Study of Rauch Low-Pass Filters using Pascal's Triangle

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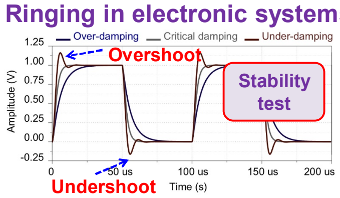
1. Research Objective

Solving the stability test problems:

- Overshoot, undershoot, ringing
- Loop gain, Nyquist diagram, Nichols chart

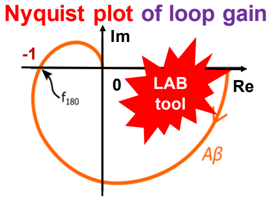
Ringing in electronic systems



Merits of Rauch low-pass filters

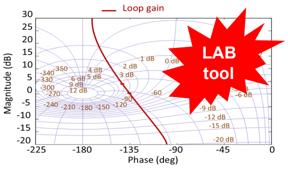
○ Easiest selection of circuit components
○ Simplest design in fully differential forms and complex topologies

Nyquist plot of loop gain



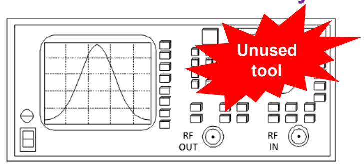
(Unclear operating region)

Nichols plot of loop gain



(Very complicate)

Nichols chart in Network Analyzer?



(Technology limitations)

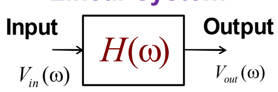
Innovation of this work

○ Nichols chart of self-loop function
→ A useful tool for stability test of feedback networks
○ Use of Pascal's triangle
→ Fast ringing test for high-order systems

Simple tool

2. Research Background

Linear system



$V_{in}(\omega)$, $V_{out}(\omega)$: periodic signals with angular frequency variable

$A(\omega)$: Numerator function

Transfer function

$$H(\omega)=\frac{V_{out}(\omega)}{V_{in}(\omega)}=\frac{A(\omega)}{1+L(\omega)}$$

Self-loop function

$$L(\omega)=\frac{A(\omega)}{H(\omega)}-1$$

○Polar chart → Nyquist chart
○Magnitude-frequency plot
○Angular-frequency plot
(Bode plots)
○Magnitude-angular diagram → Nichols diagram

Characteristics of Pascal's triangle

$$(j\omega+1)^n=a_0(j\omega)^n+a_1(j\omega)^{n-1}+\ldots+a_{n-1}j\omega+1$$

n = 1				1		1			
n = 2			1		2		1		
n = 3		1		3		3		1	
n = 4	1		4		6		4		1

• Under-damping: 1 : 1 : 1

$$H_1(\omega)=\frac{1}{(j\omega)^2+j\omega+1}\Rightarrow |H_1(\omega=1)|=1>\frac{1}{2}=-6dB$$

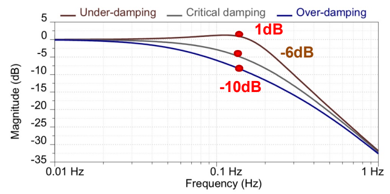
• Critical damping: 1 : 2 : 1

$$H_2(\omega)=\frac{1}{(j\omega)^2+2j\omega+1}\Rightarrow |H_2(\omega=1)|=\frac{1}{2}=-6dB$$

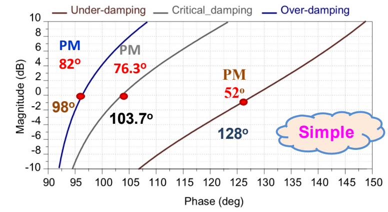
• Over-damping: 1 : 3 : 1

$$H_3(\omega)=\frac{1}{(j\omega)^2+3j\omega+1}\Rightarrow |H_3(\omega=1)|=0.316<\frac{1}{2}=-6dB$$

Bode plot of transfer function

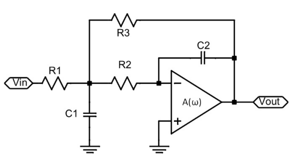


Nichols plot of self-loop function

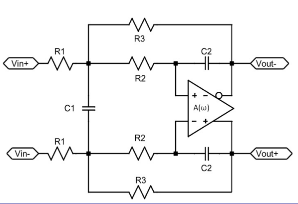


3. Stability test for Rauch LPF

Single ended Rauch LPF



Fully differential Rauch LPF



Transfer function

$$H(\omega)=-\frac{b_0}{a_0(j\omega)^2+a_1 j\omega+1};$$

Self-loop function

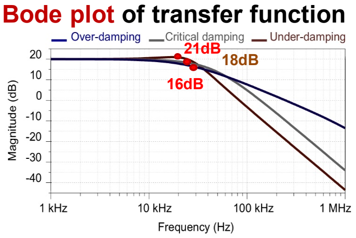
$$L(\omega)=a_0(j\omega)^2+a_1 j\omega;$$

$$b_0=\frac{R_3}{R_1};\ a_0=R_2R_3C_1C_2;\ a_1=\left(R_2+R_3+\frac{R_2R_3}{R_1}\right)C_2;$$

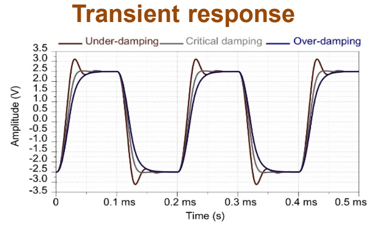
R1= R2 = 1 kΩ, R3 = 10 kΩ, C1 = 6 nF, C2 = 200 pF, at f_0 = 20 kHz.

- Over-damping (C1 = 6 nF),
- Critical damping (C1 = 2 nF), and
- Under-damping (C1 = 0.1 nF).

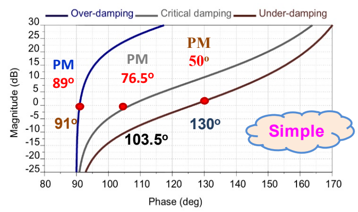
Bode plot of transfer function



Transient response



Nichols plot of self-loop function

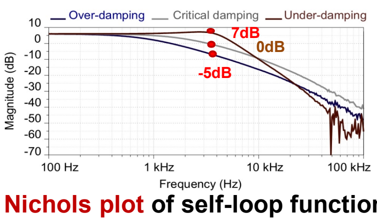


Simulated results

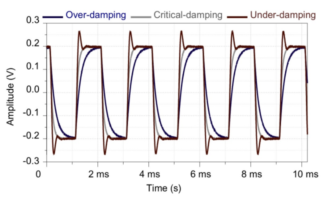
Over-damping:
→Phase margin is 89 degrees.
Critical damping:
→Phase margin is 76.5 degrees.
Under-damping:
→Phase margin is 50 degrees.

4. Experimental Results

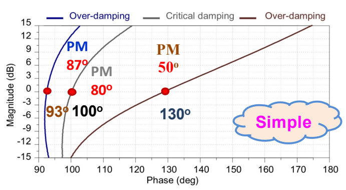
Bode plot of transfer function



Transient response



Nichols plot of self-loop function



Measured results

Over-damping:
→Phase margin is 87 degrees.
Critical damping:
→Phase margin is 80 degrees.
Under-damping:
→Phase margin is 50 degrees.

5. Conclusion

Ringing test for Rauch low-pass filters using Pascal's triangle

→ Observation of coefficients and phase margin can help us determine the operating regions of high-order systems.

→ Theoretical concepts of stability test are verified by SPICE simulations and practical measurements.

Future work: Stability test for parasitic components in printed circuit boards, physical layout layers, transmission lines…

References

[1] M. Tran, A. Kuwana, H. Kobayashi, *"Ringing Test for Tow-Thomas Low-Pass Filters"*, Int. Conf. on Promising Electronic Technologies (ICPET 2020), Jerusalem and Gaza City, Palestine, Dec. 2020.
[2] M. Tran, A. Kuwana, H. Kobayashi, *"Study of Behaviors of Electronic Amplifiers using Nichols Chart"*, IEEE the 3rd Int. Conf. on Electronics and Communication Engineering (ICECE 2020), Xi'an, China, Dec. 2020.
[3] M. Tran, A. Kuwana, H. Kobayashi, *"Ringing Test for 2nd-order Sallen-Key Low-Pass Filters"*, IEEE 2nd Int. Conf. on Circuits and Systems (ICCS 2020), Chengdu, China, 10-13, Dec. 2020.
[4] M. Tran, A. Kuwana, H.Kobayashi *"Ringing Test for Negative Feedback Amplifiers"* 11th IEEE Annual Information Technology, Electronics and Mobile Communication Conf. (IEMCON 2020), Canada, Nov. 2020.