Metallic Ratio Equivalent-Time Sampling: A Highly Efficient Waveform Acquisition Method

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Gunma University
ROHM Semiconductor

OUTLINE

- Research Objective
- Equivalent-Time Sampling
- Metallic Ratio Sampling Efficiency
  - Golden and Metallic Ratio Sampling
  - Efficiency Definition
  - Efficiency Periodicity Rule
  - Highest Efficiency Point
  - Efficiency Degradation Point
- Conclusion and Future work

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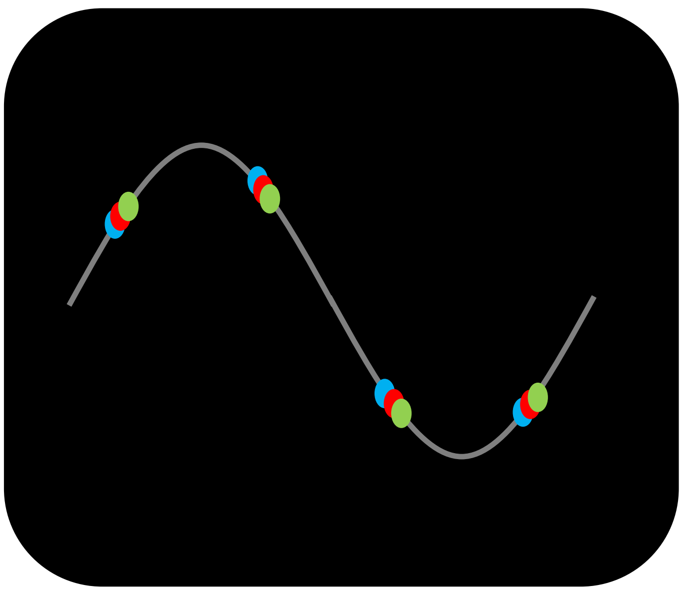
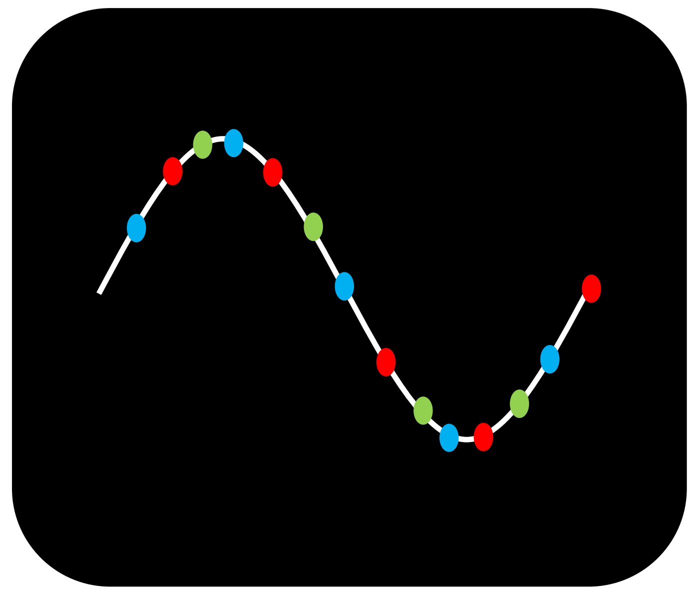
Research Objective

Objective: For efficient IC testing, high efficiency waveform acquisition with equivalent-time sampling.



Sampling points: localized



Sampling points: distributed

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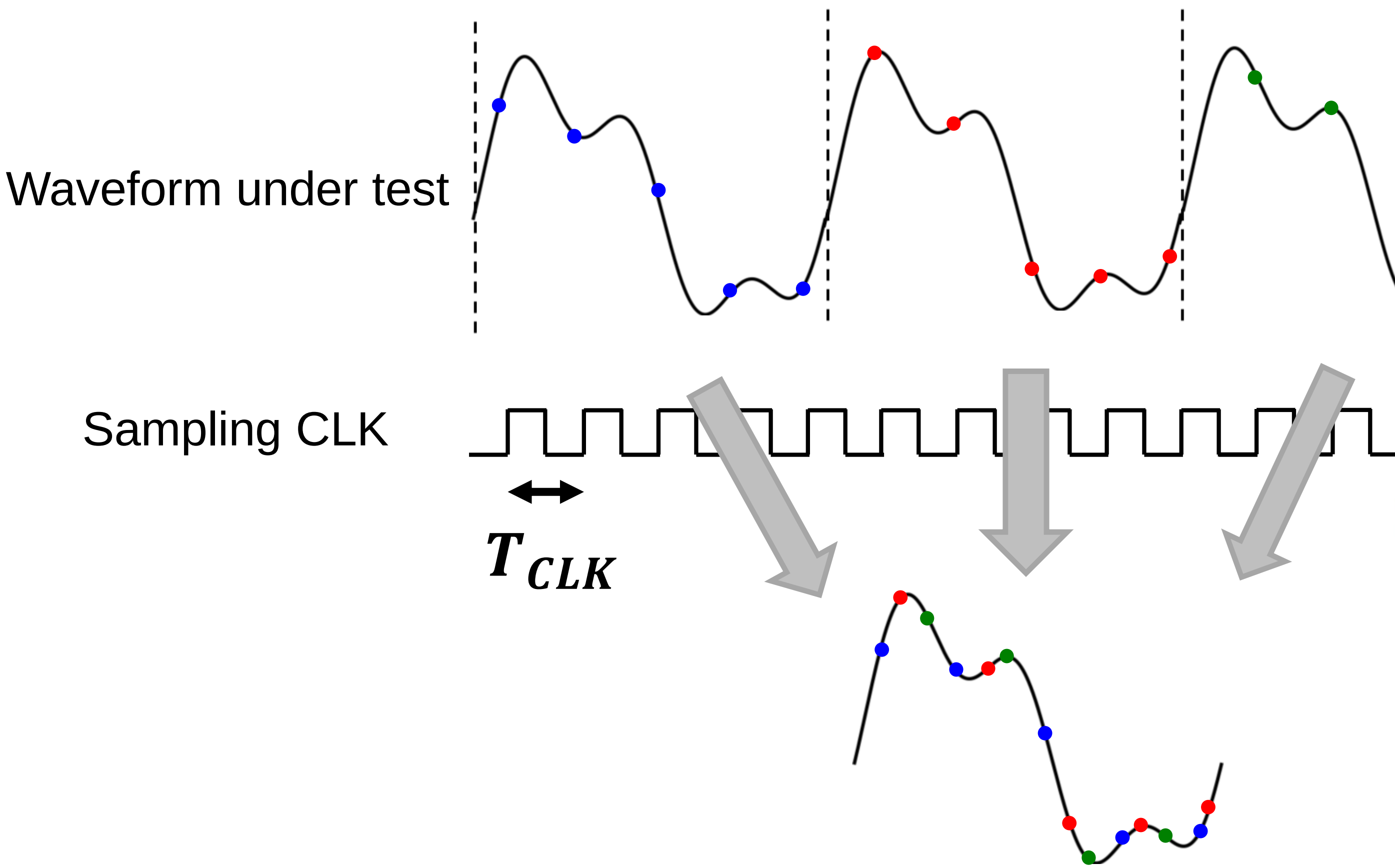
Equivalent-Time Sampling

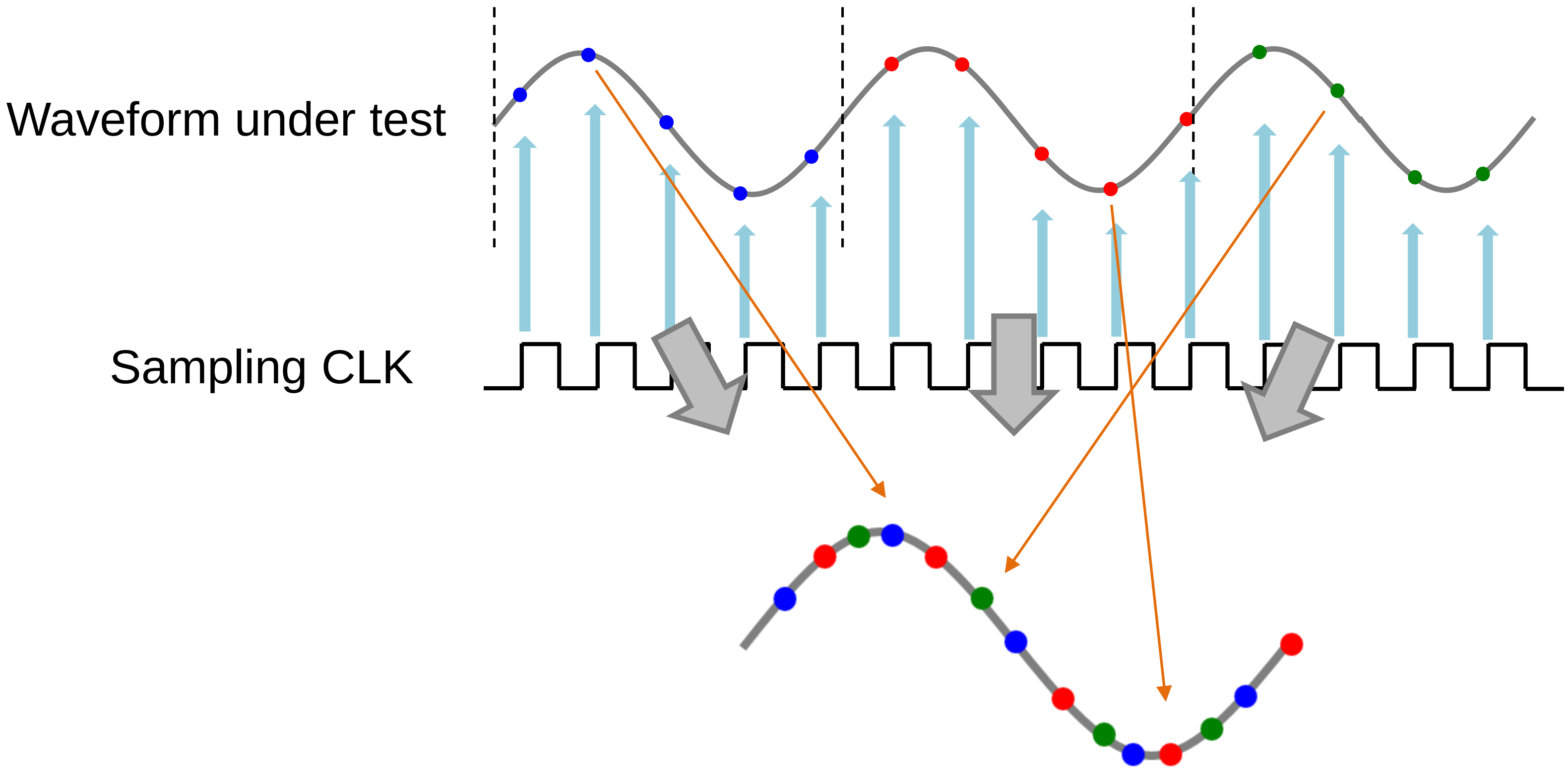
- Technique for sampling repetitive waveform
- Used in sampling oscilloscope



Sampling Oscilloscope

https://teledynelecroy.com/japan/products/scopes/we/default.asp

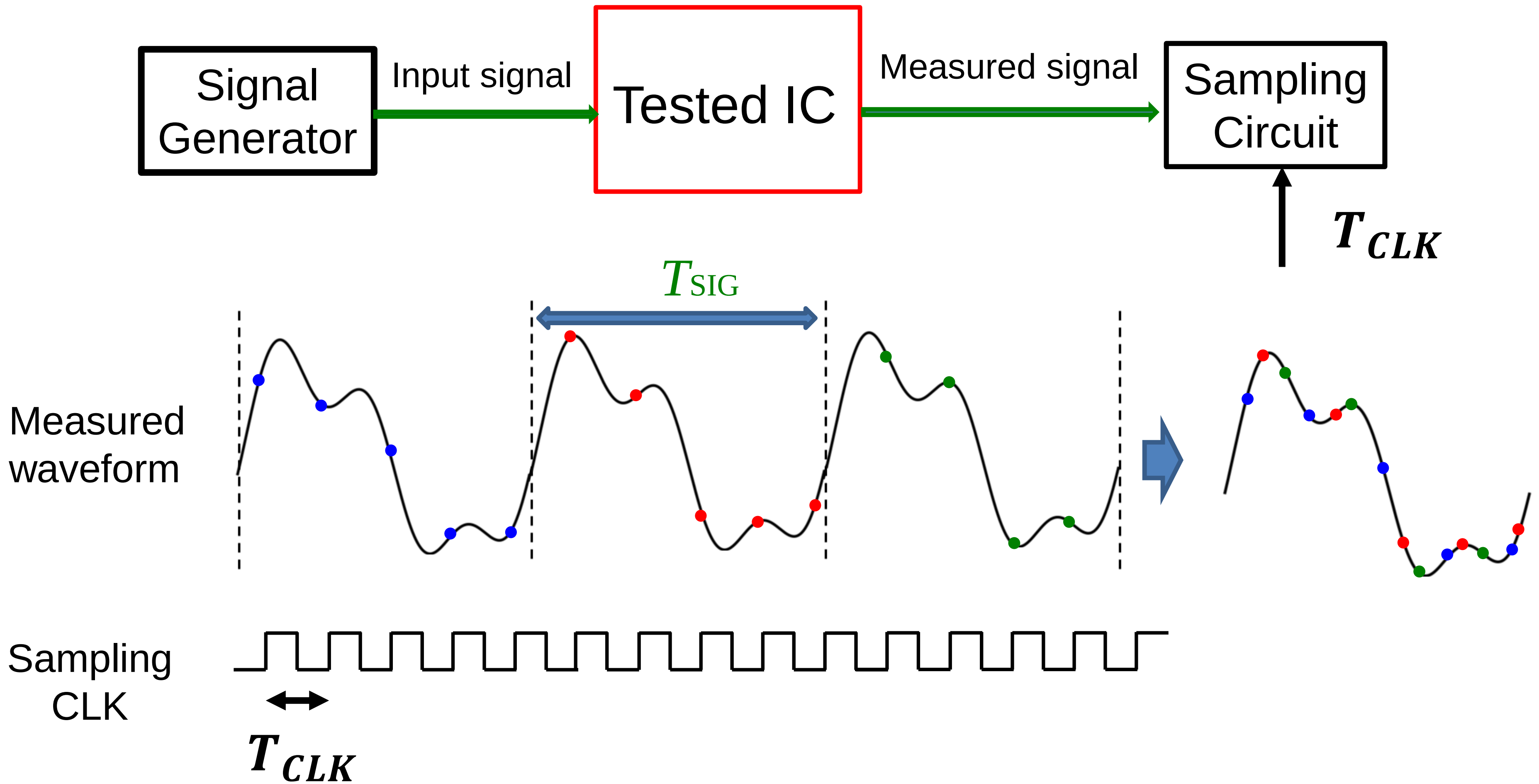
Random Sampling



Sampling repetitive waveform with asynchronous CLK
➡ Construct one-period waveform

IC Testing and Equivalent-Time Sampling

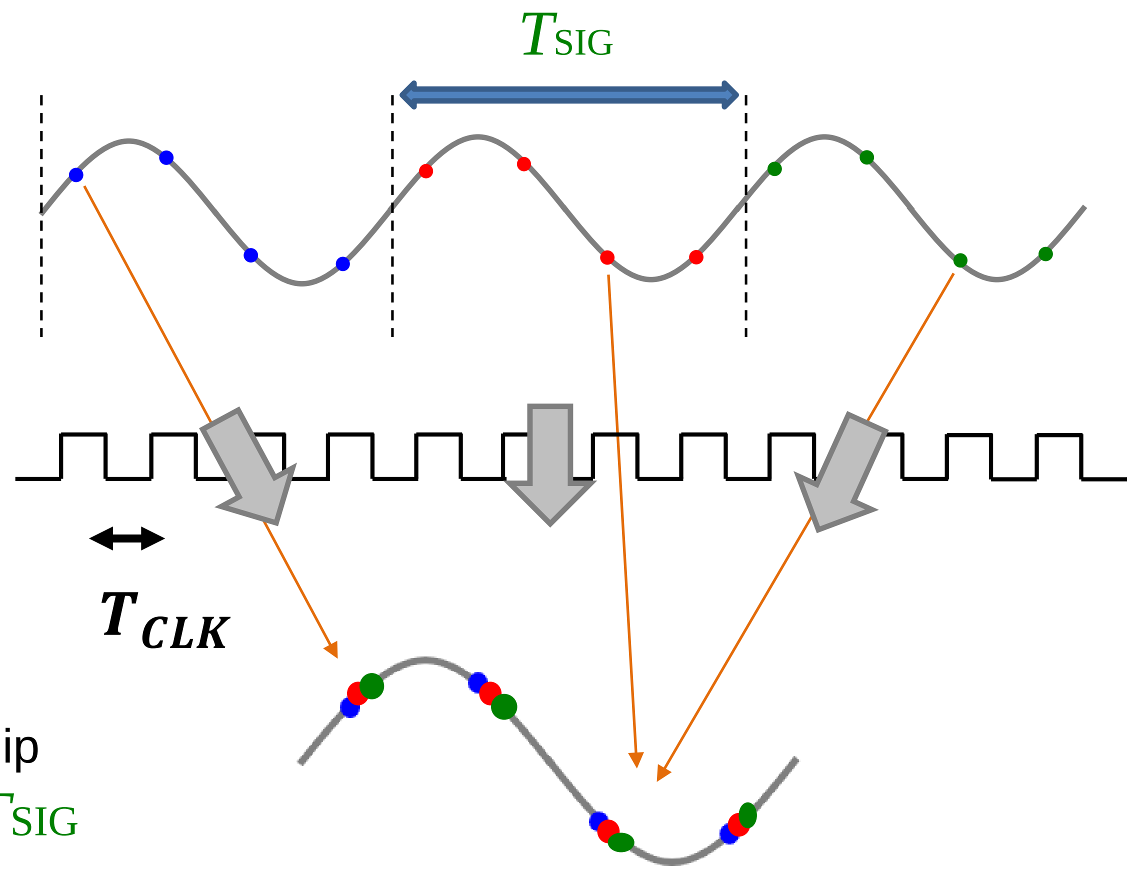
- Input signal → Controlled during IC testing
  Input signal period T_{SIG} → Output signal period T_{SIG}



Waveform Missing Phenomena

Waveform under test

Sampling CLK



Caused by relationship
between T_{CLK} and T_{SIG}

A lot of data → reconstruct one period

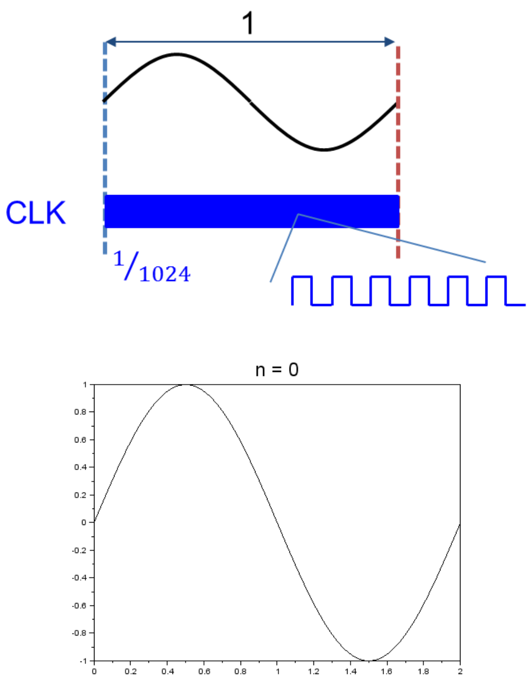
Test time : LONG

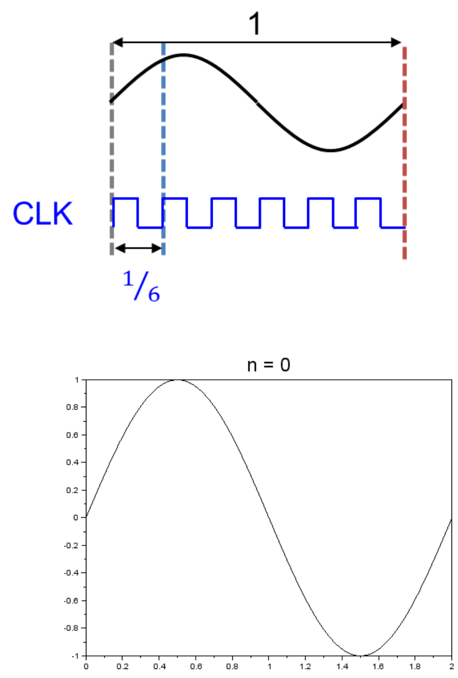
Waveform Missing Condition

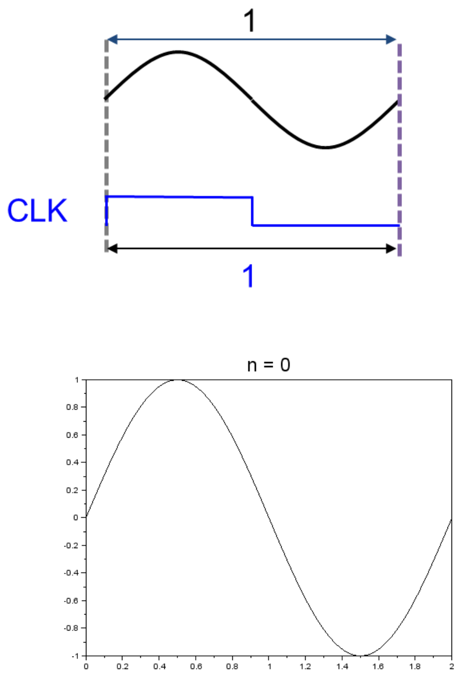
$f_{CLK} \gg f_{sin}$

$f_{CLK} \approx \frac{1}{\alpha} f_{sin} \left(\alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \cdots, \frac{1}{6}, \cdots\right)$

$f_{CLK} \approx f_{sin}$







Sampling points: Localized

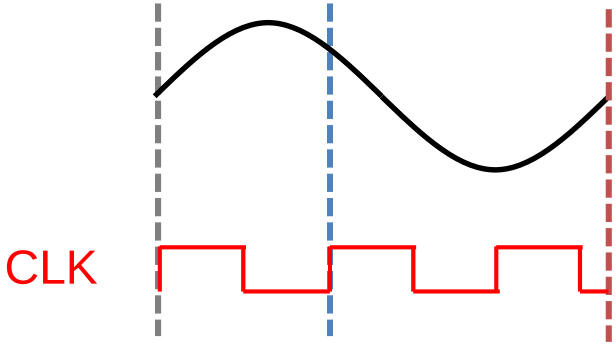
➡ Distance ratio between adjacent sampling points: Large

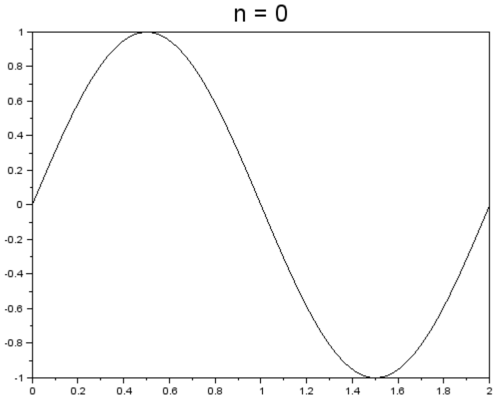
Efficient Waveform Acquisition Condition

Proper CLK

Sampling points: distributed

High efficiency waveform acquisition





Sampling points: Distributed

➡ Distance ratio between adjacent sampling points : Small

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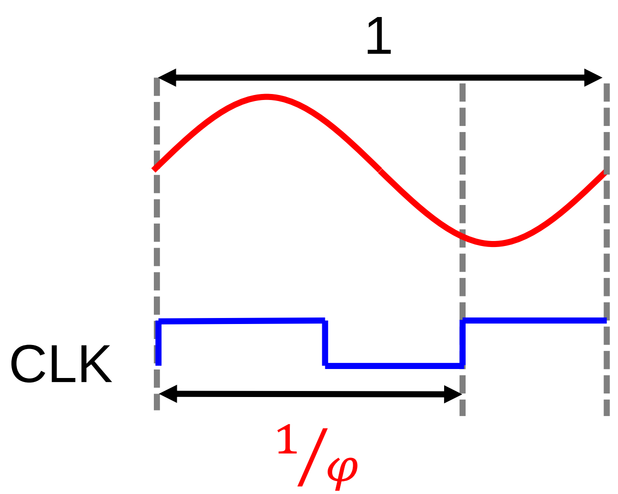
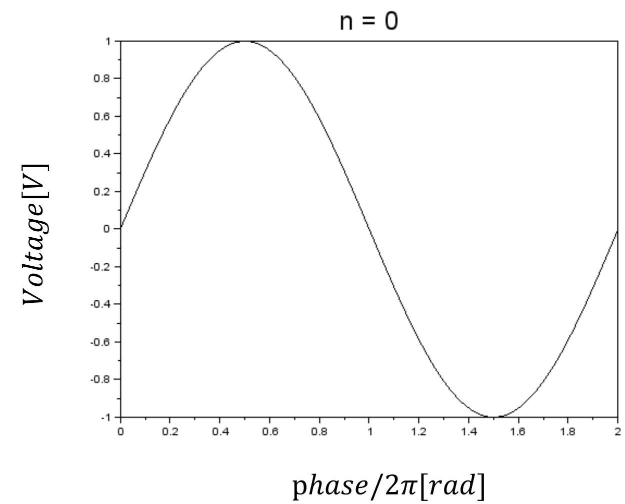
Golden Ratio Sampling

$$f_{CLK} = \varphi \times f_{sig}$$

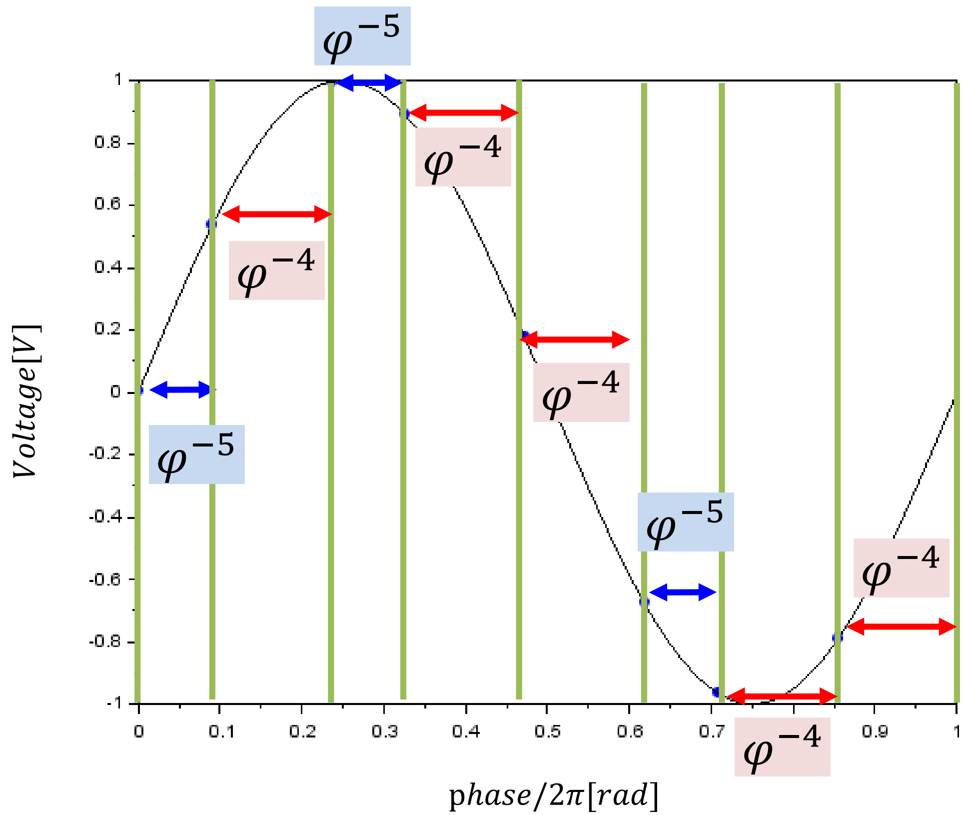
φ : golden ratio (= 1.6180339887…)





Sampling points ➡ Always distributed evenly in phase

Distance of Adjacent Sampling Points



$\boldsymbol{\varphi}$: golden ratio (= 1.6180339887…)

Maximum distance／Minimum distance $= \varphi$ or φ^2

➡ Sampling points : Nothing too close & Nothing too far

Metallic Ratios

Metallic ratios

$$1 : \frac{n + \sqrt{n^2 + 4}}{2} \qquad (n = 1, 2, 3 \cdots)$$

M : Metallic number

$n=1$: Golden ratio (M =1.6180…)

$n=2$: Silver ratio （ M = 2.4142…)

$n=3$: Bronze ratio (M = 3.3027…)

⋮

$n=m$: 1 : M

Difference from reciprocal

$M - \frac{1}{M}$ = Natural Number

Continued fraction

$$M = n + \cfrac{1}{n + \cfrac{1}{n + M}}$$

Limit of adjacent term ratio

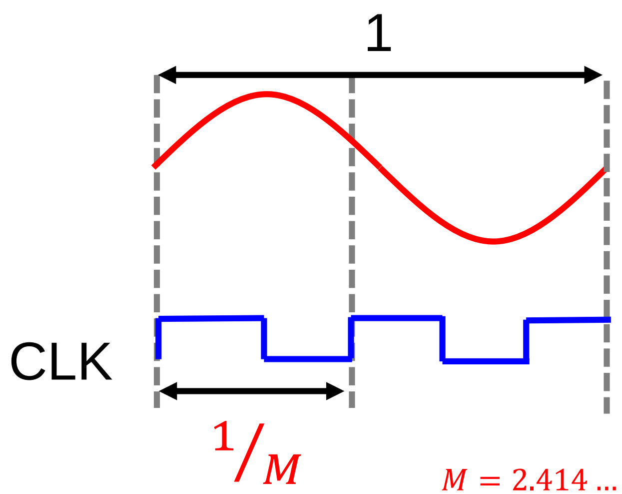
$$F_0 = 0, F_1 = 1, F_{n+2} = nF_{n+1} + F_n$$

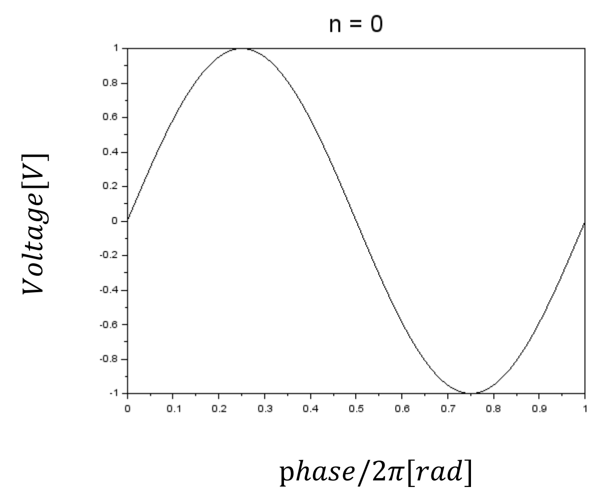
Metallic Ratio Sampling

Fixed f_{CLK} $\rightarrow$ Test ADC with various f_{sig}

$$f_{CLK} = M \times f_{sig}$$

M : Metallic ratio





In the case of silver ratio

Sampling points ➡ Always distributed evenly in phase

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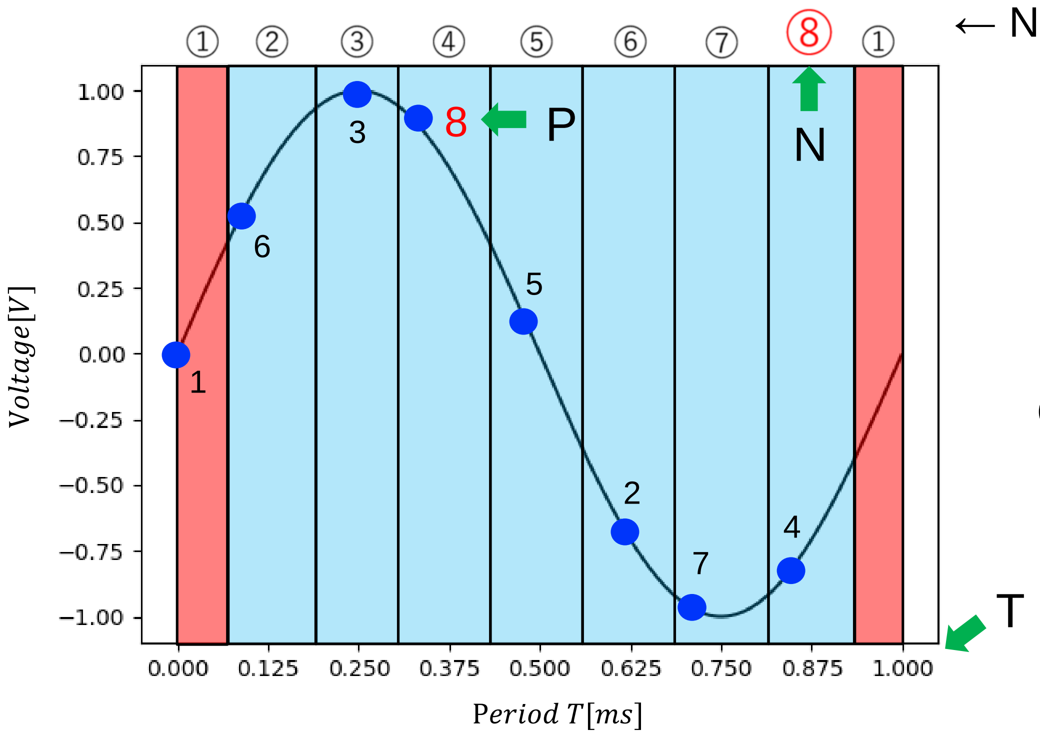
Efficiency Definition

N : Number of divisions in period T E : Sampling efficiency

P : Number of points → All divisions have at east one point in them.

$$E = \frac{N}{P}$$



In case of golden ratio sampling 8 divisions.

← Number to identify segmented area

● Sampling points and order

Difference between adjacent sampling points $< \frac{2T}{N}$

Golden ratio sampling 8 divisions.

$P = 8, N = 8, \ T = 1.0$

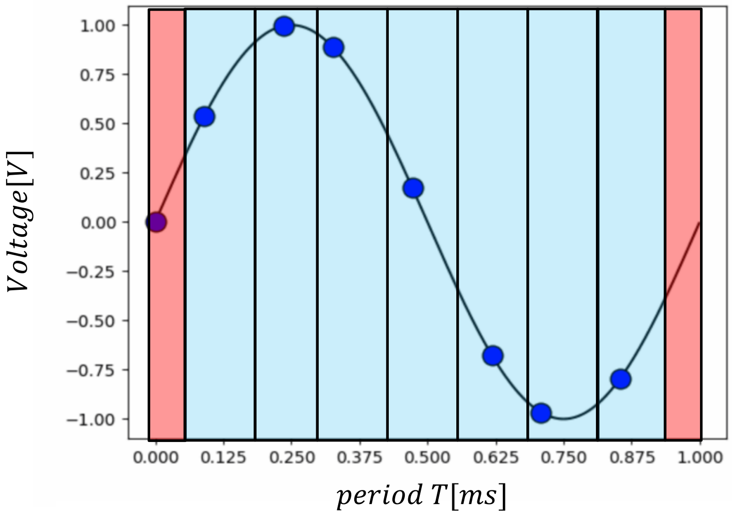
$\therefore E = \frac{8}{8} = 1.0$

Difference between adjacent sampling points $\rightarrow < \frac{2}{8}$

Efficiency by Metallic Ratios

In case of 8 divided sections

$n = 1$ ($M = 1.6180..$)

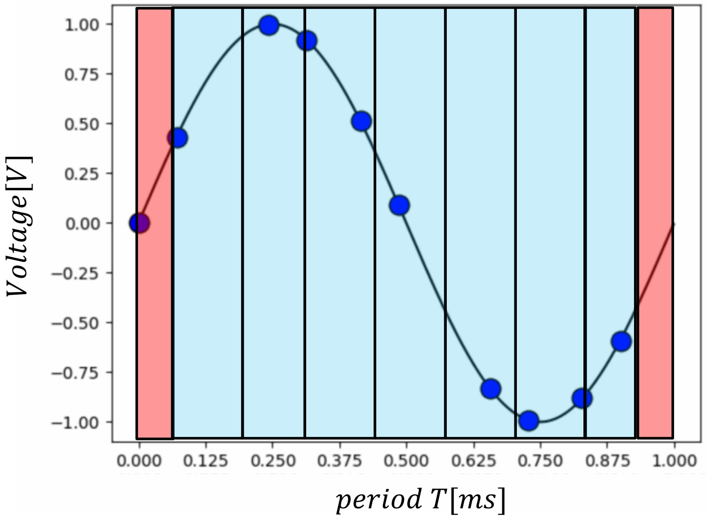


Need 8 points

$$P = 8, N = 8, T = 1.0$$

$$\therefore E = \frac{8}{8} = 1.0$$

$n = 2$ ($M = 2.4142..$)



Need 10 points

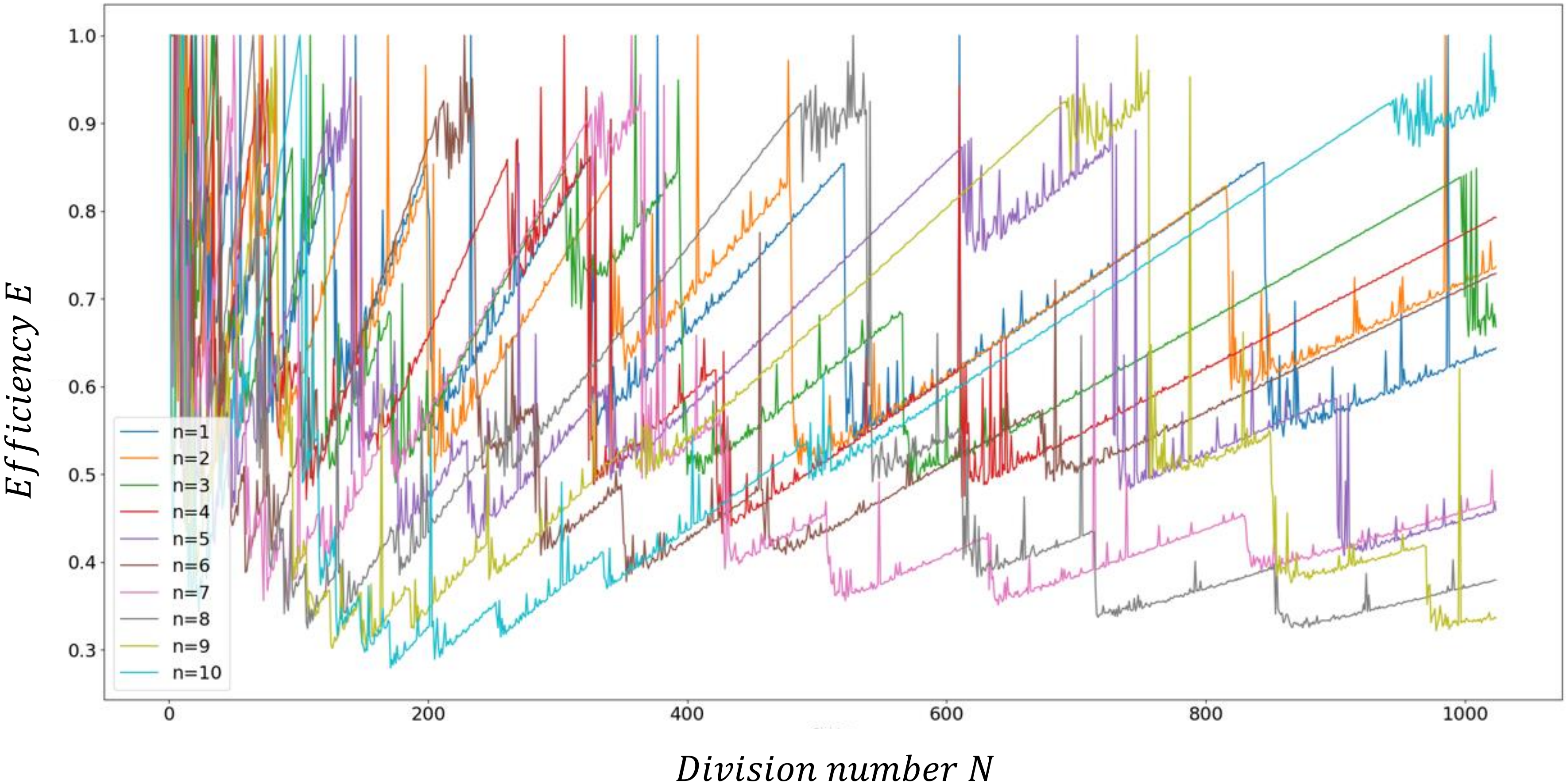
$$P = 10, N = 8, T = 1.0$$

$$\therefore E = \frac{8}{10} = 0.8$$

Efficiency → varies by metallic ratio

Efficiency with Each Metallic Ratio

n-th metallic ratio ($n = 1 \sim 10$), Division number $N = 1 \sim 1024$



Efficiency varies with metallic ratio.
Graph → Sawtooth wave shape.

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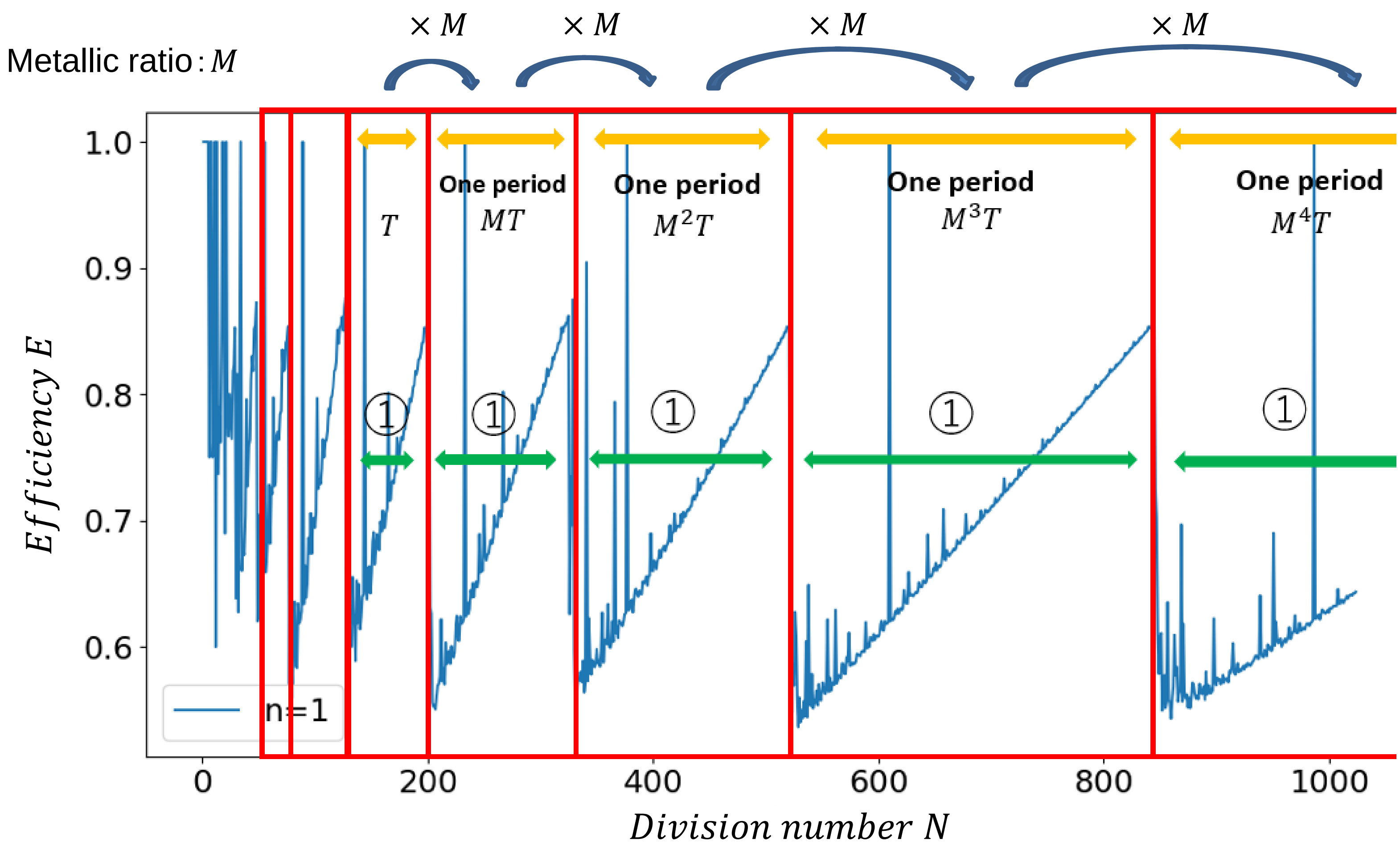
Efficiency Periodicity (Golden Ratio)

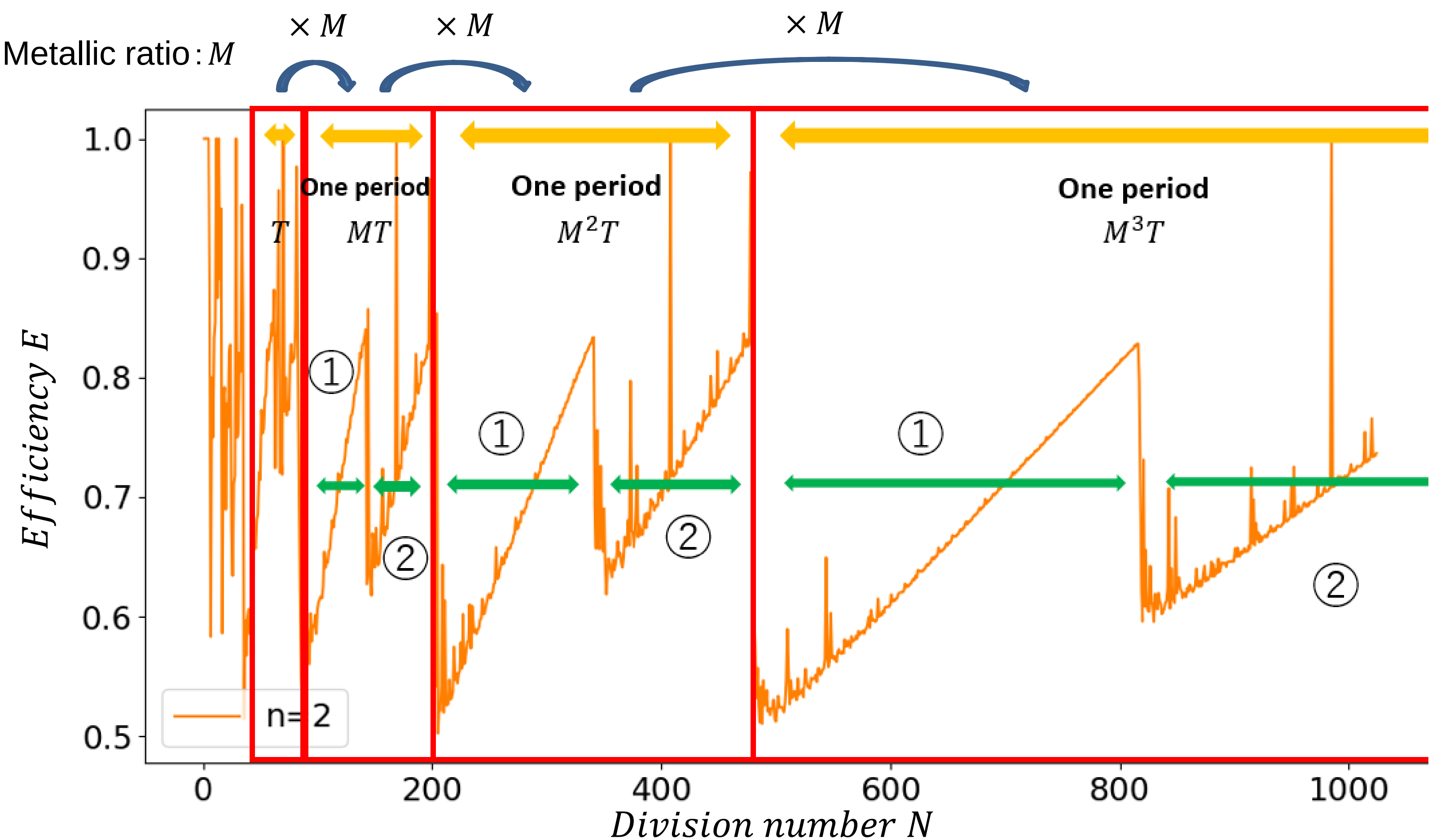
n-th metallic ratio $n = 1$, Division Number $N = 1 \sim 1024$



If one period is sawtooth wave, similar waveform is repeated.
Cycle length is multiplied by golden ratio for each cycle.

Efficiency Periodicity (Silver ratio)

n-th metallic ratios $n = 2$, Division number $N = 1 \sim 1024$



If one period is two sawtooth waves, similar waveform is repeated.
Cycle length is multiplied by silver ratio for each cycle.

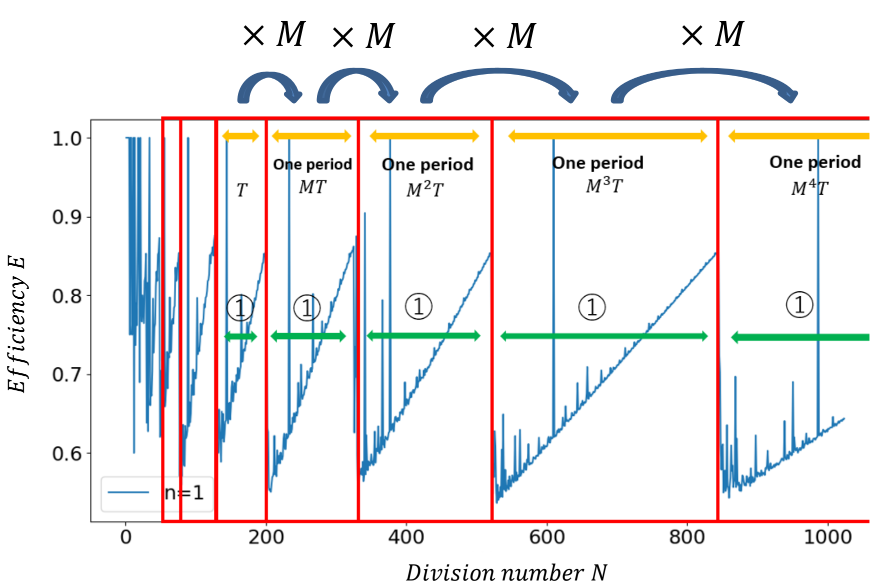
Efficiency Periodicity

One period in n-th metallic ratio → Viewed as n sawtooth waves → Periodicity.

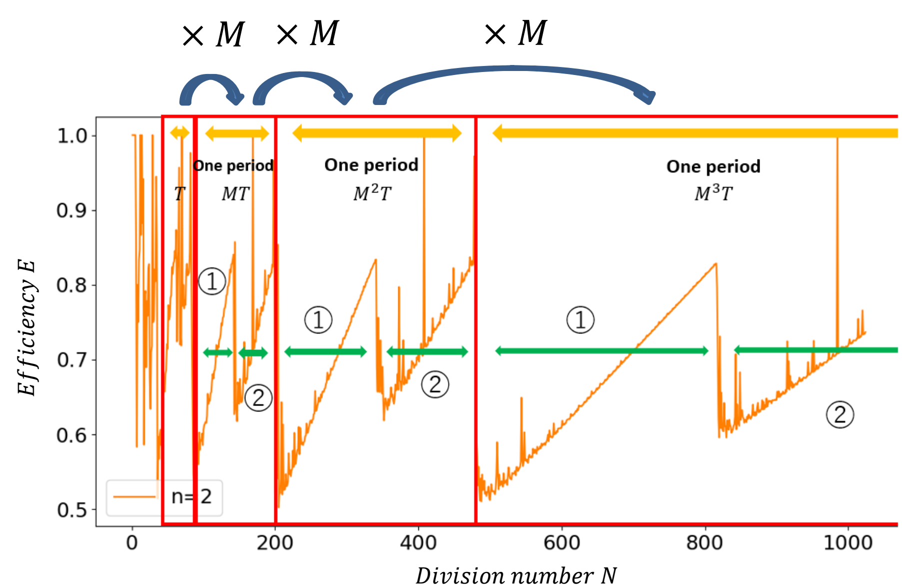
Length of efficiency period → Multiplied by metallic ratio every cycle.

T_L : L-th cycle M : metallic number

$$T_L = MT_{L-1}$$



n-th metallic ratios $n = 1$, Division number $N = 1$~1024



n-th metallic ratio $n = 2$ Division number $N = 1$~1024

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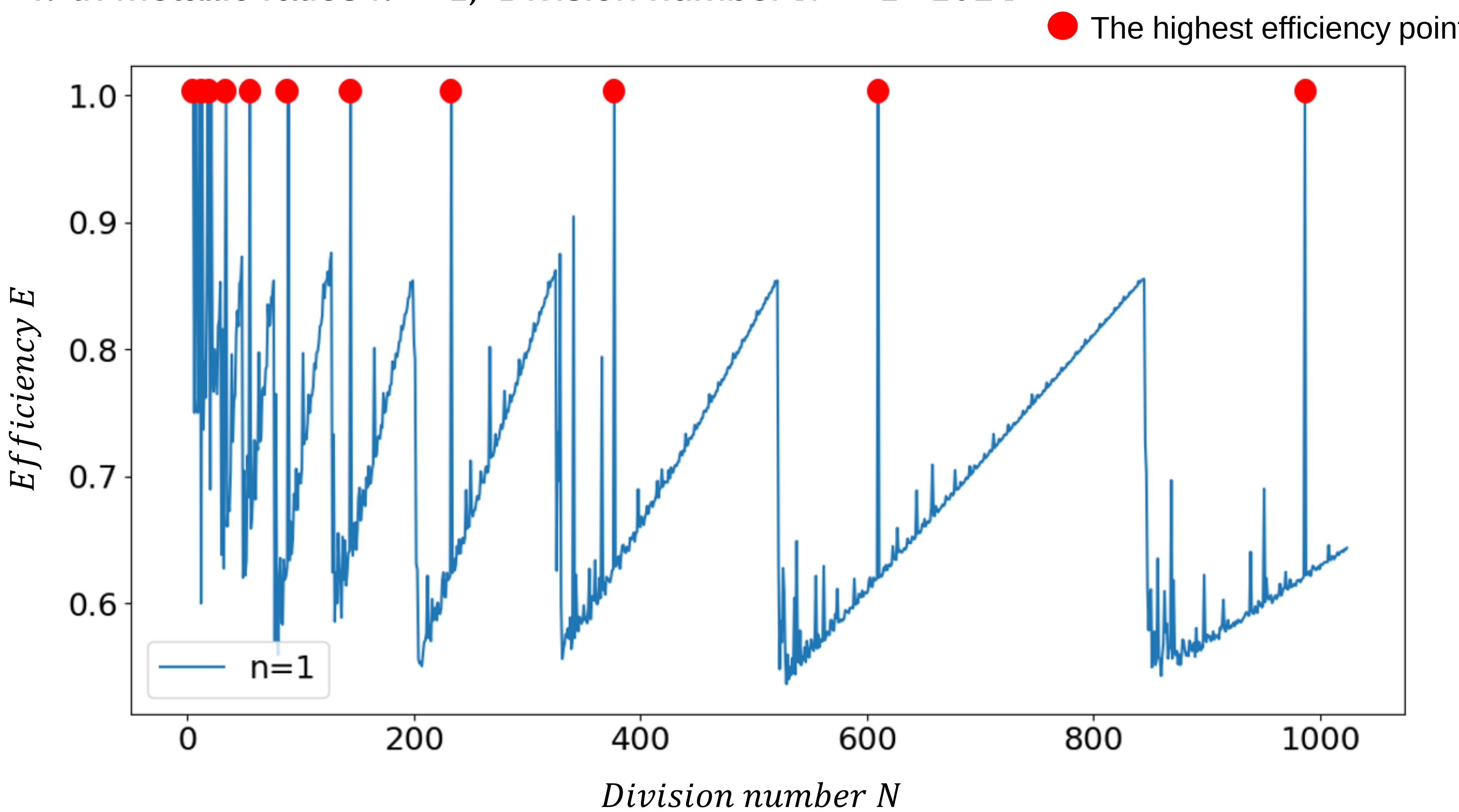
Number of Divisions for Highest Efficiency

n-th metallic ratios $n = 1$, Division number $N = 1{\sim}1024$



Periodic number of divisions for the highest efficiency

Rule of Highest Efficiency Points

Golden ratio ($n = 1, M = 1.6180339887\ldots$)

1, 2, 3, 4, 5, 7, 8, 11, 13, 18, 21, 34, 55, 89, 144, 233, 377, 610, 987,…

$987 \div 610 = 1.6180327\ldots$

Silver ratio ($n = 2, M = 2.4142135623\ldots$)

1, 2, 3, 4, 5, 12, 14, 29, 70, 169, 408, 985,…

$985 \div 408 = 2.4142156\ldots$

Bronze ratio ($n = 3, M = 3.3027756377\ldots$)

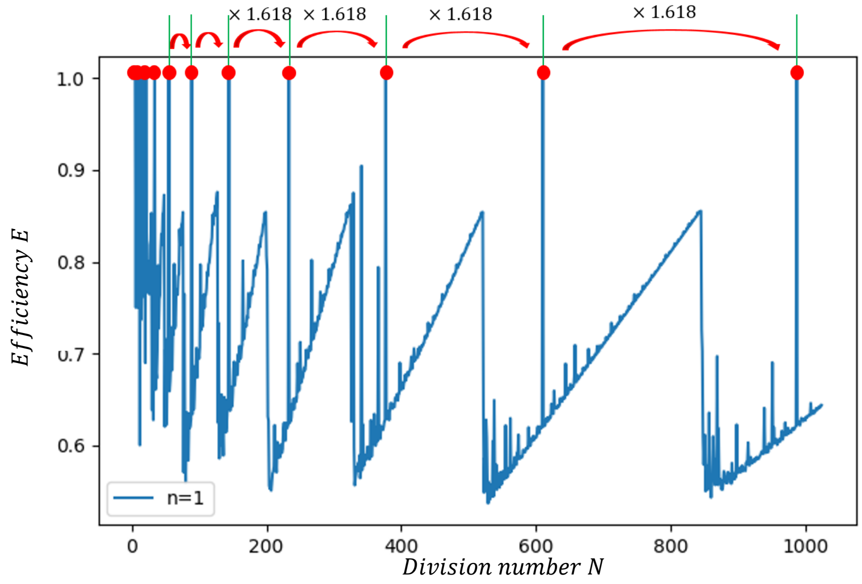
1, 2, 3, 4, 5, 6, 10, 20, 33, 35, 109, 360,…

$360 \div 109 = 3.3027522\ldots$

Sequence of highest efficiency points

$$F_0 = 0, F_1 = 1, F_{m+2} = nF_{m+1} + F_m$$

F_m : m-th highest efficiency point



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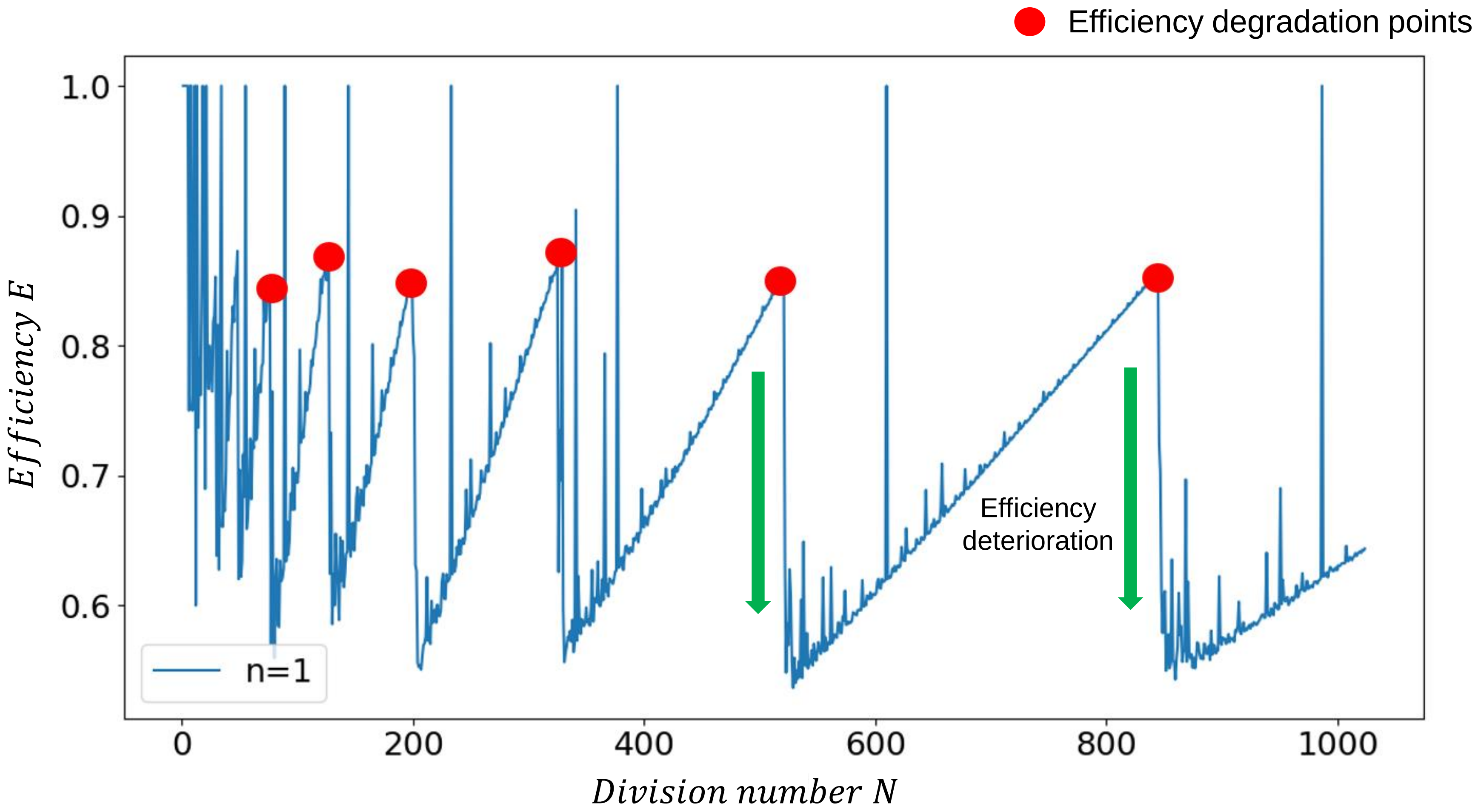
Efficiency Degradation Points

n-th metallic ratio $n = 1$, Division number $N = 1 \sim 1024$



Periodic number of divisions for efficiency degradation

Rule of Efficiency Degradation Points

Golden ratio ($n = 1, M = 1.6180339887\ldots$)

… 48, 76, 127, 199, 325, 521, 845, …

$845 \div 521 = 1.6218809 \ldots$

Silver raito ($n = 2, M = 2.4142135623\ldots$)

…83, 142, 199, 341, 479, 816…

$816 \div 341 = 2.3929618 \ldots$, $479 \div 199 = 2.4070351 \ldots$

Bronze ratio（ $n = 3, M = 3.3027756377\ldots$)

…95, 120, 170, 306, 394, 566, 997…

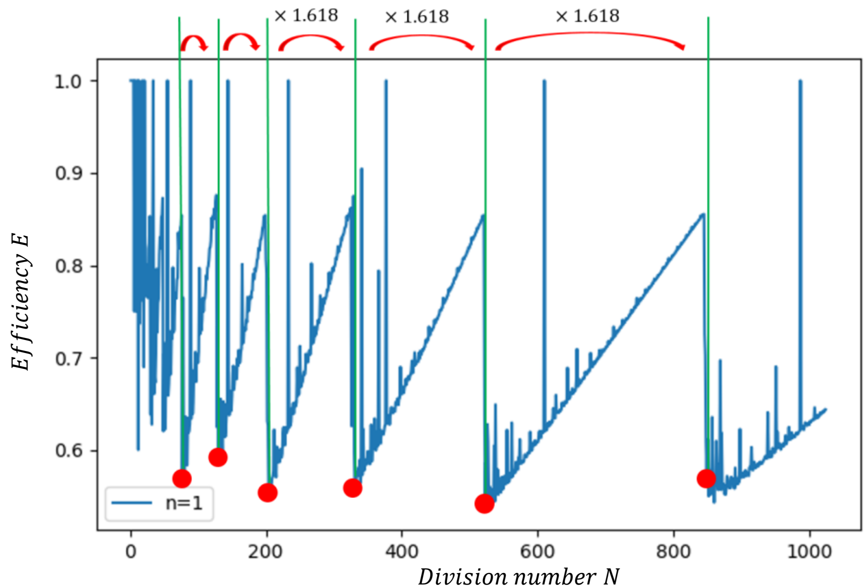
$997 \div 306 = 3.2581699 \ldots$, $566 \div 170 = 3.3294117 \ldots$

Efficiency degrading point of one n-th of the whole

$$F_0 = 0, F_1 = 1, F_{m+2} = nF_{m+1} + F_m$$
$$G_m = F_{m+2} + F_m$$

Relationship between G_m and M

$$G_m : G_{m+n} = 1 : M$$



G_m : m-th efficiency deterioration points

F_m : number sequence: $F_{m+1}/F_m = M_n$

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Conclusion

- In metallic ratio sampling, the most efficient metallic ratio for waveform acquisition depends on number of divisions.

- Discovering rule of efficiency of metallic ratio sampling
  - Efficiency periodicity

    T_L : L-th period M : metallic number

$$T_L = MT_{L-1}$$

  - Rule of the highest efficiency points

    F_m : m-th highest efficiency point

$$F_0 = 0, F_1 = 1, F_{m+2} = nF_{m+1} + F_m$$

  - Rule of efficiency degradation points

    G_m : m-th efficiency deterioration point M : n-th metallic number

$$G_m = F_{m+2} + F_m$$
$$G_m : G_{m+n} = 1 : M$$

Future works

- Theoretical proof of discovered rules
- Finding out efficiency formula
  for given metallic ratio sampling and number of divisions.
- Identification of the most efficient metallic ratio
  for any number of divisions.

“ Number theory is queen of mathematics.”

Carl Friedrich Gauss

Kobayashi Laboratory

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Thank you for your kind attention !!

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Kobayashi Lab.
Gunma University

Q&A

Q. I wonder if you already estimate the implementation cost strategy.

A. I haven’t studied that in detail. It is a future work.

Q. you have mention about IC testing, what is use.

A. When we do IC testing, we know what the output waveform will look like in relation to the input waveform. Therefore, it is necessary to sample the output waveform to confirm that it is as designed, and this technology is used as a means of sampling.