



# IEMTRONICS

## International Conference

21<sup>st</sup> - 24<sup>th</sup> April 2021

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## Limitations of Loop Gain in Motion Models of Physical Systems

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# Outline

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## 1. Research Background

- Motivation, objectives and achievements
- Self-loop function in a transfer function

## 2. Limitations of Loop Gain

- Demerits of loop gain and Nyquist stability criterion

## 3. Behaviors of Feedback Amplifier Networks

- Stability test for high-order inverting amplifiers

## 4. Ringing Test for Adaptive Feedback Networks

- Phase margin of power-stage of DC-DC buck converter

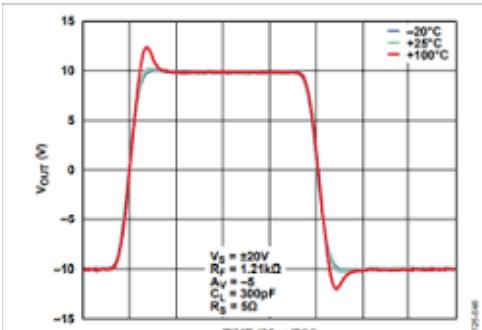
## 5. Conclusions

# 1. Research Background

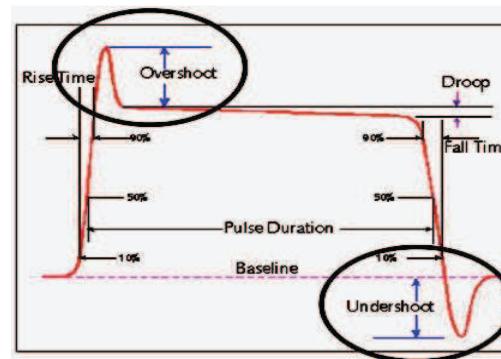
## Motivation on Limitations of Loop Gain

### ① Research papers and commercial devices

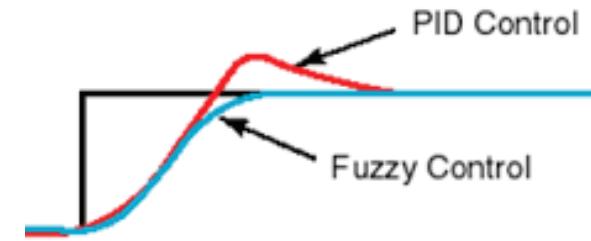
Op amp ADA4870



Feedback systems



Control systems



### ② Theoretical analysis for adaptive feedback systems

- Steady-state oscillations → Barkhausen theorem
- Nyquist criterion → Left side of complex s plain (-1, 0)
- Routh–Hurwitz, Jury stability criterion, ...
- Loop gain of adaptive feedback systems,
- Middlebrook's measurement of loop gain,

# 1. Research Background

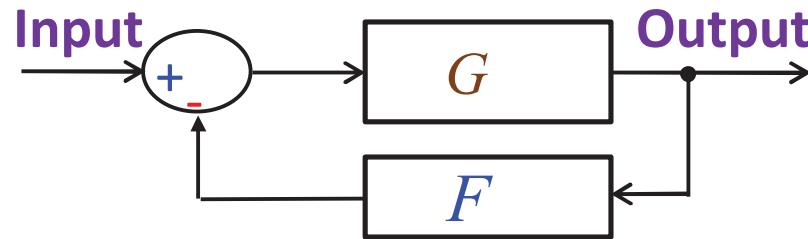
## Objectives of This Study

- Study of limitations of loop gain in motion models of mechatronic systems
- Mechanical systems and electronic circuits are usually expressed by complex functions.
  - The properties of transfer function and self-loop function in these systems are the same.
  - Investigation of phase margin at unity gain to determine operating regions of high-order systems
- Measurement of phase margin in an adaptive feedback system (power-stage of DC-DC converter)

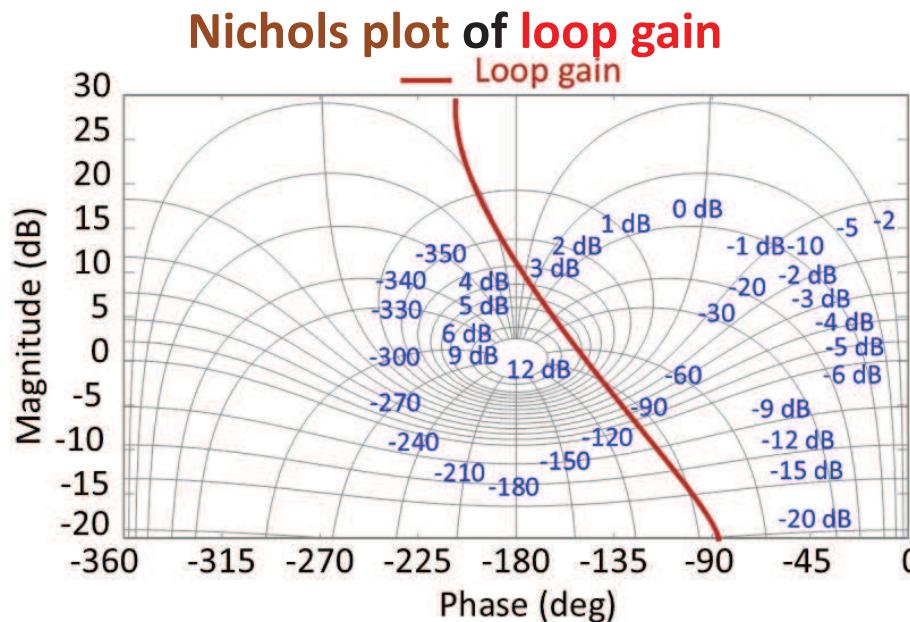
# 1. Research Background

## Conventional Concepts of Loop Gain

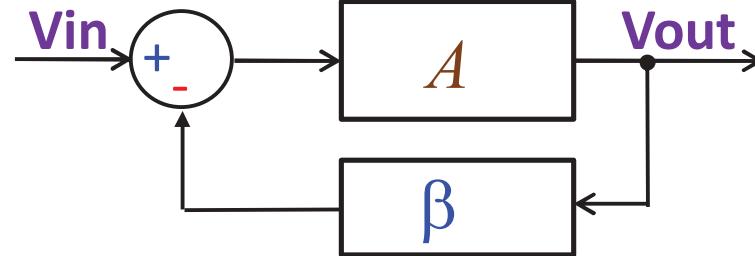
**Adaptive feedback system**



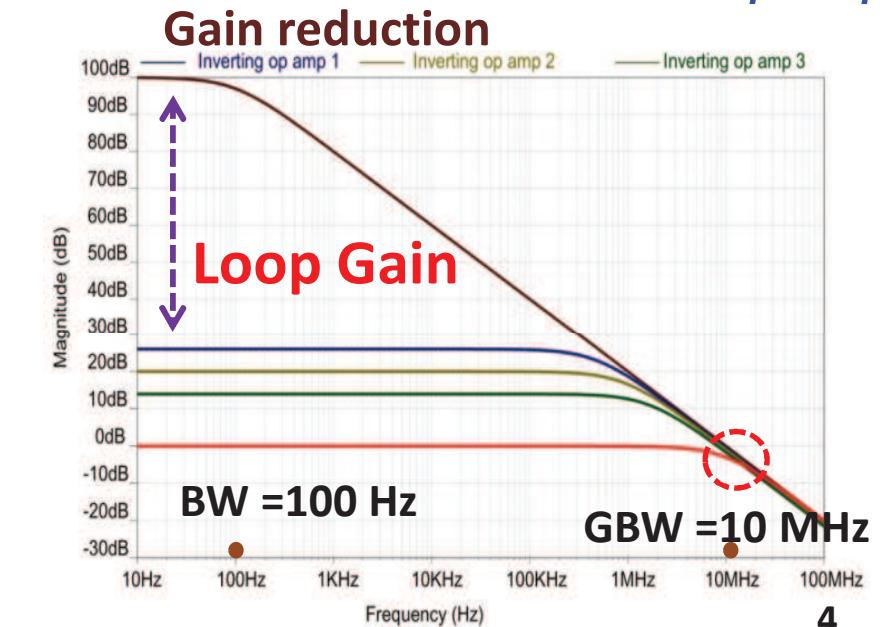
**Transfer function**  $H = \frac{G}{1 + GF} \approx 1$   
**GF : loop gain**



**Inverting amplifier**



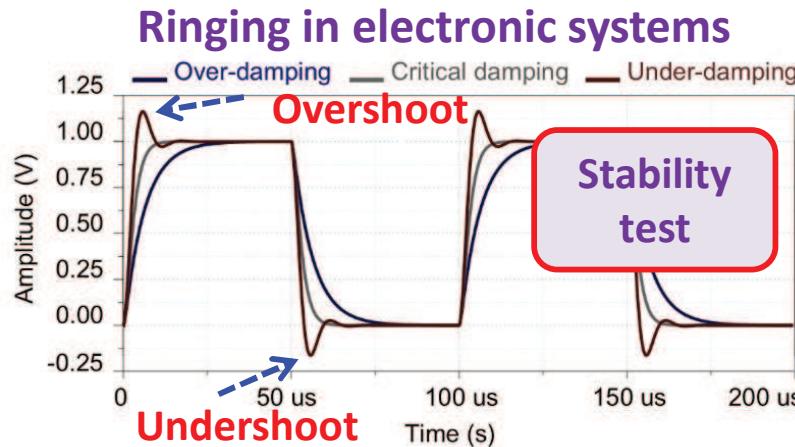
**Transfer function**  $H = \frac{A}{1 + A\beta} \approx \frac{1}{\beta}$   
**Aβ : loop gain**



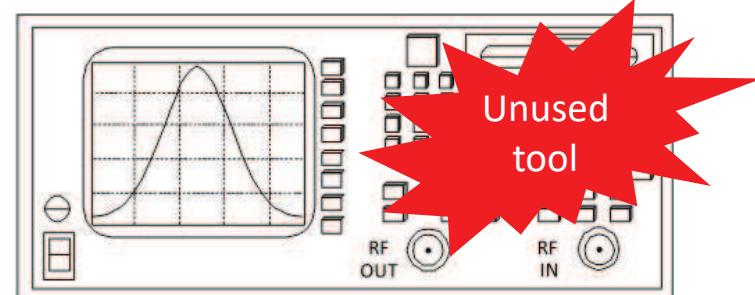
# 1. Research Background

## Stability Test for Mechatronic Systems

→ Loop gain **cannot** be used to perform the **ringing test**.

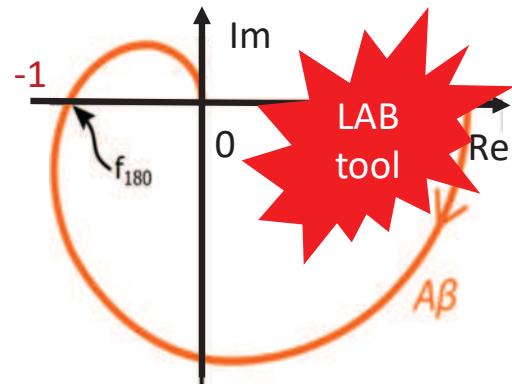


**Nichols chart in Network Analyzer?**



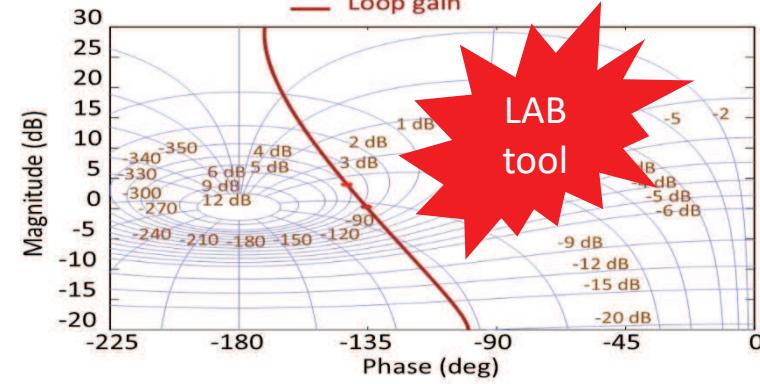
(Technology limitations)

**Nyquist plot of loop gain**



(Unclear operating region)

**Nichols plot of loop gain**



(Very complicated)

# 1. Research Background

## Self-loop Function in A Transfer Function

Linear system



Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

Motion model of a linear system

$$H(\omega) = \frac{b_0(j\omega)^n + \dots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \dots + a_{n-1}(j\omega) + a_n}$$

$A(\omega)$  : Numerator function

$H(\omega)$  : Transfer function

$L(\omega)$  : Self-loop function

Variable: angular frequency ( $\omega$ )

- Polar chart → Nyquist chart
  - Magnitude-frequency plot
  - Angular-frequency plot
  - Magnitude-angular diagram → Nichols diagram
- Bode plots

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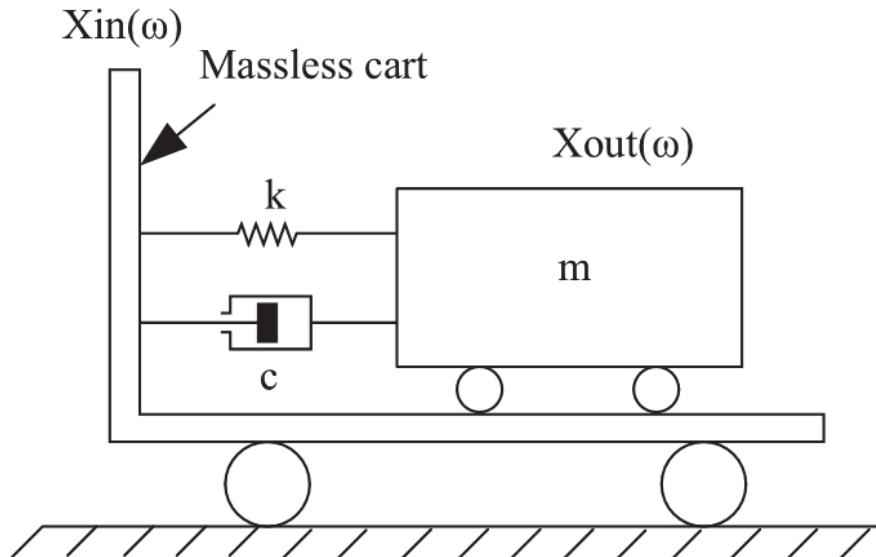
- Phase margin of power-stage of DC-DC buck converter

## 5. Conclusions

## 2. Limitations of Loop Gain Behaviors of a 2<sup>nd</sup> -Order Mechanical System

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**Model of spring-damper-mass system**



**Transfer function and self-loop function**

$$H(\omega) = \frac{X_{out}(\omega)}{X_{in}(\omega)} = \frac{b_0 j\omega + 1}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

**Loop gain cannot be applied for a 2<sup>nd</sup>-order mechanical system**

Apply **superposition** at the node  $X_{out}$ , we have

$$\left[ j\omega m + c + \frac{k}{j\omega} \right] X_{out}(\omega) = \left[ c + \frac{k}{j\omega} \right] X_{in}(\omega);$$

**Where,**

$$b_0 = \frac{c}{k}; a_0 = \frac{m}{k}; a_1 = \frac{c}{k}$$

## 2. Limitations of Loop Gain

### Characteristics of 2<sup>nd</sup>-order Self-loop Function

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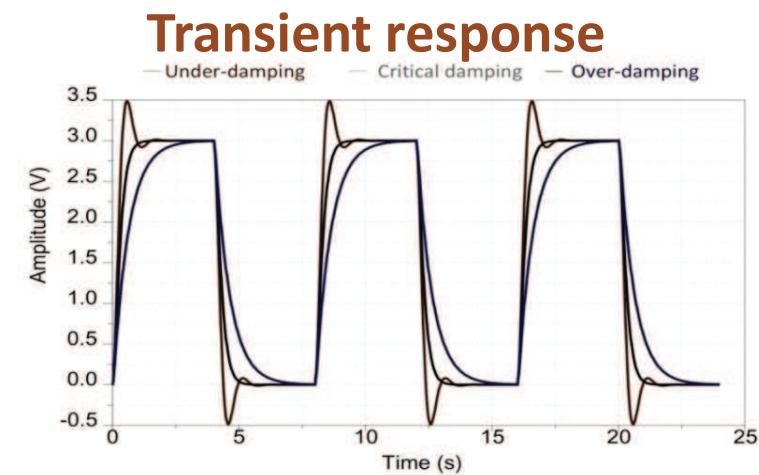
**Second-order self-loop function:**  $L(\omega) = j\omega [a_0 j\omega + a_1]$

Case	Over-damping	Critical damping	Under-damping			
<b>Delta</b> ( $\Delta$ )	$\Delta = a_1^2 - 4a_0 > 0$	$\Delta = a_1^2 - 4a_0 = 0$	$\Delta = a_1^2 - 4a_0 < 0$			
$ L(\omega) $	$\omega \sqrt{(a_0\omega)^2 + a_1^2}$	$\omega \sqrt{(a_0\omega)^2 + a_1^2}$	$\omega \sqrt{(a_0\omega)^2 + a_1^2}$			
$\theta(\omega)$	$\frac{\pi}{2} + \arctan \frac{a_0\omega}{a_1}$	$\frac{\pi}{2} + \arctan \frac{a_0\omega}{a_1}$	$\frac{\pi}{2} + \arctan \frac{a_0\omega}{a_1}$			
$\omega_1 = \frac{a_1}{2a_0} \sqrt{\sqrt{5}-2}$	$ L(\omega_1)  > 1$	$\pi - \theta(\omega_1) > 76.3^\circ$	$ L(\omega_1)  = 1$	$\pi - \theta(\omega_1) = 76.3^\circ$	$ L(\omega_1)  < 1$	$\pi - \theta(\omega_1) < 76.3^\circ$
$\omega_2 = \frac{a_1}{2a_0}$	$ L(\omega_2)  > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^\circ$	$ L(\omega_2)  = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^\circ$	$ L(\omega_2)  < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^\circ$
$\omega_3 = \frac{a_1}{a_0}$	$ L(\omega_3)  > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^\circ$	$ L(\omega_3)  = 4\sqrt{2}$	$\pi - \theta(\omega_3) = 45^\circ$	$ L(\omega_3)  < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^\circ$

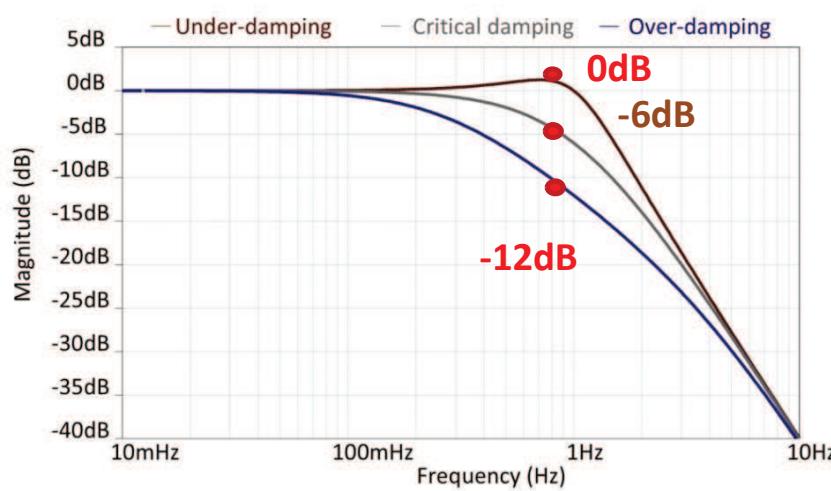
## 2. Limitations of Loop Gain

### Operating Regions of 2<sup>nd</sup>-Order System

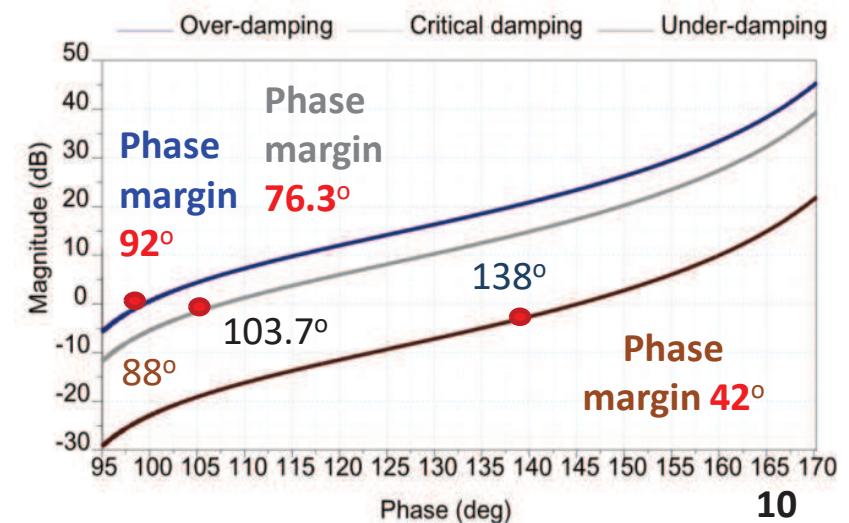
- Under-damping:**  $H_1(\omega) = \frac{1}{(j\omega)^2 + j\omega + 1};$   
 $L_1(\omega) = (j\omega)^2 + j\omega;$
- Critical damping:**  $H_2(\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1};$   
 $L_2(\omega) = (j\omega)^2 + 2j\omega;$
- Over-damping:**  $H_3(\omega) = \frac{1}{(j\omega)^2 + 3j\omega + 1};$   
 $L_3(\omega) = (j\omega)^2 + 3j\omega;$



Bode plot of transfer function



Nichols plot of self-loop function

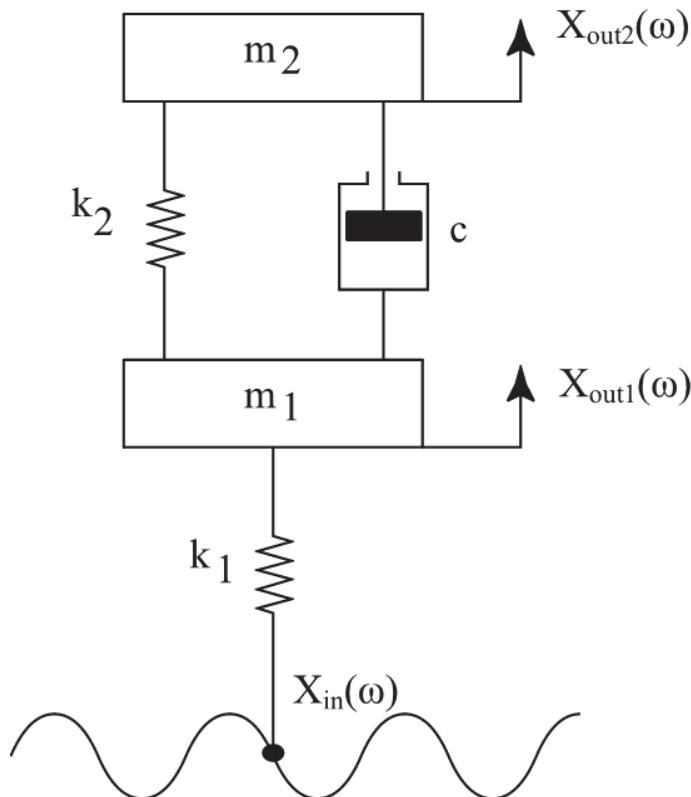


## 2. Limitations of Loop Gain Behaviors of a 4<sup>th</sup> -Order Mechanical System

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**Loop gain cannot be applied for a 4<sup>th</sup>-order mechanical system**

**Model of 4<sup>th</sup>-order  
spring-damper-mass system**



**Apply superposition at the nodes  $X_{out1}$  and  $X_{out2}$ , we have**

$$\left( \frac{k_1}{j\omega} + m_1 j\omega + c + \frac{k_2}{j\omega} \right) X_1(\omega) = \left( c + \frac{k_2}{j\omega} \right) X_2(\omega) + \frac{k_1}{j\omega} X_{in}(\omega);$$

$$\left( m_2 j\omega + c + \frac{k_2}{j\omega} \right) X_2(\omega) = \left( c + \frac{k_2}{j\omega} \right) X_1(\omega);$$

**Transfer function and self-loop function**

$$H_1(\omega) = \frac{X_1(\omega)}{X_{in}(\omega)} = \frac{b_0(j\omega)^2 + b_1 j\omega + 1}{a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega + 1};$$

$$H_2(\omega) = \frac{X_2(\omega)}{X_{in}(\omega)} = \frac{b_1 j\omega + 1}{a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega + 1};$$

$$L(\omega) = a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3 j\omega;$$

## 2. Limitations of Loop Gain

### Operating Regions of 4<sup>th</sup>-Order System

Pascal's Triangle					
n = 2	1	2	1		
n = 3	1	3	3	1	
n = 4	1	4	6	4	1
n = 5	1	5	10	10	5 1

•Under-damping: 1 : 2 : 3 : 2 : 1

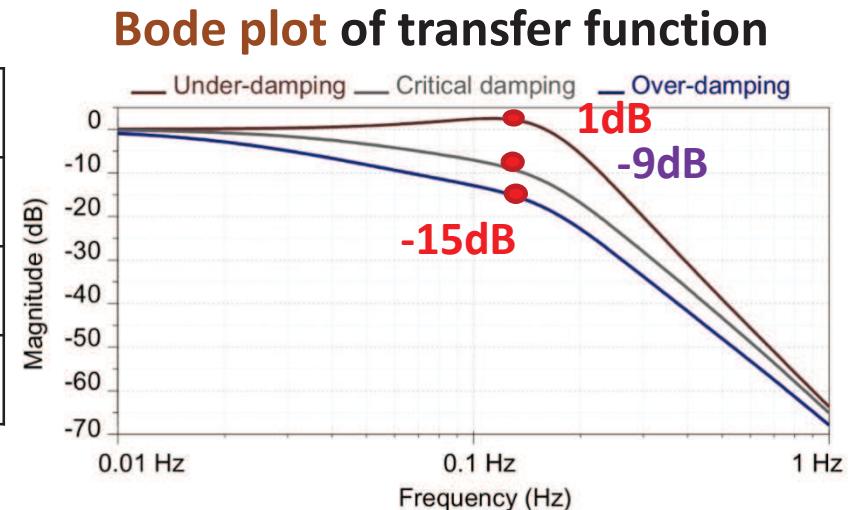
$$H_1(\omega) = \frac{1}{(j\omega)^4 + 2(j\omega)^3 + 3(j\omega)^2 + 2j\omega + 1}$$

•Critical damping: 1 : 4 : 6 : 4 : 1

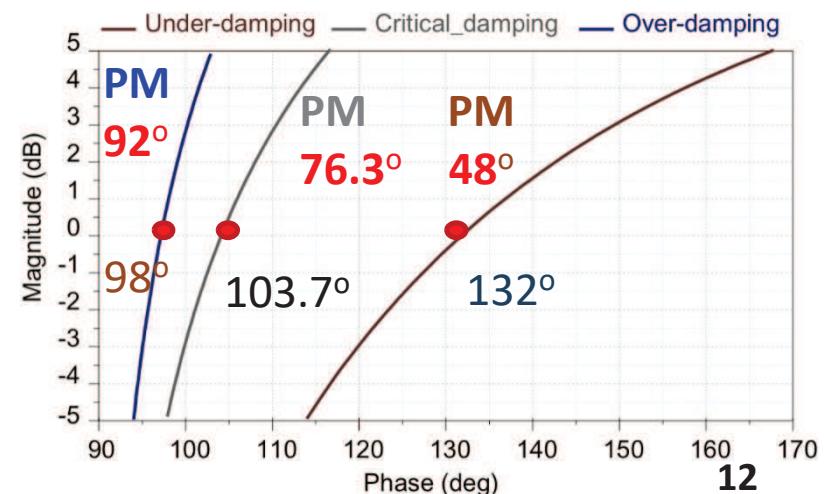
$$H_2(\omega) = \frac{1}{(j\omega)^4 + 4(j\omega)^3 + 6(j\omega)^2 + 4j\omega + 1}$$

•Over-damping: 1 : 9 : 10 : 9 : 1

$$H_3(\omega) = \frac{1}{(j\omega)^4 + 9(j\omega)^3 + 10(j\omega)^2 + 9j\omega + 1}$$



**Nichols plot of self-loop function**



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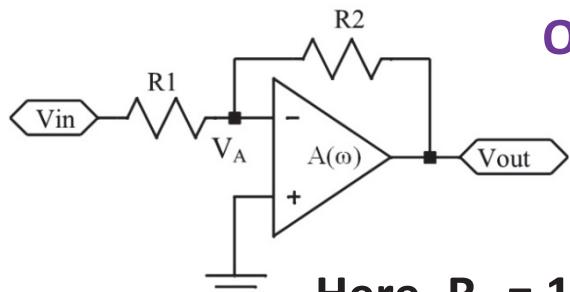
- Phase margin of power-stage of DC-DC buck converter

## 5. Conclusions

# 3. Behaviors of Feedback Amplifier Networks

## Self-loop Function of Inverting Amplifiers

### Single-ended inverting amplifier

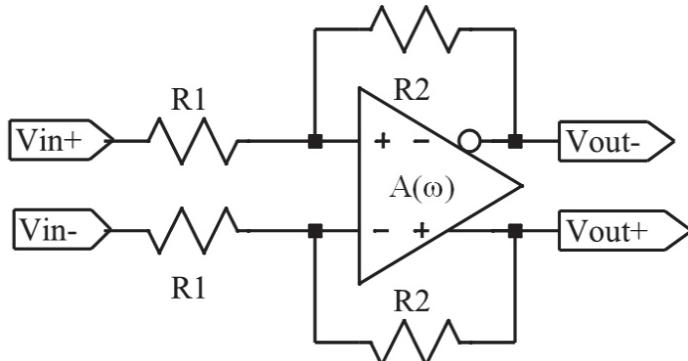


### Open-loop function

$$A(\omega) = \frac{10^5}{1 + j \frac{\omega}{2 \cdot 10^2 \pi}};$$

Here,  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 10 \text{ k}\Omega$

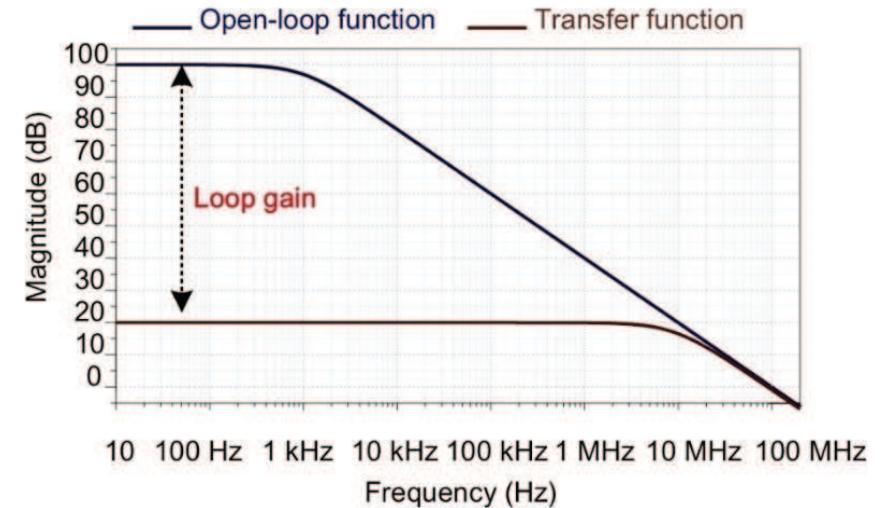
### Fully differential inverting amplifier



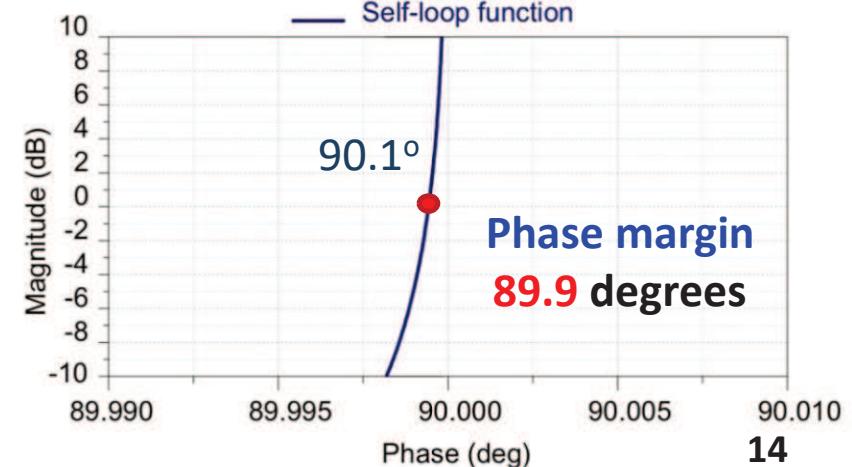
### Transfer function and self-loop function

$$H(\omega) = \frac{-\frac{R_2}{R_1}}{1 + L(\omega)} \approx -\frac{R_2}{R_1}; L(\omega) = \frac{1}{A(\omega)} \left( 1 + \frac{R_2}{R_1} \right);$$

### Bode plot of transfer function



### Nichols plot of self-loop function

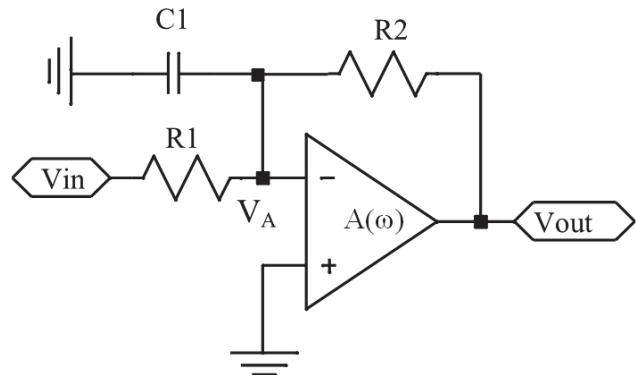


### 3. Behaviors of Feedback Amplifier Networks

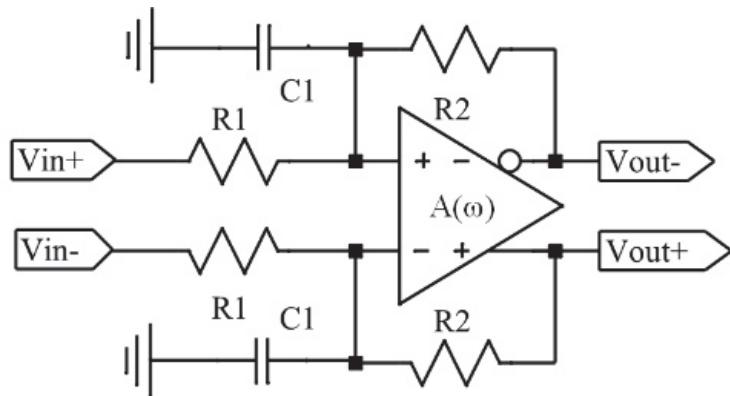
#### Analysis of Inverting Amplifier with Parasitic Capacitor

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**Single-ended inverting amplifier with parasitic capacitor**



**Fully-differential inverting amplifier with parasitic capacitors**



Apply **superposition** at the node  $V_A$ , we have

$$V_A \left( \frac{1}{R_1} + \frac{1}{R_2} + j\omega C_1 \right) = \frac{V_{out}}{R_2}; V_{out} = A(\omega)(V_{in} - V_A);$$

**Transfer function and self-loop function**

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{b_0 j\omega + b_1}{a_0 (j\omega)^2 + a_1 j\omega + a_2 + 1};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega + a_2;$$

**Where,**

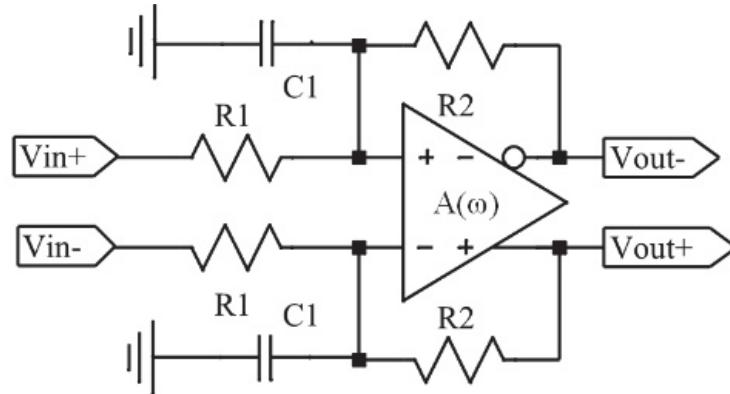
$$b_0 = \frac{R_2}{R_1}; a_1 = \frac{1}{10^5} \left[ R_2 C_1 + \frac{1}{2 * 10^2 \pi} \left( 1 + \frac{R_2}{R_1} \right) \right];$$

$$a_0 = \frac{R_2 C_1}{2 * 10^7 \pi}; a_2 = \frac{1}{10^5} \left( 1 + \frac{R_2}{R_1} \right);$$

### 3. Behaviors of Feedback Amplifier Networks

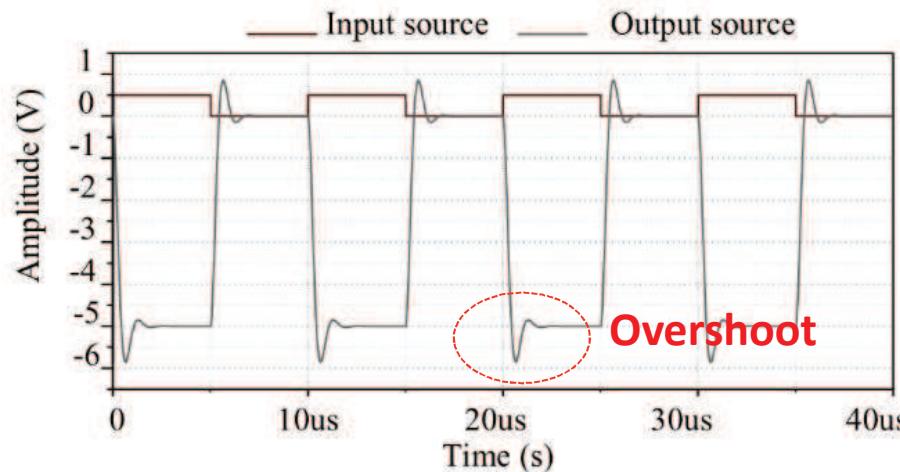
#### Self-loop Function of A 2<sup>nd</sup> - Order Inverting Amplifier

Inverting amplifier **with parasitic capacitors**

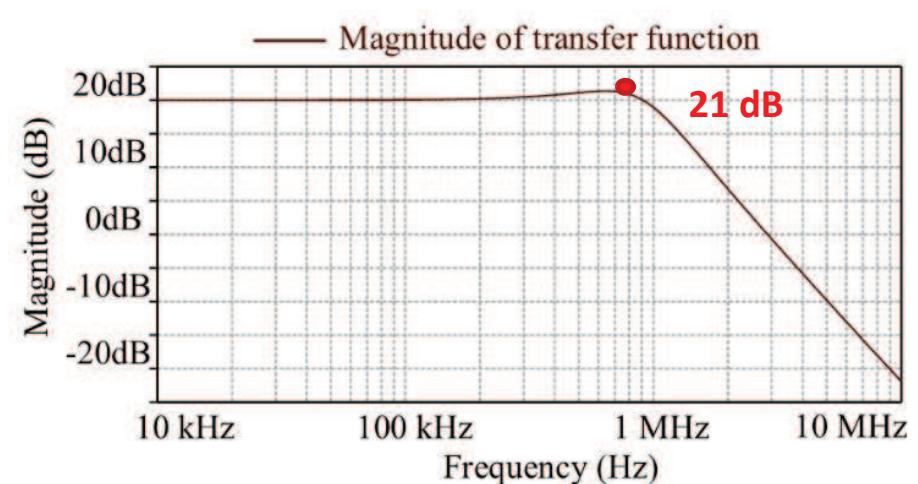


Here,  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 10 \text{ k}\Omega$

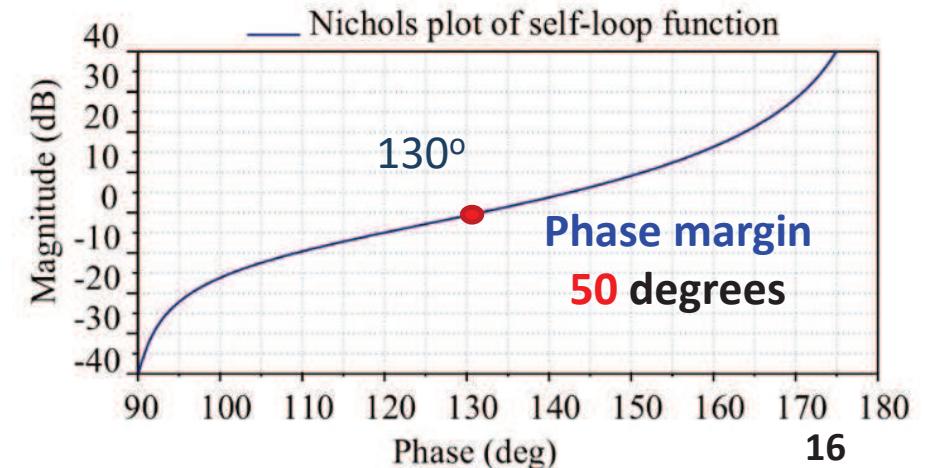
**Simulated transient response**



**Bode plot of transfer function**



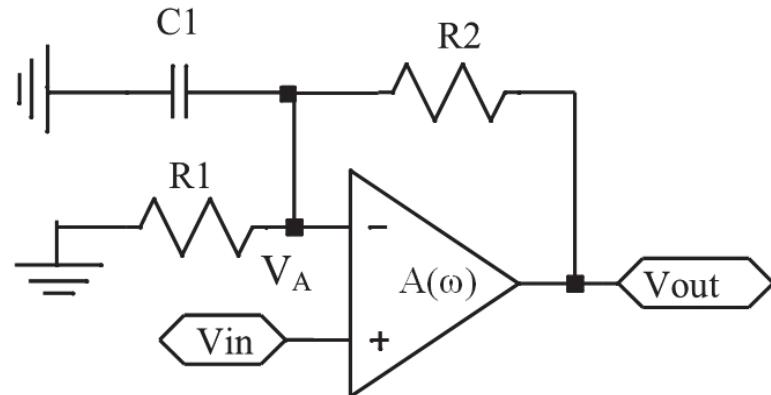
**Nichols plot of self-loop function**



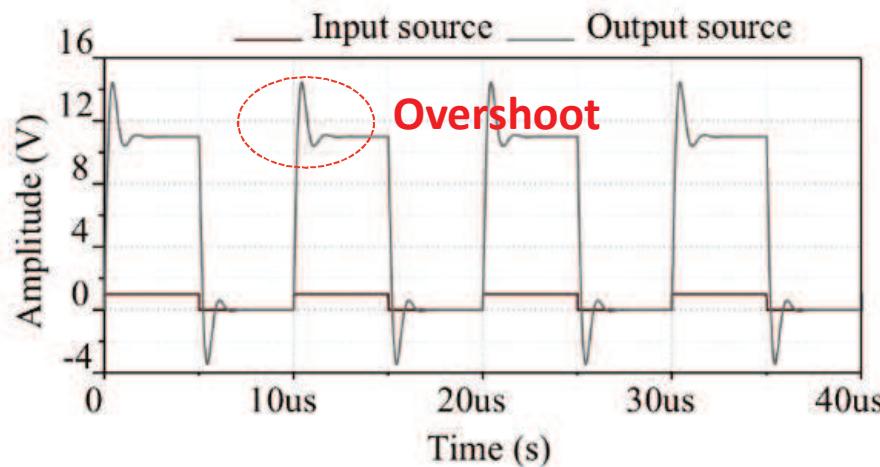
# 3. Behaviors of Feedback Amplifier Networks

## Self-loop Function of 2<sup>nd</sup>-Order Non-Inverting Amplifier

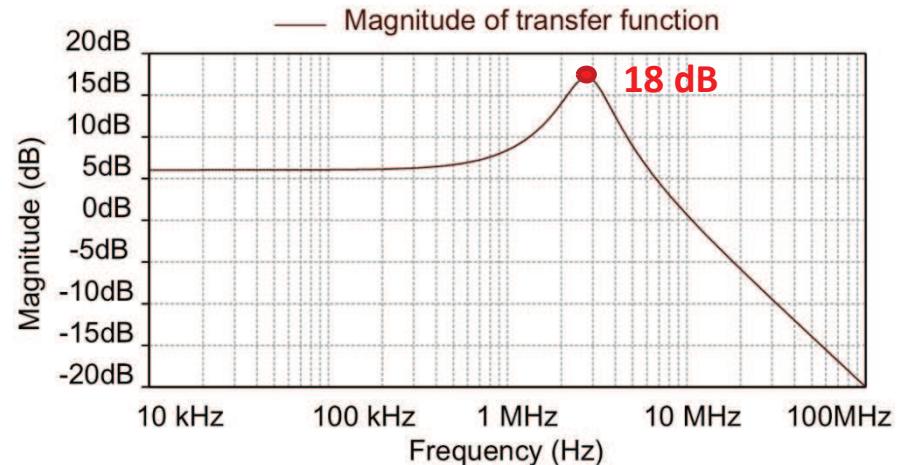
Non-inverting amplifier **with a parasitic capacitor**



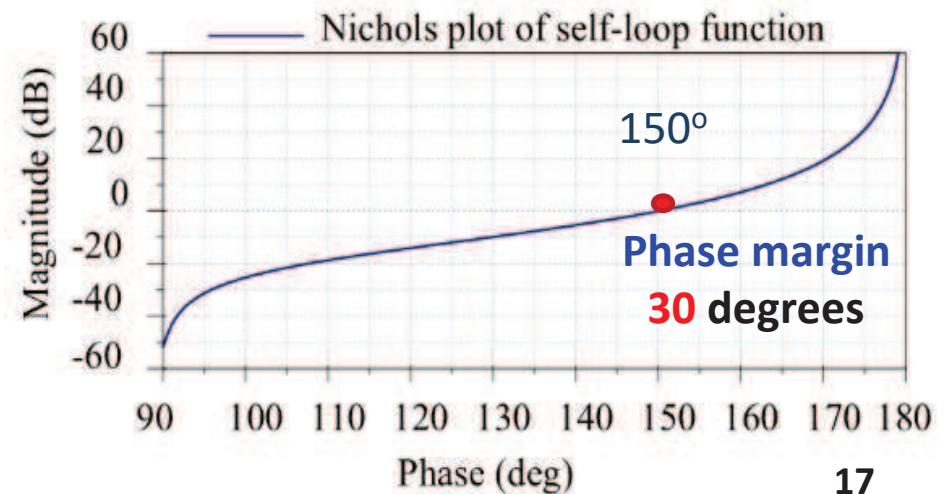
**Simulated transient response**



**Bode plot of transfer function**



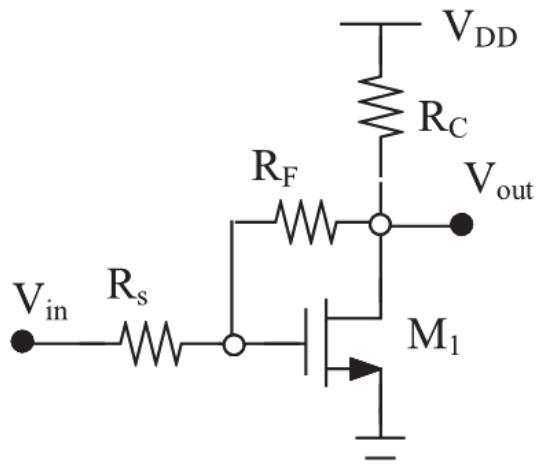
**Nichols plot of self-loop function**



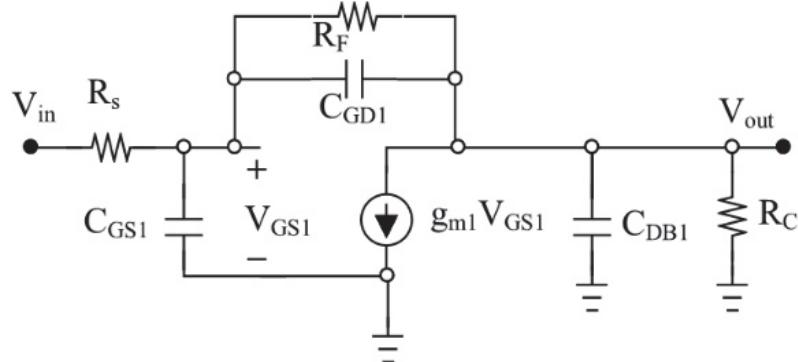
# 3. Behaviors of Feedback Amplifier Networks

## Analysis of Shunt-Shunt CMOS Feedback Amplifier

**Shunt-shunt MOS feedback amplifier**



**Small signal model**



**Apply superposition at the nodes  $V_\pi$  and  $V_{out}$ , we have**

$$V_{GS1} \left( \frac{1}{R_s} + \frac{1}{Z_{CGS1}} + \frac{1}{R_F} + \frac{1}{Z_{CGD1}} \right) = \frac{V_{in}}{R_s} + V_{out} \left( \frac{1}{R_F} + \frac{1}{Z_{CGD1}} \right);$$

$$V_{out} \left( \frac{1}{Z_{CGD1}} + \frac{1}{Z_{CDB1}} + \frac{1}{R_F} + \frac{1}{R_C} \right) = V_{GS1} \left( \frac{1}{R_F} + \frac{1}{Z_{CGD1}} - g_{m1} \right);$$

**Transfer function and self-loop function**

$$H(\omega) = \frac{b_0 j\omega + b_1}{a_0 (j\omega)^2 + a_1 j\omega + 1}; L(\omega) = a_0 (j\omega)^2 + a_1 j\omega$$

**Where,**

$$b_0 = R_F R_C C_{GD1}; b_1 = R_C - R_F R_C g_{m1};$$

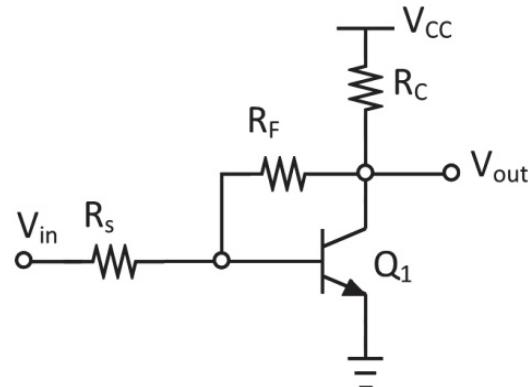
$$a_0 = R_S R_F R_C (C_{GD1} C_{GS1} + C_{GD1} C_{DB1} + C_{DB1} C_{GS1});$$

$$a_1 = (R_C R_F^2 + R_S R_C R_F g_{m1} + R_S R_F^2) C_{GD1} \\ + R_C R_F (R_S + R_F) C_{DB1} + R_S R_F (R_C + R_F) C_{GS1};$$

# 3. Behaviors of Feedback Amplifier Networks

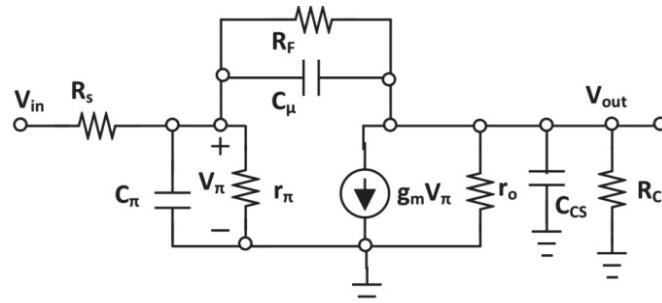
## Behaviors of Shunt-Shunt BJT Feedback Amplifier

### Shunt-shunt BJT feedback amplifier



Here,  $R_f = 1 \text{ k}\Omega$ ,  $R_C = 10 \text{ k}\Omega$ ,  $R_S = 950 \Omega$ .

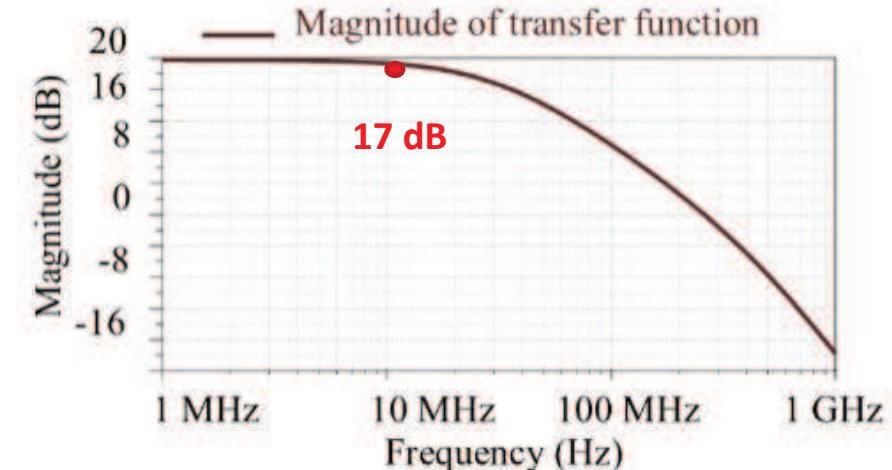
### Small signal model



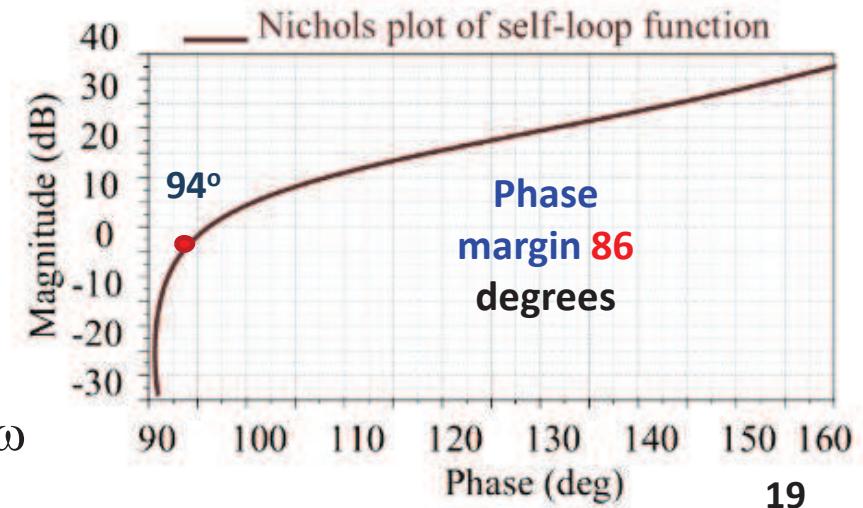
### Transfer function and self-loop function

$$H(\omega) = \frac{b_0 j\omega + b_1}{a_0 (j\omega)^2 + a_1 j\omega + 1}; L(\omega) = a_0 (j\omega)^2 + a_1 j\omega$$

### Bode plot of transfer function



### Nichols plot of self-loop function



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## 2. Limitations of Loop Gain

- Demerits of loop gain and Nyquist stability criterion

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- Stability test for high-order inverting amplifiers

## 4. Ringing Test for Adaptive Feedback Networks

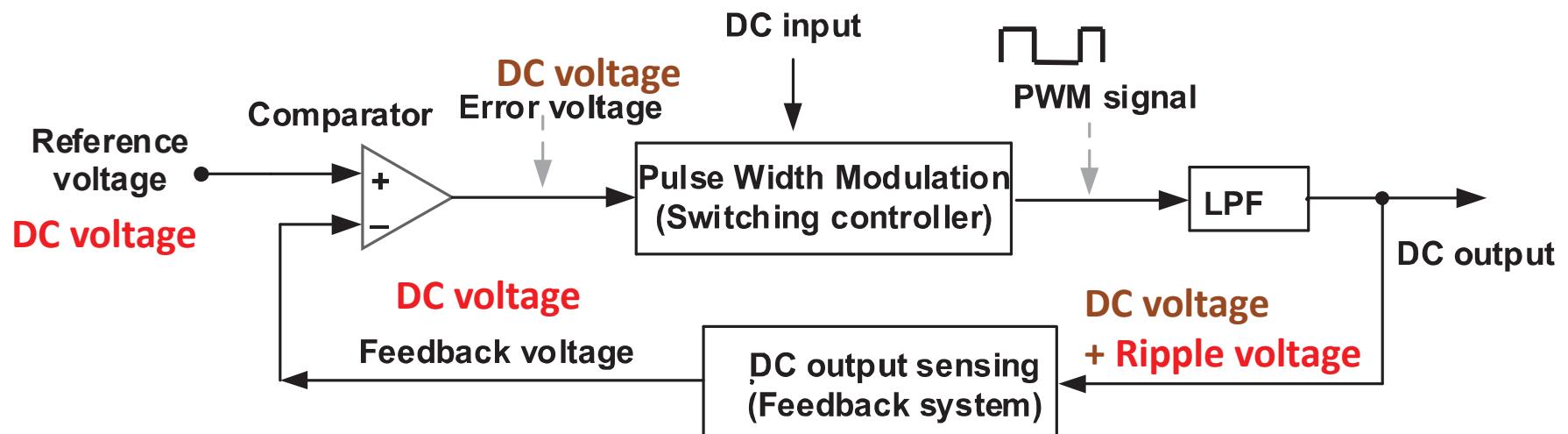
- Phase margin of power-stage of DC-DC buck converter

## 5. Conclusions

## 4. Ringing Test for Adaptive Feedback Networks

### Characteristics of Adaptive Feedback System

Block diagram of a typical adaptive feedback system



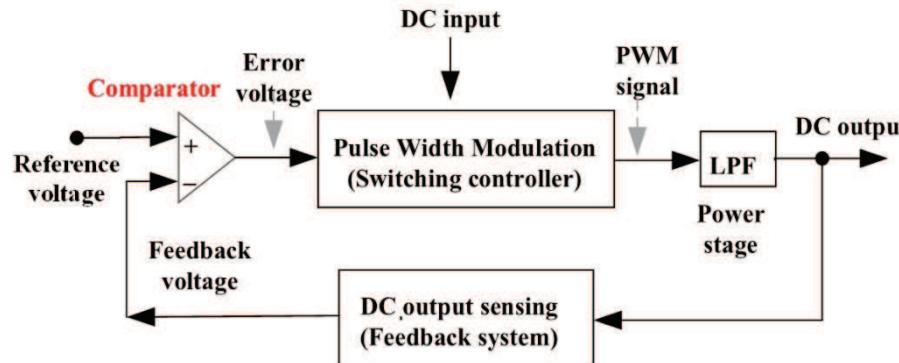
Adaptive feedback is used to control the output voltage along with the reference voltage (**DC-DC Buck converter**).

- Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).
- Loop gain is an approximation value.

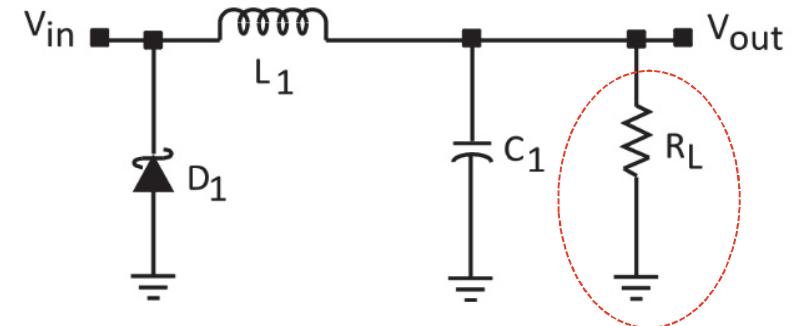
# 4. Ringing Test for Adaptive Feedback Networks

## Behaviors of Power-Stage of DC-DC Converter

Block diagram of DC-DC Converter



Simplified power-stage



Transfer function and self-loop function

$$H(\omega) = \frac{1}{a_0(j\omega)^2 + a_1 j\omega + 1}; L(\omega) = a_0(j\omega)^2 + a_1 j\omega; a_0 = L_1 C_1; a_1 = \frac{L_1}{R_L};$$

Operating regions

- Over-damping:  $\frac{1}{LC} < \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| < R/2$
- Critical damping:  $\frac{1}{LC} = \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| = R/2$
- Under-damping:  $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| > R/2$

Max power propagation condition

$|Z_L| = |Z_C| = 2R$   
 Balanced charging-discharging time condition

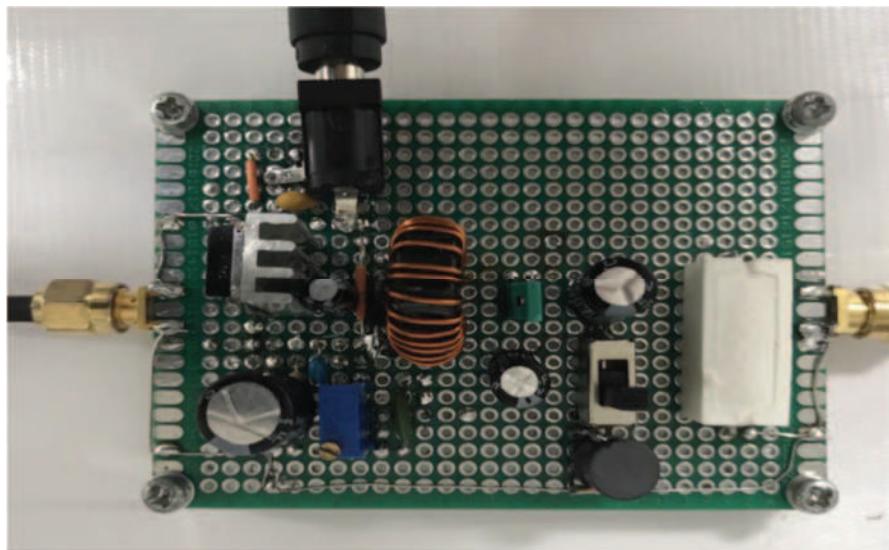
## 4. Ringing Test for Adaptive Feedback Networks

### Implemented Circuit for DC-DC Buck Converter

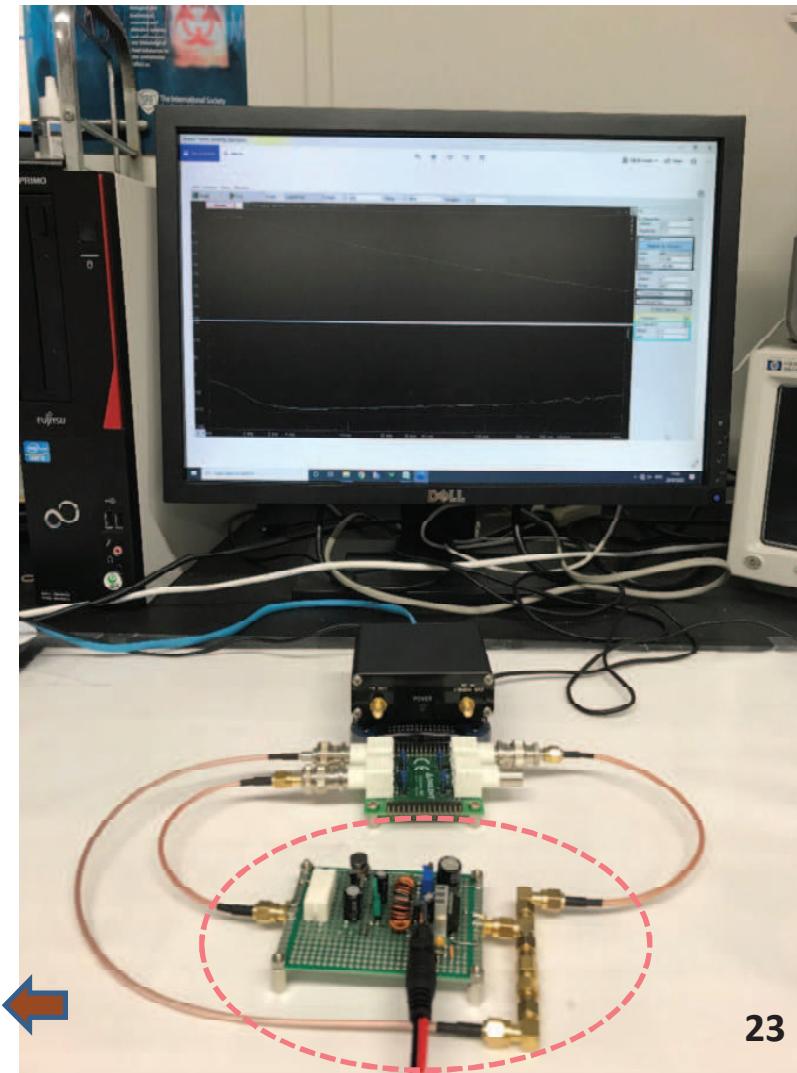
Design parameters

Input voltage (Vin)	12 V
Output voltage (Vo)	5.0 V
Output current (Io)	1 A
Clock frequency (Fck)	180 kHz
Output ripple	< 10 mVpp

Implemented circuit



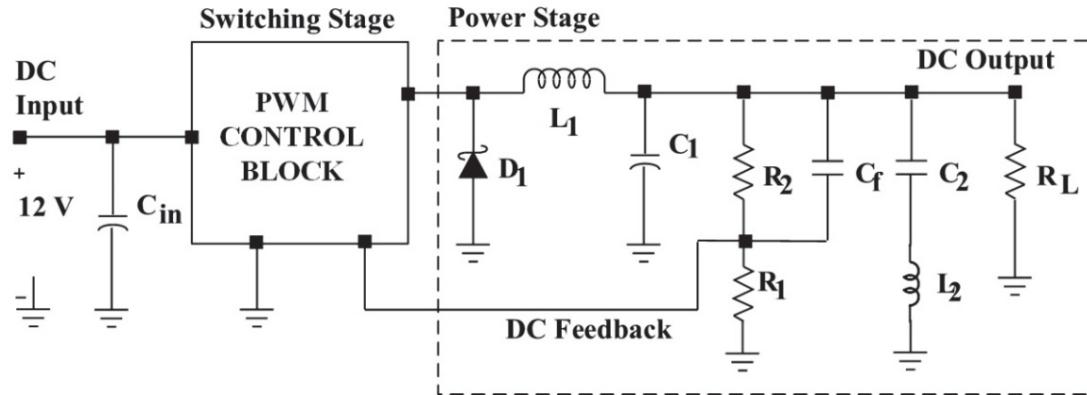
Measurement set up



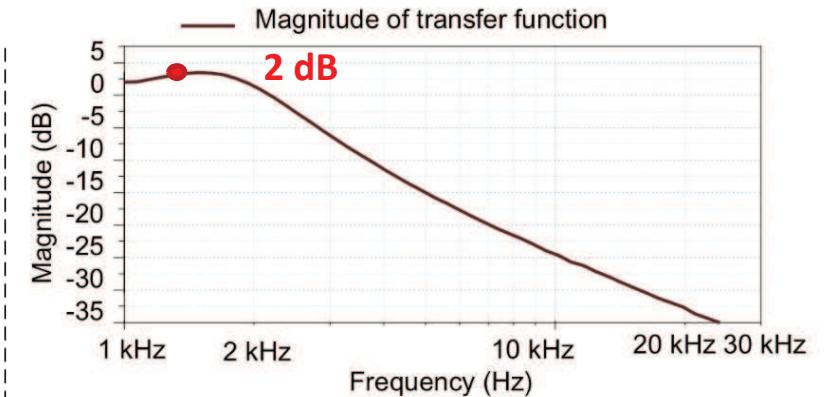
# 4. Ringing Test for Adaptive Feedback Networks

## Phase Margin of Power-Stage of DC-DC Converter

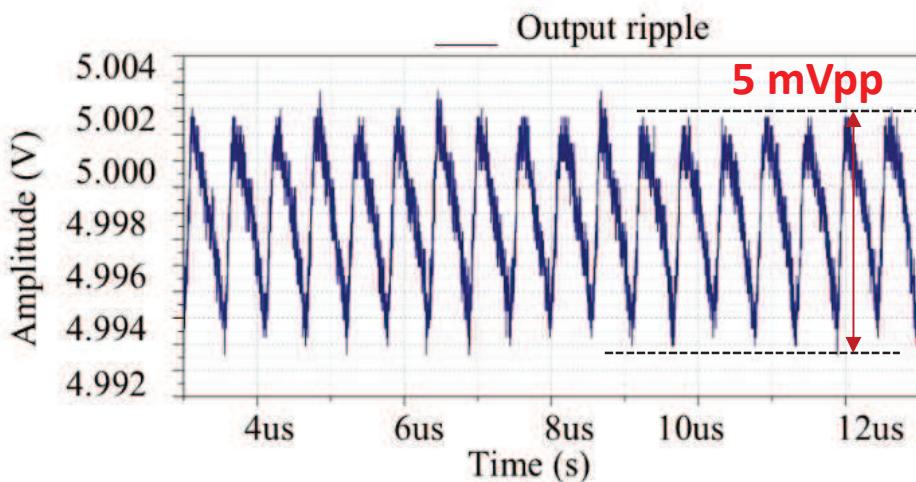
Schematic diagram of DC-DC converter



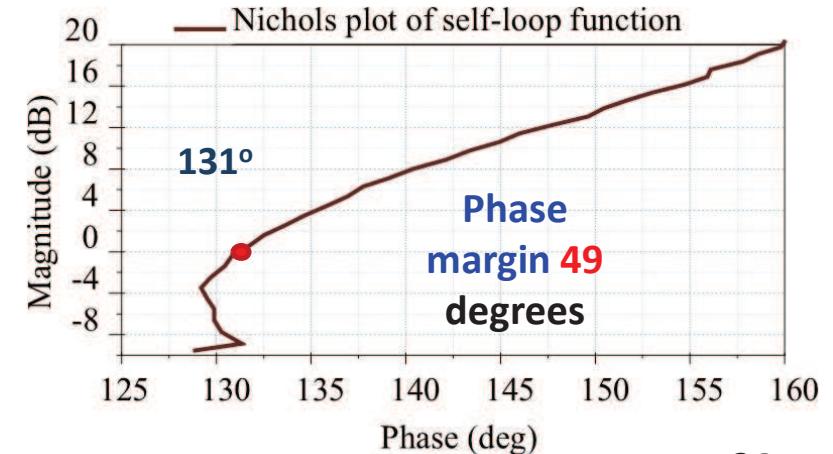
Bode plot of transfer function



Measured transient response



Nichols plot of self-loop function



# Outline

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## 1. Research Background

- Motivation, objectives and achievements
- Self-loop function in a transfer function

## 2. Limitations of Loop Gain

- Demerits of loop gain and Nyquist stability criterion

## 3. Behaviors of Feedback Amplifier Networks

- Stability test for high-order inverting amplifiers

## 4. Ringing Test for Adaptive Feedback Networks

- Phase margin of power-stage of DC-DC buck converter

## 5. Conclusions

## 5. Conclusions

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### This work:

- Study of some limitations of loop gain in motion models of mechatronic systems such as spring-damper-mass systems, inverting amplifier, and power-stage of DC-DC buck converter.  
→ Observation of self-loop function can help us optimize the behaviors of high-order mechatronic systems easily.
- Implementation of circuit and measurements of self-loop functions for adaptive feedback systems.  
→ Theoretical concepts of stability test are verified by laboratory simulations and practical experiments.

### Future work:

- Stability test for dynamic load systems and other mechatronic systems.

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# IEMTRONICS

## International Conference

21<sup>st</sup> - 24<sup>th</sup> April 2021

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Thank you very much!

