

# Digital-to-Analog Converter Configuration Based on Non-uniform Current Division Resistive-Ladder

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# OUTLINE

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- Research Background and Objective
- R-2R DAC Configuration
  - R-2R Current-steering DAC
- N-ary DAC Configuration
  - Resistive Ladder with Current Division
- Connecting Resistor Ladders
  - Connection conditions
  - Proposed DAC configuration and simulation
- Conclusion and Future work

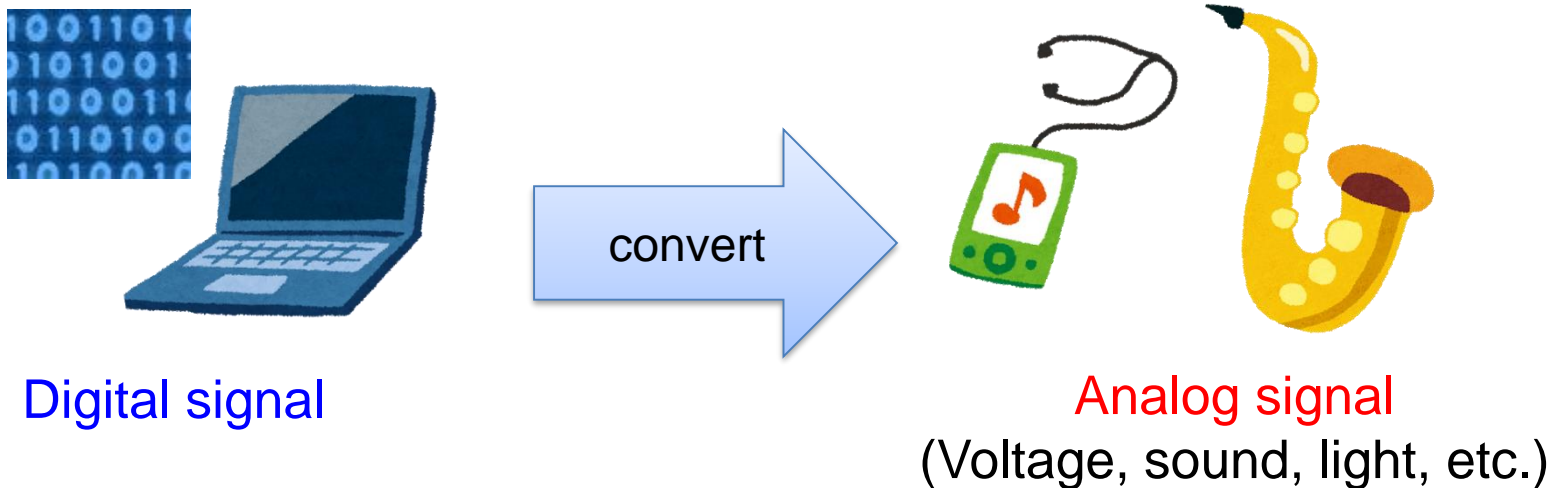
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# Research Background

- Digital-to-Analog Converter (DAC)
  - Output of digital signal processing results



- Internal circuit of ADC

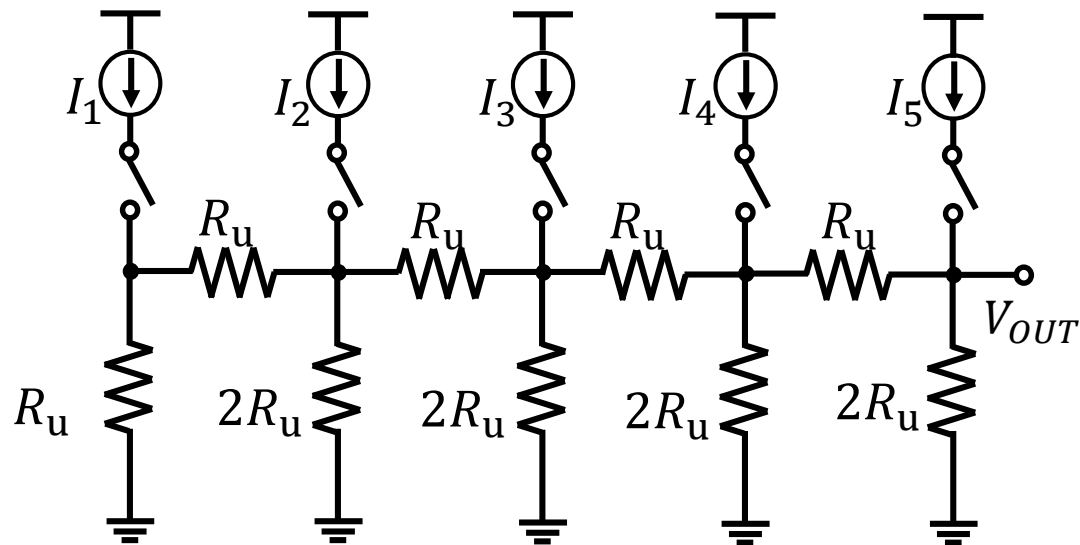
Objective : Investigation of **resistor ladder and current-steering DAC architectures**

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# R-2R Current-Steering DAC



5-bit R-2R current-steering DAC

- Using binary weighting with a well-known R-2R ladder
  - Identical currents  $I_1, \dots, I_5$

## Good

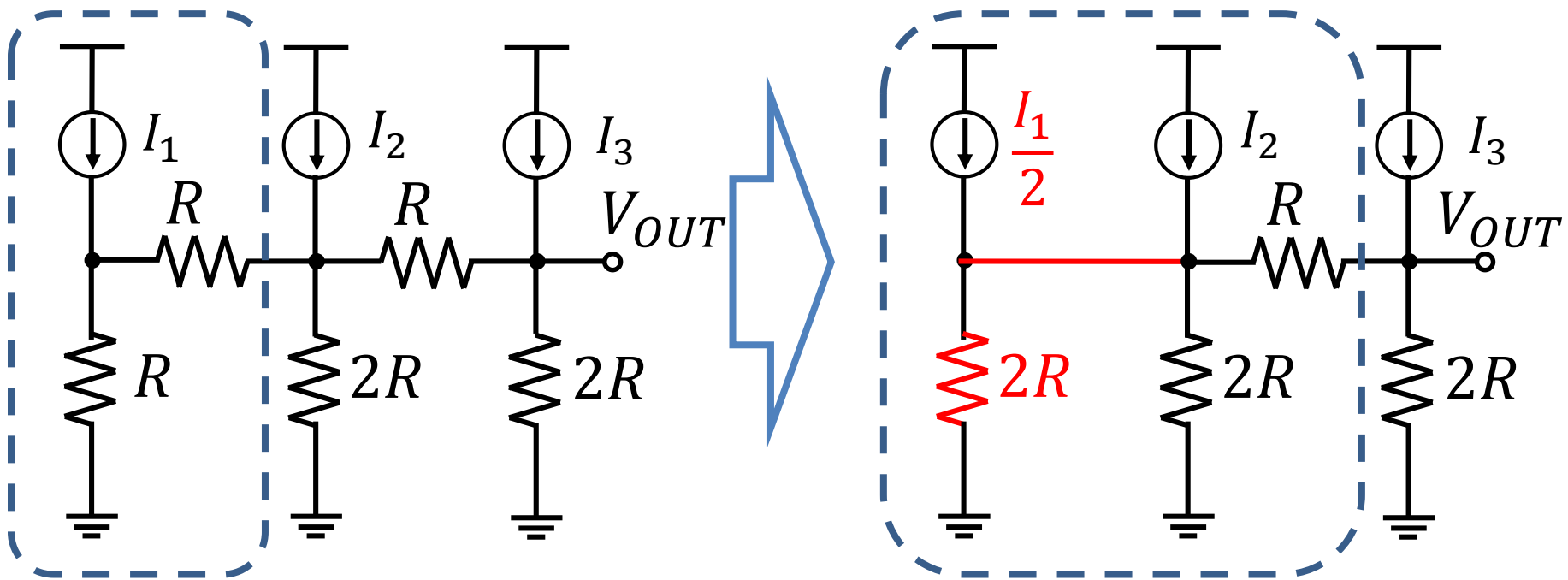
- No decoder
- Fast operation with current sources

## Bad

- DNL degradation due to element mismatches
- Large glitches

# Principle of operation of R-2R DAC

- Replacing a current source and resistors with equivalent circuit, using Norton theorem
  - Internal current is **weighted twice** for each stage

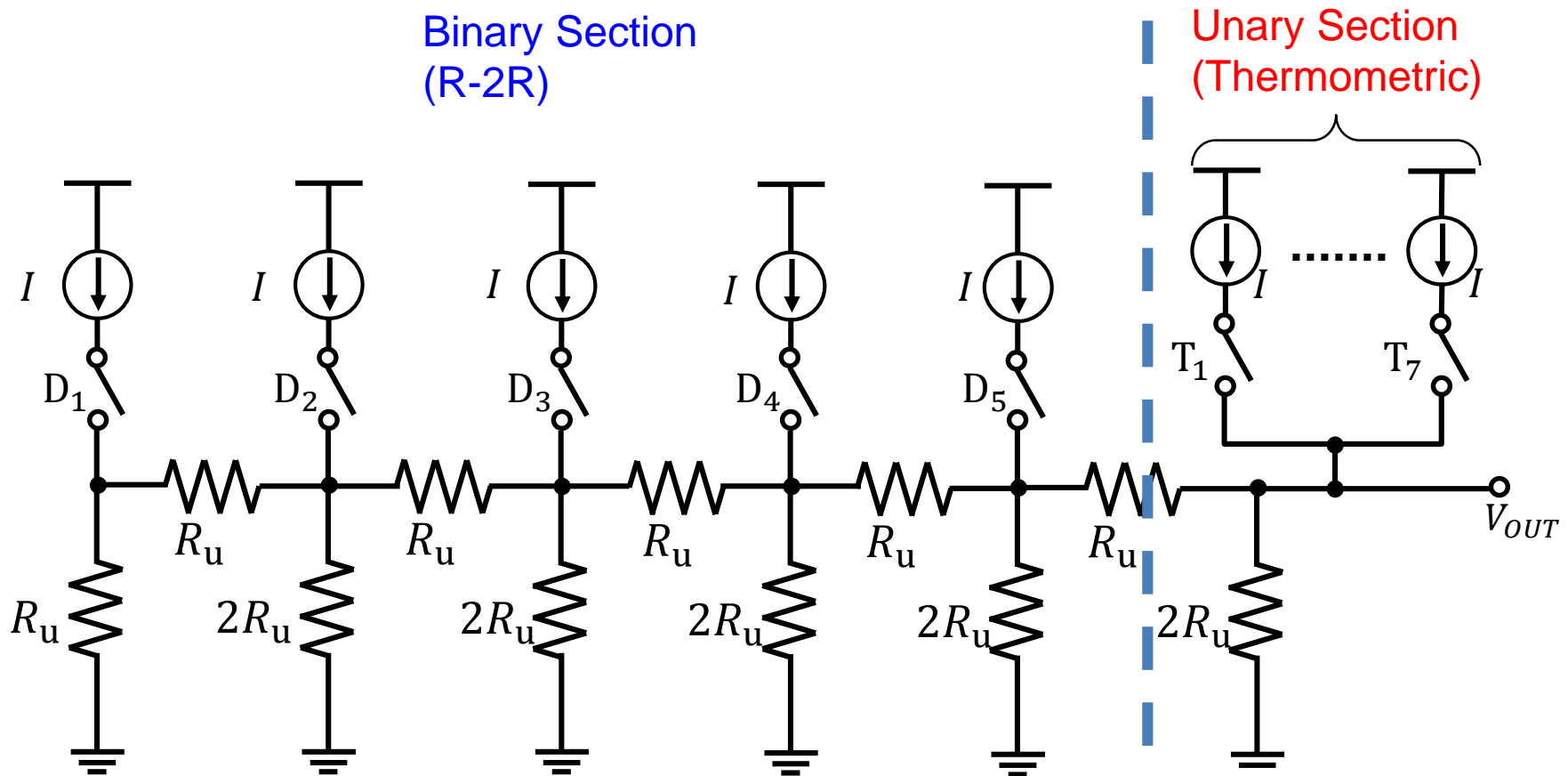


3-bit R-2R DAC

Replacement to equivalent circuit

# Segmented R-2R DAC

- Glitch reduction, DNL reduction
  - Need binary-to-thermometer decoder for unary section



8-bit segmented R-2R DAC (Upper 3-bit for unary codes)

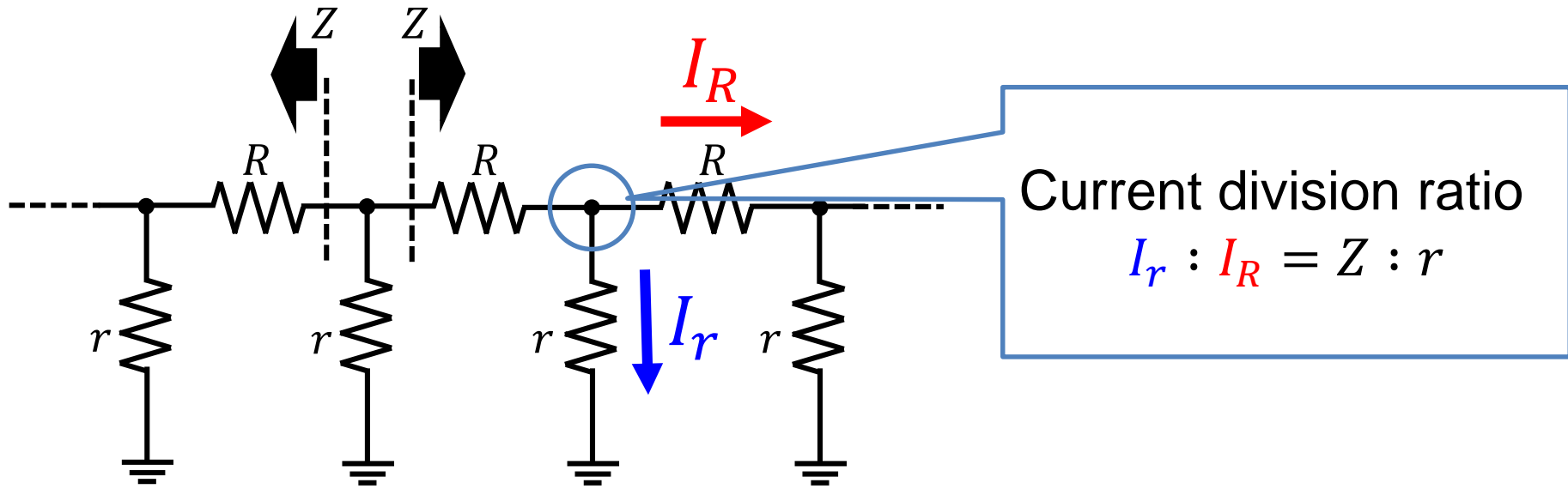


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# Current Division with Resistive Ladder



- Composite resistance  $Z$

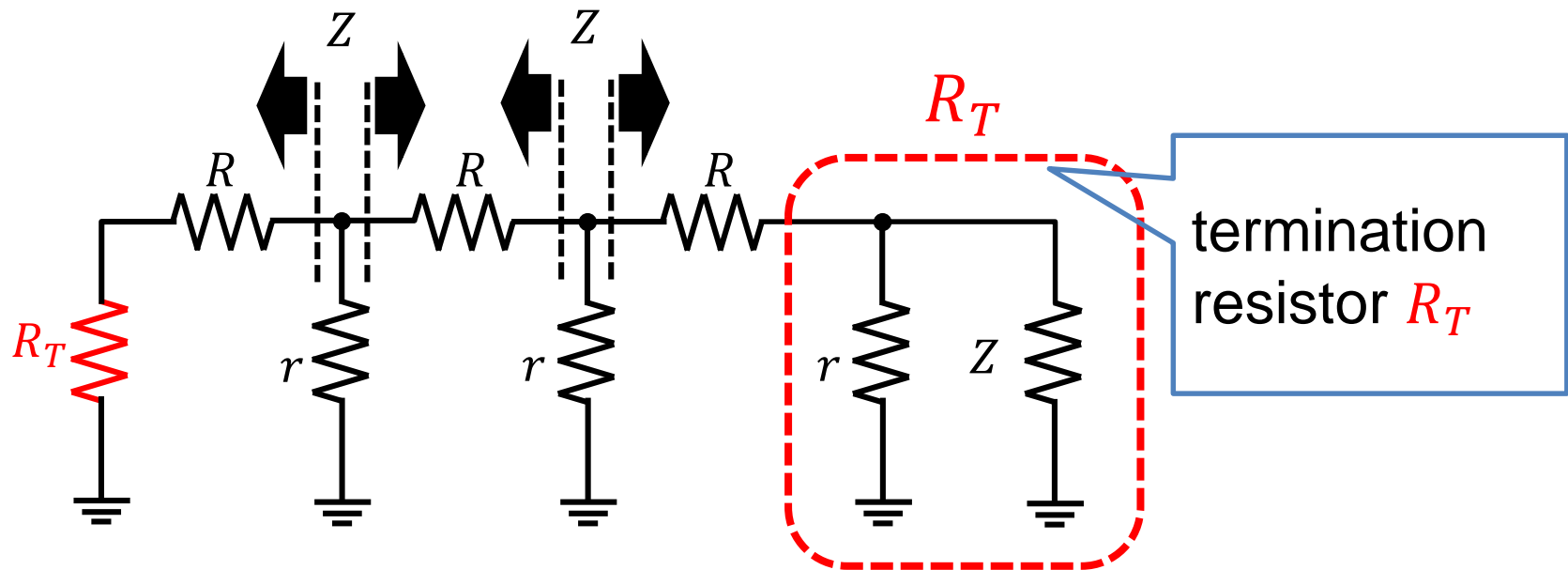
$$Z = \frac{R}{2} + \frac{\sqrt{R(R + 4r)}}{2}$$

- For current division ratio  $N - 1 : 1$  ( $N = 2, 3, 4, \dots$ )

$$I_r : I_R = Z : r = N - 1 : 1$$

$$\Leftrightarrow R : r = (N - 1)^2 : N$$

# Finite Length of Resistor Ladder



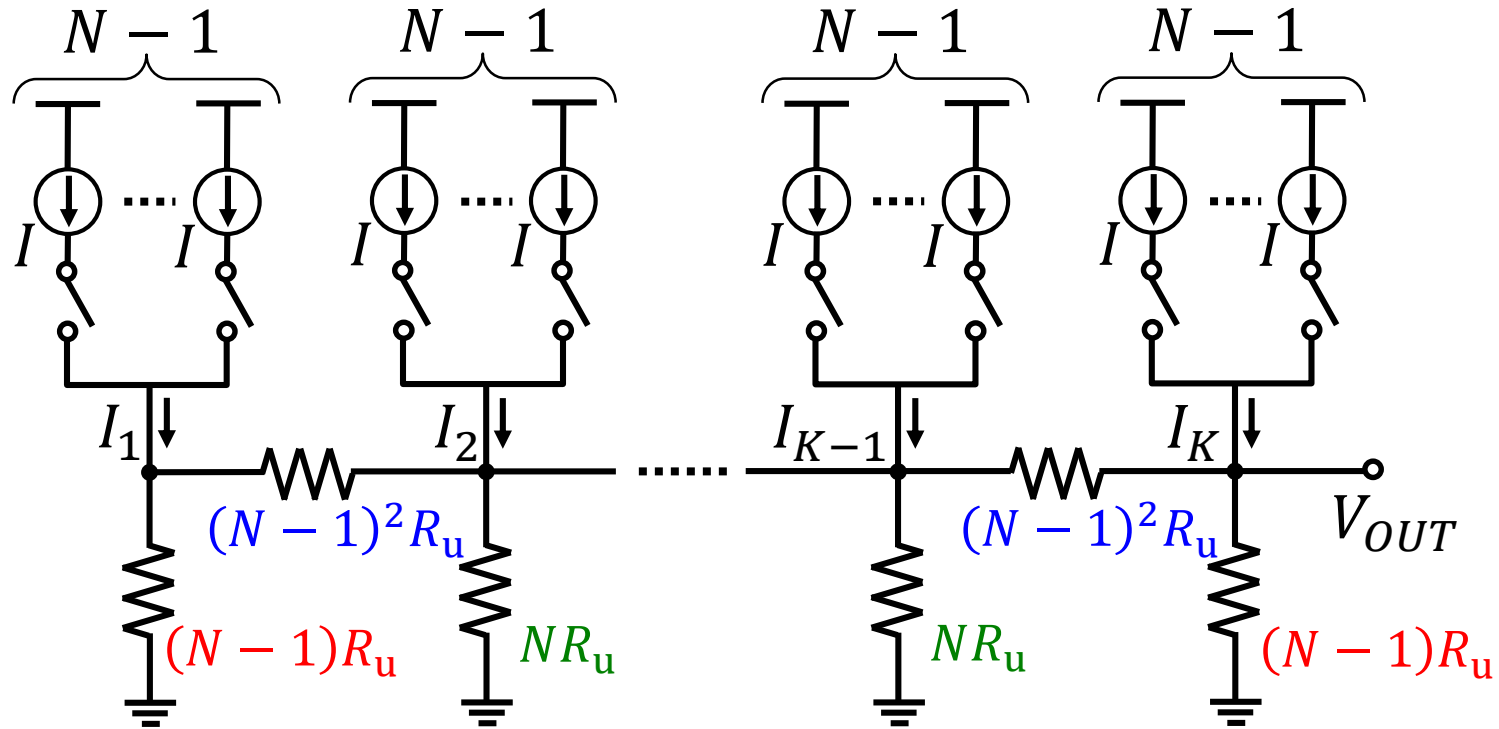
- Termination resistor  $R_T$  with current division ratio  $N - 1 : 1$

$$R_T = Z - R = \frac{R}{N - 1}$$

- A new interpretation** of the R-2R ladder and N-ary ladder

$$R : r : R_T = (N - 1)^2 : N : N - 1$$

# N-ary Resistance Ladder Configuration



$N$  : Current division ratio

$R_u$  : Reference resistance

$I_j$  : Current flowing into  $j$ -th node

$K$  : Number of ladder stages

$I$  : Unit current

- For  $N = 2 \Rightarrow$   **$K$ -bit R-2R DAC**

# Output Voltage and Steps

- Output voltage

$$V_{\text{OUT}}(I_1, \dots, I_K, R_u, N, K) = (N - 1)R_u \sum_{j=1}^K \left( \frac{I_j}{N^{K-j}} \right)$$

- Maximum output voltage

$$V_{\text{MAX}}(I, R_u, N, K) = R_u I \cdot N(N - 1) \cdot \left( 1 - \frac{1}{N^K} \right)$$

- Output voltage minimum step

$$V_{\text{MIN}}(I, R_u, N, K) = (N - 1)R_u I \cdot \frac{1}{N^{K-1}}$$

- Number of output voltages

$$N^K - 1$$

$N$  : Current division ratio

$R_u$  : Reference resistor

$I_j$  : Current into  $j$ -th node

$K$  : Number of ladder stages

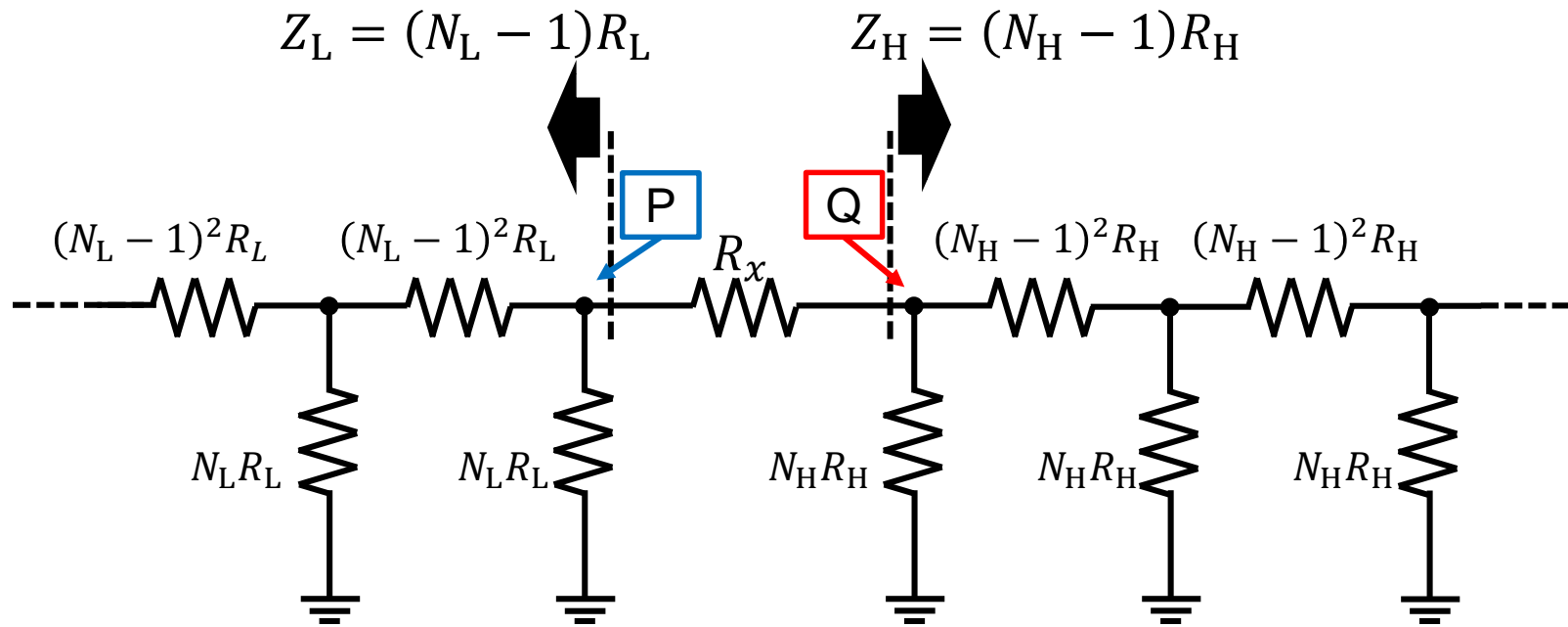
$I$  : Unit current

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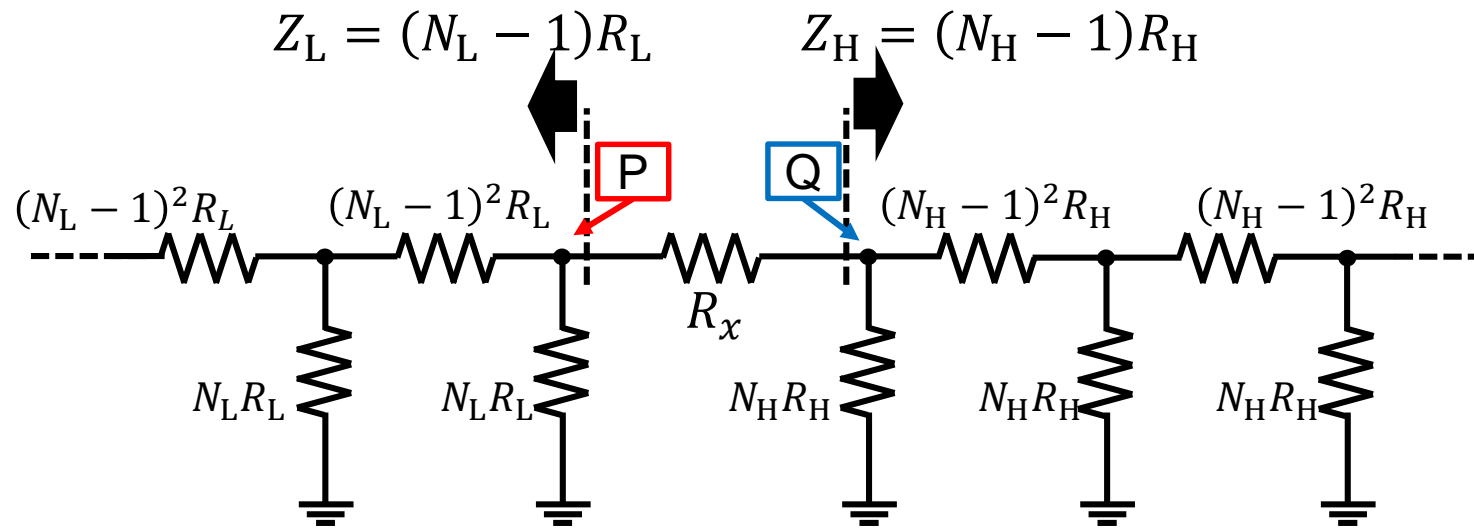
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# Connection of Resistive Ladders



- Parameters
  - $R_x$  : Connection resistor
  - $N_H/N_L$  : Upper/Lower-side current division parameter
  - $R_H/R_L$  : Upper/Lower-side reference resistor
  - $Z_H/Z_L$  : Resistance from node Q/P of view

# Connection Conditions



- Connection conditions

1. To the right of **P**  $\Rightarrow$  Characteristics of  $N_H$ -ary resistor ladder
2. To the left of **Q**  $\Rightarrow$  Upper side  $N_H$ -ary characteristic is not broken

$$\begin{cases} R_x + Z_H = N_H Z_L \\ R_x + Z_L = N_H Z_H \end{cases}$$



$$R_H = \frac{N_L - 1}{N_H - 1} R_L, \quad R_x = (N_H - 1)(N_L - 1)R_L$$



# OUTLINE

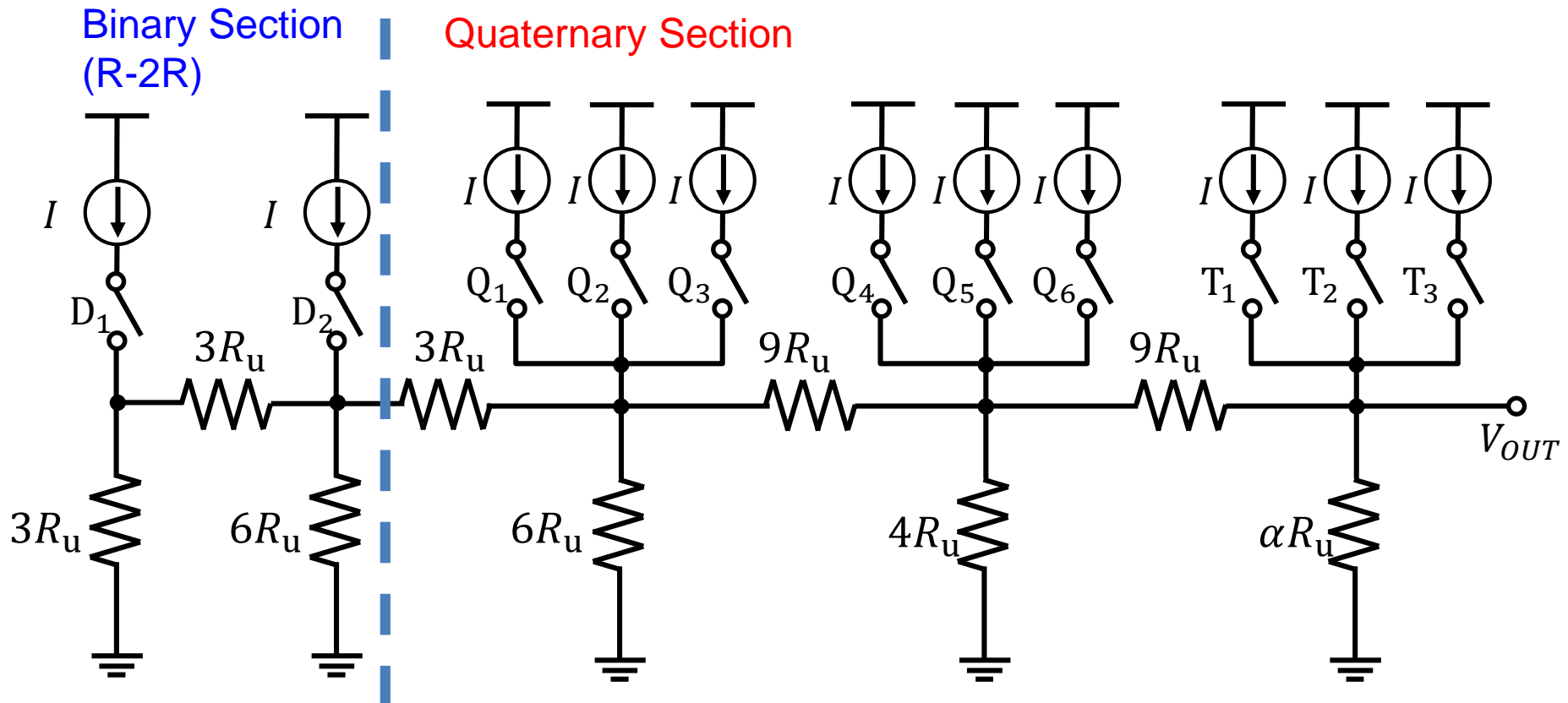
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# Configuration Example

- 8-bit R-2R and 9R-4R connected resistive ladder DAC

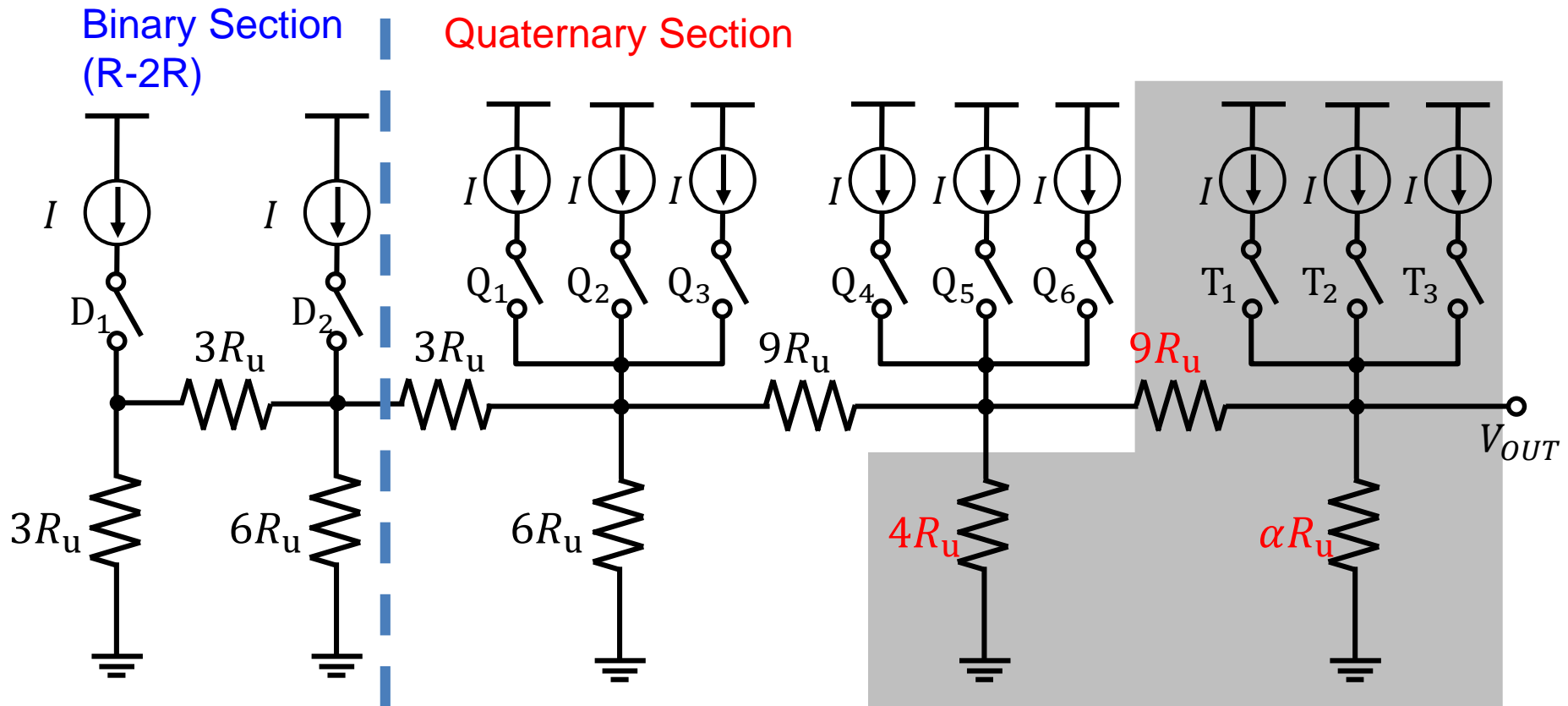
$$R_L = 3R_u, R_H = R_u, R_x = 9R_u$$



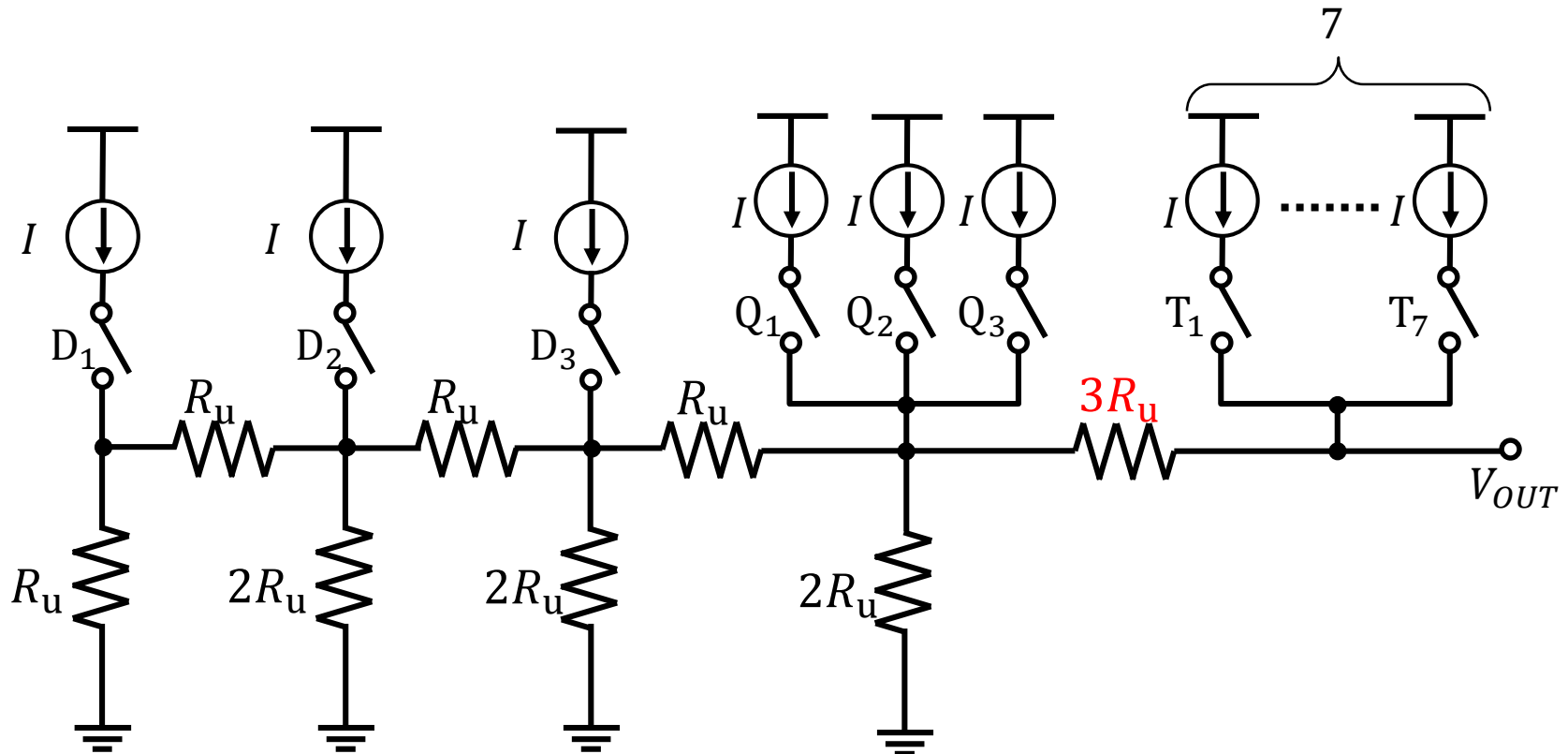
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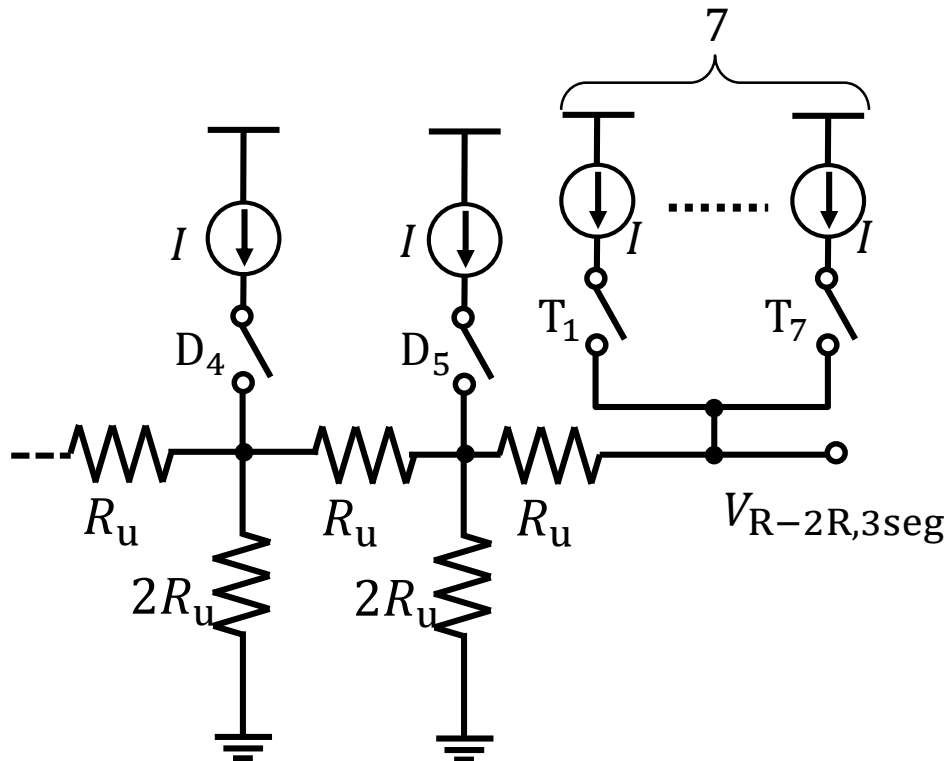


# 2-4-unary Connected Resistor Ladder

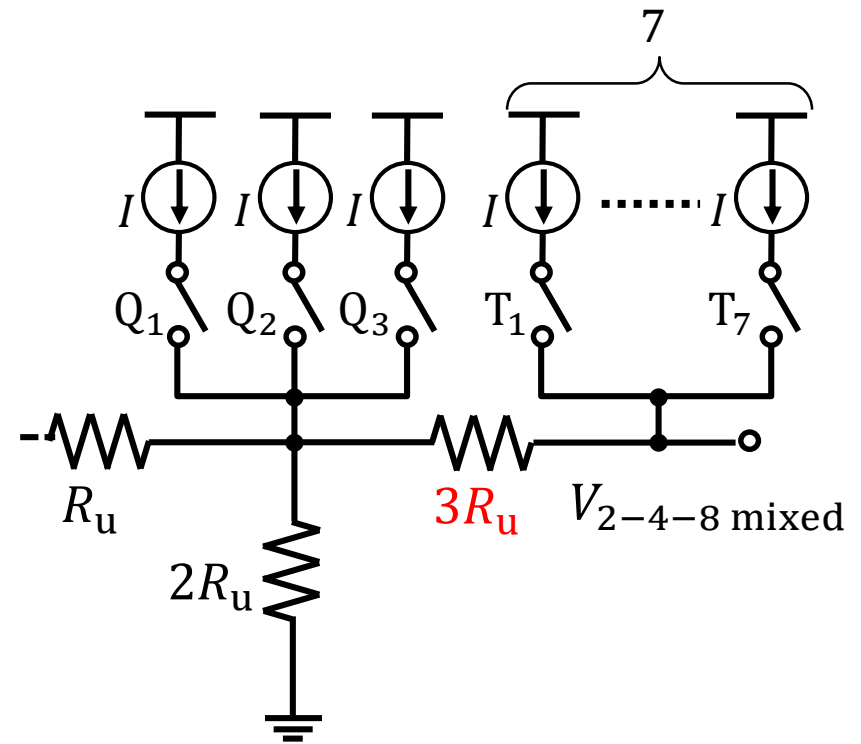


- $3R_u$  between upper and quaternary parts
  - Circuit area is the same as segmented R-2R
  - Additional thermometer decoder for 2 bits of  $D_4, D_5$

# Comparison



segmented R-2R current-steering DAC



binary-quaternary-unary connected resistor ladder

$$V_{R-2R,3seg.} = \frac{1}{16} R_u I \cdot \left\{ \sum_{p=1}^5 (D_p \cdot 2^{p-1}) + 32 \sum_{r=1}^7 T_r \right\}$$

$$V_{2-4-8 \text{ mixed}} = \frac{1}{8} R_u I \cdot \left\{ \sum_{p=1}^3 (D_p \cdot 2^{p-1}) + 8 \sum_{q=1}^3 Q_q + 32 \sum_{r=1}^7 T_r \right\}$$

Double the gain with the same circuit area

# DNL Calculation

- DNL Definition

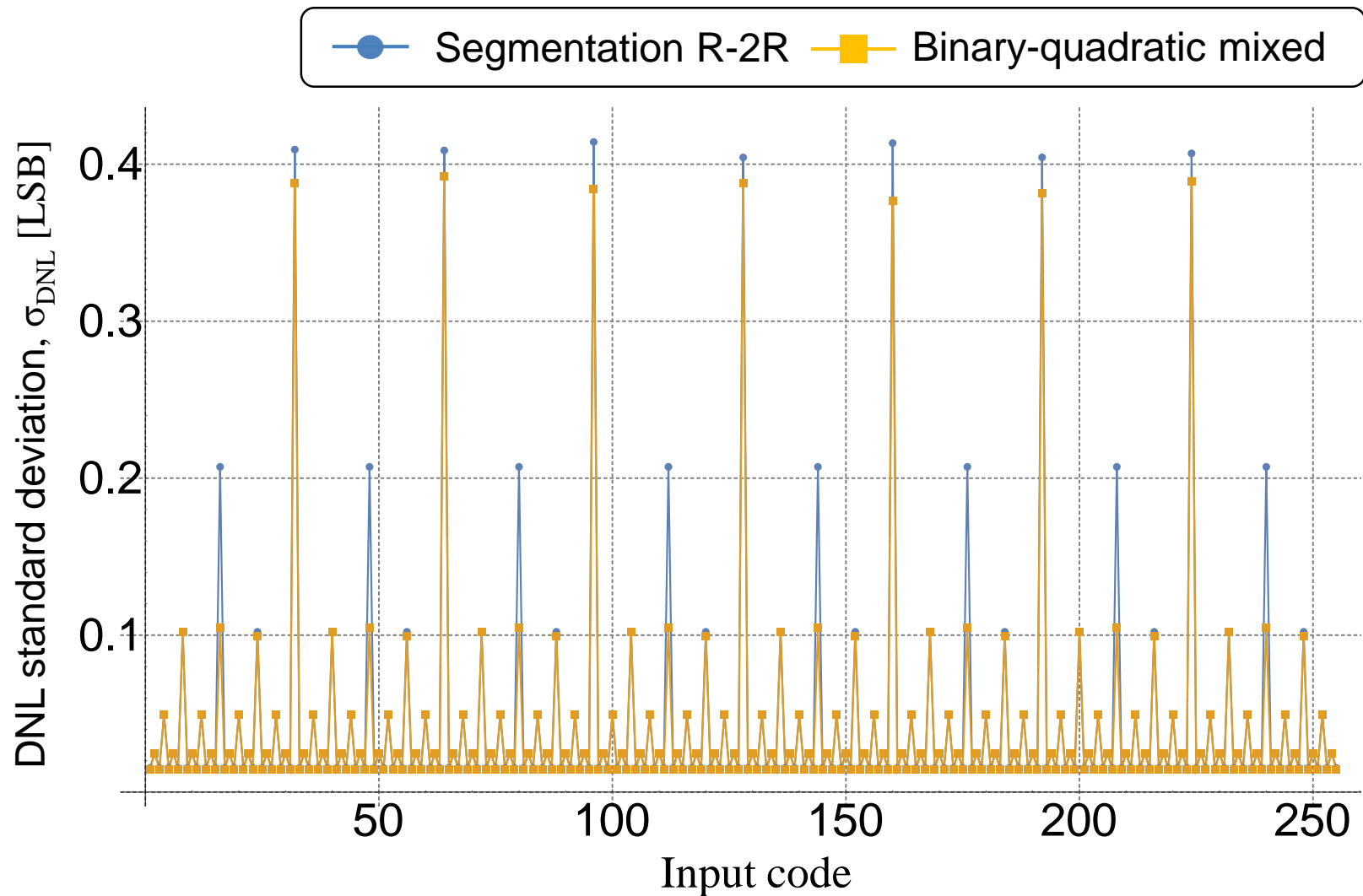
$$DNL(n) = \frac{V_{OUT}(n) - V_{OUT}(n - 1)}{V_{LSB}} - 1 \quad [\text{LSB}]$$

$V_{LSB}$  : 1 LSB voltage

- Simulation conditions

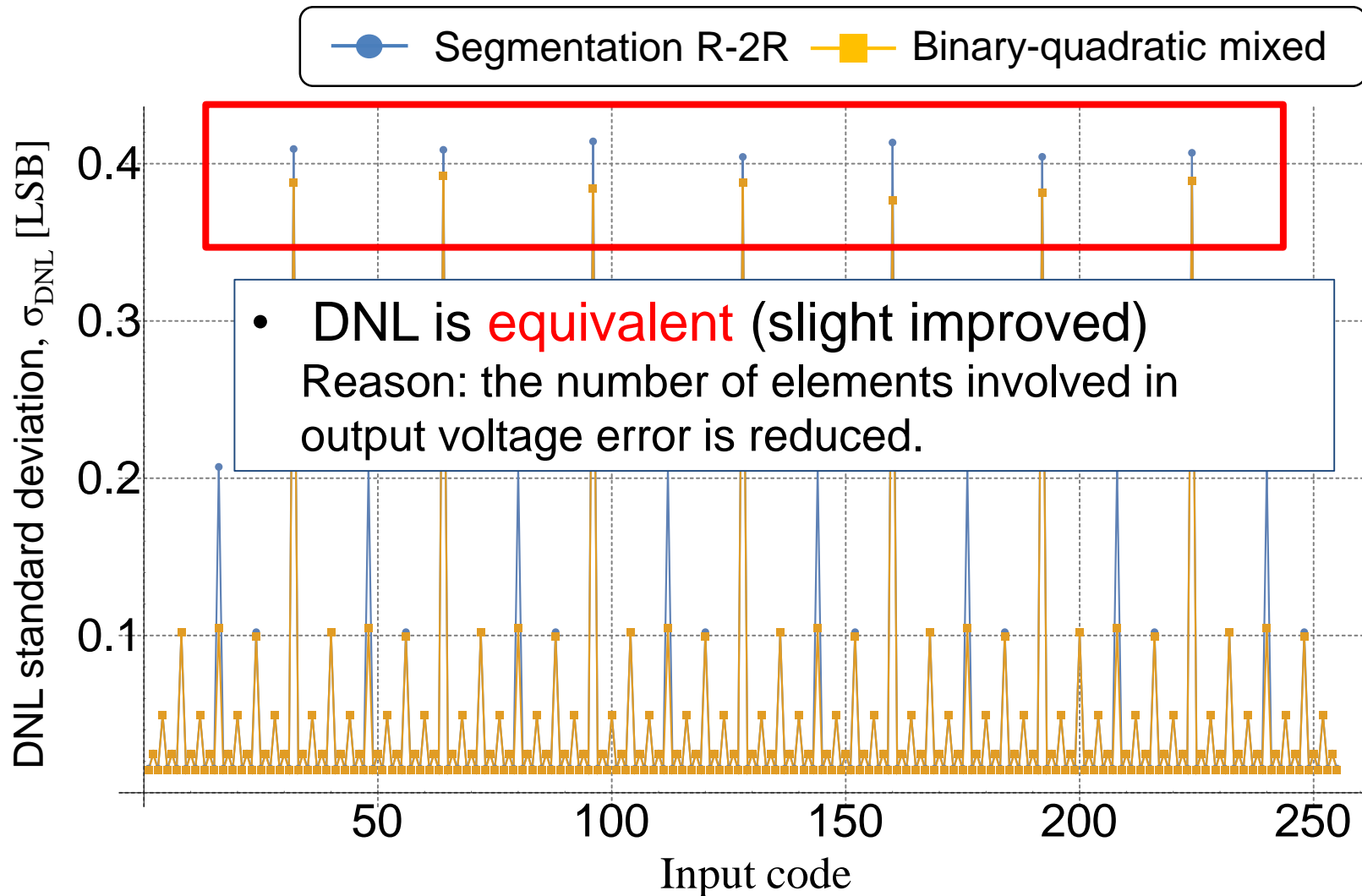
- Monte Carlo simulation assuming a linear circuit model consisting of resistors and current sources.
- Number of simulation sets : 3000
- Assume variations with Gaussian distribution in unit resistance  $R_u$  and unit current  $I$
- Standard deviation  $\sigma \Rightarrow 1\%$  of the mean value

# DNL Standard Deviation



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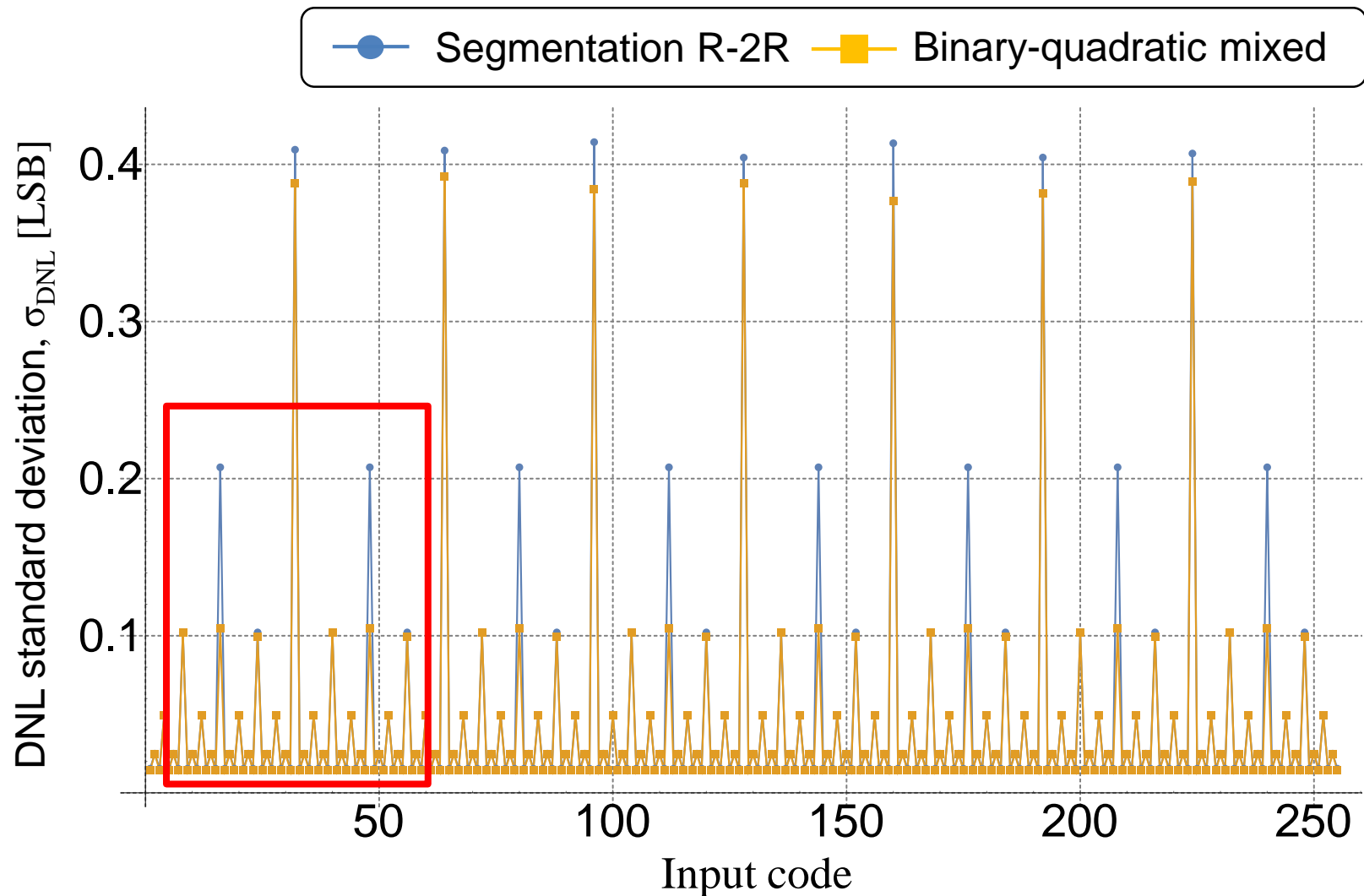
- Maximum DNL





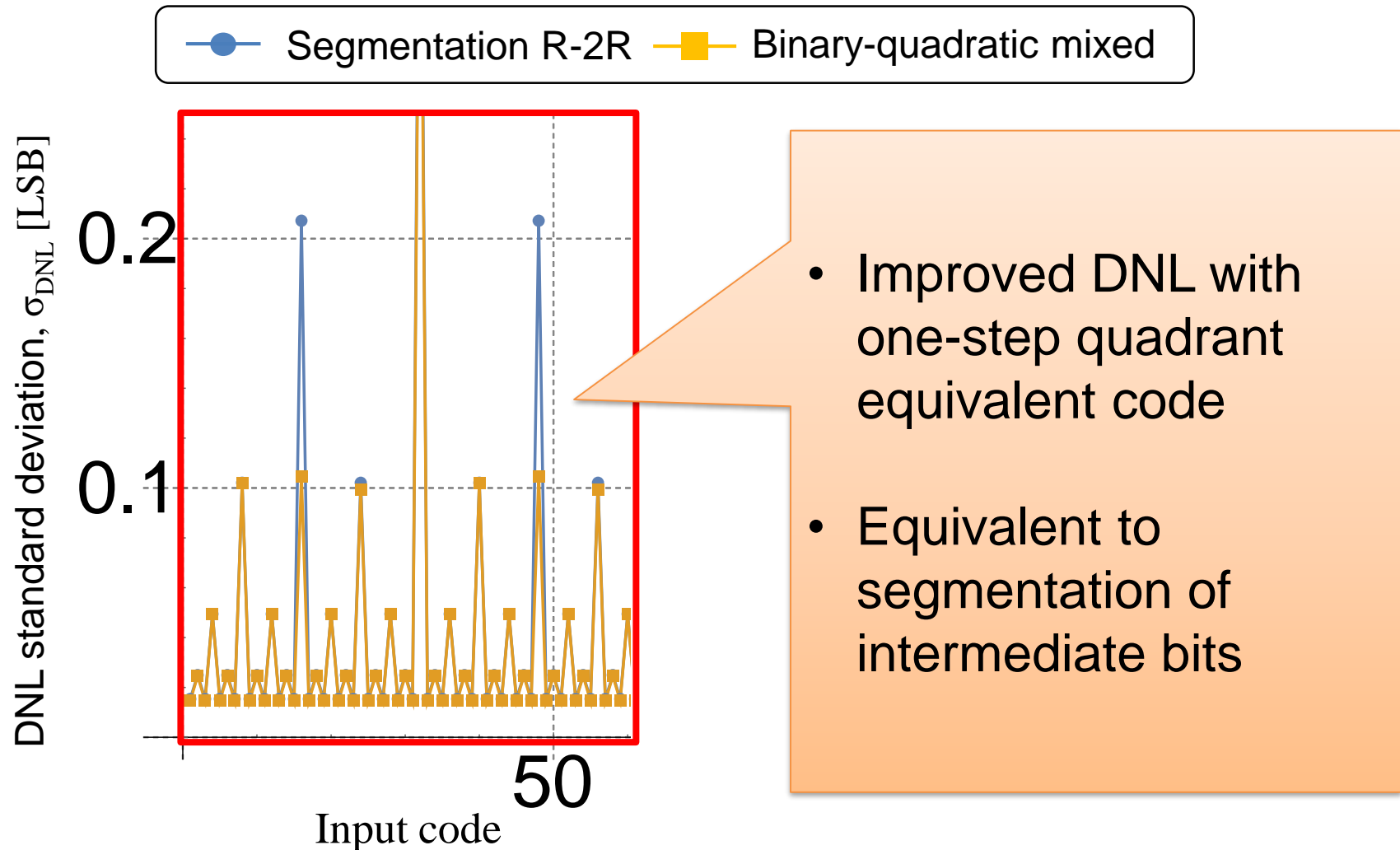
# DNL Standard Deviation

- DNL with codes driving quadratic part



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# Conclusion

- Summary
  - DACs with resistor ladders with non-uniform current sharing ratios
  - Segmentation between upper and lower bits is possible in a DAC using a resistor ladder
  - Binary-quaternary-unary connected resistor ladder DAC can increase gain with equivalent size and DNL to a conventional segmented R-2R DAC
- Challenges
  - Dynamic characteristics of the design should be evaluated including associated circuits

# Q&A

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- Q. ゲインの値は最大どこまで取れますか、また制限はありますか。  
消費電力や安定性、線形性の制限はありますか。
- A. 質問頂いた内容に関して、検討を行っていないため今後の検討課題と致します。