



Study of Helix Functions and Multi-Source Rauch Filters

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Outline

1. Research Background

- Motivation, objectives and achievements
- Helix functions and superposition formula

2. Ringing Test for Rauch Low-Pass Filter

- Behaviors of fully differential op amp
- Stability test for fully differential Rauch LPF

3. Analysis of Rauch Complex Filter

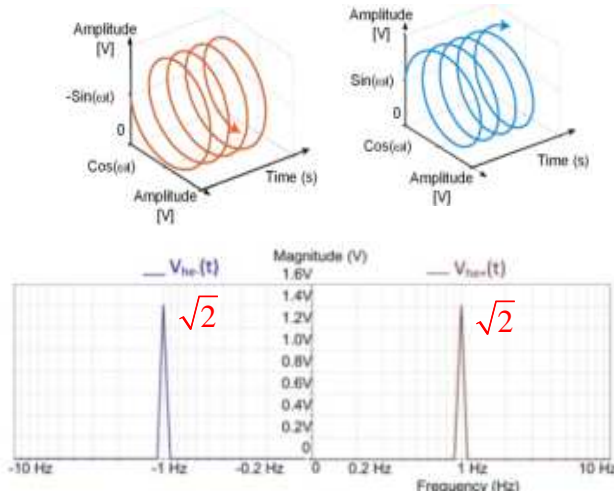
- Spectrum of polyphase signals
- Design of 4th-order Rauch complex filter

4. Conclusions

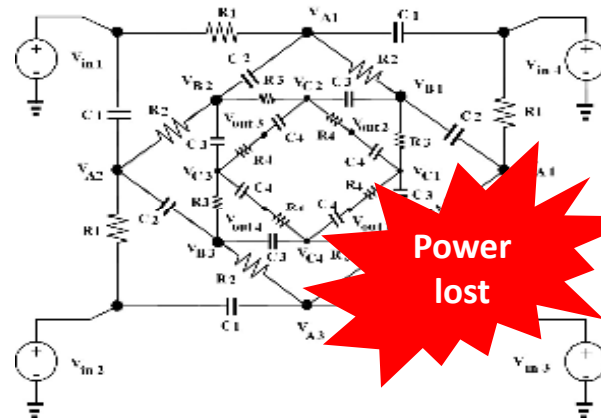
1. Research Background

Motivation of Helix Waves and Multi-Source Circuits

Definition of Helix functions?



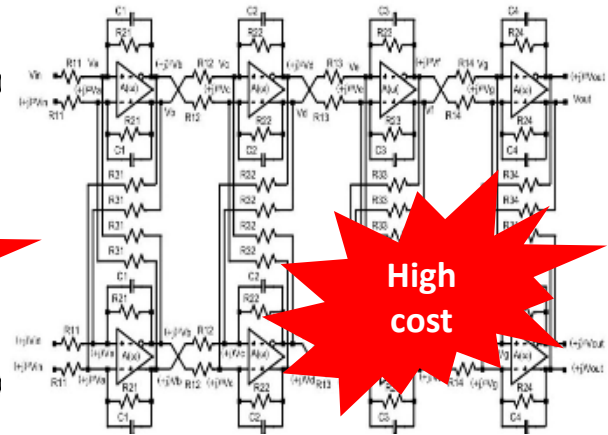
Polyphase filter



Power lost

(Very complicated)

Complex filter



High cost

(High power consumption)

Spectrum?

- Fully differential Rauch LPF
- Rauch complex filter

New architectures

Undefined concepts of multi-source signals

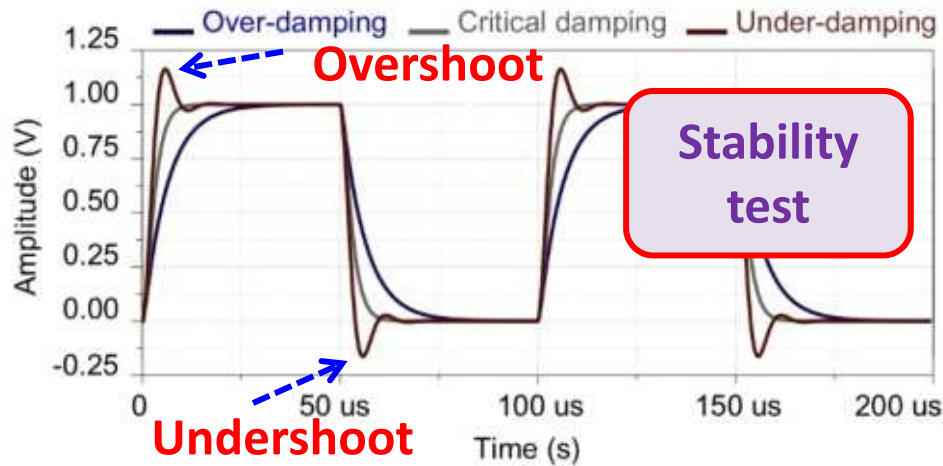
- Negative and positive frequencies?
- Negative and positive complex signals?
- Negative and positive polyphase signals?
- Spectrum of multi-source signals?

A general formula for multi-source networks?

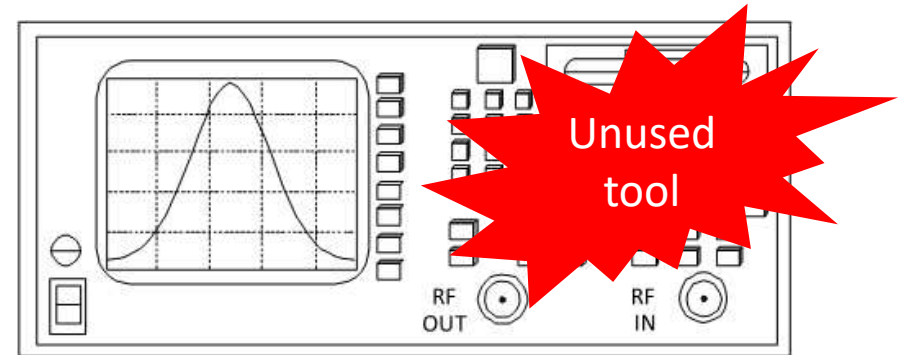
1. Research Background

Stability Test for Electronic Systems

Ringling in electronic systems

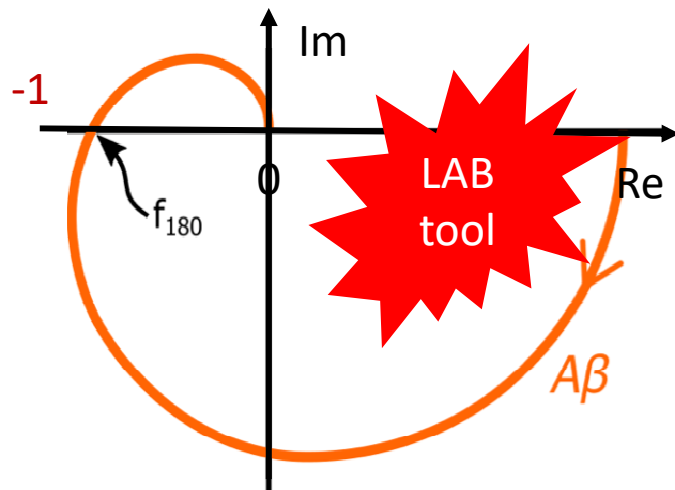


Nichols chart in Network Analyzer?



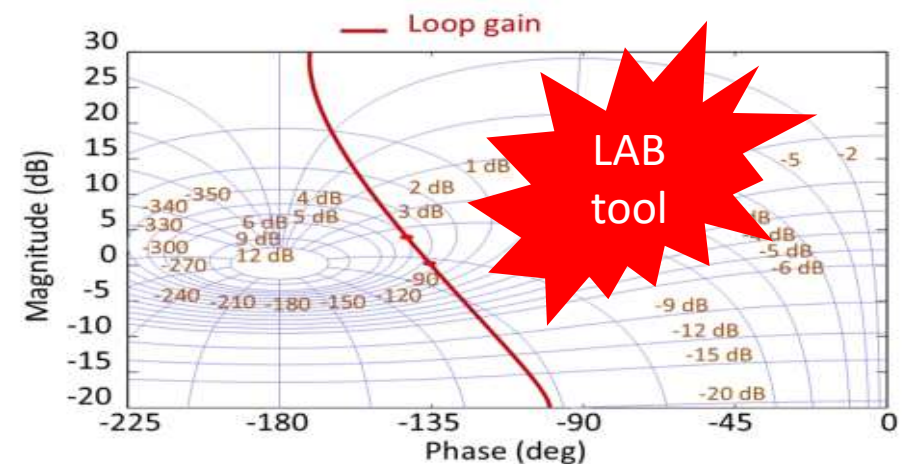
(Technology limitations)

Nyquist plot of loop gain



(Unclear operating region)

Nichols plot of loop gain



(Very complicated)

1. Research Background

Innovation of This Work

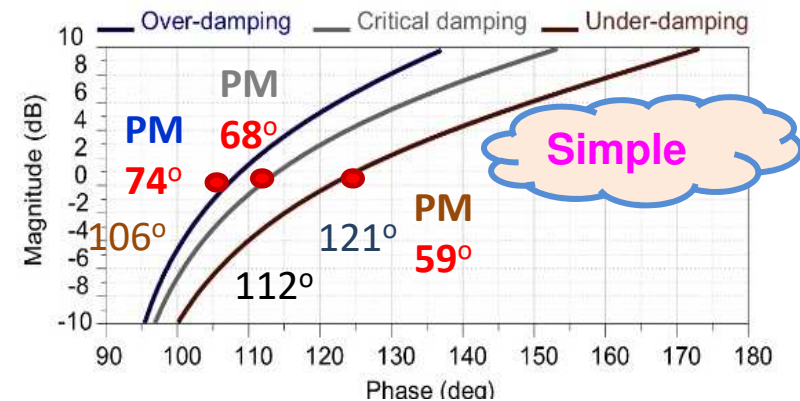
Merits of complex numbers

- Use of complex functions
 - Fundamental model of motion in both time and frequency domains
- Superposition formula for multi-source networks
- Nichols chart of self-loop function
 - A useful tool for stability test
 - Easily integrated in network analyzers

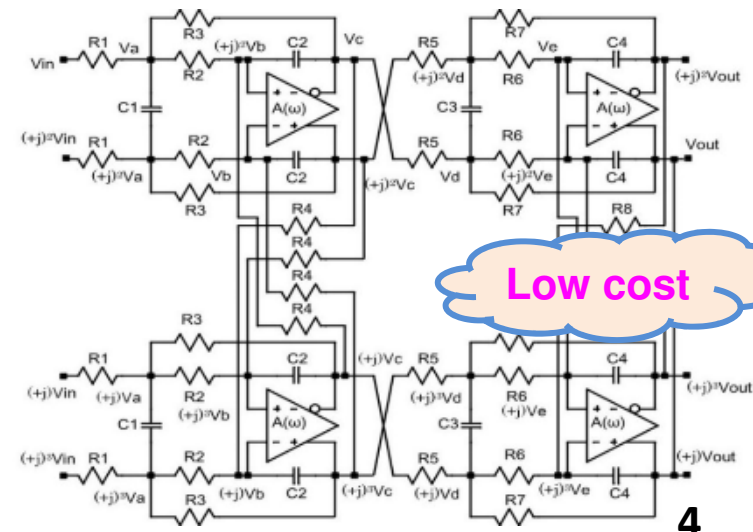
Merits of Rauch filters

- Easiest selection of components
- Lowest power consumption
- Simplest design in fully differential forms and complex topologies

Nichols plot of self-loop function



Rauch complex filter



1. Research Background

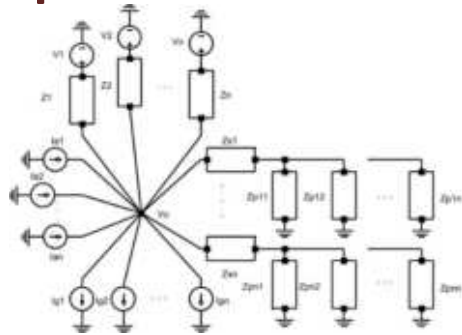
Objectives of Study

- **Definitions of helix waves and polyphase signals**
- **Derivation of transfer functions in multi-source networks using superposition formula**
 - **Investigation of operating regions of 4th-order fully differential Rauch low-pass filter**
 - **Over-damping (high delay in rising time)**
 - **Critical damping (max power propagation)**
 - **Under-damping (overshoot and ringing)**
- **Derivation of transfer function and image rejection ratio for 4th-order Rauch complex filter**

1. Research Background

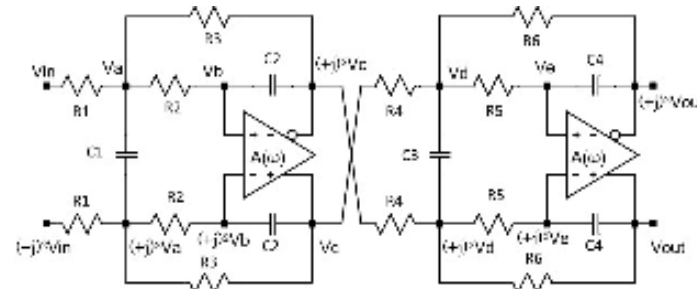
Achievements of Study

Superposition formula for multi-source networks



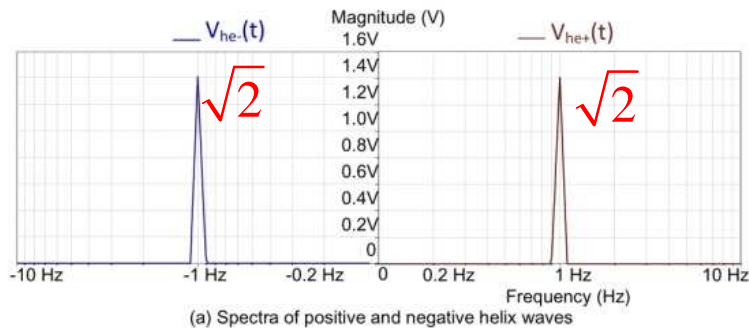
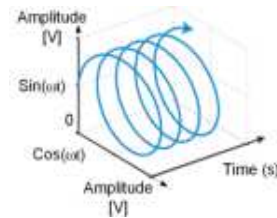
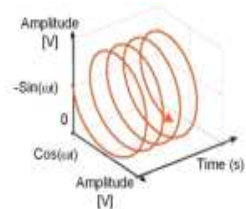
$$V_o(t) \sum_{i=1}^n \frac{I}{Z_i} + V_o(t) \sum_{i=1}^n \frac{1}{Z_{si} + \frac{1}{\sum_{k=1}^n \frac{1}{Z_{pik}}}} = \sum_{i=1}^n \left(\frac{V_i(t)}{Z_i} + I_{ai}(t) - I_{gi}(t) \right)$$

Stability Test for a 4th-order Rauch LPF

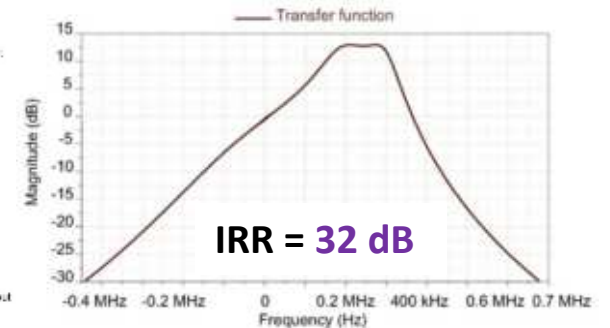
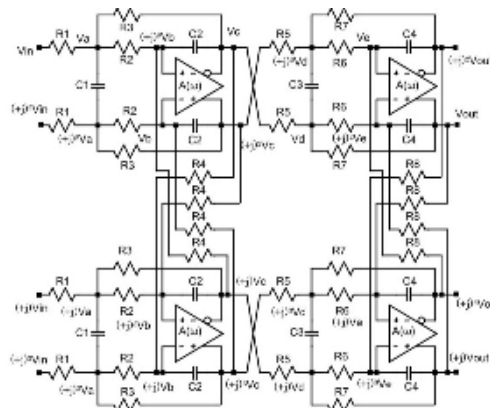


Definitions of helix waves

$$V_{he-}(t) = Ahe(-\omega_0 t - \theta_0) \quad V_{he+}(t) = Ahe(\omega_0 t + \theta_0)$$



Analysis of a 4th-order Rauch complex filter



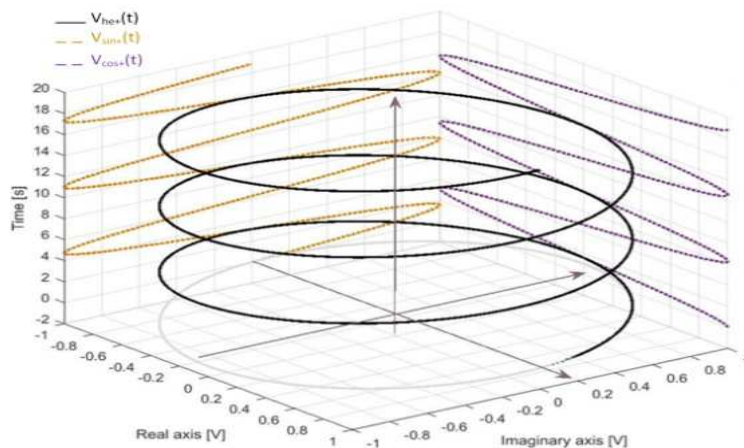
1. Research Background

Helix and Sinusoidal Waves

Positive helix function

$$V_{he+}(t) = Ahe(\omega_0 t + \theta_0) = A\sqrt{2}e^{j(\omega_0 T_0 + \theta_0)}$$

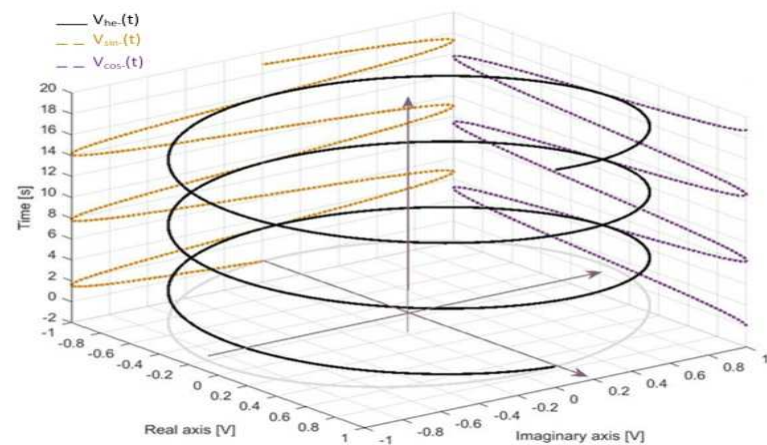
$$= A\cos(\omega_0 t + \theta_0) + jA\sin(\omega_0 t + \theta_0)$$



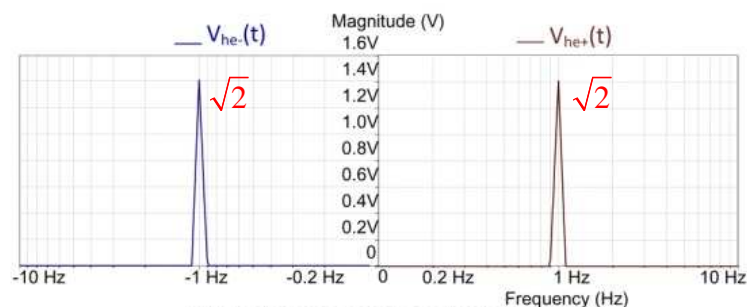
Negative helix function

$$V_{he-}(t) = Ahe(-\omega_0 t - \theta_0) = A\sqrt{2}e^{j(-\omega_0 T_0 - \theta_0)}$$

$$= A\cos(-\omega_0 t - \theta_0) + jA\sin(-\omega_0 t - \theta_0)$$

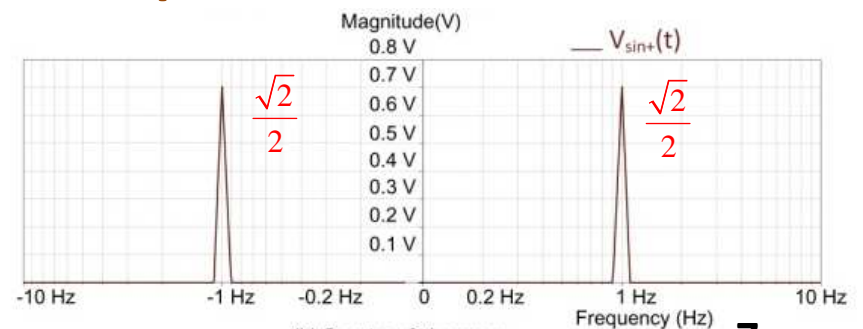


Spectrum of helix waves



(a) Spectra of positive and negative helix waves

Spectrum of sine wave



(b) Spectra of sine wave

1. Research Background

Superposition Theorem for Multi-Source Systems

Superposition formula:

$$V_o(t) \left(\sum_{i=1}^n \frac{1}{Z_i} + \sum_{i=1}^n \frac{1}{Z_{si}} + \frac{1}{\sum_{k=1}^n \frac{1}{Z_{pik}}} \right) = \sum_{i=1}^n \left(\frac{V_i(t)}{Z_i} + I_{ai}(t) - I_{gi}(t) \right)$$

$V_o(t)$: Voltage at one node

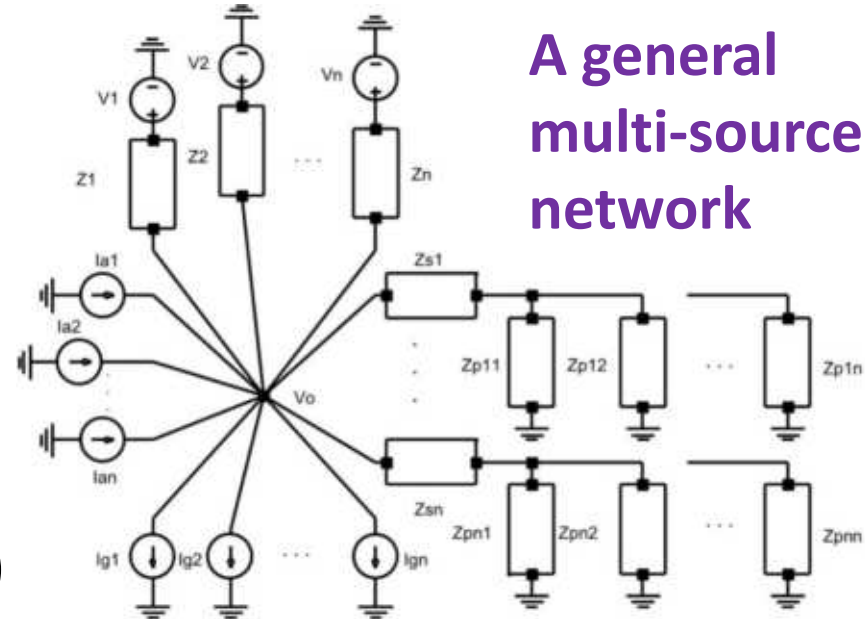
$V_i(t)$: Input voltage sources

$I_{ai}(t)$: Ahead-toward current sources

$I_{gi}(t)$: Ground-toward current sources

$Z_i, s_i, p_i, (t)$: Impedances at each branch

- Multi-source systems, feedback networks (op amps, amplifiers, polyphase filters, complex filters...)



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- **Stability test for fully differential Rauch LPF**

3. Analysis of Rauch Complex Filter

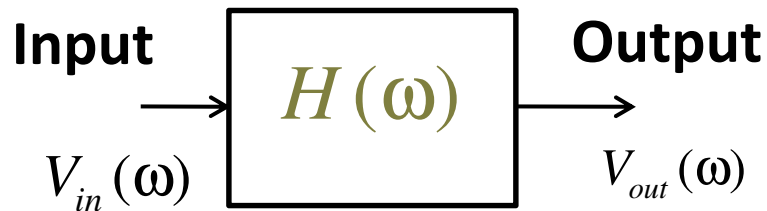
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4. Conclusions

2. Ringing Test for Rauch Low-Pass Filter

Self-loop Function in A Transfer Function

Linear system



Model of a linear system

$$H(\omega) = \frac{b_0(j\omega)^n + \dots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \dots + a_{n-1}(j\omega) + a_n}$$

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

$A(\omega)$: Numerator function

$H(\omega)$: Transfer function

$L(\omega)$: Self-loop function

Variable: angular frequency (ω)

○ Polar chart → Nyquist chart

○ Magnitude-frequency plot

○ Angular-frequency plot

○ Magnitude-angular diagram → Nichols diagram

Bode plots

2. Ringing Test for Rauch Low-Pass Filter Operating Regions of 4th-Order System

Pascal's Triangle

n = 2		1	2	1		
n = 3		1	3	3	1	
n = 4		1	4	6	4	1
n = 5	1	5	10	10	5	1

• **Under-damping:** **1 : 2 : 3 : 2 : 1**

$$H_1(\omega) = \frac{1}{(j\omega)^4 + 2(j\omega)^3 + 3(j\omega)^2 + 2j\omega + 1}$$

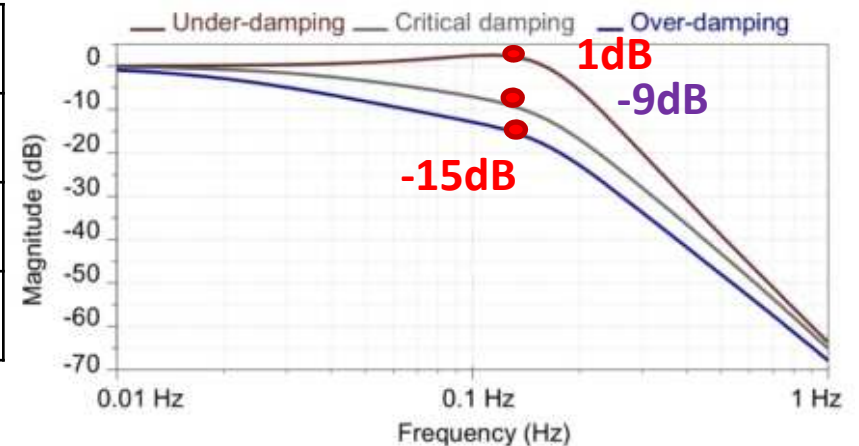
• **Critical damping:** **1 : 4 : 6 : 4 : 1**

$$H_2(\omega) = \frac{1}{(j\omega)^4 + 4(j\omega)^3 + 6(j\omega)^2 + 4j\omega + 1}$$

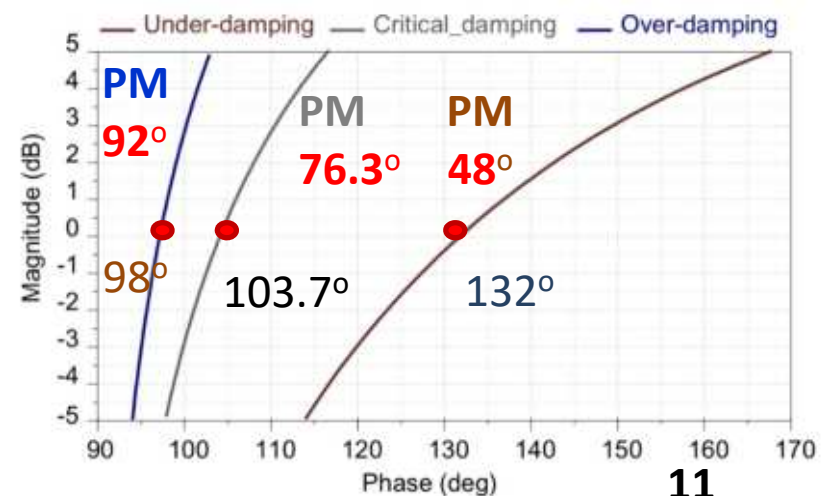
• **Over-damping:** **1 : 9 : 10 : 9 : 1**

$$H_3(\omega) = \frac{1}{(j\omega)^4 + 9(j\omega)^3 + 10(j\omega)^2 + 9j\omega + 1}$$

Bode plot of transfer function

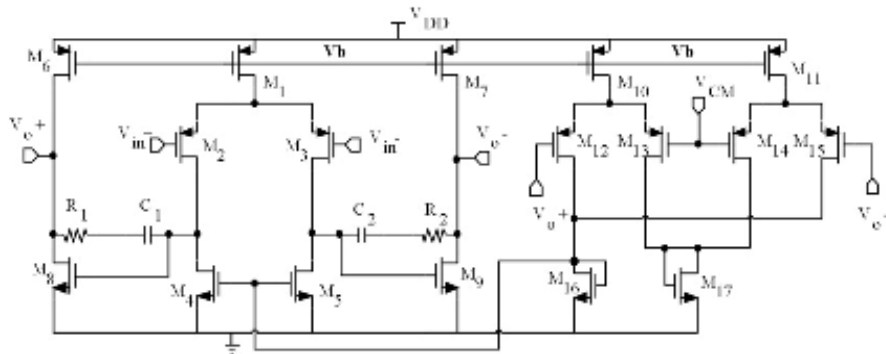


Nichols plot of self-loop function

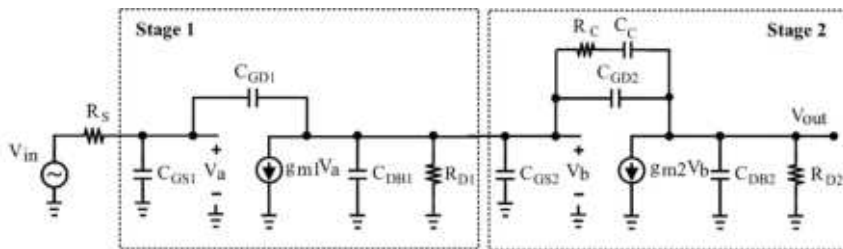


3. Ringing Test for Rauch Low-Pass Filter Behaviors of Fully Differential Op Amp

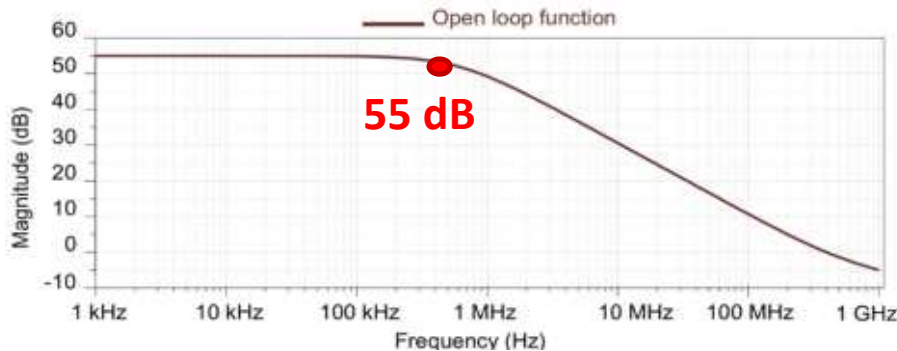
Fully differential two-stage op amp



Small signal model



Bode plot of open-loop function A(ω)



Applying superposition formula at each node

$$V_a \left[\frac{1}{R_s} + j\omega(C_{GS1} + C_{GD1}) \right] = \frac{V_{in}}{R_s} + V_b j\omega C_{GD1};$$

$$V_b \left[j\omega(C_{GD1} + C_{DB1} + C_{GS2} + C_{GD2}) + \frac{1}{R_{D1}} + \frac{j\omega C_c}{1 + j\omega R_c C_c} \right] = V_a (j\omega C_{GD1} - g_{m1}) + V_{out} \left(j\omega C_{GD2} + \frac{j\omega C_c}{1 + j\omega R_c C_c} \right);$$

$$V_{out} \left[j\omega(C_{GD2} + C_{DB2}) + \frac{j\omega C_c}{1 + j\omega R_c C_c} + \frac{1}{R_{D2}} \right] = V_b \left(j\omega C_{GD2} + \frac{j\omega C_c}{1 + j\omega R_c C_c} - g_{m2} \right);$$

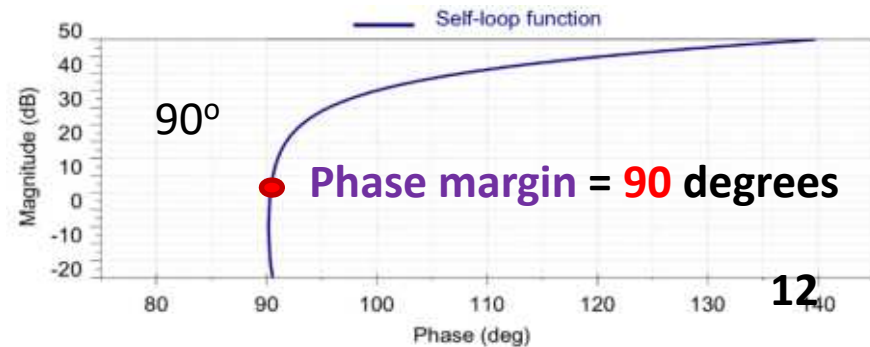
Open-loop function A(ω)

$$A(\omega) = \frac{b_0 (j\omega)^4 + b_1 (j\omega)^3 + b_2 (j\omega)^2 + b_3 (j\omega)^1 + b_4}{a_0 (j\omega)^5 + a_1 (j\omega)^4 + a_2 (j\omega)^3 + a_3 (j\omega)^2 + a_4 j\omega + 1};$$

Self-loop function L(ω)

$$L(\omega) = a_0 (j\omega)^5 + a_1 (j\omega)^4 + a_2 (j\omega)^3 + a_3 (j\omega)^2 + a_4 j\omega;$$

Nichols plot of self-loop function L(ω)

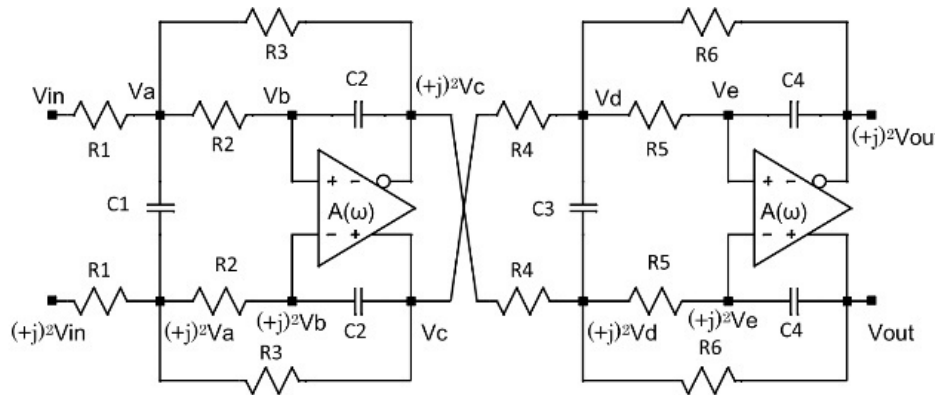


3. Ringing Test for Rauch Low-Pass Filter

Analysis of Fully Differential Rauch Low-Pass Filter

Fully differential Rauch LPF

Applying superposition formula at each node



$$V_a \left(\frac{1}{R_1} + j\omega C_1 + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{in}}{R_1} + \frac{V_b}{R_2} + \frac{(+j)^2 V_c}{R_3} + (+j)^2 V_a j\omega C_1;$$

$$V_b \left(\frac{1}{R_2} + j\omega C_2 \right) = \frac{V_a}{R_2} + (+j)^2 V_c j\omega C_2; V_c = \left(V_b - (+j)^2 V_b \right) A(\omega);$$

$$V_d \left(\frac{1}{R_4} + j\omega C_3 + \frac{1}{R_5} + \frac{1}{R_6} \right) = \frac{V_c}{R_4} + \frac{V_e}{R_5} + \frac{(+j)^2 V_{out}}{R_6} + (+j)^2 V_d j\omega C_3;$$

$$V_e \left(\frac{1}{R_5} + j\omega C_4 \right) = \frac{V_d}{R_5} + (+j)^2 V_{out} j\omega C_4; V_{out} = \left(V_e - (+j)^2 V_e \right) A(\omega);$$

Transfer function $H(\omega)$ and self-loop function $L(\omega)$

$$H(\omega) = \frac{b_0}{1 + a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega};$$

$$L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega;$$

Here, parameters are given as

$$a_1 = R_2 R_3 \left(R_5 + R_6 + \frac{R_5 R_6}{R_4} \right) C_1 C_2 C_4 + R_5 R_6 \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) C_2 C_3 C_4; a_3 = \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) C_2 + \left(R_5 + R_6 + \frac{R_5 R_6}{R_4} \right) C_4;$$

$$a_2 = R_2 R_3 C_1 C_2 + R_5 R_6 C_3 C_4 + \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) \left(R_5 + R_6 + \frac{R_5 R_6}{R_4} \right) C_2 C_4; b_0 = \frac{R_3 R_6}{R_1 R_4}; a_0 = R_2 R_3 R_5 R_6 C_1 C_2 C_3 C_4;$$

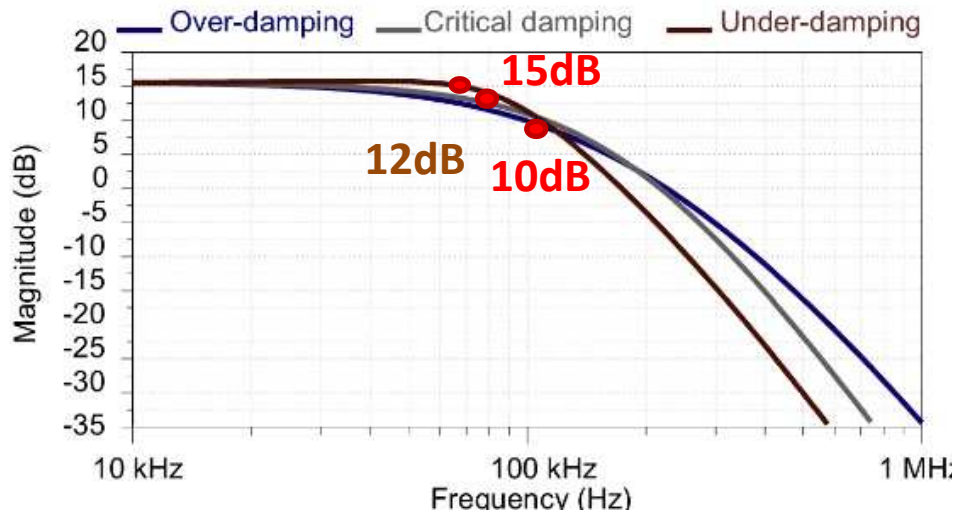
Operating regions

- Over-damping ($C_3 = 0.1$ nF),
- Critical damping ($C_3 = 0.5$ nF),
- Under-damping ($C_3 = 1.4$ nF).

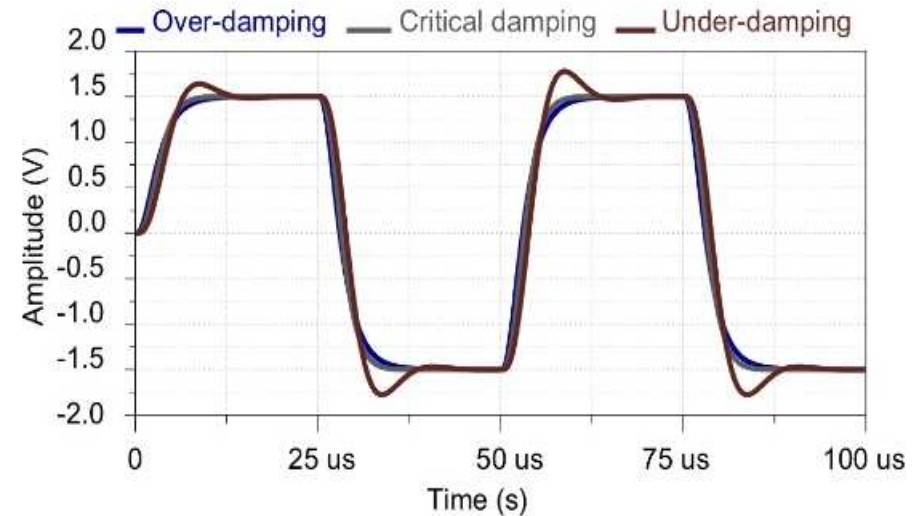
3. Ringing Test for Rauch Low-Pass Filter

Behaviors of Fully Differential Rauch Low-Pass Filter

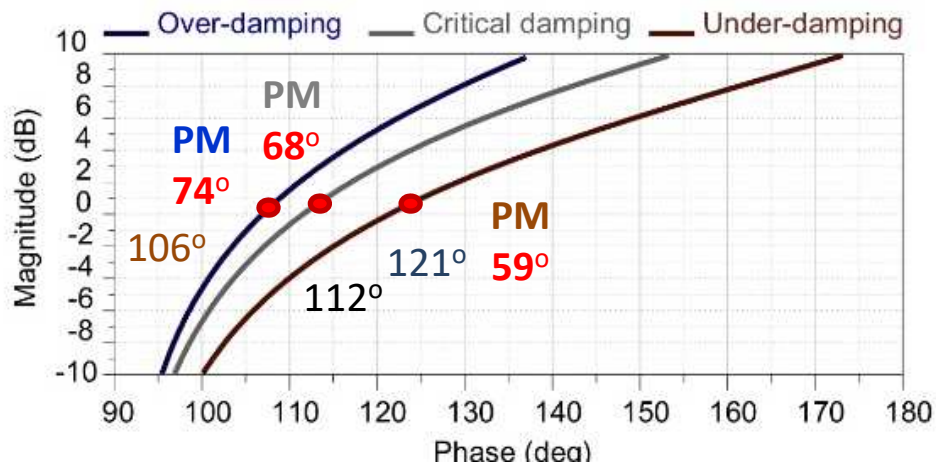
Bode plot of transfer function



Transient response



Nichols plot of self-loop function



Over-damping:

→ Phase margin is **74** degrees.

Critical damping:

→ Phase margin is **68** degrees.

Under-damping:

→ Phase margin is **59** degrees.

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- Spectrum of polyphase signals
- Design of 4th-order Rauch complex filter

4. Conclusions

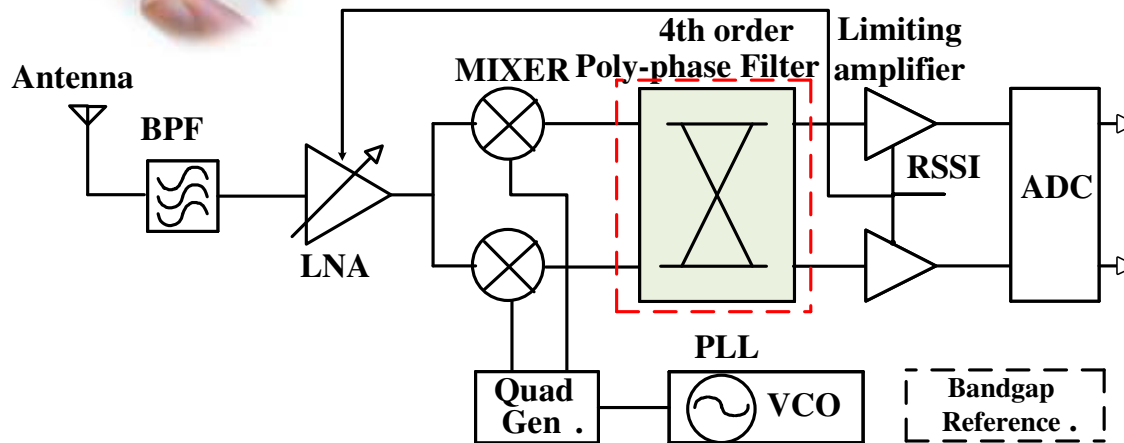
3. Analysis of Rauch Complex Filter

Characteristics of Low-IF Receiver

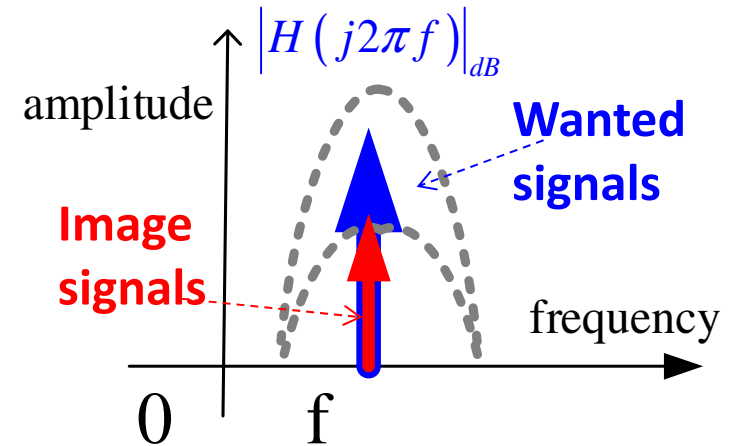


Block diagram of Low-IF receiver

This Work



Spectrum of received signals



Step-down frequency converter

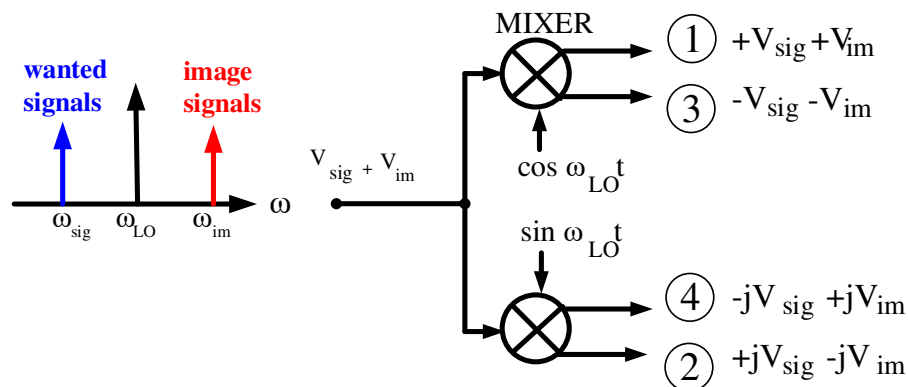
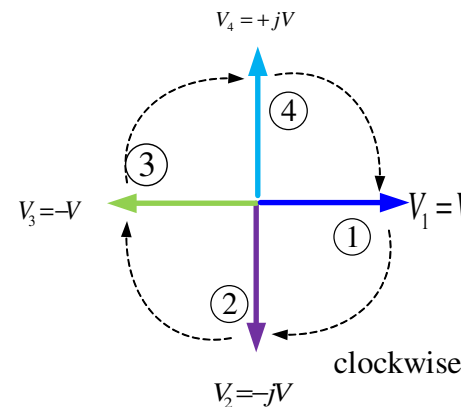
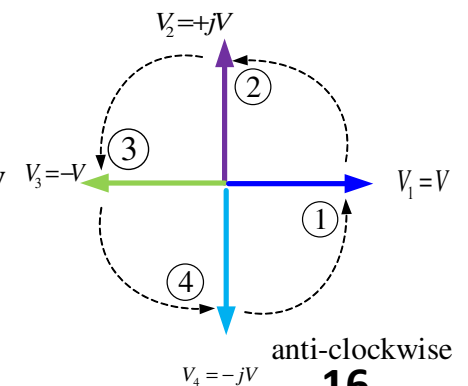


Image Signals



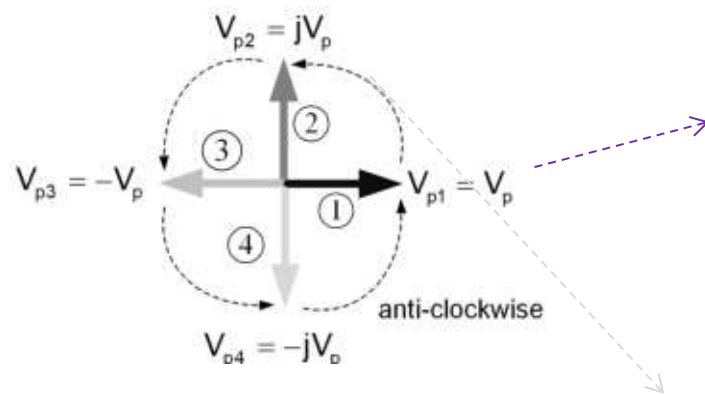
Wanted Signals



3. Analysis of Rauch Complex Filter

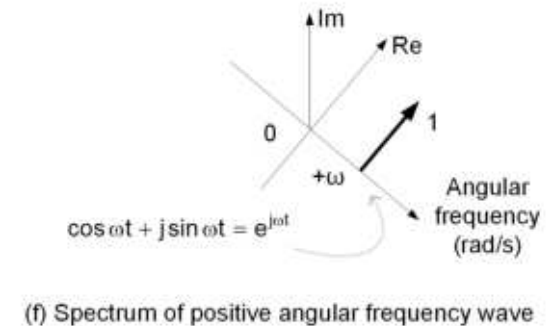
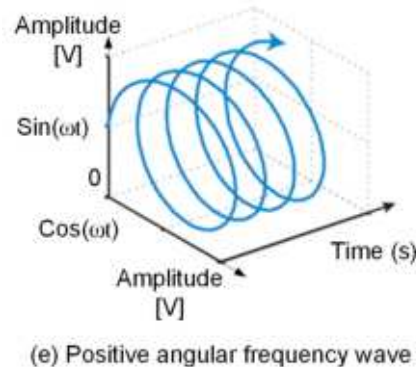
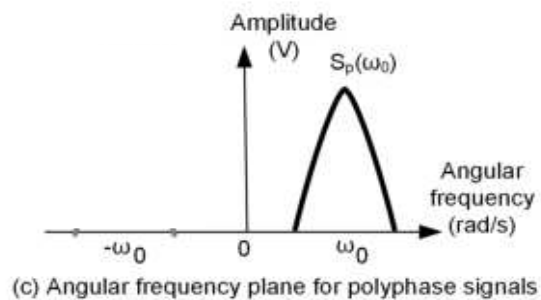
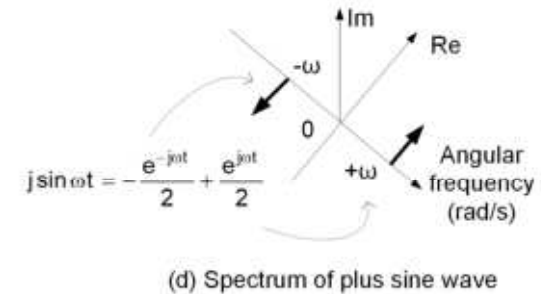
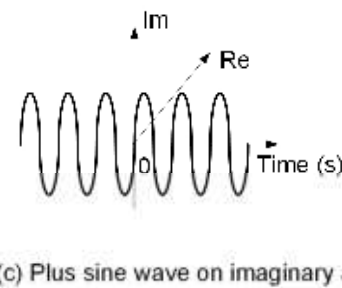
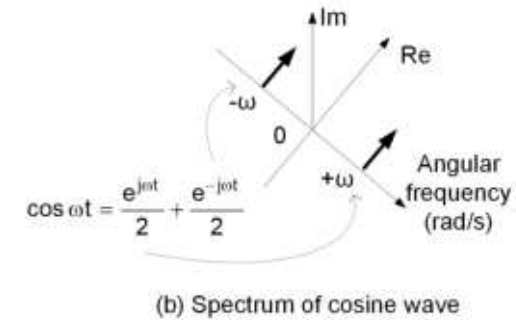
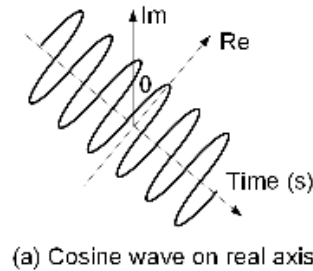
Positive Polyphase Signals on Frequency Domain

Positive polyphase signals



$$S_{Pos_poly} \{V_1(t); V_2(t); V_3(t); V_4(t)\}$$

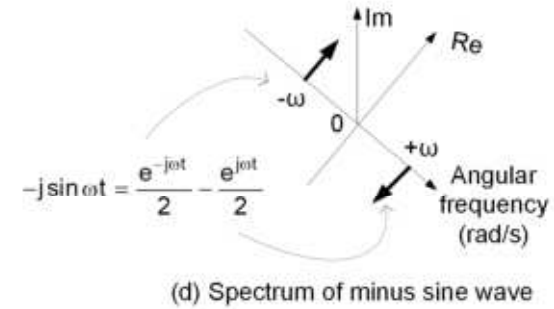
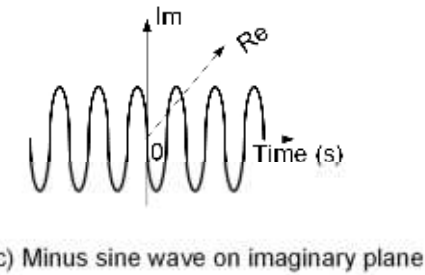
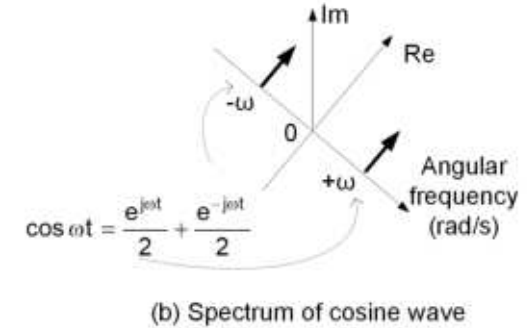
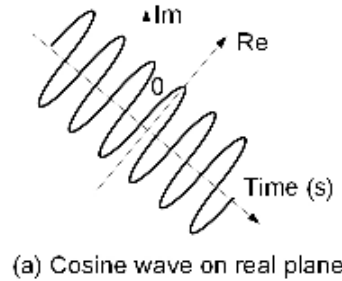
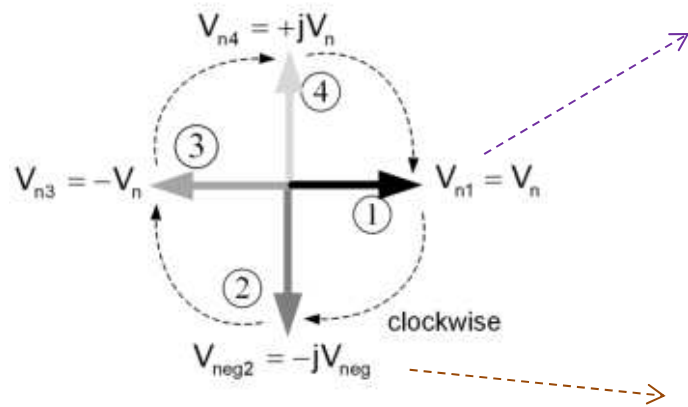
$$= \{1; +j; (+j)^2; (+j)^3\} V_{pos}(t)$$



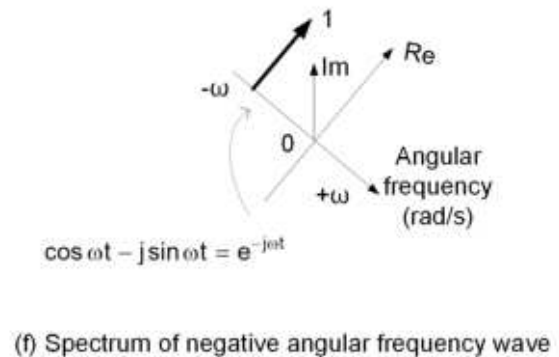
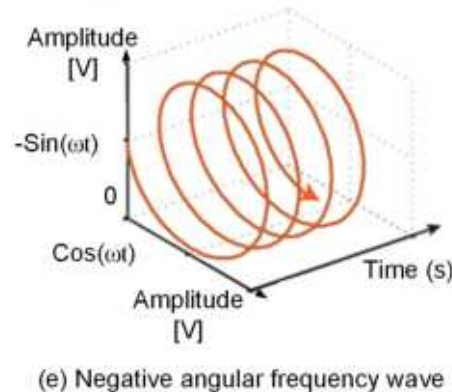
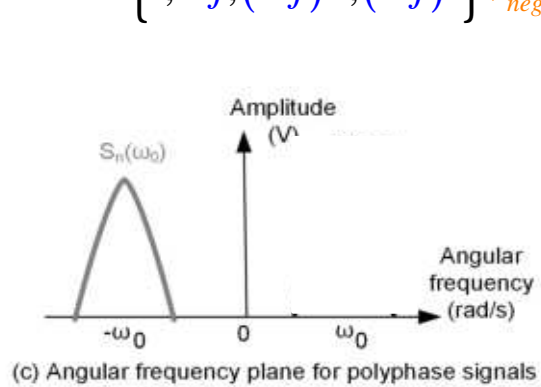
3. Analysis of Rauch Complex Filter

Negative Polyphase Signals on Frequency Domain

Negative polyphase signals

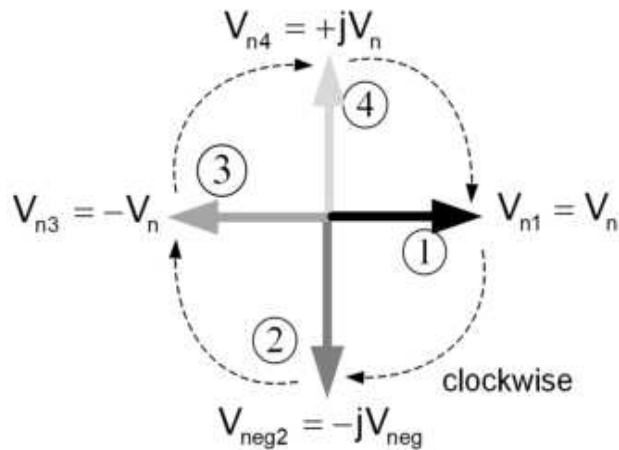


$$S_{Neg_poly} \{V_1(t); V_2(t); V_3(t); V_4(t)\} = \{1; -j; (-j)^2; (-j)^3\} V_{neg}(t)$$



3. Analysis of Rauch Complex Filter Polyphase Signals in All Frequency Domains

Negative polyphase signals



Complex number

Imaginary unit j
= 90° phase shift

$$(+j)^2 = e^{+j\pi} = -1$$

$$(-j)^2 = e^{-j\pi} = -1$$

Positive polyphase signals

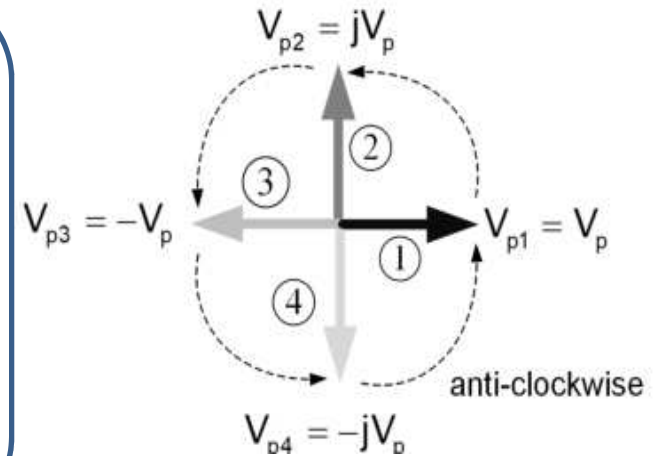
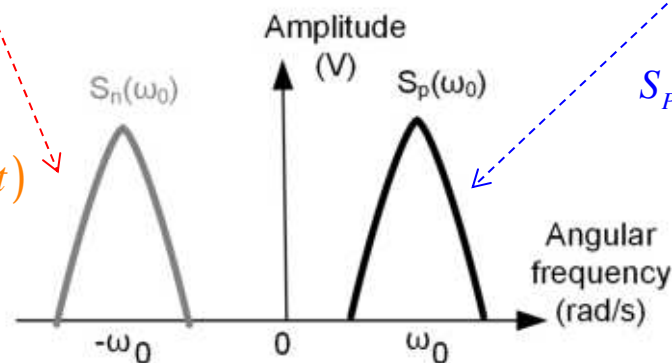


Image signals

$$S_{Neg_poly} \{V_1(t); V_2(t); V_3(t); V_4(t)\} \\ = \{1; -j; (-j)^2; (-j)^3\} V_{neg}(t)$$



Wanted signals

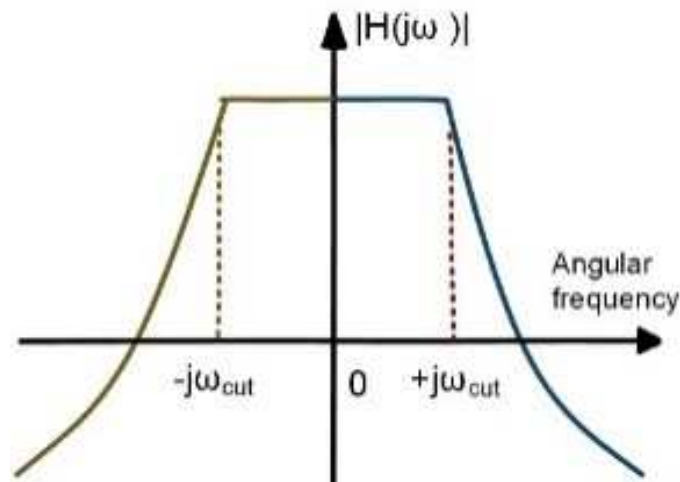
$$S_{Pos_poly} \{V_1(t); V_2(t); V_3(t); V_4(t)\} \\ = \{1; +j; (+j)^2; (+j)^3\} V_{pos}(t)$$

3. Analysis of Rauch Complex Filter

Design Principle for Complex Filter Networks

Frequency shifting of real low-pass filter in all frequency domains \rightarrow an active complex filter

Low-pass filter (LPF)



$$H_{LPF}(\omega) = -\frac{A_{21}}{\left(j\frac{\omega}{\omega_{cut}} + 1\right)}$$

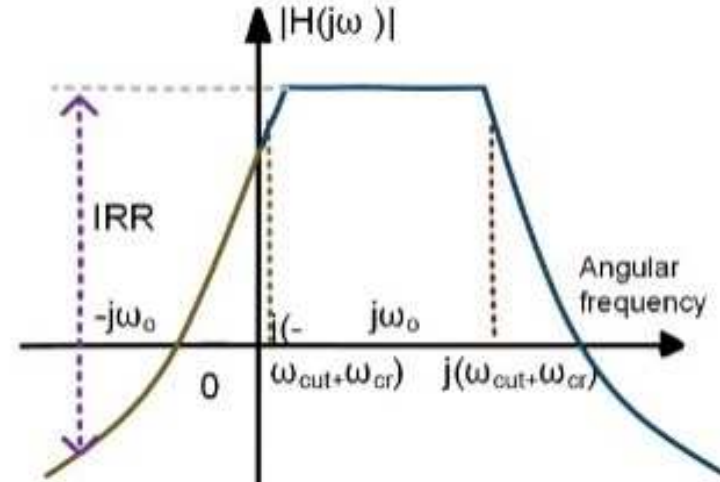
Angular frequency shifting



$j(\omega \cdot \omega_{cr})$

$$\omega_{cr} = \frac{1}{R_3 C_1}$$

Complex filter (CF)



$$H_{CF}(\omega) = -\frac{A_{21}}{\left(j\frac{\omega - \omega_{cr}}{\omega_{cut}} + 1\right)}$$

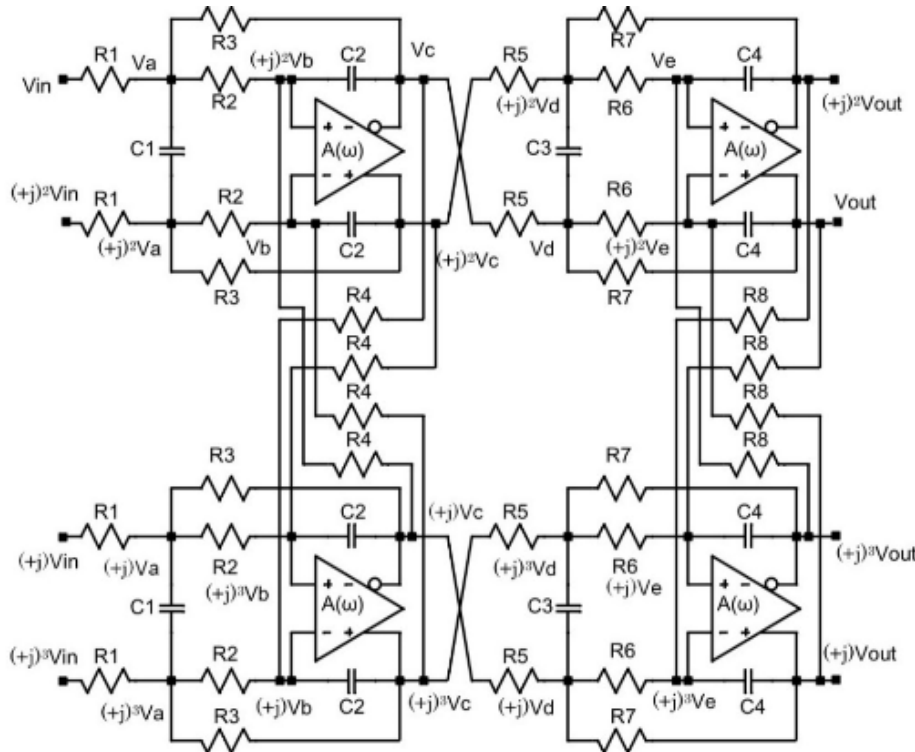
ω_{cr} : cross angular frequency

3. Analysis of Rauch Complex Filter

Analysis of 4th-Order Rauch Complex Filter

4th-order Rauch complex filter

Applying superposition formula at each node



$$V_a \left(\frac{1}{R_1} + j\omega C_1 + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{in}}{R_1} + (+j)^2 V_a j\omega C_1 + \frac{V_b}{R_2} + \frac{(+j)^2 V_c}{R_3};$$

$$V_b \left(\frac{1}{R_2} + j\omega C_2 + \frac{1}{R_4} \right) = \frac{V_a}{R_2} + (+j)^2 V_c j\omega C_2 + \frac{(+j)^3 V_c}{R_4};$$

$$V_c = [V_b - (+j)^2 V_b] A(\omega); V_{out} = [V_e - (+j)^2 V_e] A(\omega);$$

$$V_d \left(\frac{1}{R_5} + j\omega C_3 + \frac{1}{R_6} + \frac{1}{R_7} \right) = \frac{V_c}{R_5} + (+j)^2 V_d j\omega C_3 + \frac{V_e}{R_6} + \frac{(+j)^2 V_{out}}{R_7};$$

$$V_e \left(\frac{1}{R_6} + j\omega C_4 + \frac{1}{R_8} \right) = \frac{V_d}{R_6} + (+j)^2 V_{out} j\omega C_4 + \frac{(+j)^3 V_{out}}{R_8};$$

Positive transfer function

$$H_P(\omega) = \left(\frac{b_0}{a_0(j\omega)^2 + a_{P1}j\omega + a_{P2}} \right) \left(\frac{b_1}{a_3(j\omega)^2 + a_{P4}j\omega + a_{P5}} \right);$$

Negative transfer function

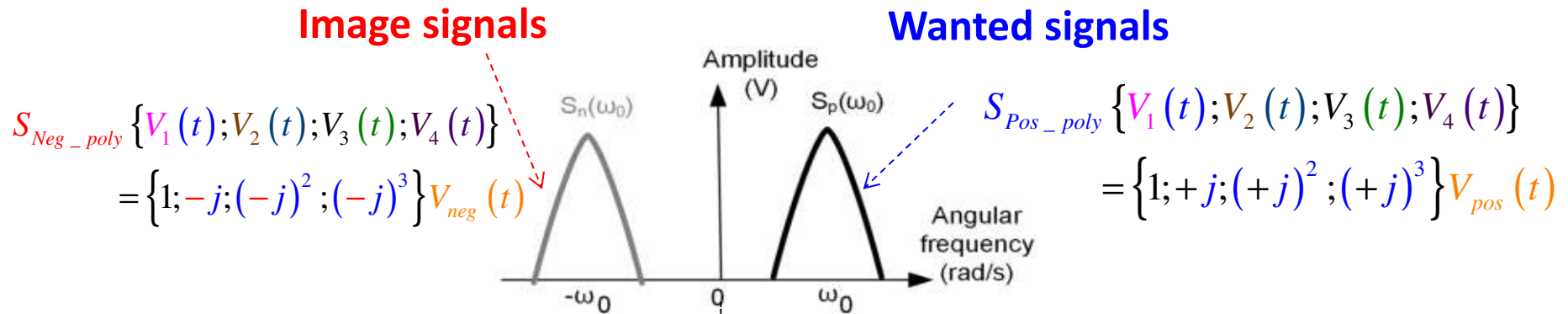
$$H_N(\omega) = \left(\frac{b_0}{a_0(j\omega)^2 + a_{N1}j\omega + a_{N2}} \right) \left(\frac{b_1}{a_3(j\omega)^2 + a_{N4}j\omega + a_{N5}} \right);$$

Image rejection ratio

$$IRR(\omega) = \left(\frac{a_0(j\omega)^2 + a_{N1}j\omega + a_{N2}}{a_0(j\omega)^2 + a_{P1}j\omega + a_{P2}} \right) \left(\frac{a_3(j\omega)^2 + a_{N4}j\omega + a_{N5}}{a_3(j\omega)^2 + a_{P4}j\omega + a_{P5}} \right);$$

3. Analysis of Rauch Complex Filter

Behaviors of 4th-Order Rauch Complex Filter

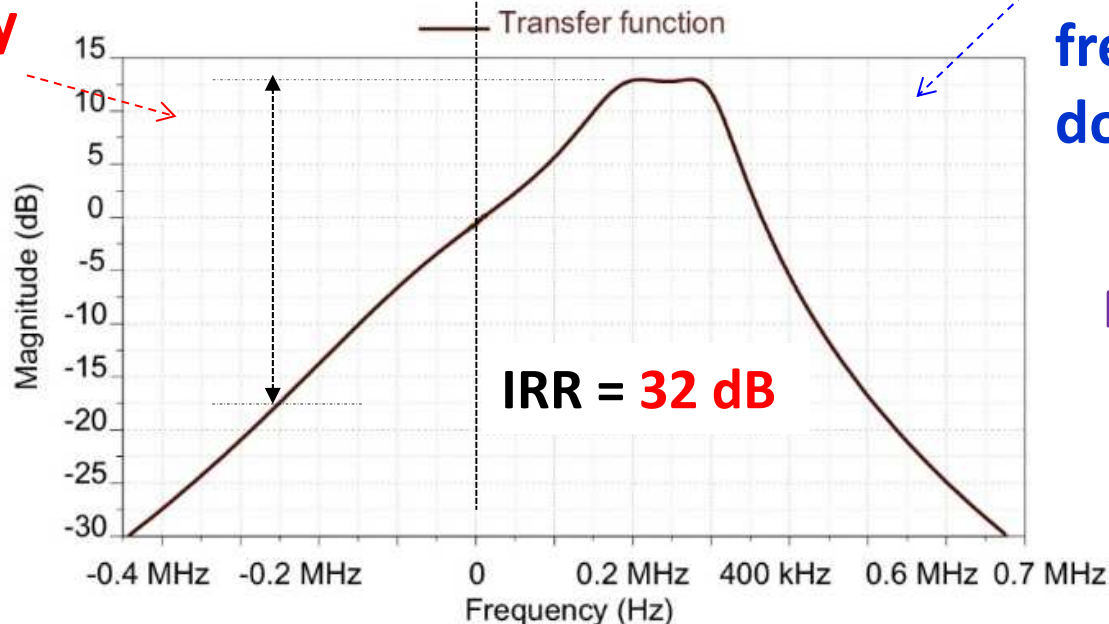


Negative frequency domain

Bode plot of transfer function

Positive frequency domain

Bandwidth = 200 kHz



Outline

1. Research Background

- Motivation, objectives and achievements
- Helix functions and superposition formula

2. Ringing Test for Rauch Low-Pass Filter

- Behaviors of fully differential op amp
- Stability test for fully differential Rauch LPF

3. Analysis of Rauch Complex Filter

- Spectrum of polyphase signals
- Design of 4th-order Rauch complex filter

4. Conclusions

4. Comparison

Features	Proposed formula	Conventional Superposition	Millan's theorem
Effects of all actuating sources	At one time	Several times	At one time
Transfer function accuracy	Yes	No	No
Single-input network analysis	Yes	Yes	Yes
Polyphase network analysis	Yes	No	No
Complex network analysis	Yes	No	No
Image rejection ratio accuracy	Yes	No	No

4. Discussions

- **Ringings test for electronic networks using Nichols chart of self-loop function**
 - **Observation of phase margin can help us determine the operating regions of high-order systems.**
- **Transfer function and image rejection ratio give useful information about the behaviors of polyphase filters and complex filters.**
- **Superposition formula: fundamental network analysis theory for multi-source systems**
 - **Compute** the effects of all sources at one time,
 - **Get** the transfer function faster, and
 - **Reduce** the network complexity.

4. Conclusions

This work:

- Definitions of **helix functions** and **polyphase signals**
- Proposal of **superposition formula** for deriving **transfer function** in multi-source networks
- **Stability test** for **4th-order fully differential Rauch LPF**
- **Derivation of transfer function and image rejection ratio for 4th-order Rauch complex filter**
 - **Theoretical concepts of stability test** are verified by **laboratory simulations.**

Future work:

- **Stability test for parasitic components** in transmission lines, printed circuit boards, physical layout layers
- **Investigation of DC offset and IQ mismatches** in **polyphase filters and complex filters**

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Thank you very much!

