

フラクタルを用いた集積回路での 小チップ面積での抵抗実現法

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Resistor Implementation Algorithm with Small Chip Area based on Fractals
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Today in the realm of computer science and engineering, fractal geometry is a brand-new subject of study. It can be used in a variety of ways. Fractals in nature are so intricate and irregular that modeling them with traditional geometry objects is futile. The Fractal Geometry of Nature, by Benoit Mandelbrot, the father of fractal geometry, from his book published in 1982. This study examines numerous fractal topics, such as fractal dimension, various fractal generation techniques, their properties, and their applicability in real life.

キーワード：抵抗, 小チップ面積, フラクタル,
(Resistor, small chip area, fractals)

1. Introduction

Throughout my research year in Kobayashi Lab from Gunma University which is a lab that focus mainly on electronic circuits we have discovered that using the concept of fractals to design better circuits in small chips. Hence the implementation of resistor within small chip area are brought into discussion. The general concept of fractal albeit a little complex to comprehend agrees that this is the next step forward in our research in addition a significant notice in the study of electronic circuit analysis.

2. Fractal

A fractal is a form of infinitely complicated mathematical shape. In essence, a fractal is a pattern that repeats indefinitely, and every section of the Fractal, no matter how zoomed in or out, appears to be identical to the entire image. Fractals are used in a variety of fields, including economics, geography, medical imaging, and art. People believe that figures like as lines, circles, conic sections, polygons, spheres, and quadratic surfaces can be used to generate or explain natural objects. Natural objects that are irregular cannot be modeled using classical geometry. Gaston Julia, a French mathematician, studied the iteration process of a complex function extensively in 1918 and discovered the Julia set, which is a milestone in the field of fractal geometry. The

Julia set is the dividing line between the prisoner and escape sets. The prisoner set is made up of points that are inside the Mandelbrot set, whereas the escape set is made up of points that are outside the orbit of the Mandelbrot set. The Julia set is the location where the complex function's chaotic behavior happens.

3. Well-known Fractal

Fractals can now be constructed as mathematical formulas rather than finite shapes, which has the advantage of allowing users to fully investigate the implications of Fractal equations. Examining the whole concept of fractals, we manage to conclude that although the area is finite but the perimeter of a fractal is infinite. Take the Koch Curve fractal for an example when investigating the properties of said fractal can be proven to be a good solution for the study of small chips. Due to the small area in small chips the amount of resistors we assume that small resistors can only fit the space provided in integrated circuits. Although this may be the case the small area is compensated by the infinite perimeter which can result in a greater resistor for each use. To put in details, figure 1 shows how a fractal have limited area but infinite perimeter. At each step, the total length of the curve gets longer approaching infinity. The length of the curve increases the more closely you measure it.

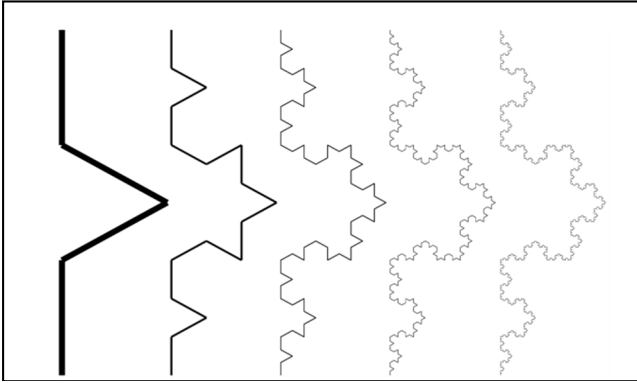


Fig.1. Koch Curve.

First, let's consider the lengths of the edges in order as shown in Fig. 2. If we begin with an equilateral triangle with side length 1, then the length of a side is:

$$S_n = \frac{S_{n-1}}{3} = \frac{1}{3^n}$$

For iterations 0 to 3, length = $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$.

For each iteration, one side of the figure from the previous stage becomes four sides in the following stage. Since we begin with three sides, the formula for the number of sides in the Koch Curve is:

$$N_n = N_{n-1} \cdot 4 = 3 \cdot 4^n$$

For iterations 0, 1, 2 and 3, the number of sides are 3, 12, 48 and 192, respectively.

Since all the sides in every iteration of the Koch Curve is the same the perimeter is simply the number of sides multiplied by the length of a side.

$$P_n = N_n \cdot S_n = 3 \cdot \left(\frac{4}{3}\right)^n$$

for the first 4 iterations (0 to 3) the perimeter is $1, \frac{4}{3}, \frac{16}{9}, \frac{64}{27}$.

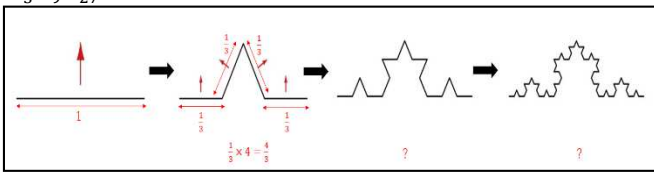


Fig.2. The iterations for each creation of new and smaller triangles.

Next, we will discuss about the area of Koch Curve using the equations and Fig. 3. In each iteration a new triangle is added on each side of the previous iteration, so the number of new triangles added in iteration n is:

$$T_n = T_{n-1} = 3 \cdot 4^{n-1} = \frac{3}{4} \cdot 4^n$$

The area of each new triangle added in an iteration is $\frac{1}{9}$ of the area of each triangle added in the previous

iteration, so the area of each triangle added in iteration n is:

$$a_n = \frac{a_{n-1}}{9} = \frac{a_0}{9^n}$$

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The area of each new triangle added in an iteration is $\frac{1}{9}$ of the area of each triangle added in the previous iteration, so the area of each triangle added in iteration n is:

$$a_n = \frac{a_{n-1}}{9} = \frac{a_0}{9^n}$$

where a_0 is the area of the original triangle. The total new area added in iteration n is therefore:

$$b_n = T_n \cdot a_n = \frac{3}{4} \cdot \left(\frac{4}{9}\right)^n \cdot a_0$$

The total area of the curve after n iterations is:

$$\begin{aligned} A_n &= a_0 + \sum_{k=1}^n b_k = a_0 \left(1 + \frac{3}{4} \sum_{k=1}^n \left(\frac{4}{9}\right)^k\right) \\ &= a_0 \left(1 + \frac{1}{3} \sum_{k=0}^{n-1} \left(\frac{4}{9}\right)^k\right) \\ &= a_0 \left(1 + \frac{3}{5} \left(1 - \left(\frac{4}{9}\right)^n\right)\right) \\ &= \frac{a_0}{5} \left(8 - 3 \left(\frac{4}{9}\right)^n\right) \end{aligned}$$

If the length of one side of the first triangle is 1, $A_0 = \frac{\sqrt{3}}{4}$, then, $A_n = \frac{1\sqrt{3}}{5 \cdot 4} \left(8 - 3 \left(\frac{4}{9}\right)^n\right)$.

For iterations 0, 1, 2 and 3, the total areas are $\frac{\sqrt{3}}{4}$, $\frac{\sqrt{3}}{3}$, $\frac{10\sqrt{3}}{27}$ and $\frac{94\sqrt{3}}{243}$, respectively.

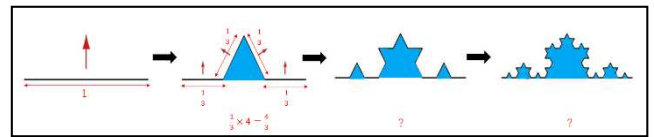


Fig. 3. Area of New Triangles.

When iteration n is set to infinity, the perimeter becomes infinite and the total area converges to $\frac{2\sqrt{3}}{5}$. Thus, the characteristic of a fractal is that the length of the sides can be infinite within a limited area.

To highlight that exactly on how area works in fractals using computer animation is one of the way to clearly see the beauty behind fractals. Using turtle module based from Python Programming we have been able to create magnificent images of fractals.

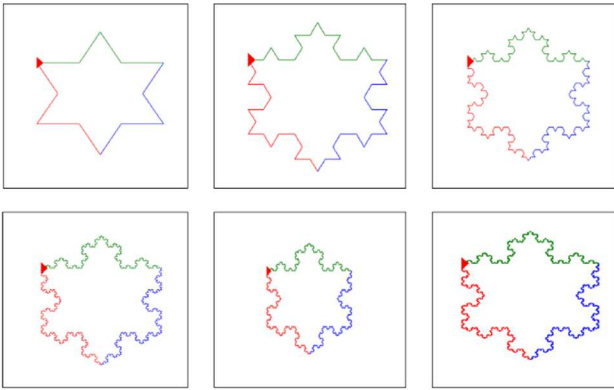


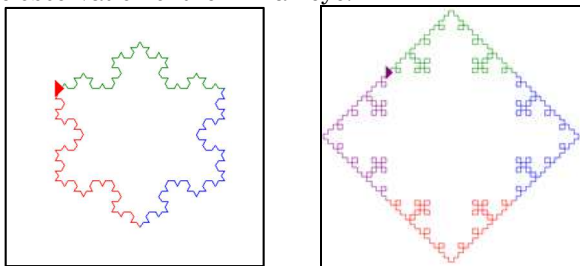
Fig. 4. Python Animation of Koch Curve.

As shown in Fig. 4 a snowflake images are formed. The image is from the first iterations of triangles to the sixth iterations. With close observations at each iteration a lot more of smaller triangles are formed creating longer lines hence proving that with more iterations within the small area provided perimeter of the Koch Curve increases significantly. The final iteration looks the same as the previous iteration but only thicker in lines as the naked eyes can only process that much of information. Producing more iterations does not improve the shape itself because the lines are getting only thicker due to the huge amount of small shapes taking process compiled into the limited area.

4. New fractal

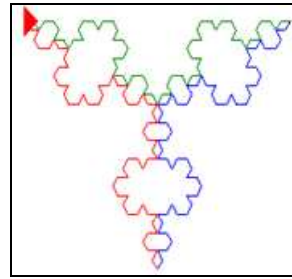
Fig. 5. shows a few example of my testing the outcomes of fractal imaging for the third iteration for each shape. No. 1 is the Koch Curve introduced in the previous chapter. In the same way, we developed new fractals from No.2 to No.11. They are also named based on their shapes. When new shapes are generated facing inward, we gave it the name "reverse".

Although some fractals have area inside them but another cases of fractals also exist where the inside of the shape developed smaller area until none can be seen by the observation of the human eye.

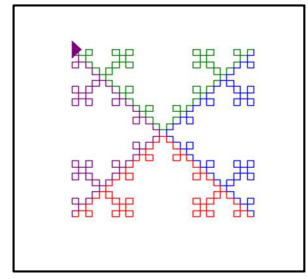


(No.1) Triangle

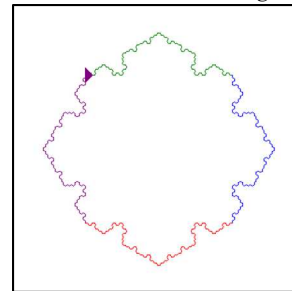
(No. 2) Square



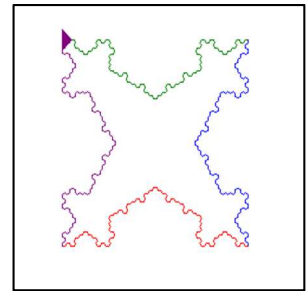
(No. 3) Reverse Triangle



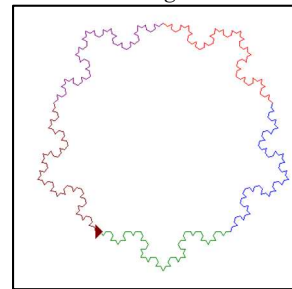
(No. 4) Reverse Square



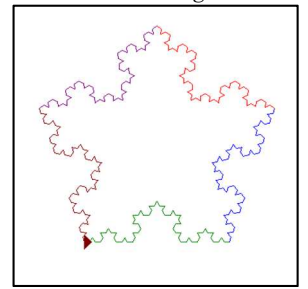
(No. 5) Square-Half-Hexagon



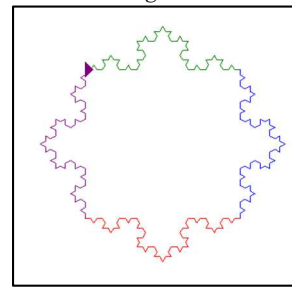
(No. 6) Reverse Square-Half-Hexagon



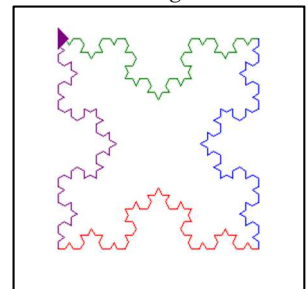
(No. 7) Pentagon-Triangle



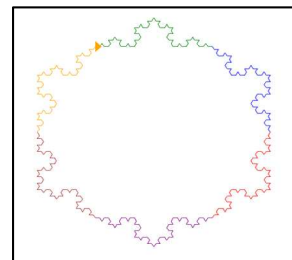
(No. 8) Reverse Pentagon-Triangle



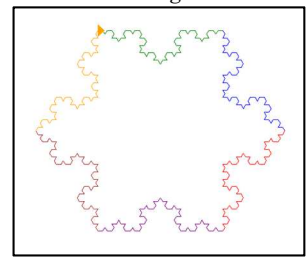
(No. 9) Square-Triangle



(No.10) Reverse Square-Triangle



(No.11) Hexagon-Triangle



(No.12) Reverse Hexagon-Triangle

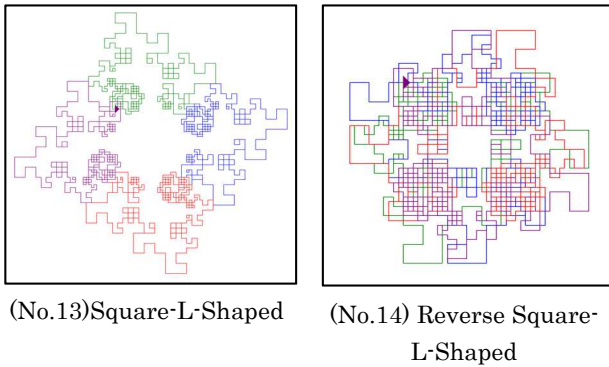


Fig. 5. Python Animations of New Fractals.

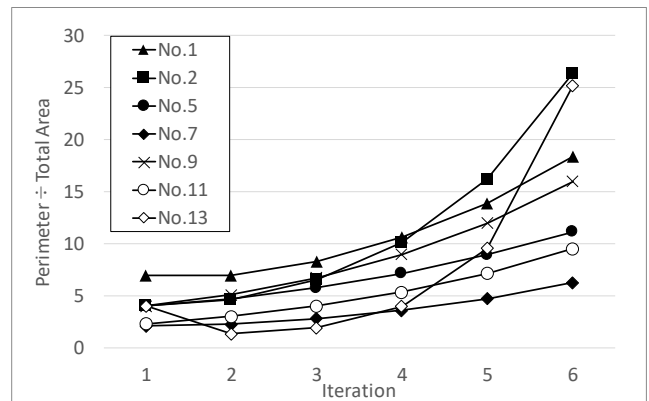
In the same way as in the previous chapter, We derived equations for the length and area of the perimeters of each fractal, and the length is divided by the area to compare the 14 fractals quantitatively. Since the shapes of No. 13 and No. 14 were too complicated, approximate values were used for the area when the iteration was more than two. Fig. 6. shows the result. We divided the results into reverse and non-reverse because there are a lot of fractals to compare once. The horizontal axis is the number of iterations. If the larger values, longer perimeters can be achieved with a limited area. The largest one is No.4, but this is due to its near-zero area, so it could be a problem when actually making the chip.

5. Conclusion

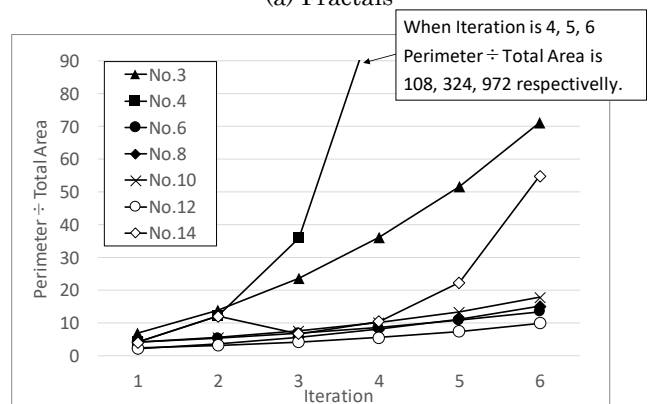
Within our discovery we will be implementing the resistor value based on fractal concept during this development of our awareness regarding this theme. Comparing the previous triangled-shaped fractals that we have created with Python we were able to create plethora of choices of shapes to display areas within fractals that are crucial for space consuming theories. The theory provided within this particular research showed that based on the gist concept of fractal providing a progress in the electronic circuit output value when tempering with resistor value aside from the finite areas compiled within small chips. We tried a few more types of shapes of fractals troubleshooting the small area circumstances resulting in the addition of resistor value due to large perimeter gain.

Acknowledgment

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(a) Fractals



(b) Reverse Fractals

Fig. 6. Perimeter divided by Total Area.

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