Keynote Lecture 03 (Abstract)

Smart Mathematics Leads to Sophisticated Analog/Mixed-Signal Circuit

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Our Statement

Beautiful mathematics

⇨ good analog/mixed-signal circuit

Besides transistor level design

- Control theory
- Integer theory
- Coding theory
- Statistics
- Modulation
- Signal processing algorithm

⇩

Enhance analog/mixed-signal circuit performance

Integer Theory and Electronic Circuit Design

Many interesting properties of Integers

Currently
No Link

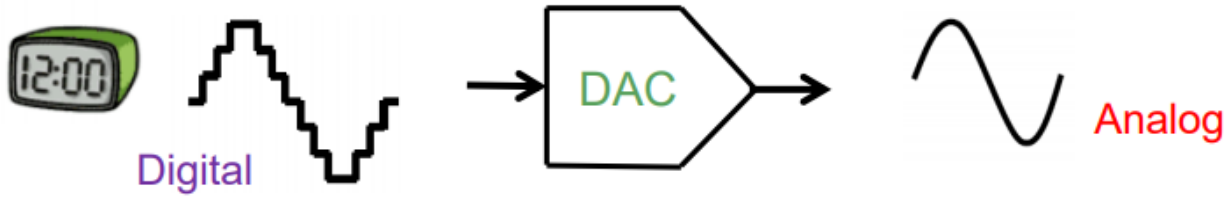
Electronic circuit designs

Our research here makes their links !

Carolus Fridericus Gauss
(1777-1855)

Integer theory is
Queen of Mathematics

DACs are Everywhere !



Communication equipment

Electronic measuring instrument

Audio systems

DAC: Digital-to-Analog Converter

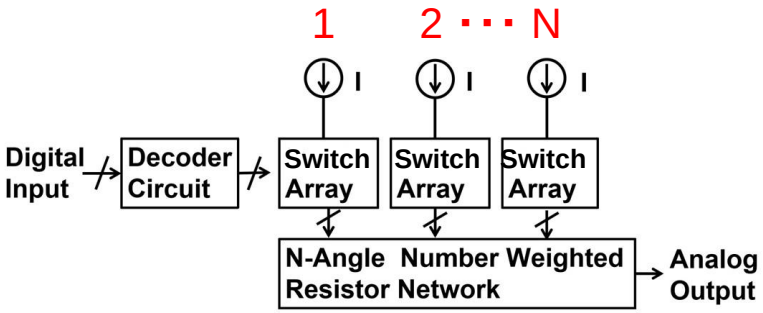
Research 1: Polygonal Number DAC

- **Interesting properties of polygonal numbers**

- **New configurations of DAC**

Polygonal number DAC

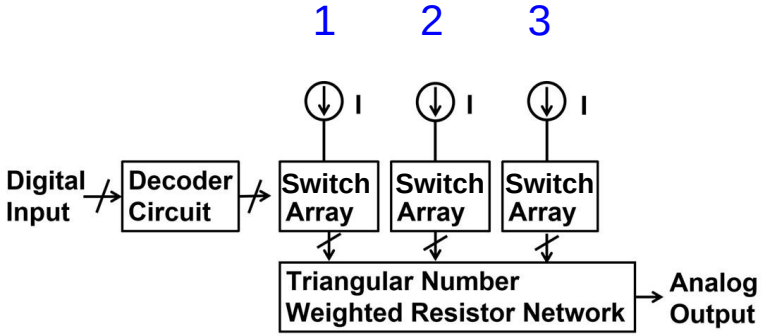
N current sources
N switch arrays



N=3, 4, 5, ….

Triangular number DAC

3 current sources
3 switch arrays

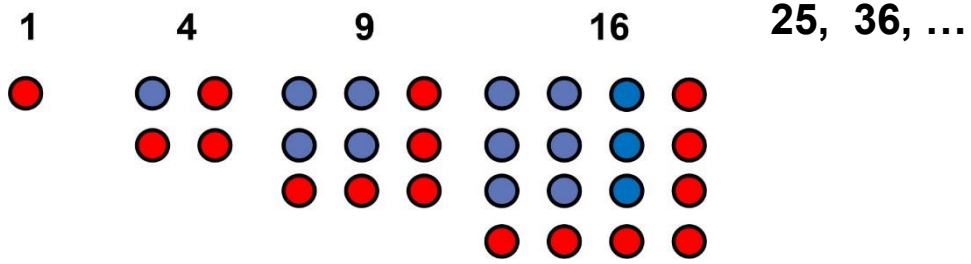


What is Polygonal Number ?

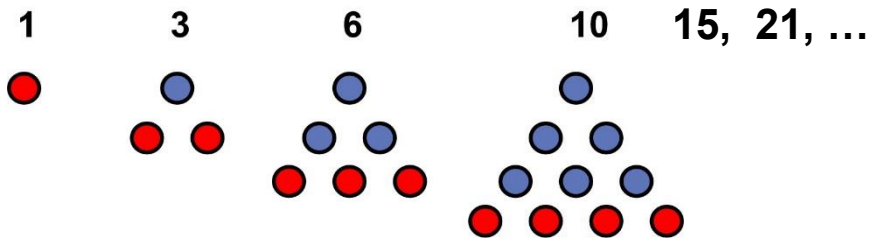
Polygonal Number

Represented as dots arranged in regular polygon shaper.

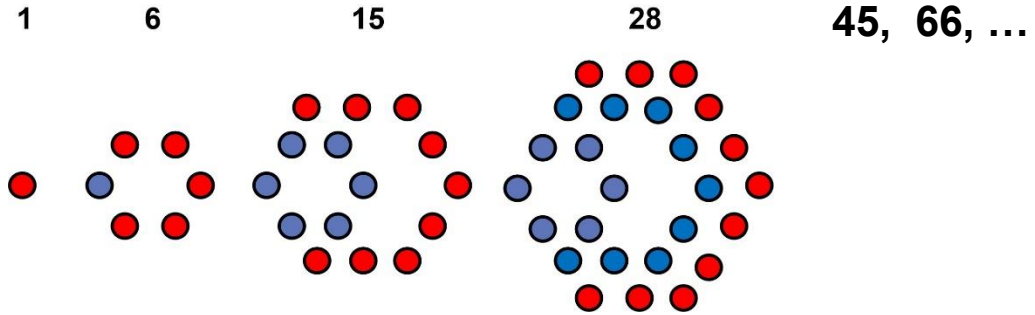
Square numbers.



Triangular numbers.



Hexagonal numbers.



Fermat Polygonal Number Theorem

Any natural number

⬇ **expressed by**

Sum of N N-angular numbers

French mathematicians

Conjecture

Pierre de Fermat
1607 – 1665

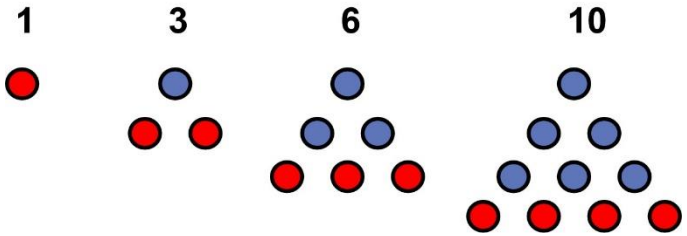
Augustin-Louis Cauchy
1789 –1857

Proved in 1813

Triangular Number Case

Any natural number ➡ Sum of 3 triangular numbers

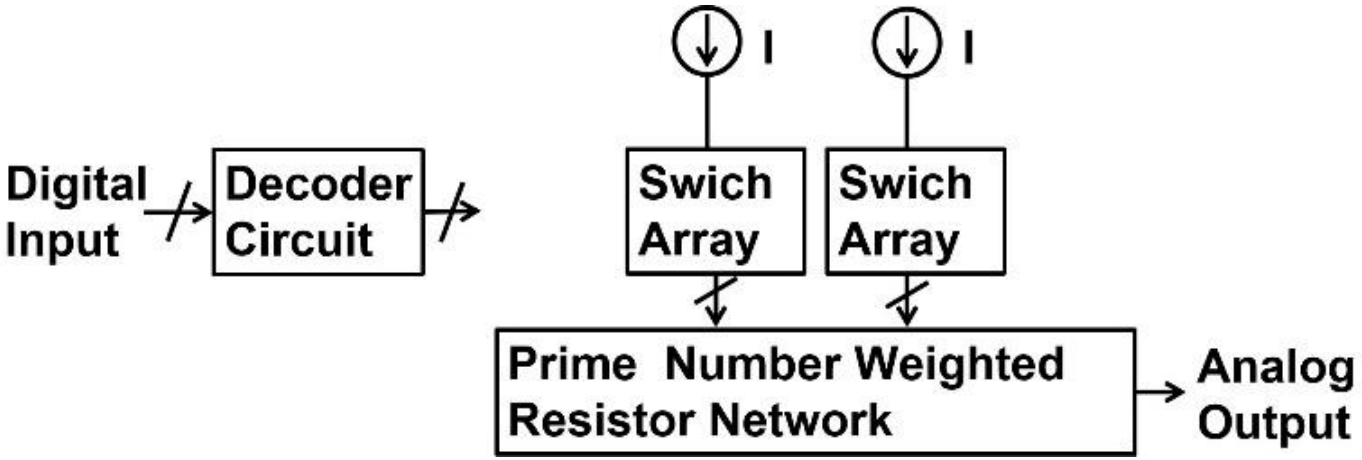
expressed by



1:	1
2:	1+1
3:	3
4:	1+3
5:	1+1+3
6:	6
7:	1+6
8:	1+1+6
9:	3+6
10:	10
11:	1+10
12:	1+1+10
13:	3+10
14:	1+3+10
15:	15
16:	1+15
17:	1+1+15
18:	3+15
19:	1+3+15
20:	10+10
21:	21
22:	1+21
23:	1+1+21
24:	3+21
25:	1+3+21
26:	1+10+15
27:	6+21
28:	28
29:	1+28
30:	1+1+28
31:	3+28
32:	1+3+28
33:	6+6+21
34:	6+28
35:	1+6+28
36:	36
37:	1+36
38:	1+1+36
39:	3+36
40:	1+3+36
41:	3+10+28
42:	6+36
43:	1+6+36
44:	6+10+28
45:	45
46:	1+45
47:	1+1+45
48:	3+45
49:	1+3+45
50:	1+21+28
51:	15+36
52:	1+6+45
53:	10+15+28
54:	3+6+45
55:	55
56:	1+55
57:	1+1+55
58:	3+55
59:	1+3+55
60:	15+45

Research 2: Prime Number DAC

- Prime number DAC architecture
  with only 2 current sources (minimum analog circuitry)
  for any resolution
  ⇨ Suitable for advanced digital-oriented CMOS ULSI.
- Composed of a prime number weighted resistor network,
  2 current sources, 2 switch arrays, a decoder.



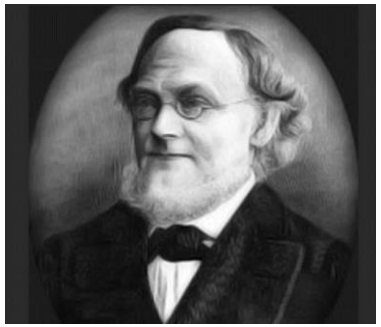
Goldbach's Conjecture

All even numbers can be represented
by sum of two prime numbers.

+	2	3	5	7	11	13	17	19
2	4	5	7	9	13	15	19	21
3	5	6	8	10	14	16	20	22
5	7	8	10	12	16	18	22	24
7	9	10	12	14	18	20	24	26
11	13	14	16	18	22	24	28	30
13	15	16	18	20	24	26	30	32
17	19	20	22	24	28	30	34	36
19	21	22	24	26	30	32	36	38

Prime number

Prime number



Christian Goldbach
1690-1764

Not proven yet !

Prime Numbers and Even Numbers

Prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……

All even numbers are represented by two prime numbers

2:	**2**
4:	**2+2**
6:	**3+3**
8:	**3+5**
10:	**3+7**
12:	**5+7**
14:	**7+7**
16:	**5+11**
18:	**7+11**
20:	**7+13**
22:	**11+11**
24:	**11+13**
26:	**13+13**
28:	**11+17**
30:	**13+17**
32:	**13+19**
34:	**17+17**
36:	**17+19**
38:	**19+19**
40:	**17+23**
42:	**19+23**
44:	**13+31**
46:	**23+23**
48:	**19+29**
50:	**19+31**
52:	**23+29**
54:	**23+31**
56:	**19+37**
58:	**29+29**
60:	**29+31**

Digital Input with Sum of 2 Prime Numbers

- All the digital inputs of DAC can correspond to an even number by sum of two prime numbers.

DAC input

0:0=0+0	16:32=13+19→16	32:64=23+41→32	48:96=43+53→48
1:2=2+0→1	17:34=17+17→17	33:66=23+43→33	49:98=31+67→49
2:4=1+3→2	18:36=17+19→18	34:68=31+37→34	50:100=41+59→50
3:6=3+3→3	19:38=19+19→19	35:70=29+41→35	51:102=43+59→51
4:8=3+5→4	20:40=17+23→20	36:72=29+43→36	52:104=43+61→52
5:10=3+7→5	21:42=19+23→21	37:74=31+43→37	53:106=53+53→53
6:12=5+7→6	22:44=13+31→22	38:76=29+47→38	54:108=47+61→54
7:14=7+7→7	23:46=23+23→23	39:78=31+47→39	55:110=43+67→55
8:16=5+11→8	24:48=19+29→24	40:80=19+61→40	56:112=53+59→56
9:18=7+11→9	25:50=19+31→25	41:82=41+41→41	57:114=53+61→57
10:20=7+13→10	26:52=23+29→26	42:84=41+43→42	58:116=43+73→58
11:22=11+11→11	27:54=23+31→27	43:86=43+43→43	59:118=47+71→59
12:24=11+13→12	28:56=19+37→28	44:88=41+47→44	60:120=59+61→60
13:26=13+13→13	29:58=29+29→29	45:90=43+47→45	61:122=61+61→61
14:28=11+17→14	30:60=29+31→30	46:92=31+61→46	62:124=53+71→62
15:30=13+17→15	31:62=31+31→31	47:94=41+53→47	63:126=53+73→63

Lesson from Integer Theory DACs

We could come up with
completely new DAC architectures
based on integer theory.

Polygonal numbers, Prime numbers

Euclid

Leonhard Euler
1707 - 1783

Srinivasa Aiyangar Ramanujan
1887 - 1920

Research 3: Gray code DAC

Objective

- DAC architecture for clean signal generation

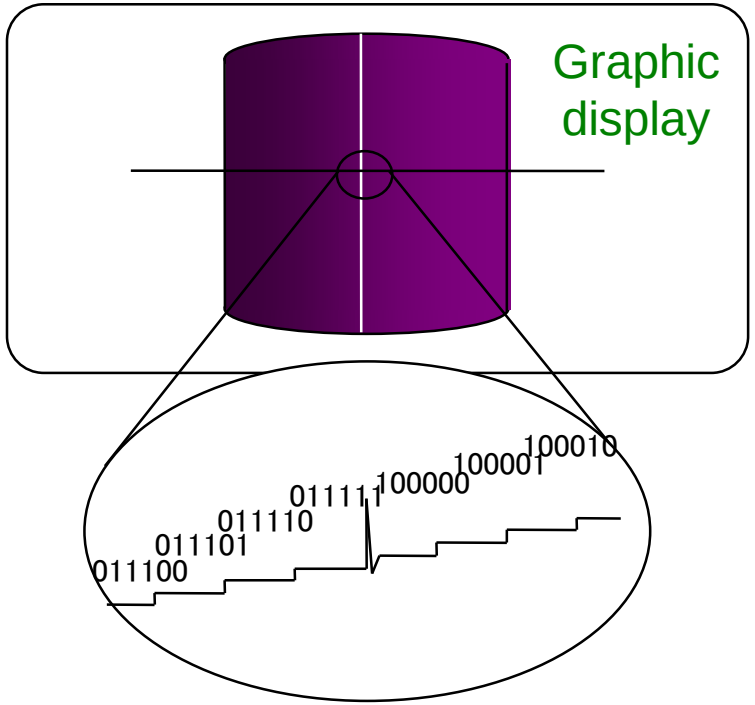
Approach

- Glitch reduction with Gray-code input topology

Glitch Problem and Remedy

Glitch

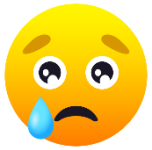
Serious deterioration of
graphic display and video applications



Remedy

- High-order reconstruction filter
- Track/hold circuitry at the DAC output

} Extra chip area & power



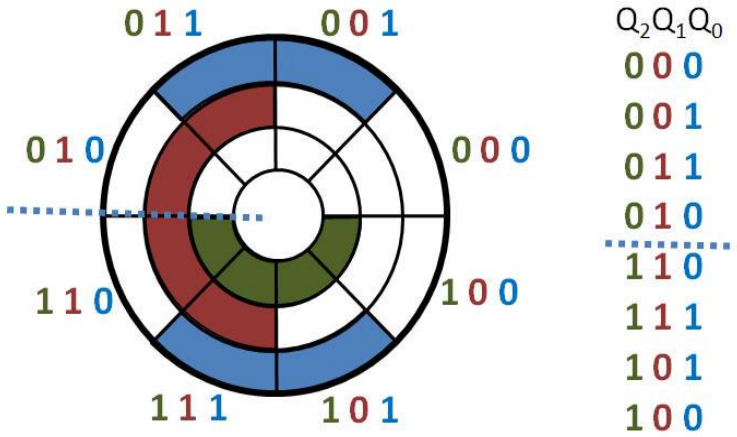
- Gray-code input DAC topology

What is Gray code ?

Gray code is a binary numeral system
where two successive values differ in only one bit.

4-bit Gray code vs. 4-bit Natural Binary Code

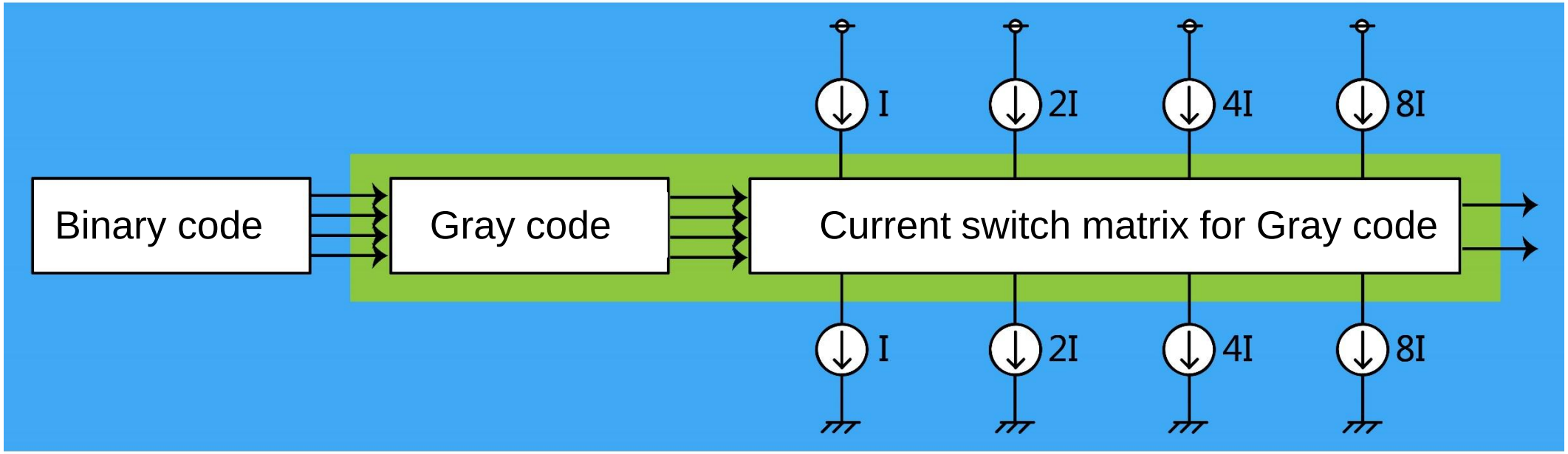
Decimal numbers	Natural Binary Code	4-bit Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000



Frank Gray and A. L. Johnsrud in television booth. Behind the glass panels at sides and top are the photo-electric cells.

Gray code was invented by Frank Gray at Bell Lab in 1947.

Proposed DAC Architecture

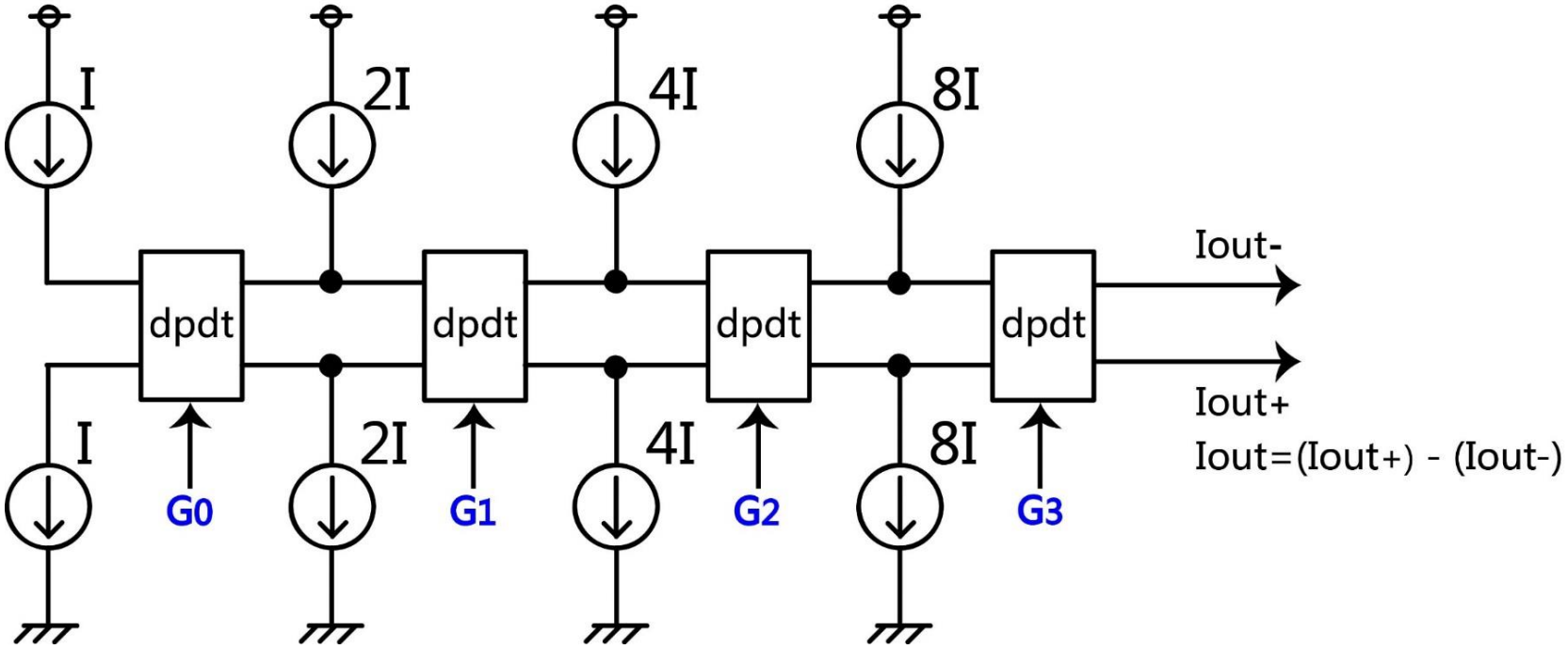


Binary code domain

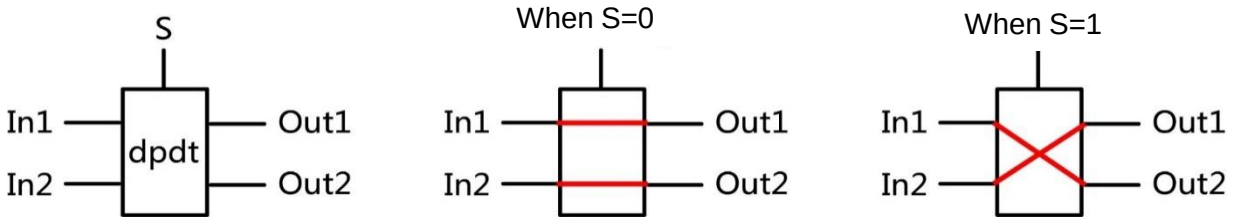
Gray code domain

A long time ago, an analog IC design authority said,
“There is no Gray-code input DAC.”

Proposed Gray-code Input Current-Steering DAC



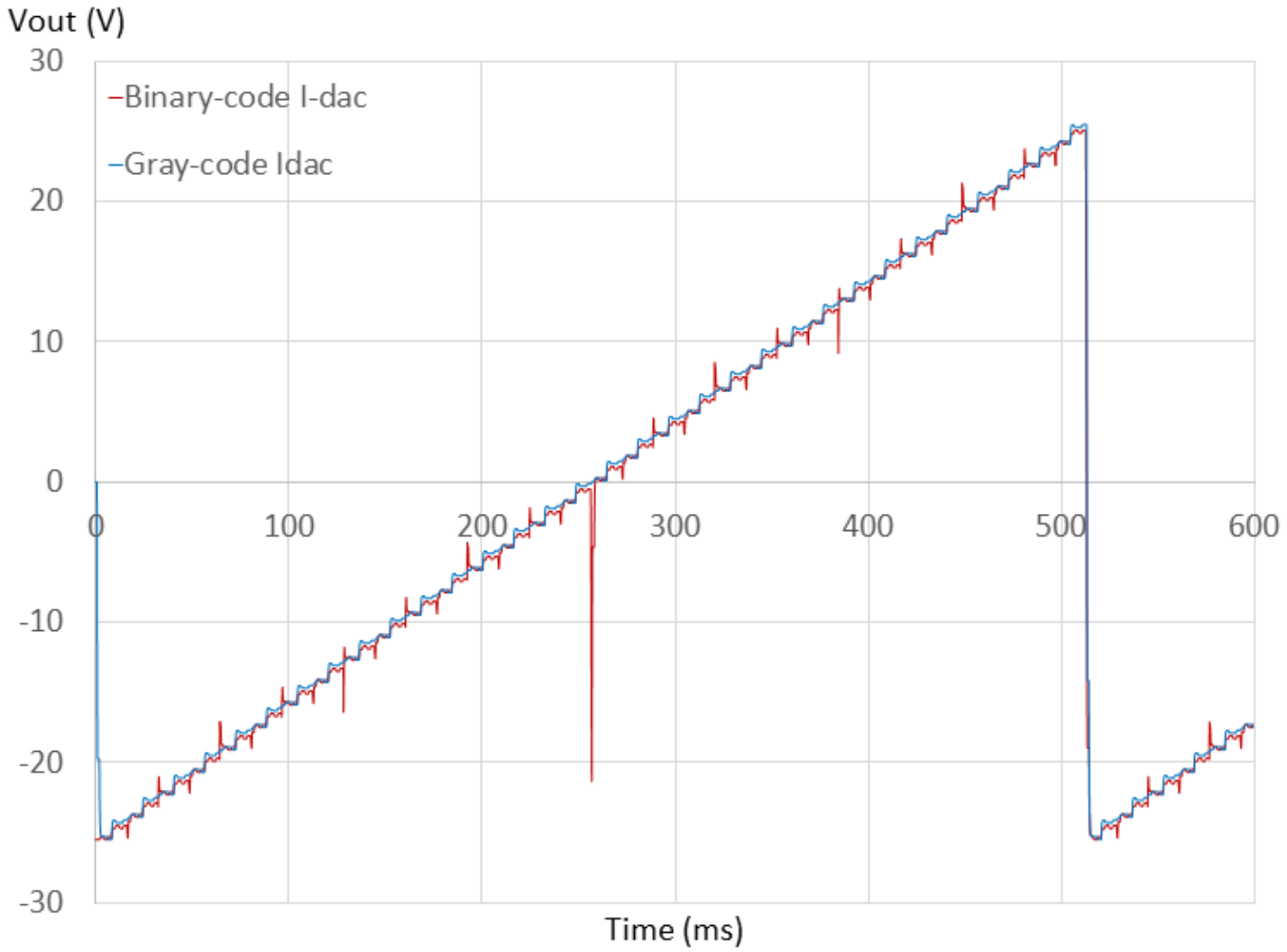
DPDT (Double-Pole Double-Throw) Switch



Parallel connection

Cross connection

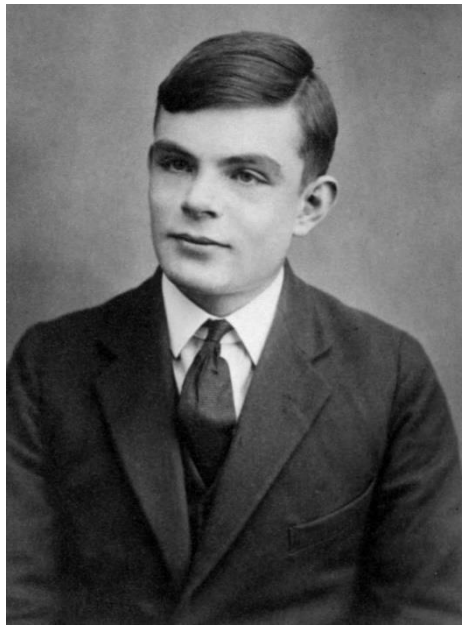
Simulation Result (Random Switching Delay)



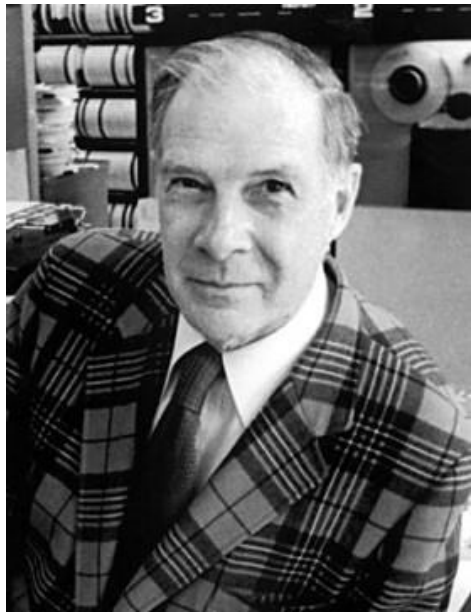
Binary-input Current-Steering DAC vs. Gray-code input current-steering DAC

Lesson from Gray-code Input DAC

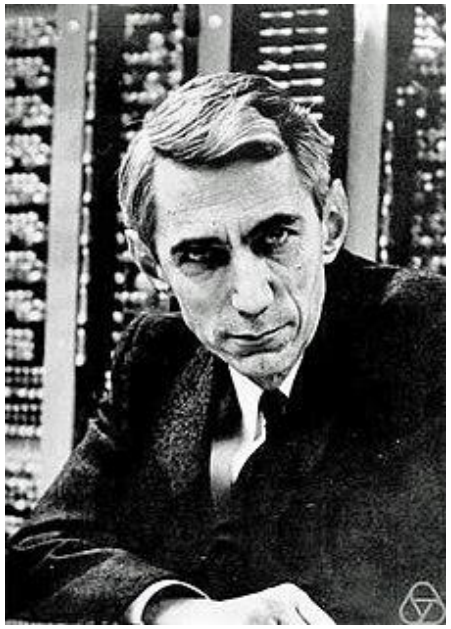
Coding theory leads to robust mixed-signal circuit design.



Alan Mathison Turing
1912 - 1954



Richard Wesley Hamming
1915 - 1998



Claude Elwood Shannon
1916- 2001

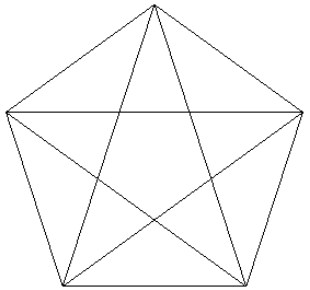
Conclusion

Analog / mixed-signal IC designers can obtain
new circuit design hints from classical mathematics.

Both circuit design and mathematics are fun !

Number is within of all things.

Pythagoras



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