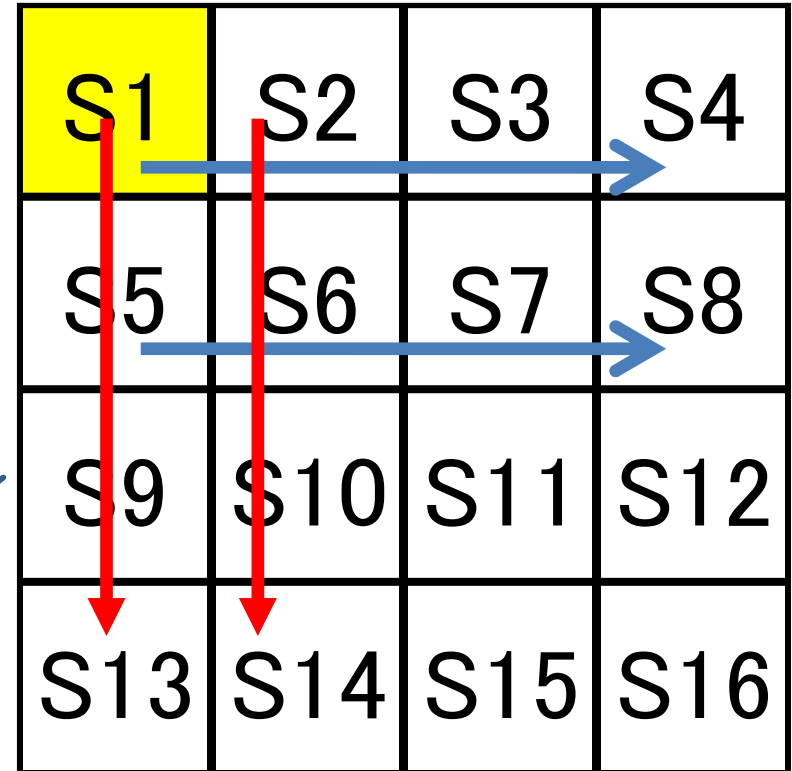
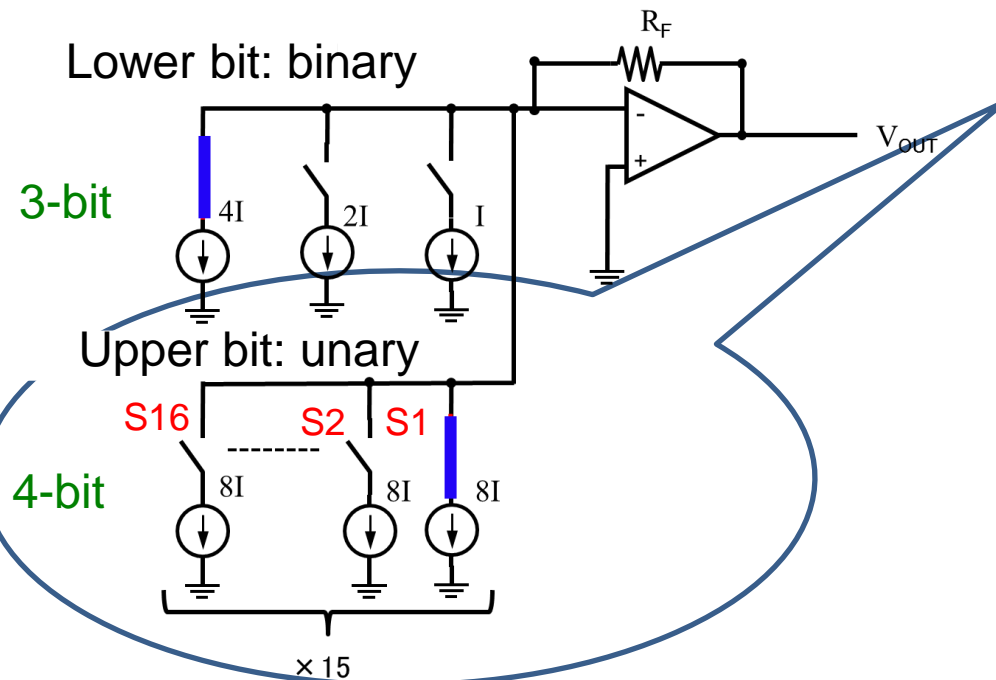


# Unary DAC Current Cells Layout

✓ 7bit DAC

(0001100)

$$V_{out} = 12IR_F$$



2D array of  
unit current cells (8I)

Regular layout

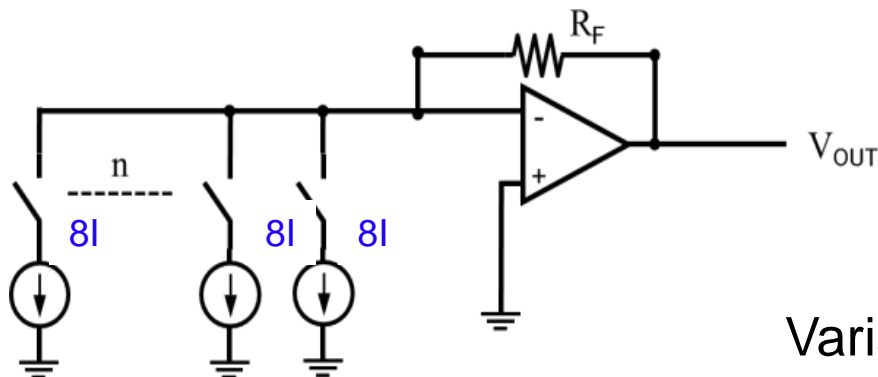


# Unary DAC Features

- Identical current sources
- Small glitch
- Inherent monotonicity



- Large circuits
  - Decoder
  - Many switches and current sources



Variations ( $e_{16}, \dots, e_2, e_1$ )  
among current sources

 Real chip

8I	8I	8I
+	+	+
$e_{16}$	$e_2$	$e_1$



DAC nonlinearity

# Problem of Regular Layout

Error  $e$  depends on place

➔ Systematic variation

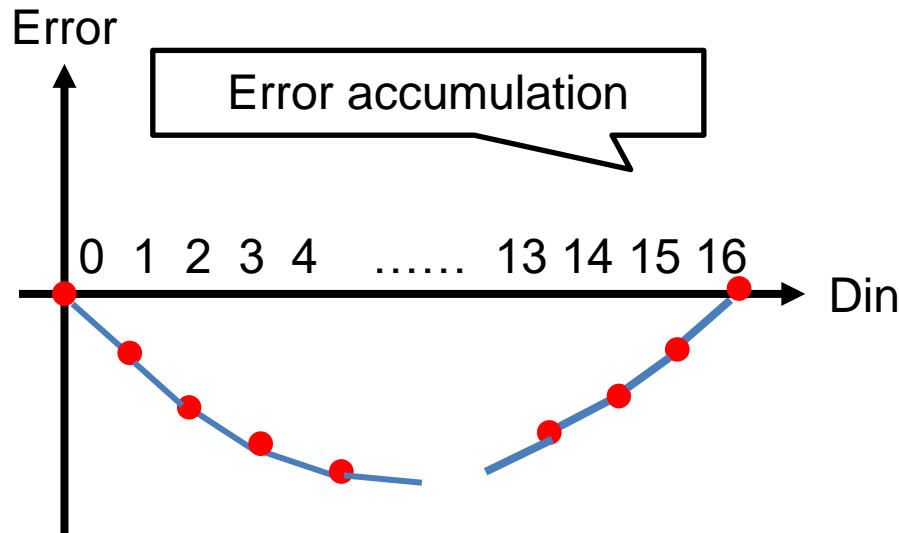
$$V(0) = 0$$

$$V(1) = 8l + e_1$$

$$V(2) = 16l + e_1 + e_2$$

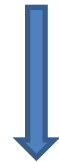
$$V(3) = 32l + e_1 + e_2 + e_3$$

⋮



small ➔  $e$  ➔ large

small



$e$



large

S1	S2	S3	S4
S5	S6	S7	S8
S9	S10	S11	S12
S13	S14	S15	S16

2D array of  
unit current cells ( $8l$ )

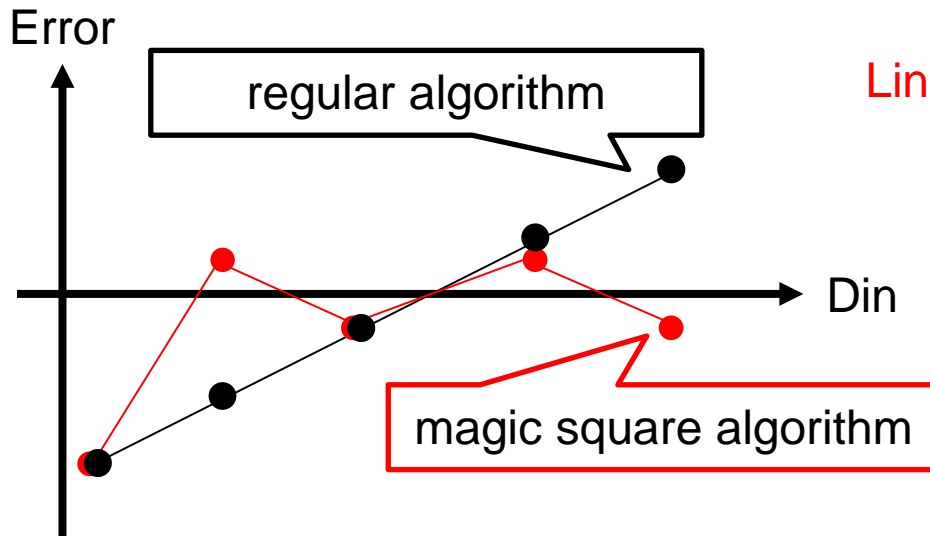
Regular layout

# Cell Layout and Systematic Mismatch

- In modern ULSIs, systematic mismatches exist.
- Changing the unit cell layout order  
Cancellation of

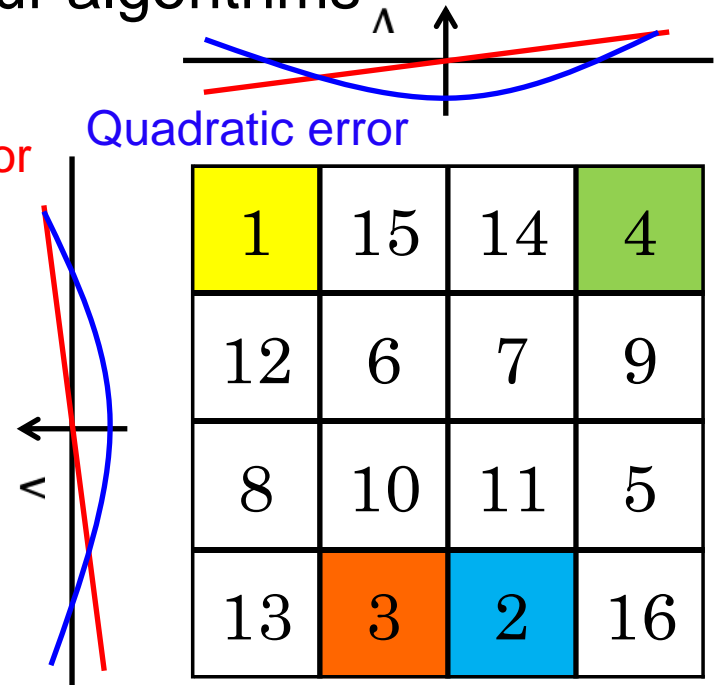
➡ systematic mismatch effects

- Magic square and Euler's Knight Tour algorithms



Linear error

Quadratic error



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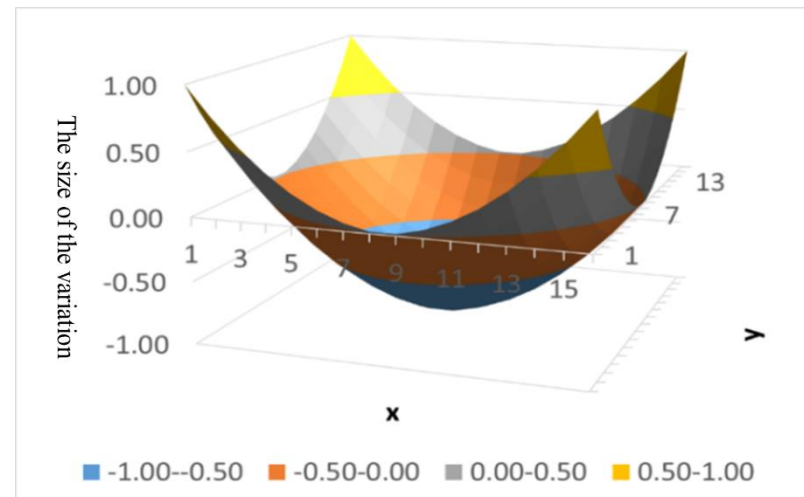
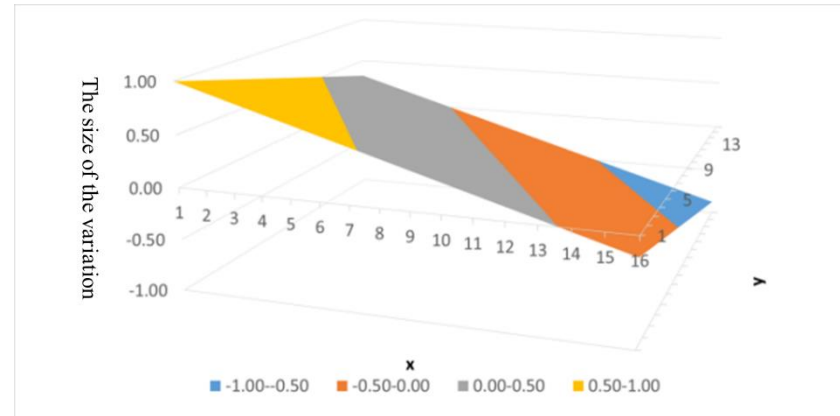
# Variation in Circuit Element Characteristics

## ◆ Systematic variations

- ✓ Voltage drop
- ✓ Thickness of oxide film
- ✓ Doping
- ✓ Mechanical stress
- ✓ Temperature distribution
- ✓ In wafer plane

**Linear**  
error

**Quadratic**  
error



**Joint Error** (Sum of both)

# Systematic Variation Model

**Linear** Error

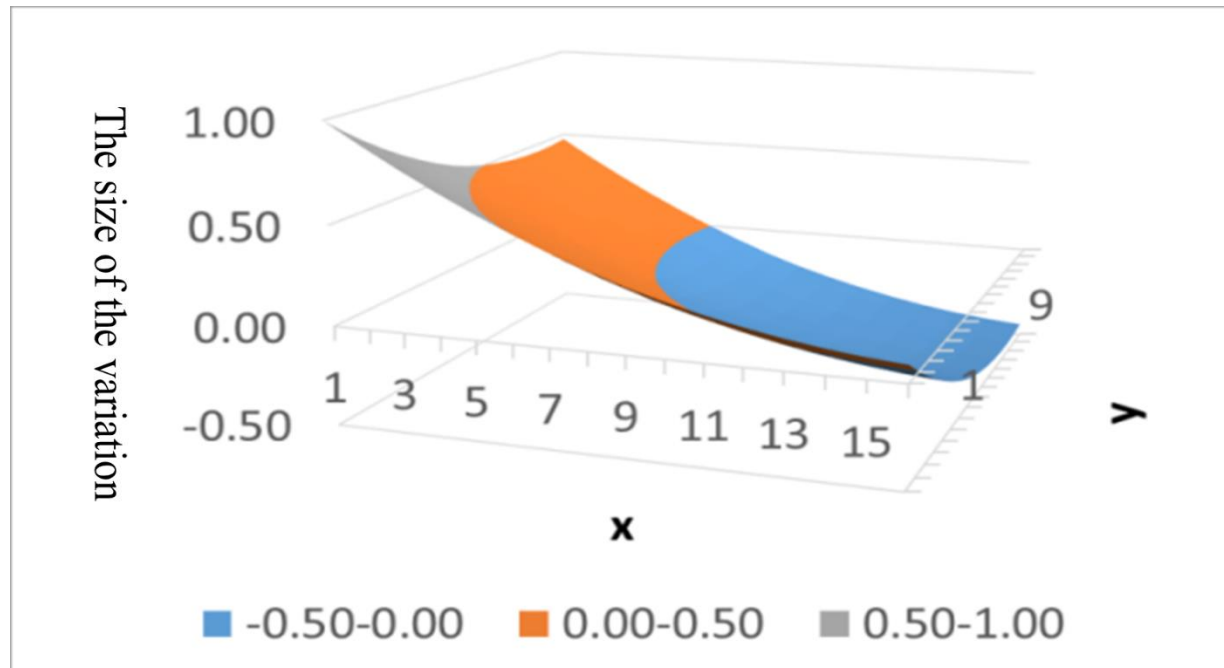
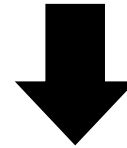
$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

**Quadratic** Error

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

**Joint** Errors

$$\varepsilon_j(x, y) = \varepsilon_l(x, y) + \varepsilon_q(x, y)$$



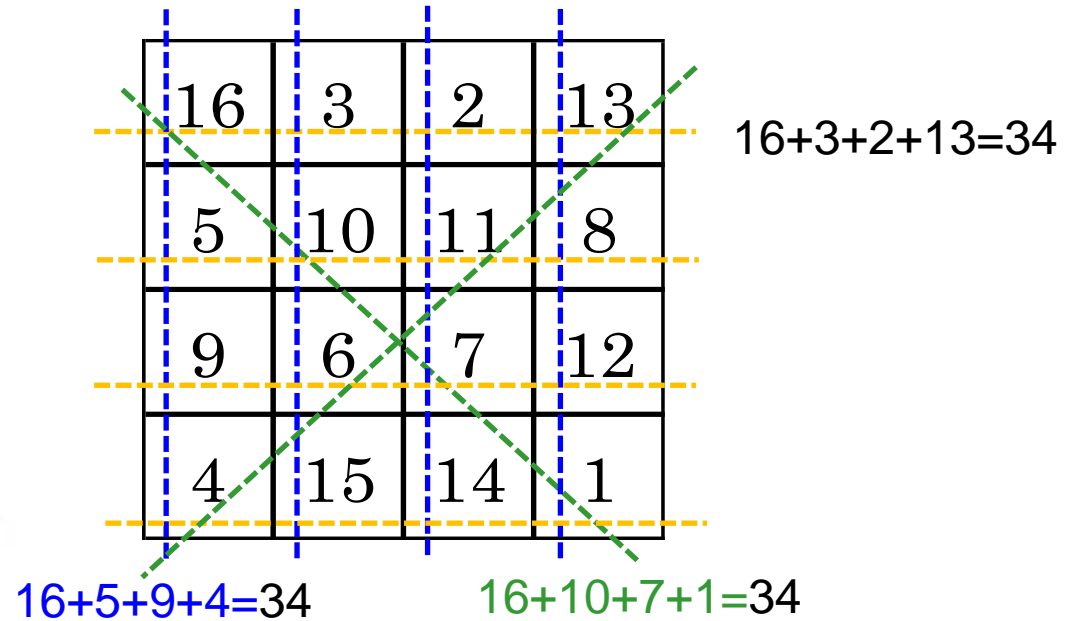
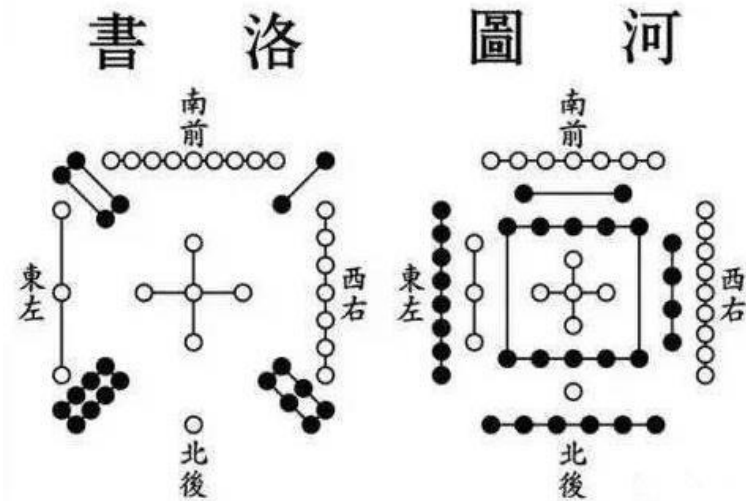
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10	5	3	16
15	4	6	9
8	11	13	2
1	14	12	7

# What is Magic Square ?

- Origin from Chinese academia
- “Constant sum” characteristics
- Varieties of magic squares



**Constant Sum**

Row, Column, Diagonal



**Good balance**





# 16\*16 Magic Square

255	2	3	253	252	6	7	249	248	10	11	245	244	14	15	241
17	239	238	20	21	235	234	24	25	231	230	28	29	227	226	32
33	223	222	36	37	219	218	40	41	215	214	44	45	211	210	48
208	50	51	205	204	54	55	201	200	58	59	197	196	62	63	193
192	66	67	189	188	70	71	185	184	74	75	181	180	78	79	177
81	175	174	84	85	171	170	88	89	167	166	92	93	163	162	96
97	159	158	100	101	155	154	104	105	151	150	108	109	147	146	112
144	114	115	141	140	118	119	137	136	122	123	133	132	126	127	129
128	130	131	125	124	134	135	121	120	138	139	117	116	142	143	113
145	111	110	148	149	107	106	152	153	103	102	156	157	99	98	160
161	95	94	164	165	91	90	168	169	87	86	172	173	83	82	176
80	178	179	77	76	182	183	73	72	186	187	69	68	190	191	65
64	194	195	61	60	198	199	57	56	202	203	53	52	206	207	49
209	47	46	212	213	43	42	216	217	39	38	220	221	35	34	224
225	31	30	228	229	27	26	232	233	23	22	236	237	19	18	240
6	242	243	13	12	246	247	9	8	250	251	5	4	254	255	1

2056

2056

2056

Constant Sum: Row, Column, Diagonal = 2056

# Contents

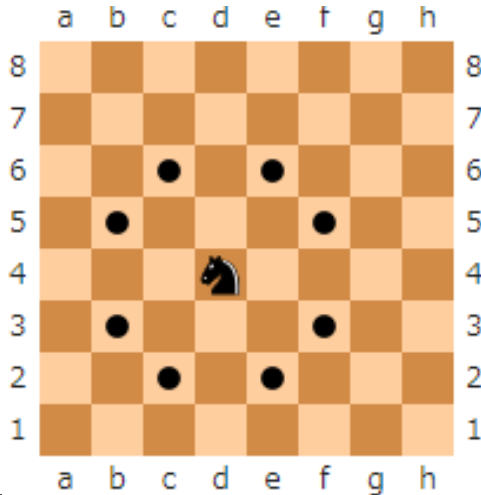
- Research Objective
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Harry Potter

# What is Knight Tour ?

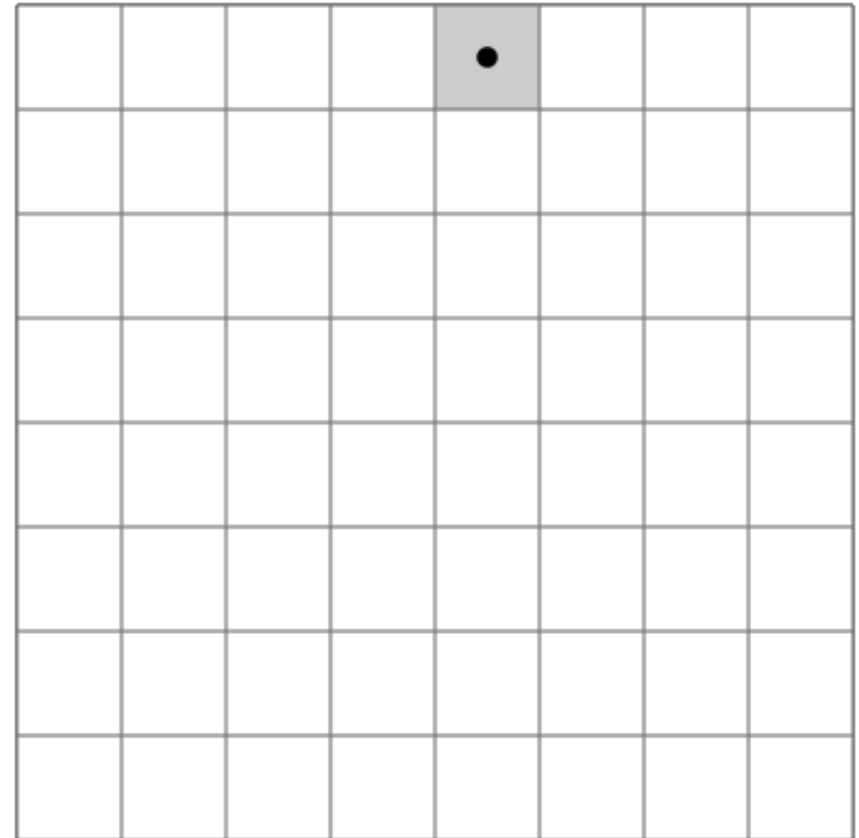
Chess



Knight



	a	b	c	d	e	f	g	h
8	15	62	19	34	1	50	31	46
7	18	35	16	63	32	47	2	49
6	61	14	33	20	51	4	45	30
5	36	17	60	13	64	29	48	3
4	11	58	21	40	5	54	27	44
3	22	37	12	59	28	41	6	53
2	57	10	39	24	55	8	43	26
1	38	23	56	9	42	25	54	7



# What is Euler's Knight Tour ?



- Found by Leonhard Euler
- Magic square  
+  
Knight tour

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

260
260

8x8 Euler's Knight Tour

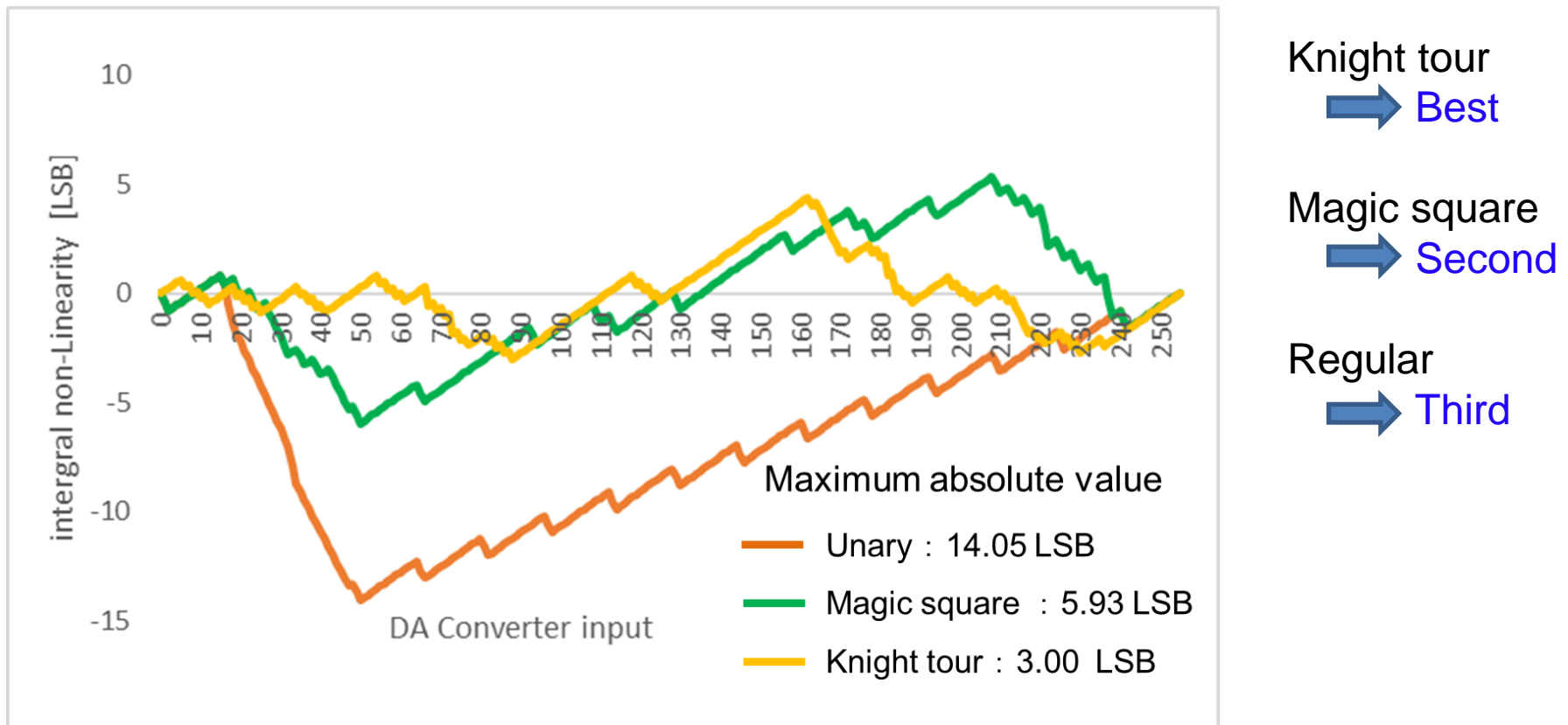
# 16x16 Euler's Knight Tour

84	217	170	75	188	219	172	77	228	37	86	21	230	39	88	25
169	74	185	218	171	76	189	220	85	20	229	38	87	24	231	40
216	183	68	167	222	187	78	173	36	227	22	83	42	237	26	89
73	168	215	186	67	174	221	190	19	84	35	238	23	90	41	232
182	213	166	69	178	223	176	79	226	33	82	31	236	43	92	27
165	72	179	214	175	66	191	224	81	18	239	34	91	30	233	44
212	181	70	163	210	177	80	161	48	225	32	95	46	235	28	93
71	164	211	180	65	162	209	192	17	96	47	240	29	94	45	234
202	13	126	61	208	15	128	49	160	241	130	97	148	243	132	103
125	60	203	14	127	64	193	16	129	112	145	242	131	102	149	244
12	201	62	123	2	207	5	113	256	159	98	143	246	147	104	133
59	124	11	204	63	114	1	194	111	144	255	146	101	134	245	150
200	9	122	55	206	3	116	51	158	253	142	99	154	247	136	105
121	58	205	10	115	54	195	4	141	110	155	254	135	100	151	248
8	199	56	119	6	197	52	117	252	157	108	139	250	153	106	137
57	120	7	198	53	118	5	106	109	140	251	156	107	138	249	152

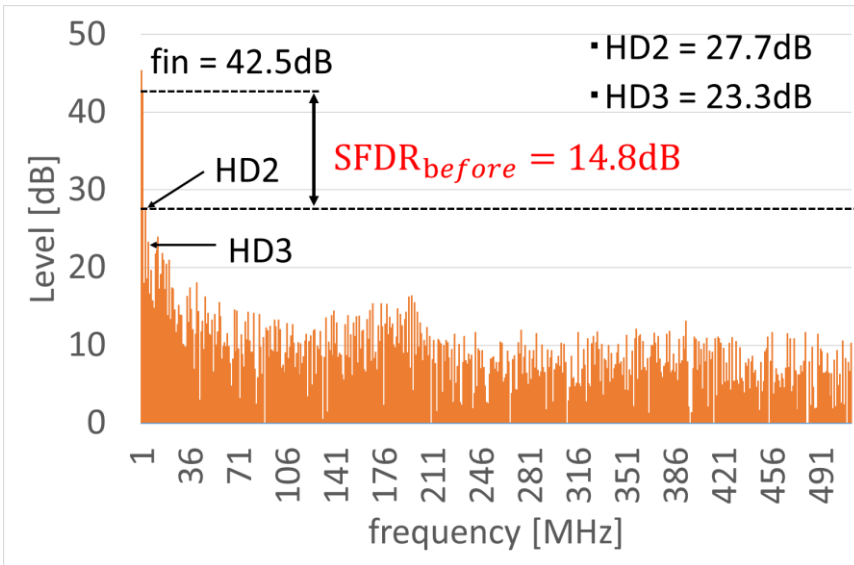
Constant Sum: Row, Column, Diagonal = 2056

# DAC Simulation Results (1)

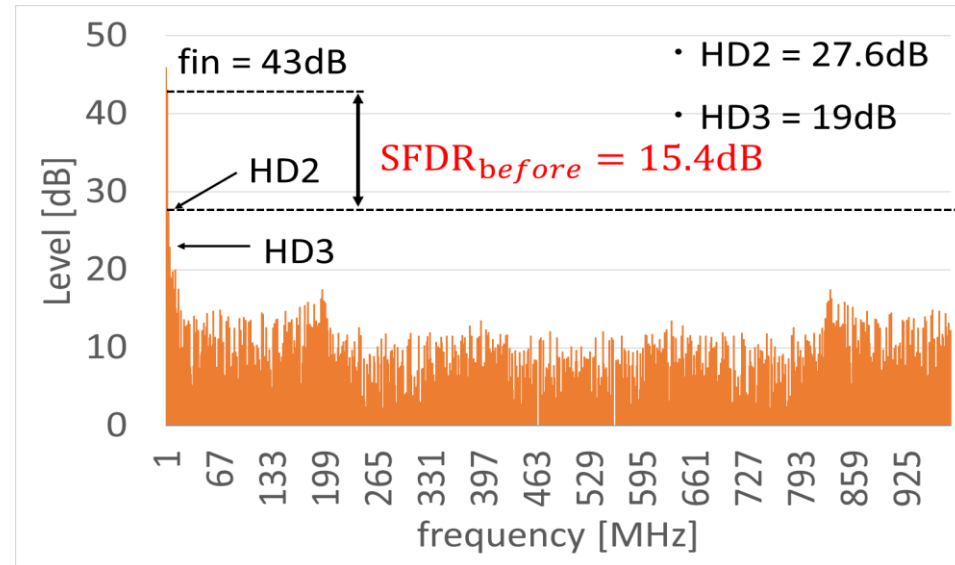
- Randomize error  $e$  by magic square / knight tour layout.
- Linear gradient  $e$  case
- DAC integral non-linearity (INL)



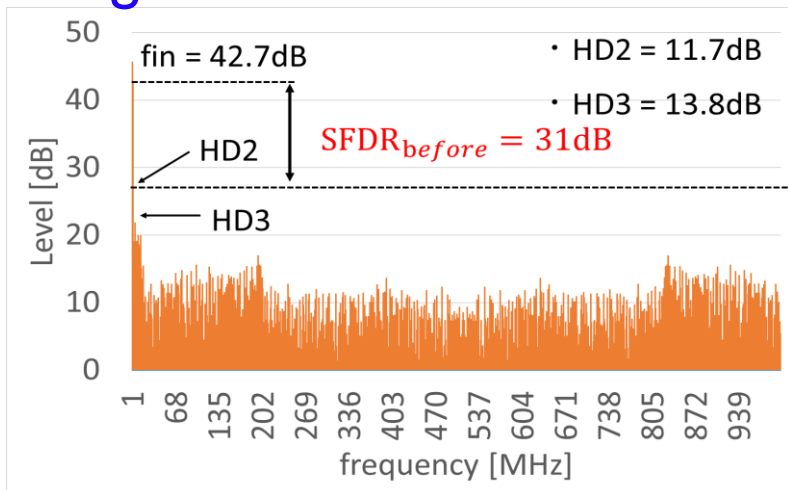
# DAC Simulation Results (2)



Regular



Magic square



Euler's Knight Tour

**SFDR:** Spurious Free Dynamic Range

Regular: 14.8 dB

⇒ Third

Magic square: 15.4 dB

⇒ Second

Knight tour : 31.0 dB

⇒ Best

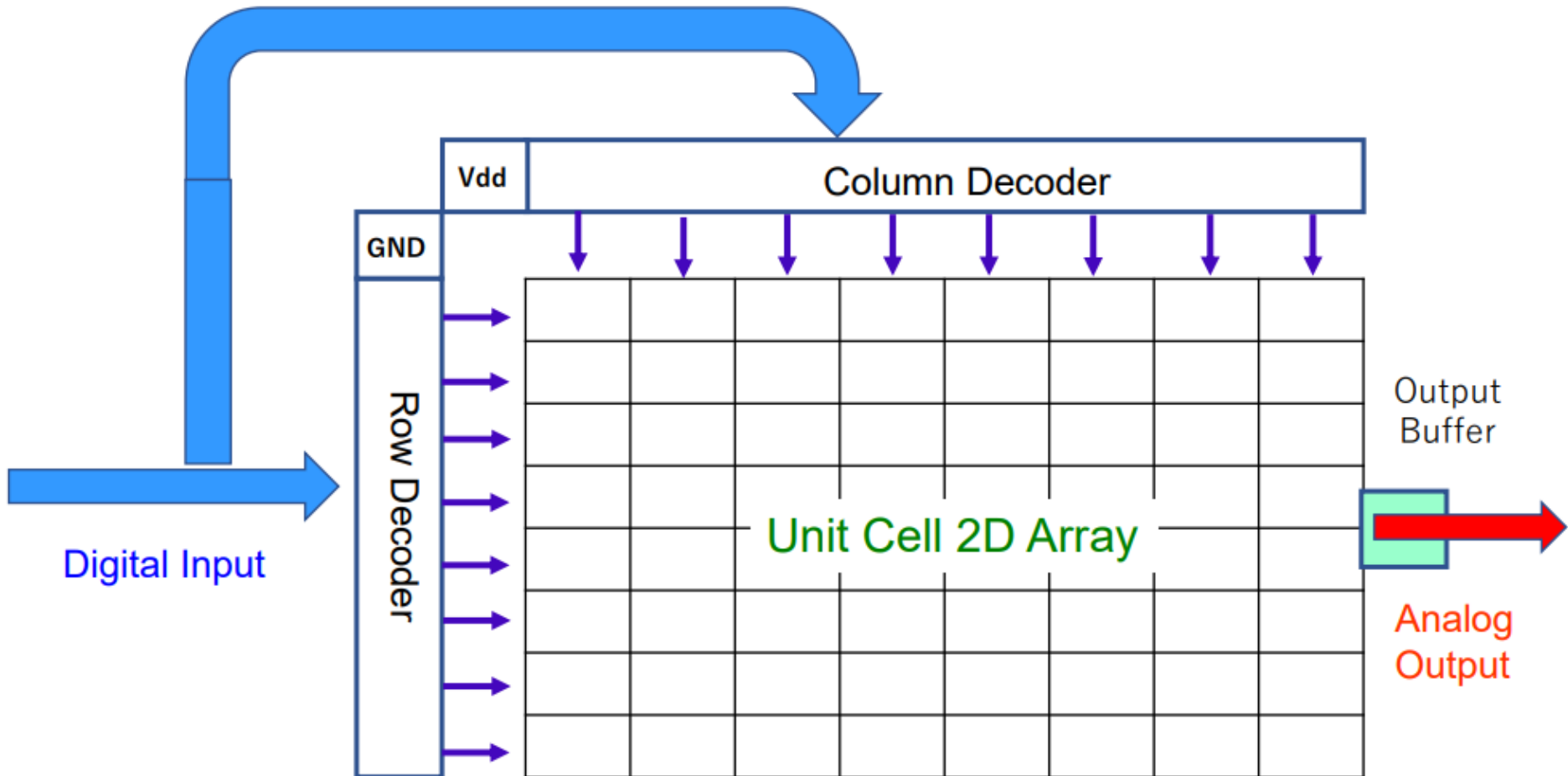
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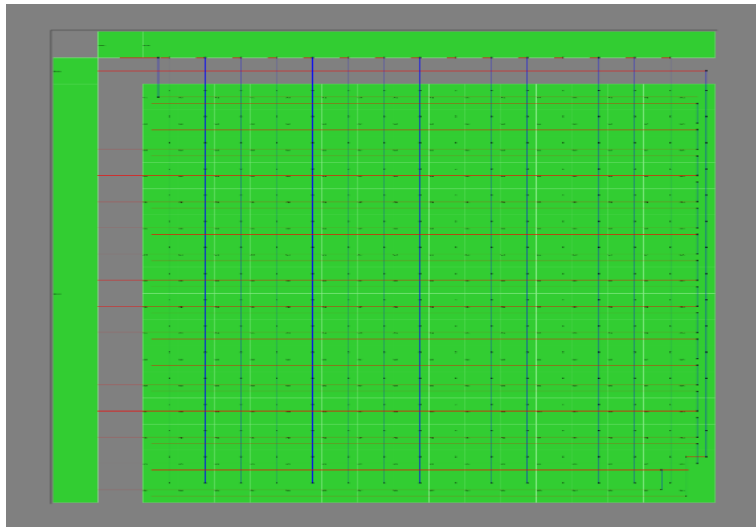
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# Unary DAC Floor Plan

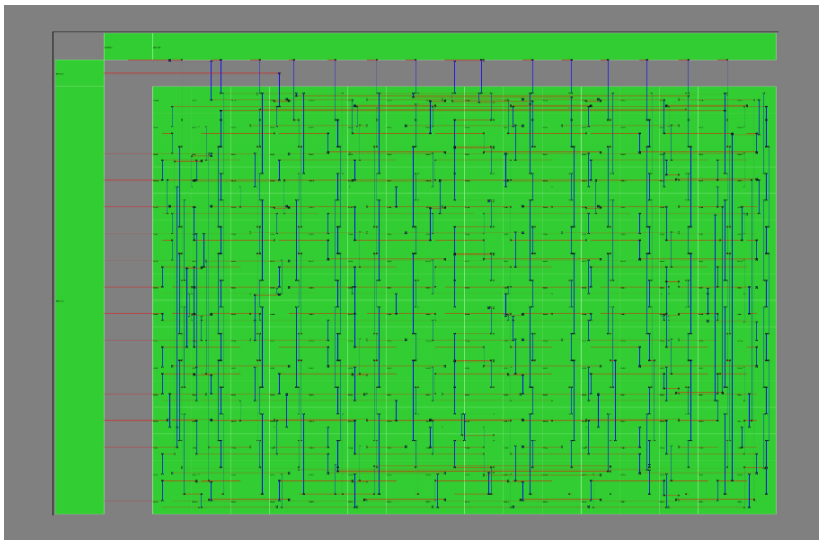


# 8bit Unary DAC Layout and Routing

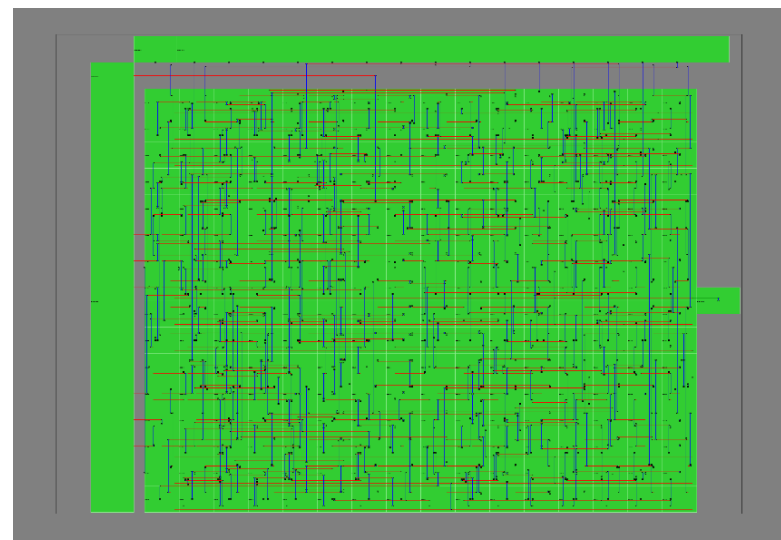


Regular

- 8-bit (16x16) unit cell  
2D layout and routing are feasible for magic square and Euler's knight tour algorithms



Magic square



Euler's Knight Tour

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# Conclusion

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- Unary DAC linearity improvement
  - Systematic mismatch effects cancellation
  - Unit cell layout algorithm  
based on **Magic Square and Euler's Knight Tour**
- Validation by DAC Simulation
  - INL improvement
  - SFDR improvement
- Feasibility of layout and routing



Magic Squares by  
French Mathematician  
Bernard Frénicle de Bessy

TABLE GENERALE  
DES  
QUARREZ DE QUATRE.

1 13 8 12	1 13 11 8	1 13 8 11	1 13 11 8	1 14 8 11
16 4 9 5	16 4 5 9	16 4 9 5	16 4 5 9	15 4 10 5
11 7 14 2	7 11 14 2	10 6 13 3	6 10 13 3	12 7 13 2
6 10 3 15	10 6 3 15	7 11 1 14	11 7 2 14	6 9 3 16
1 14 11 8	1 14 7 12	1 14 12 7	1 11 14 8	1 14 11 8
15 4 5 10	15 4 9 6	15 4 6 9	16 5 4 9	16 5 4 9
6 9 16 3	10 5 16 3	8 11 13 2	7 11 13 2	7 13 13 2
12 7 2 13	8 11 2 13	10 5 3 16	10 6 3 15	10 5 6 11
1 14 7 12	1 10 13 8	1 13 10 8	1 11 8 14	1 11 14 8

ご清聴ありがとうございました

Thank you for listening

謝謝

Merci de votre attention