Study of Helix Functions and Multi-Source Rauch Filters

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1. Research Background
   • Motivation, objectives and achievements
   • Helix functions and superposition formula
2. Ringing Test for Rauch Low-Pass Filter
   • Behaviors of fully differential op amp
   • Stability test for fully differential Rauch LPF
3. Analysis of Rauch Complex Filter
   • Spectrum of polyphase signals
   • Design of 4\textsuperscript{th}-order Rauch complex filter
4. Conclusions
1. Research Background
Motivation of Helix Waves and Multi-Source Circuits

Definition of Helix functions?

Polyphase filter

Complex filter

 Undefined concepts of multi-source signals
• Negative and positive frequencies?
• Negative and positive complex signals?
• Negative and positive polyphase signals?
• Spectrum of multi-source signals?

• Fully differential Rauch LPF
• Rauch complex filter

New architectures

A general formula for multi-source networks?
1. Research Background

Stability Test for Electronic Systems

Ringing in electronic systems

Nyquist plot of loop gain

Nichols plot of loop gain

Nichols chart in Network Analyzer?

(Overshoot)

(Unused tool)

(Unclear operating region)

(Very complicated)

(Technology limitations)
1. Research Background
Innovation of This Work

**Merits of complex numbers**
- **Use of complex functions**
  - Fundamental model of motion in both time and frequency domains
- **Superposition formula** for multi-source networks
- **Nichols chart** of self-loop function
  - A useful tool for stability test
  - Easily integrated in network analyzers

**Nichols plot** of self-loop function

**Merits of Rauch filters**
- **Easiest selection** of components
- **Lowest power consumption**
- **Simplest design** in fully differential forms and complex topologies
1. Research Background

Objectives of Study

• Definitions of helix waves and polyphase signals
• Derivation of transfer functions in multi-source networks using superposition formula
  ○ Investigation of operating regions of $4^{th}$-order fully differential Rauch low-pass filter
    → Over-damping (high delay in rising time)
    → Critical damping (max power propagation)
    → Under-damping (overshoot and ringing)
• Derivation of transfer function and image rejection ratio for $4^{th}$-order Rauch complex filter
1. Research Background

Achievements of Study

Superposition formula for multi-source networks

\[ V_o(t) \sum_{i=1}^{n} \frac{1}{Z_i} + V_o(t) \sum_{i=1}^{n} \frac{1}{Z_{si}} \sum_{k=1}^{n} \frac{1}{Z_{pk}} = \sum_{i=1}^{n} \left( \frac{V_i(t)}{Z_i} + I_{ai}(t) - I_{gi}(t) \right) \]

Definitions of helix waves

\[ V_{he-}(t) = Ahe(-\omega_0t - \theta_0) \quad \quad V_{he+}(t) = Ahe(\omega_0t + \theta_0) \]

Stability Test for a 4\textsuperscript{th}-order Rauch LPF

Analysis of a 4\textsuperscript{th}-order Rauch complex filter

IRR = 32 dB
1. Research Background

Helix and Sinusoidal Waves

Positive helix function

\[ V_{he^+}(t) = Ahe(\omega_0 t + \theta_0) = A\sqrt{2}e^{j(\omega_0 T_0 + \theta_0)} \]

\[ = A\cos(\omega_0 t + \theta_0) + jA\sin(\omega_0 t + \theta_0) \]

Negative helix function

\[ V_{he^-}(t) = Ahe(-\omega_0 t - \theta_0) = A\sqrt{2}e^{j(-\omega_0 T_0 - \theta_0)} \]

\[ = A\cos(-\omega_0 t - \theta_0) + jA\sin(-\omega_0 t - \theta_0) \]
1. Research Background

Superposition Theorem for Multi-Source Systems

Superposition formula:

\[ V_O(t)(\sum_{i=1}^{n} \frac{1}{Z_i} + \sum_{i=1}^{n} \frac{1}{Z_{si}} + \frac{1}{Z_{pik}}) = \sum_{i=1}^{n} \left( \frac{V_i(t)}{Z_i} + I_{ai}(t) - I_{gi}(t) \right) \]

\( V_O(t) \): Voltage at one node
\( V_i(t) \): Input voltage sources
\( I_{ai}(t) \): Ahead-toward current sources
\( I_{gi}(t) \): Ground-toward current sources
\( Z_i, si, pi (t) \): Impedances at each branch

- Multi-source systems, feedback networks (op amps, amplifiers, polyphase filters, complex filters...)

A general multi-source network
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   • Stability test for fully differential Rauch LPF

3. Analysis of Rauch Complex Filter
   • Spectrum of polyphase signals
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4. Conclusions
2. Ringing Test for Rauch Low-Pass Filter
Self-loop Function in A Transfer Function

Linear system

\[ H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)} \]

Transfer function

\[ H(\omega) = b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n \]
\[ a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n \]

Model of a linear system

\[ A(\omega) : \text{Numerator function} \]
\[ H(\omega) : \text{Transfer function} \]
\[ L(\omega) : \text{Self-loop function} \]

Variable: angular frequency (\( \omega \))

- Polar chart \( \rightarrow \) Nyquist chart
- Magnitude-frequency plot
- Angular-frequency plot
- Magnitude-angular diagram \( \rightarrow \) Nichols diagram

Bode plots
2. Ringing Test for Rauch Low-Pass Filter

Operating Regions of 4\textsuperscript{th}-Order System

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<td>5</td>
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<td>10</td>
<td>5</td>
</tr>
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</table>

- **Under-damping:** \[ 1 : 2 : 3 : 2 : 1 \]
  \[ H_1(\omega) = \frac{1}{(j\omega)^4 + 2(j\omega)^3 + 3(j\omega)^2 + 2j\omega + 1} \]

- **Critical damping:** \[ 1 : 4 : 6 : 4 : 1 \]
  \[ H_2(\omega) = \frac{1}{(j\omega)^4 + 4(j\omega)^3 + 6(j\omega)^2 + 4j\omega + 1} \]

- **Over-damping:** \[ 1 : 9 : 10 : 9 : 1 \]
  \[ H_3(\omega) = \frac{1}{(j\omega)^4 + 9(j\omega)^3 + 10(j\omega)^2 + 9j\omega + 1} \]
3. Ringing Test for Rauch Low-Pass Filter

Behaviors of Fully Differential Op Amp

**Fully differential two-stage op amp**

**Small signal model**

**Applying superposition formula at each node**

\[
V_a \left[ \frac{1}{R_s} + j\omega(C_{gs1} + C_{gd1}) \right] = V_{in} + V_a \cdot j\omega C_{gs1};
\]

\[
V_b \left[ j\omega(C_{gd2} + C_{db1} + C_{gs2} + C_{gd2}) + \frac{1}{R_{d1}} + j\omega C_c \right]
\]

\[
= V_a \left( j\omega C_{gd1} - g_{m1} \right) + V_{in} \left( j\omega C_{gd2} + \frac{j\omega C_c}{1 + j\omega R_c C_c} \right);
\]

\[
V_{out} \left[ j\omega(C_{gd2} + C_{db2}) + \frac{j\omega C_c}{1 + j\omega R_c C_c} + \frac{1}{R_{d2}} \right] = V_a \left( j\omega C_{gd2} + \frac{j\omega C_c}{1 + j\omega R_c C_c} - g_{m2} \right);
\]

**Open-loop function** \( A(\omega) \)

\[
A(\omega) = \frac{b_0 (j\omega)^4 + b_1 (j\omega)^3 + b_2 (j\omega)^2 + b_3 (j\omega) + b_4}{a_0 (j\omega)^5 + a_1 (j\omega)^4 + a_2 (j\omega)^3 + a_3 (j\omega)^2 + a_4 j\omega + 1}.
\]

**Self-loop function** \( L(\omega) \)

\[
L(\omega) = a_0 (j\omega)^5 + a_1 (j\omega)^4 + a_2 (j\omega)^3 + a_3 (j\omega)^2 + a_4 j\omega;
\]

**Nichols plot of self-loop function** \( L(\omega) \)

- **Magnitude (dB)**
  - 55 dB
- **Phase (deg)**
  - 90°
- **Phase margin = 90 degrees**

**Bode plot of open-loop function** \( A(\omega) \)

- **55 dB**
- **Frequency (Hz)**
3. Ringing Test for Rauch Low-Pass Filter

Analysis of Fully Differential Rauch Low-Pass Filter

**Fully differential Rauch LPF**

![Circuit Diagram]

**Operating regions**

- **Over-damping** \((C_3 = 0.1 \text{ nF})\),
- **Critical damping** \((C_3 = 0.5 \text{ nF})\),
- **Under-damping** \((C_3 = 1.4 \text{ nF})\).

**Transfer function** \(H(\omega)\) and **self-loop function** \(L(\omega)\)

\[
H(\omega) = \frac{b_0}{1 + a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega};
\]

\[
L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega;
\]

Here, parameters are given as

\[
a_1 = R_2 R_3 \left( R_5 + R_6 + \frac{R_5 R_6}{R_4} \right) C_1 C_2 C_4 + R_3 R_6 \left( R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) C_2 C_3 C_4; a_3 = \left( R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) C_2 + \left( R_5 + R_6 + \frac{R_5 R_6}{R_4} \right) C_4;
\]

\[
a_2 = R_2 R_3 C_1 C_2 + R_5 R_6 C_3 C_4 + \left( R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) \left( R_5 + R_6 + \frac{R_5 R_6}{R_4} \right) C_2 C_4; b_0 = \frac{R_3 R_6}{R_1 R_4}; a_0 = R_2 R_3 R_5 R_6 C_1 C_2 C_3 C_4;
\]
3. Ringing Test for Rauch Low-Pass Filter
Behaviors of Fully Differential Rauch Low-Pass Filter

**Bode plot of transfer function**

- **Over-damping:**
  - Phase margin is 74 degrees.
- **Critical damping:**
  - Phase margin is 68 degrees.
- **Under-damping:**
  - Phase margin is 59 degrees.

**Nichols plot of self-loop function**

- **Over-damping:**
  - Phase margin is 74 degrees.
- **Critical damping:**
  - Phase margin is 68 degrees.
- **Under-damping:**
  - Phase margin is 59 degrees.
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4. Conclusions
3. Analysis of Rauch Complex Filter

Characteristics of Low-IF Receiver

Block diagram of Low-IF receiver

This Work

Antenna

BPF

LNA

MIXER

4th order Poly-phase Filter

Limiting amplifier

PLL

BPF

Quad Gen.

VCO

RSSI

ADC

Bandgap Reference

Step-down frequency converter

Wanted Signals

V_i = +jV

V_i = -jV

anti-clockwise

V_i = jV

clockwise

Wanted signals

Image signals

Spectrum of received signals

$|H(j2\pi f)|_{dB}$

frequency

$0$. $f$

amplitude

Wanted Signals

V_i = +jV

V_i = -jV

anti-clockwise

V_i = jV

clockwise

Wanted signals

Image signals

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$|H(j2\pi f)|_{dB}$

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$|H(j2\pi f)|_{dB}$

frequency

$0$. $f$

amplitude

Wanted Signals

V_i = +jV

V_i = -jV

anti-clockwise

V_i = jV

clockwise

Wanted signals

Image signals

Spectrum of received signals

$|H(j2\pi f)|_{dB}$

frequency

$0$. $f$

amplitude
3. Analysis of Rauch Complex Filter

Positive Polyphase Signals on Frequency Domain

Positive polyphase signals

\[ S_{Pos\_poly} \{ V_1(t); V_2(t); V_3(t); V_4(t) \} = \{ 1; j; (j^2); (j^3) \} V_{pos}(t) \]
3. Analysis of Rauch Complex Filter

Negative Polyphase Signals on Frequency Domain

Negative polyphase signals

\[ S_{\text{Neg-poly}} \{ V_1(t); V_2(t); V_3(t); V_4(t) \} = \{1; -j; (-j)^2; (-j)^3\} V_{\text{neg}}(t) \]
3. Analysis of Rauch Complex Filter
Polyphase Signals in All Frequency Domains

**Negative polyphase signals**

\[ V_{n4} = +jV_n \]
\[ V_{n3} = -V_n \]
\[ V_{n1} = V_n \]
\[ V_{neg2} = -jV_{neg} \]
clockwise

**Complex number**

Imaginary unit \( j = 90^\circ \) phase shift

\[ (+j)^2 = e^{+j\pi} = -1 \]
\[ (-j)^2 = e^{-j\pi} = -1 \]

**Positive polyphase signals**

\[ V_{p2} = jV_p \]
\[ V_{p3} = -V_p \]
\[ V_{p4} = -jV_p \]
anti-clockwise

**Image signals**

\[ S_{Neg\_poly} \{ V_1(t); V_2(t); V_3(t); V_4(t) \} = \{ 1; -j(-j)^2; (-j)^3 \} V_{neg}(t) \]

**Wanted signals**

\[ S_{Pos\_poly} \{ V_1(t); V_2(t); V_3(t); V_4(t) \} = \{ 1; +j(+j)^2; (+j)^3 \} V_{pos}(t) \]
3. Analysis of Rauch Complex Filter Design Principle for Complex Filter Networks

Frequency shifting of real low-pass filter in all frequency domains → an active complex filter

**Low-pass filter (LPF)**

\[ H_{LPF}(\omega) = -\frac{A_{21}}{j \frac{\omega}{\omega_{cut}} + 1} \]

\( \omega_{cr} = \frac{1}{R_3 C_1} \)

\( \omega_{cr} : \text{cross angular frequency} \)

**Complex filter (CF)**

\[ H_{CF}(\omega) = -\frac{A_{21}}{j \frac{\omega - \omega_{cr}}{\omega_{cut}} + 1} \]
3. Analysis of Rauch Complex Filter

Analysis of 4th-Order Rauch Complex Filter

Applying superposition formula at each node

\[ V_a \left( \frac{1}{R_1} + j\omega C_1 + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{in}}{R_1} + \left( +j \right)^2 V_a j\omega C_1 + \frac{V_o}{R_2} + \left( +j \right)^2 \frac{V_o}{R_3}; \]

\[ V_b \left( \frac{1}{R_2} + j\omega C_2 + \frac{1}{R_4} \right) = \frac{V_a}{R_2} + \left( +j \right)^3 V_c j\omega C_2 + \frac{\left( +j \right)^3 V_c}{R_4}; \]

\[ V_c = \left[ V_a - \left( +j \right)^2 V_b \right] A(\omega); V_{out} = \left[ V_c - \left( +j \right)^2 V_c \right] A(\omega); \]

\[ V_d \left( \frac{1}{R_5} + j\omega C_3 + \frac{1}{R_6} + \frac{1}{R_7} \right) = \frac{V_c}{R_5} + \left( +j \right)^2 V_d j\omega C_3 + \frac{V_c}{R_6} + \frac{\left( +j \right)^2 V_{out}}{R_7}; \]

\[ V_e \left( \frac{1}{R_6} + j\omega C_4 + \frac{1}{R_8} \right) = \frac{V_d}{R_6} + \left( +j \right)^2 V_{out} j\omega C_4 + \frac{\left( +j \right)^3 V_{out}}{R_8}; \]

Positive transfer function

\[ H_p(\omega) = \frac{b_0}{a_0 \left( j\omega \right)^2 + a_{N1} j\omega + a_{N2}} \frac{b_1}{a_3 \left( j\omega \right)^2 + a_{N4} j\omega + a_{N5}}; \]

Negative transfer function

\[ H_n(\omega) = \frac{b_0}{a_0 \left( j\omega \right)^2 + a_{N1} j\omega + a_{N2}} \frac{b_1}{a_3 \left( j\omega \right)^2 + a_{N4} j\omega + a_{N5}}; \]

Image rejection ratio

\[ IRR(\omega) = \frac{a_0 \left( j\omega \right)^2 + a_{N1} j\omega + a_{N2}}{a_0 \left( j\omega \right)^2 + a_{N1} j\omega + a_{N2}} \frac{a_3 \left( j\omega \right)^2 + a_{N4} j\omega + a_{N5}}{a_3 \left( j\omega \right)^2 + a_{N4} j\omega + a_{N5}}; \]
3. Analysis of Rauch Complex Filter
Behaviors of 4th-Order Rauch Complex Filter

Image signals

\[ S_{\text{Neg}_\text{ poly}} \{V_1 (t); V_2 (t); V_3 (t); V_4 (t)\} = \{1; -j; (-j)^2; (-j)^3\} V_{\text{neg}} (t) \]

Wanted signals

\[ S_{\text{Pos}_\text{ poly}} \{V_1 (t); V_2 (t); V_3 (t); V_4 (t)\} = \{1; +j; (+j)^2; (+j)^3\} V_{\text{pos}} (t) \]

Bode plot of transfer function

Bandwidth = 200 kHz

Pass-band gain = 13 dB

IRR = 32 dB
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   • Design of 4\textsuperscript{th}-order Rauch complex filter

4. Conclusions
## 4. Comparison

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<td>Complex network analysis</td>
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<tr>
<td>Image rejection ratio accuracy</td>
<td>Yes</td>
<td>No</td>
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4. Discussions

- **Ringing test** for electronic networks using Nichols chart of self-loop function
  - Observation of **phase margin** can help us determine the operating regions of high-order systems.

- **Transfer function** and **image rejection ratio** give useful information about the behaviors of polyphase filters and complex filters.

- **Superposition formula**: fundamental network analysis theory for multi-source systems
  - Compute the effects of all sources at one time,
  - Get the transfer function faster, and
  - Reduce the network complexity.
4. Conclusions

This work:

- **Definitions** of helix functions and polyphase signals
- **Proposal** of superposition formula for deriving transfer function in multi-source networks
- **Stability test** for 4th-order fully differential Rauch LPF
- **Derivation** of transfer function and image rejection ratio for 4th-order Rauch complex filter

→ *Theoretical concepts of stability test* are verified by laboratory simulations.

Future work:

- **Stability test** for parasitic components in transmission lines, printed circuit boards, physical layout layers
- **Investigation** of DC offset and IQ mismatches in polyphase filters and complex filters
References


Thank you very much!