15 May 2022 (Sun)

Invited

Recent Innovation of Waveform Acquisition Methods: Residue Sampling and Metallic Ratio Sampling

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Self-Introduction

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Analog/Mixed-Signal IC Design and Test

B.S. from U. Tokyo, Information Physics

M.S. from U. Tokyo, Information Physics

M.S. from UCLA, Electrical Engineering

Ph.D. from Waseda U. Electrical Engineering

OUTLINE

- Introduction
- Residue Sampling
- Metallic Ratio Sampling
- Conclusion

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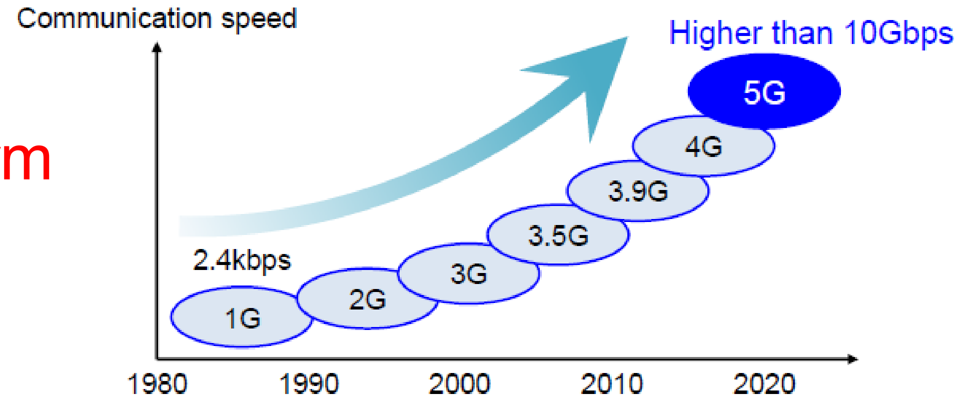
Research Motivation

Next Generation Communication System "5G"

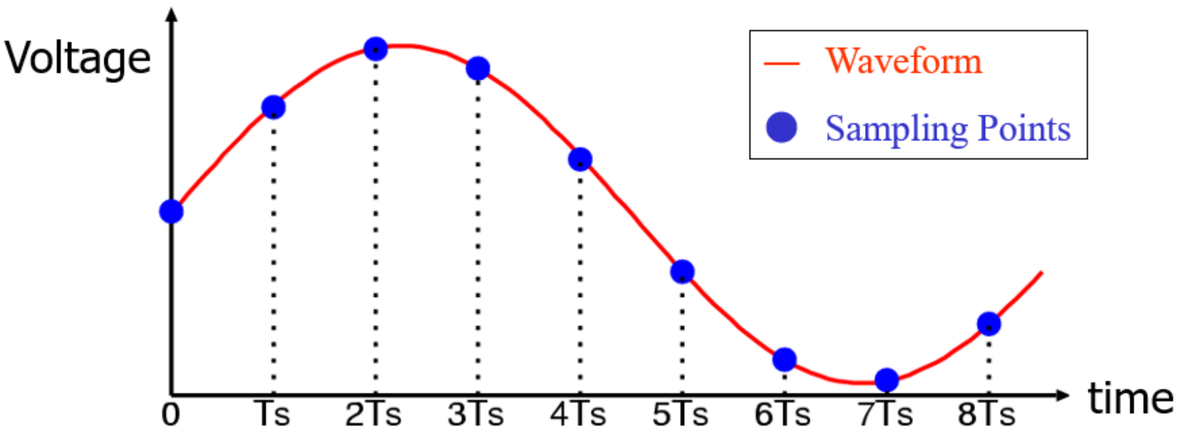
High frequencies
in communication systems

Electronic components
for high frequency bands

Their testing technology
"High frequency waveform sampling"
should be developed



Sampling for Waveform Acquisition

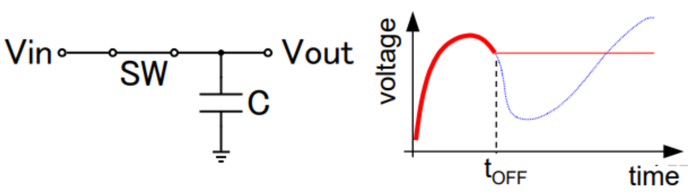


Track/Hold Circuit

- SW : ON

  Vout(t) = Vin(t)

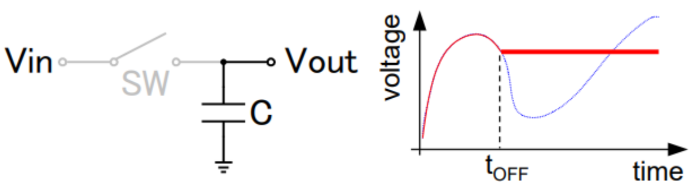
  Track mode



- SW : OFF

  Vout(t) = Vin(t_{OFF})

  Hold mode



Varieties of Sampling Technologies

Keywords:

Track/Hold Circuit
Anti-Aliasing Filter
Sampling Theorem
Spectrum Folding
Oversampling
Equivalent-Time Sampling
Coherent Sampling
Frequency Conversion by Sampling
Quadrature Sampling
Sampling Clock Jitter
Finite Aperture Time

New Concepts: Residue Sampling
Metallic Ratio Sampling
Based on Number Theorem

OUTLINE

- Introduction
- <u>Residue Sampling</u>
- Metallic Ratio Sampling
- Conclusion
- Appendix

[1] S. Katayama, H. Kobayashi, et. al., "Application of Residue Sampling to RF/AMS Device Testing“, 30th IEEE Asian Test Symposium (Nov. 2021)

[2] Y. Abe, H. Kobayashi, et. al., "Frequency Estimation Sampling Circuit Using Analog Hilbert Filter and Residue Number System", 13th IEEE International Conference on ASIC (Oct. 2019)

Research Goal

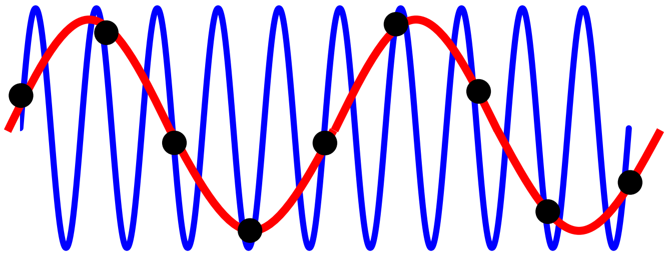
Estimate high-frequency input signal
with multiple low-frequency clock sampling circuits

High-frequency sampling circuit is difficult to realize

Our Approach :

Sampling high frequency signal with multiple low frequency clocks

Use Aliasing proactively



Analog Hilbert filter and residue number system

Chinese Remainder Theorem

Sun Tzu

孫子算經卷上

唐朝議大夫行太史令上輕車都尉臣李淳風等奉 勅注釋

度之所起起於忽欲知其忽蠶吐絲為忽十忽

為一絲十絲為一毫十毫為一釐十釐為一分

十分為一寸十寸為一尺十尺為一丈十丈為

一引五十尺為一端四十尺為一疋六尺為一

步二百四十步為一畝三百步為一里

稱之所起起於黍十黍為一絫十絫為一銖二

十四銖為一兩十六兩為一斤三十斤為一鈞

Sun Tzu calculation

Chinese arithmetic book
'Sun Tzu calculation'

孫子算経

"When dividing by 3, its residue is 2,
dividing by 5, its residue is 3,
dividing by 7,its residue is 2.
What is the original number ?"

Answer 23

Generalization

Chinese Remainder Theorem

How to use Chinese remainder theorem

He quickly found out how many soldiers were.

Sun Tzu

“Divide by 3.”

Remainder: 2

...

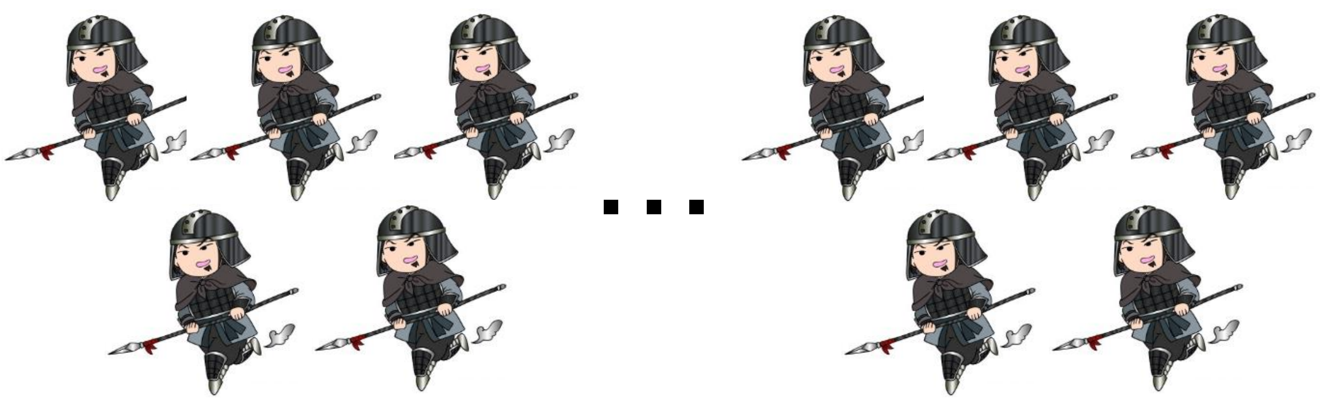
How to use Chinese remainder theorem

He quickly found out how many soldiers were.

Sun Tzu

“Divide by 5.”

Remainder: 3



How to use Chinese remainder theorem

He quickly found out how many soldiers were.

“Divide by 7.”

“There are 23 soldiers in all.”

Sun Tzu

Remainder: 2

Example of Residue Number System

23 % 3 = 2,   23 % 5 = 3,   23 % 7 = 2

- Natural numbers
  3, 5, 7 (relatively prime)
  N=3×5×7=105
- k (0 <= k <= N-1 (=104))

a : Remainder of k dividing by 3   a=mod3(k)
b : Remainder of k dividing by 5   b=mod5(k)
c : Remainder of k dividing by 7   c=mod7(k)

k ⟷ (a, b, c)

one to one

Chinese remainder theorem

a	b	c	k
0	0	1	15
1	1	2	16
2	2	3	17
0	3	4	18
1	4	5	19
2	0	6	20
0	1	0	21
1	2	1	22
2	**3**	**2**	**23**
0	4	3	24
1	0	4	25
2	1	5	26
0	2	6	27
1	3	0	28
2	4	1	29

Residue number system

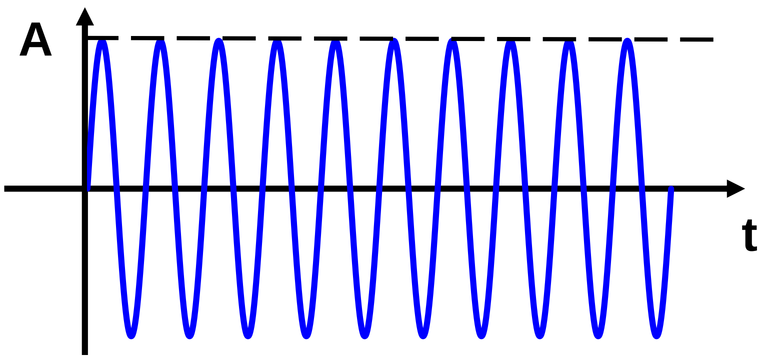
Aliasing Phenomenon

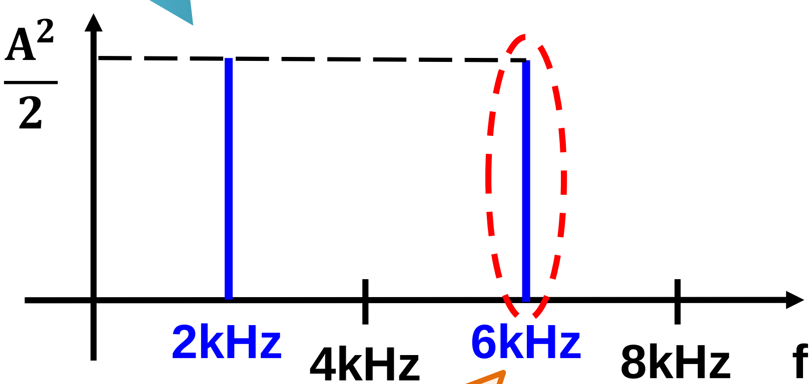
FFT

Sampling frequency : 8 kHz

Spectrum is folded
within the sampling frequency band
(sampling theorem)

Waveform frequency : 30kHz

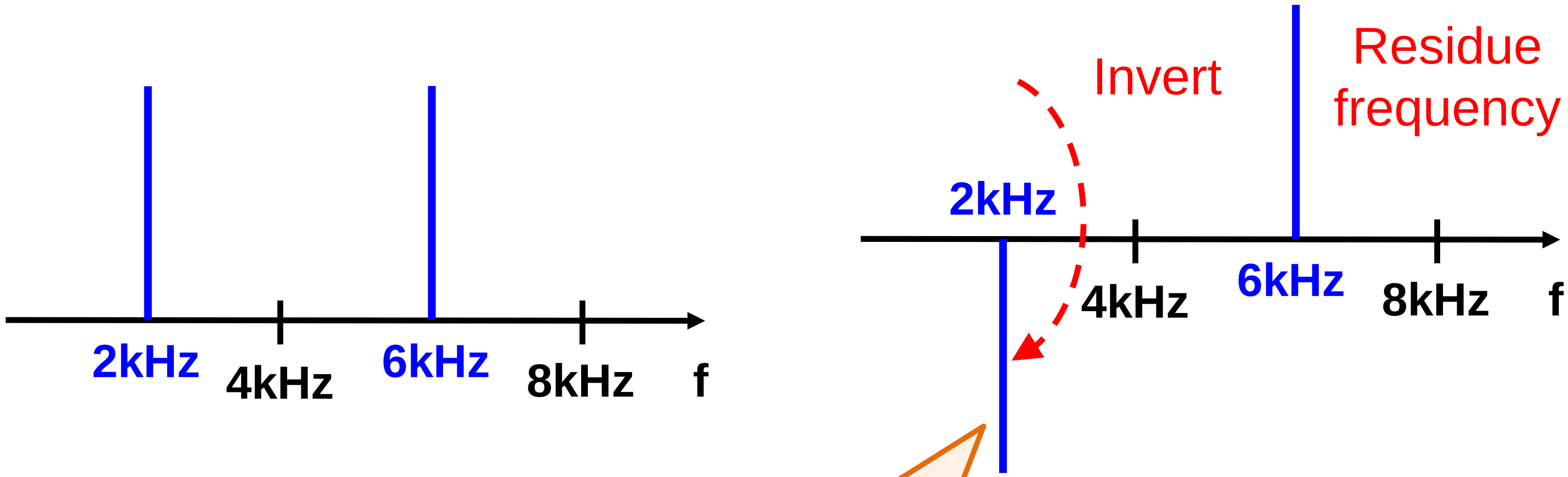




Residue frequency
(6 is the remainder of 30 divided by 8)

Complex FFT of $j \times \sin(2\pi f_{in} t)$

Complex FFT
Input frequency : 30 kHz
Sampling frequency : 8 kHz



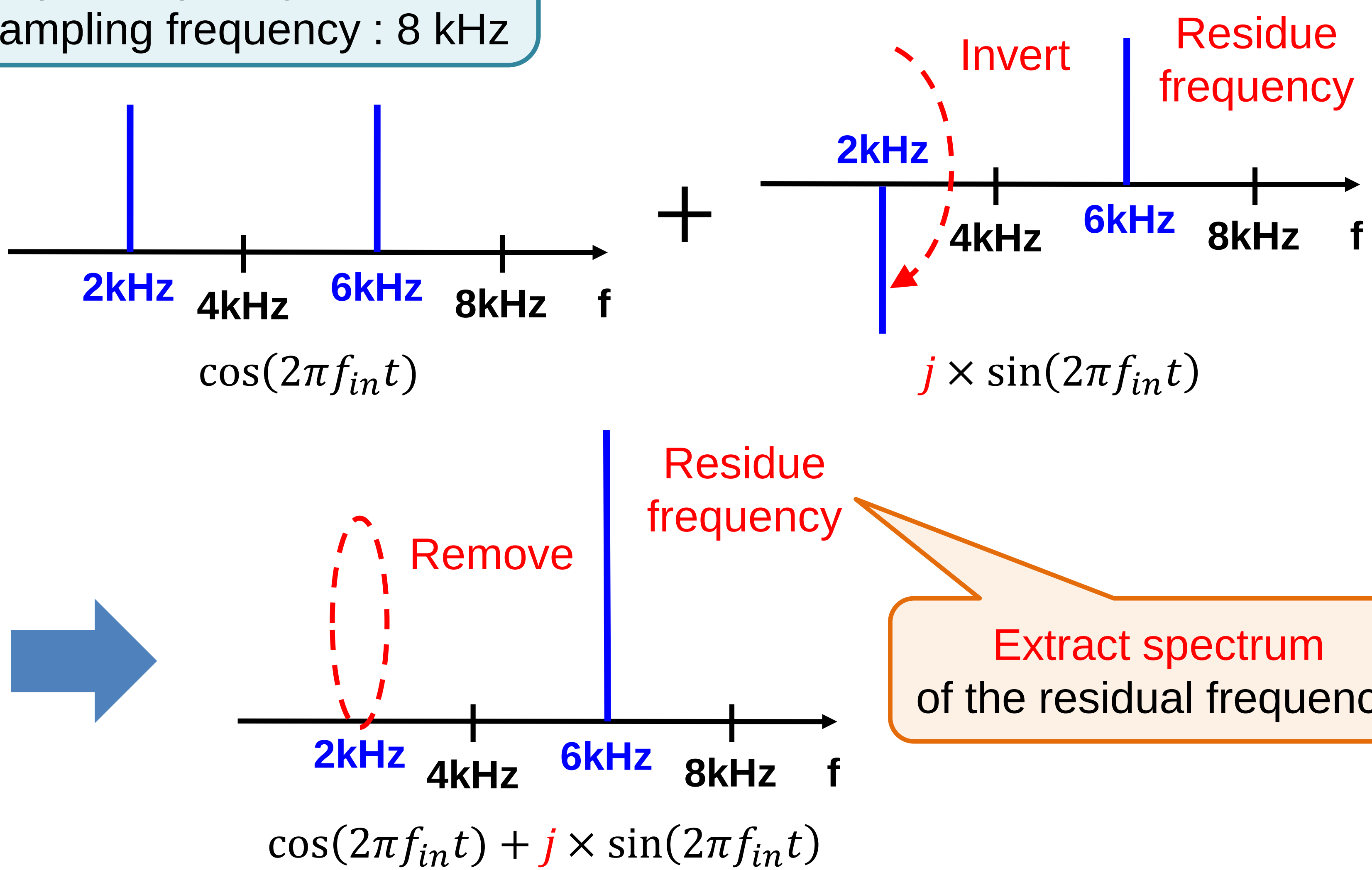
$\cos(2\pi f_{in} t)$

$j \times \sin(2\pi f_{in} t)$

Inverted spectrum
anti-symmetric at Nyquist frequency

Complex FFT of $\cos(2\pi f_{in} t) + j \times \sin(2\pi f_{in} t)$

Complex FFT
Input frequency : 30 kHz
Sampling frequency : 8 kHz



Extract spectrum
of the residual frequency

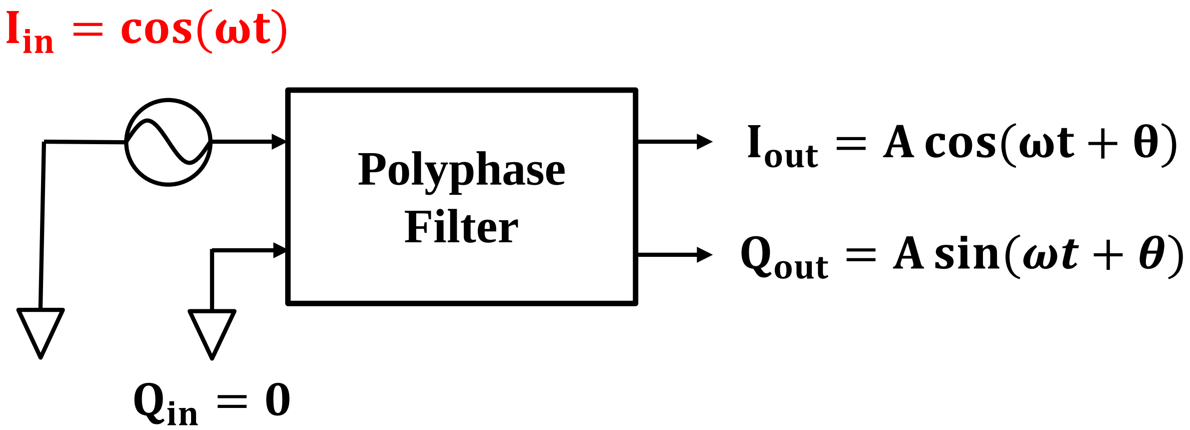
How to Generate $j \times \sin(2\pi f_{in} t)$

Use Analog Hilbert filter

↓

RC polyphase filter

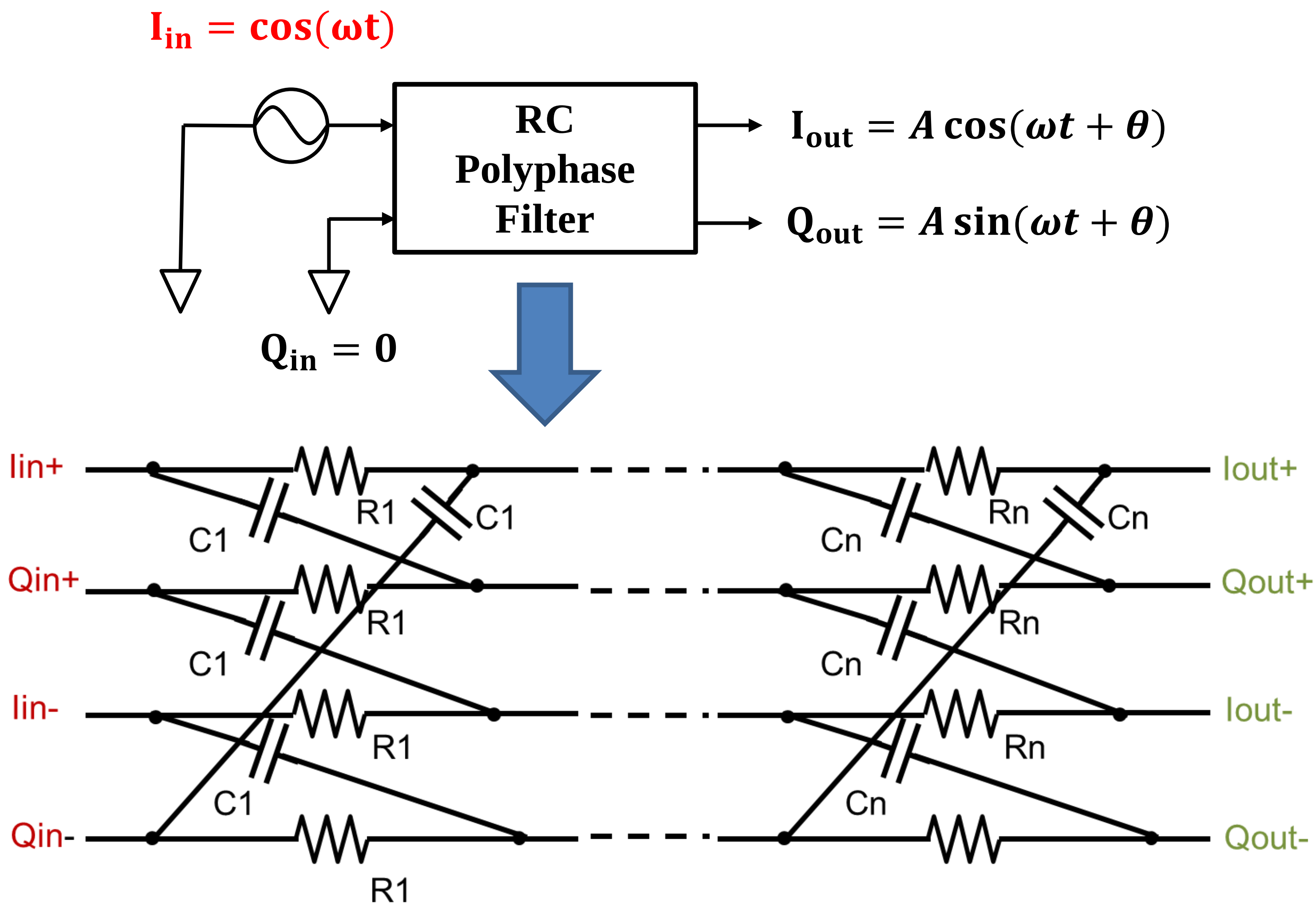
David Hilbert
(German mathematician)
1862-1943



Generate in-phase and quadrature waves from a single cosine wave

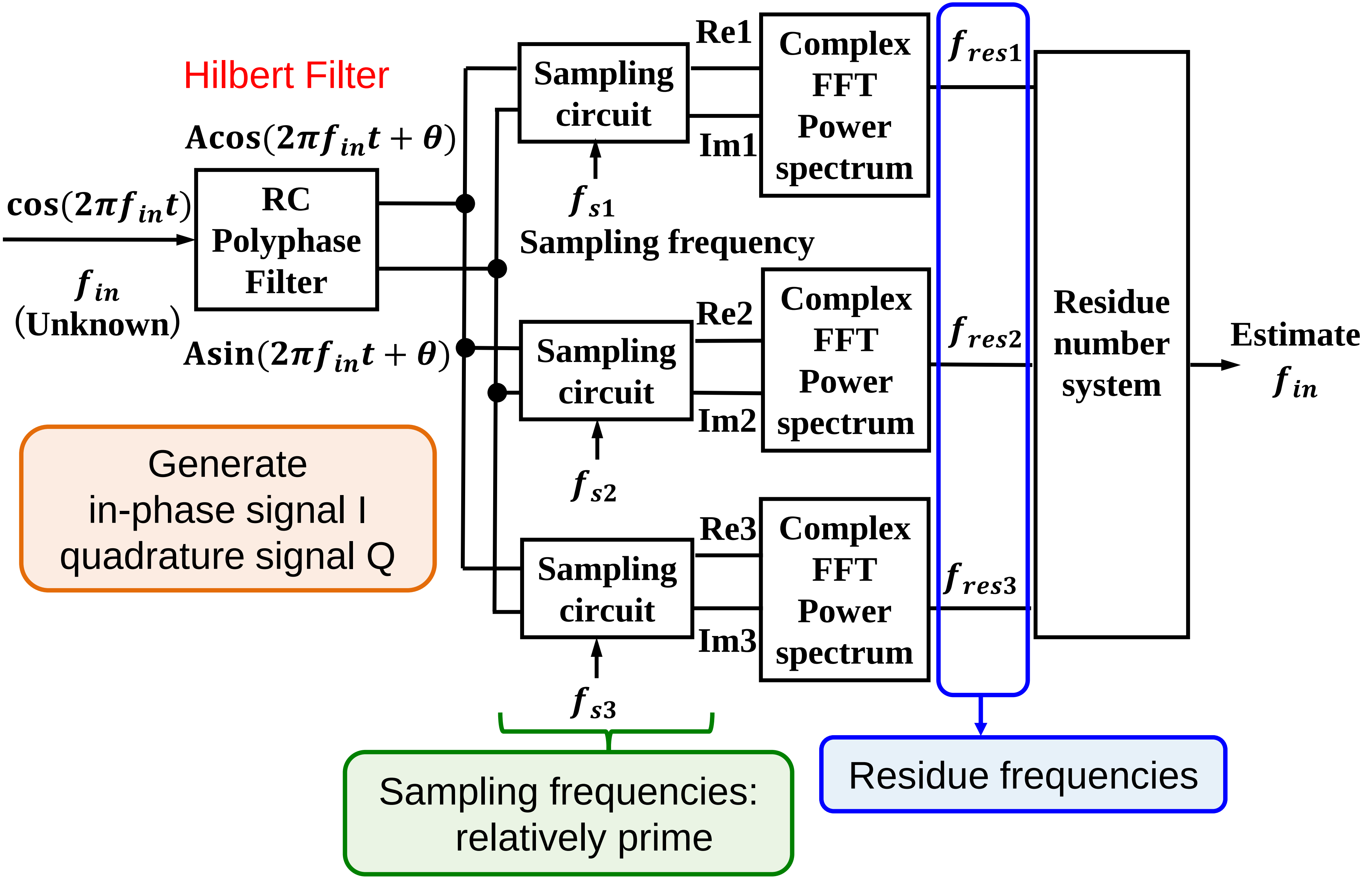
[3] Y. Tamura, K. Asami, H. Kobayashi, et. al.,
"RC Polyphase Filter as Complex Analog Hilbert Filter", IEEE ICSICT (Oct. 2016)

RC Polyphase Filter



Passive analog bandstop filter

Proposed Sampling Circuit



Simulation Setting

Complex FFT

- Input frequency : 12 GHz
- Frequency resolution : 1 kHz
- Sampling frequency : 229 kHz, 233 kHz, 239 kHz
  (Relatively prime)
- Range of measurement : 0~2080622 kHz
  (Note: 229 × 233 × 239 = 2080623)

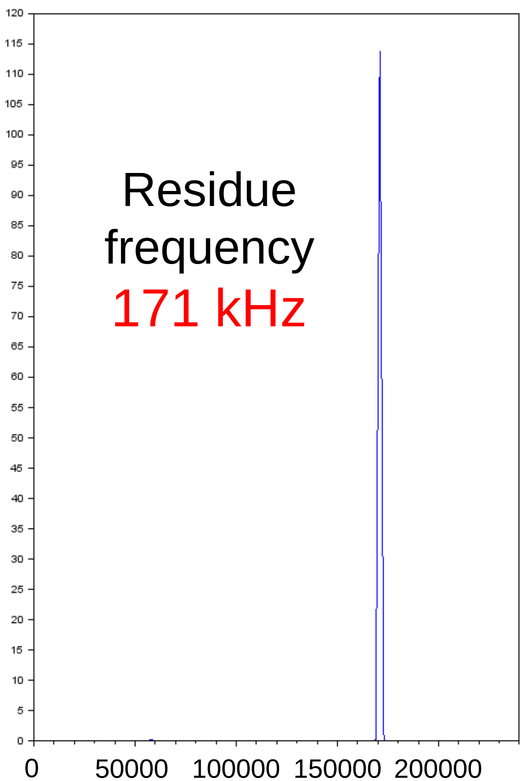
Measurement at 20 GHz
using sampling frequencies of ≒ 200 kHz

Simulation Results

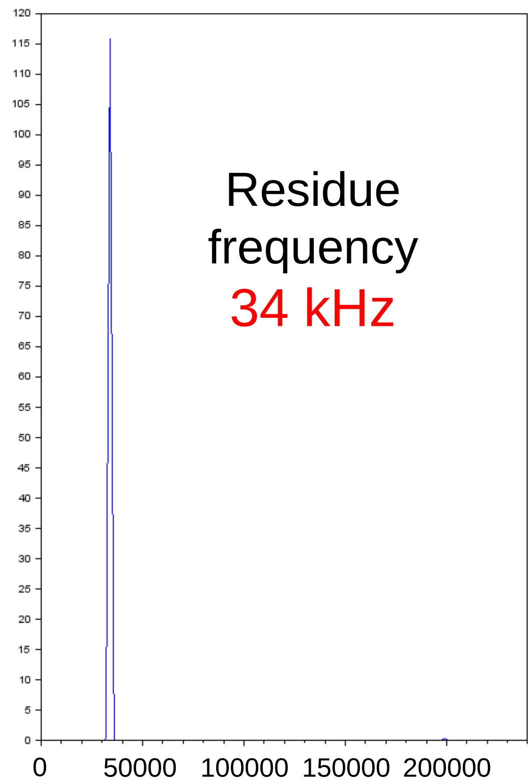
Complex FFT : $\cos(2\pi f_{in} t) + j \times \sin(2\pi f_{in} t)$

- Input frequency : 12 GHz
- Frequency resolution : 1 kHz
- Sampling frequency : 229 kHz 233 kHz 239 kHz

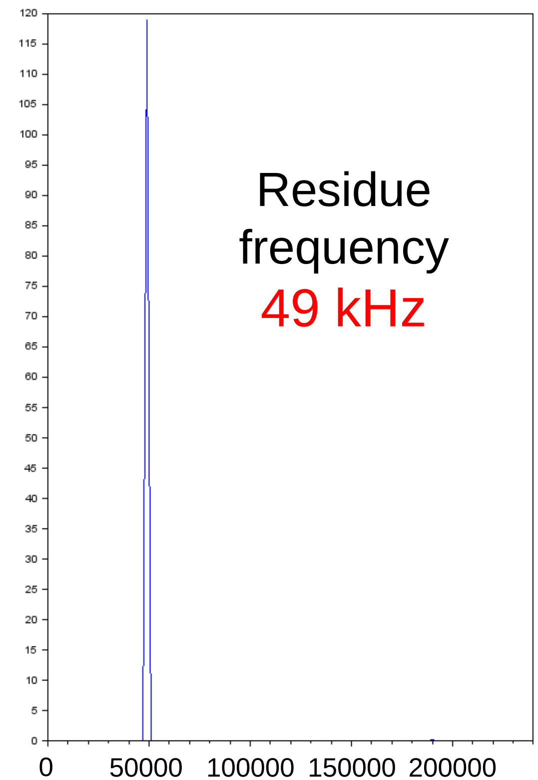
229 kHz Sampling



233 kHz Sampling



239 kHz Sampling



Frequency Estimation by Residue Number System

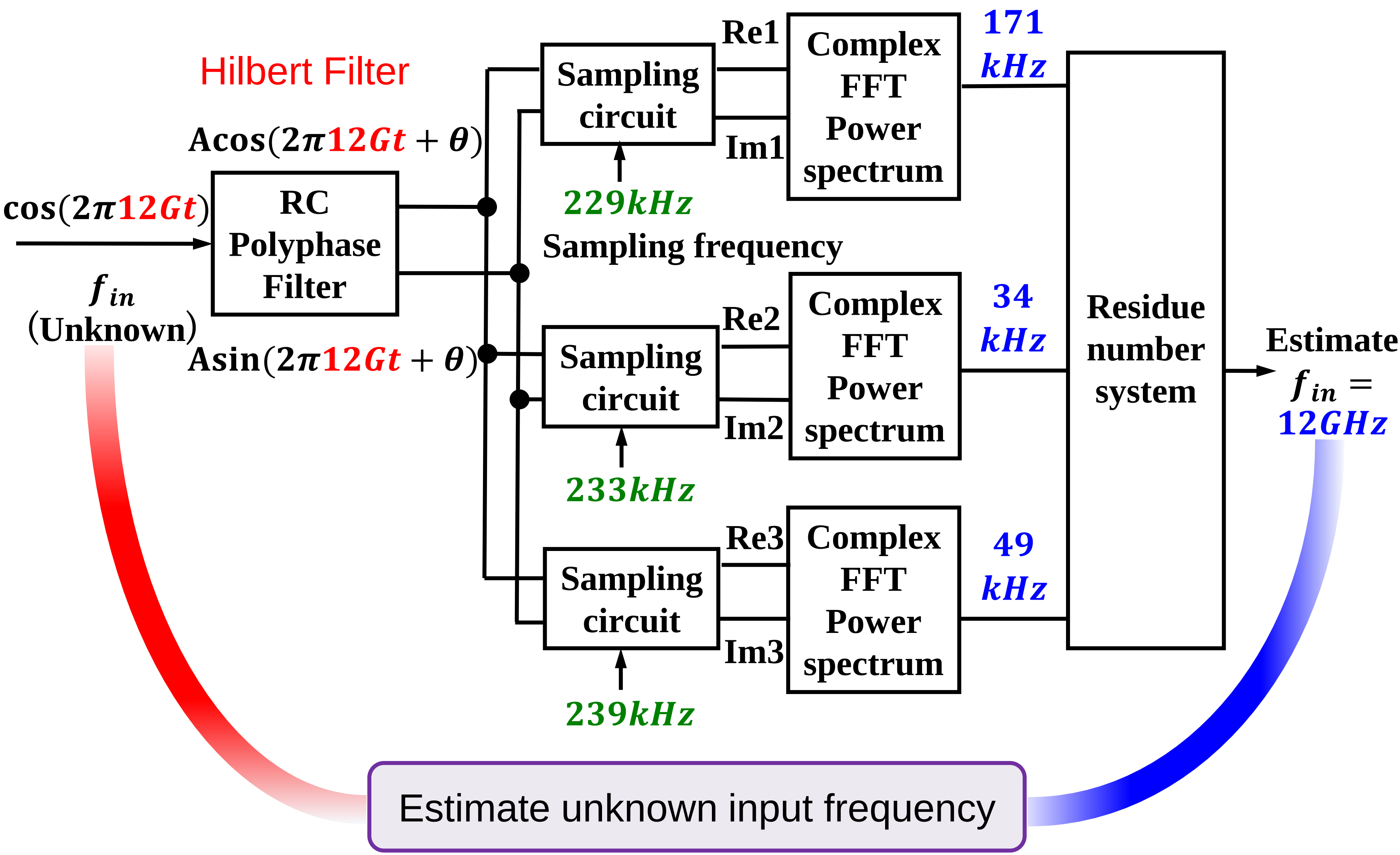
Residue frequencies
171 kHz, 34 kHz, 49 kHz

Input frequency estimation using residue frequencies and residue number system

Estimate input frequency 12GHz

a [kHz]	b [kHz]	c [kHz]	k [kHz]
0	0	0	0
1	1	1	1
2	2	2	2
⋮	⋮	⋮	⋮
169	[illegible]	47	11999998
170	33	48	11999999
171	34	49	12000000
172	35	50	12000001
173	36	51	12000002
⋮	⋮	⋮	⋮
226	230	[illegible]	12752320
227	231	237	12752321
228	232	238	12752322

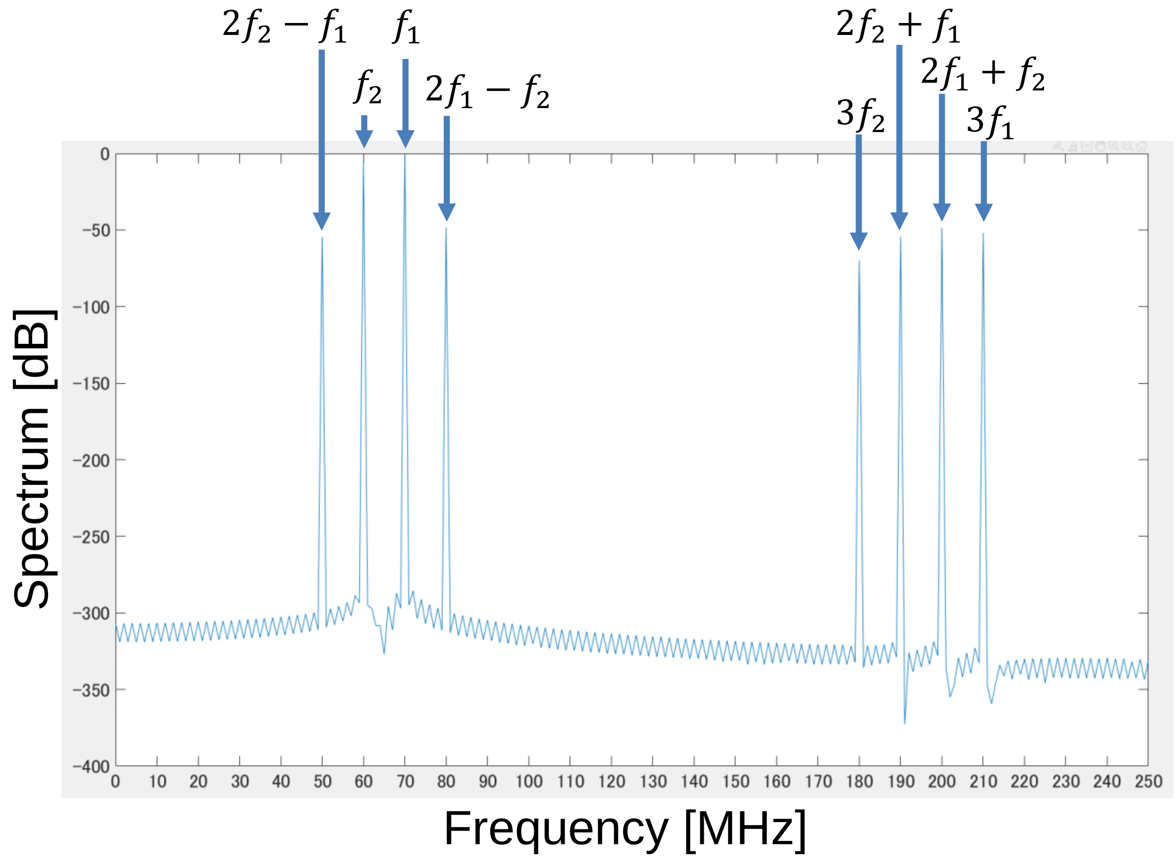
Simulation Result Overview



Two-Tone Test by Residue Sampling

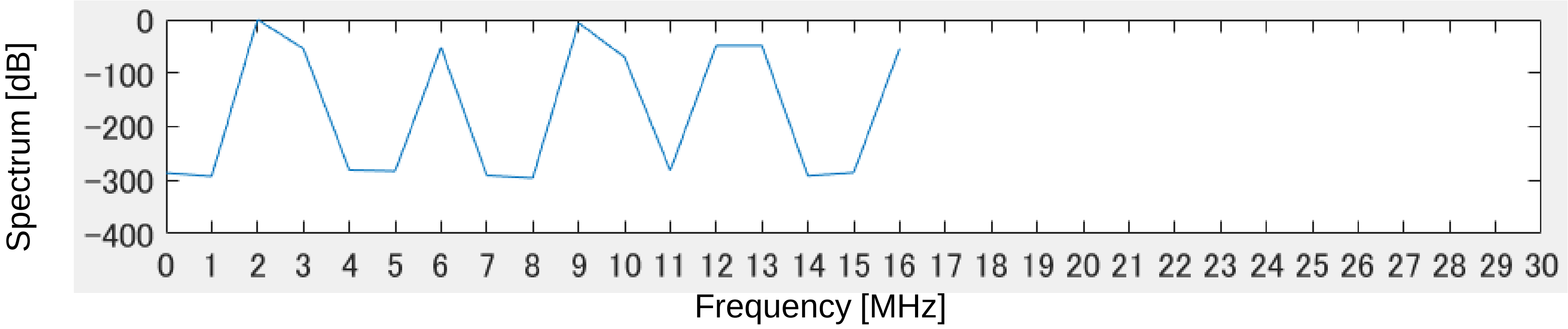
Input: $x(t) = \cos(2\pi f_1 t) + 0.5\cos(2\pi f_2 t), f_1 = 70\text{ MHz}, f_2 = 60\text{ MHz}$

Output: $y(t) = x(t) - 0.01\, x(t)^3$



Two-Tone Test Simulation ($f_{s1} = 17$ MHz)

Sampling Frequency: $f_{s1} = 17$ MHz

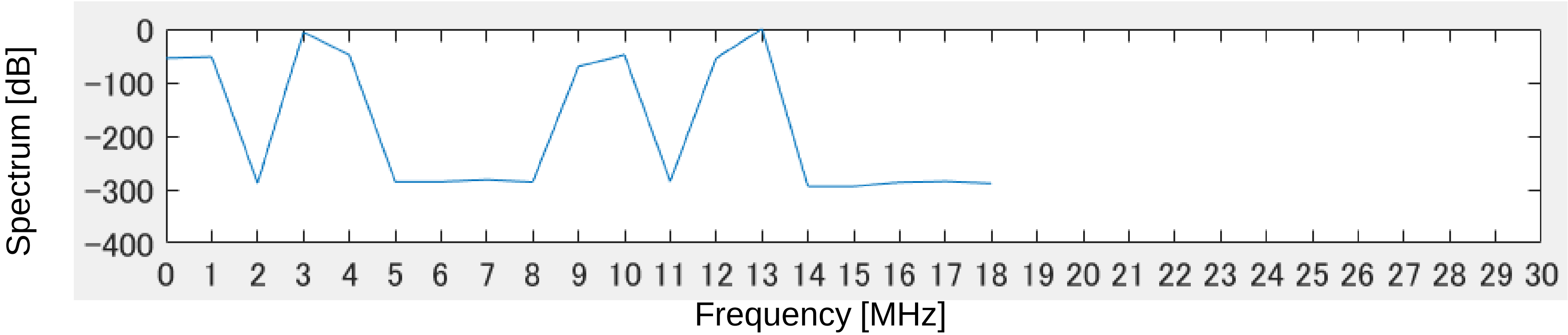


Theory			**Simulation**	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	2	0.00
f_2	60	-6.07	9	-6.07
$3f_1$	210	-51.9	6	-51.9
$3f_2$	180	-70.0	10	-70.0

Theory			**Simulation**	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	12	-48.4
$2f_2 - f_1$	50	-54.4	16	-54.4
$2f_1 + f_2$	200	-48.4	13	-48.4
$2f_2 + f_1$	190	-54.4	3	-54.4

Two-Tone Test Simulation $(f_{s2} = 19\ \mathrm{MHz})$

Sampling Frequency: $f_{s2} = 19\ \mathrm{MHz}$

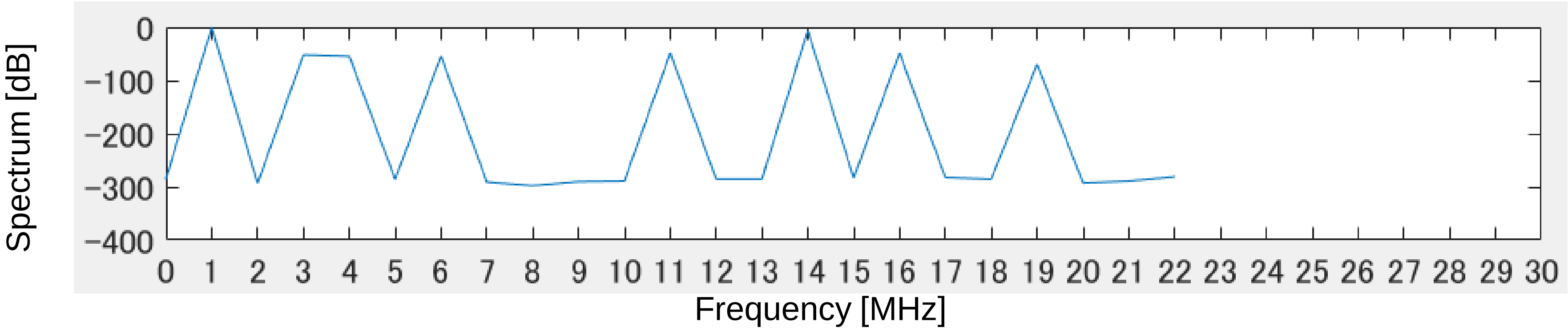


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	13	0.00
f_2	60	-6.07	3	-6.07
$3f_1$	210	-51.9	1	-51.9
$3f_2$	180	-70.0	9	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	4	-48.4
$2f_2 - f_1$	50	-54.4	12	-54.4
$2f_1 + f_2$	200	-48.4	10	-48.4
$2f_2 + f_1$	190	-54.4	0	-54.4

Two-Tone Test Simulation ($f_{s3} = 23\ \mathrm{MHz}$)

Sampling Frequency: $f_{s3} = 23\ \mathrm{MHz}$

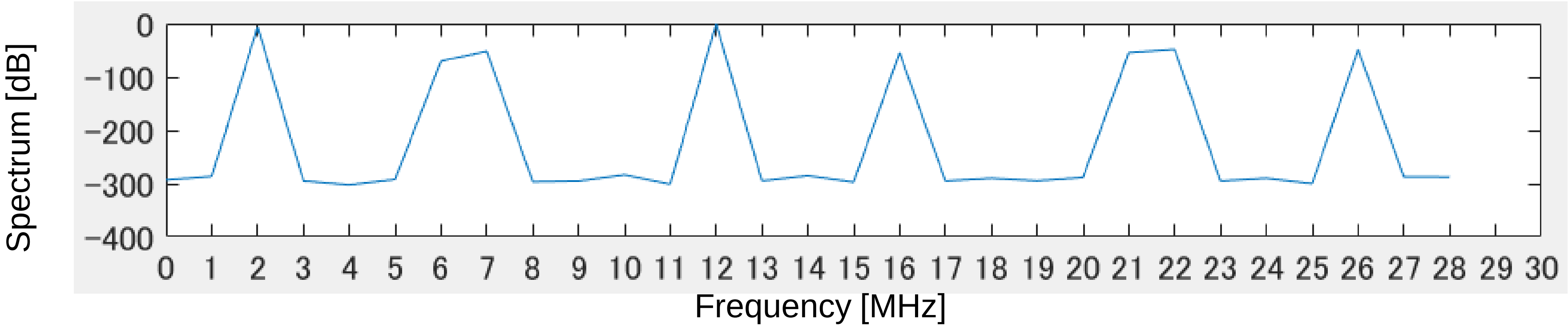


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	1	0.00
f_2	60	-6.07	14	-6.07
$3f_1$	210	-51.9	3	-51.9
$3f_2$	180	-70.0	19	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	11	-48.4
$2f_2 - f_1$	50	-54.4	4	-54.4
$2f_1 + f_2$	200	-48.4	16	-48.4
$2f_2 + f_1$	190	-54.4	6	-54.4

Two-Tone Test Simulation ($f_{s4} = 29\ \mathrm{MHz}$)

Sampling Frequency: $f_{s4} = 29\ \mathrm{MHz}$

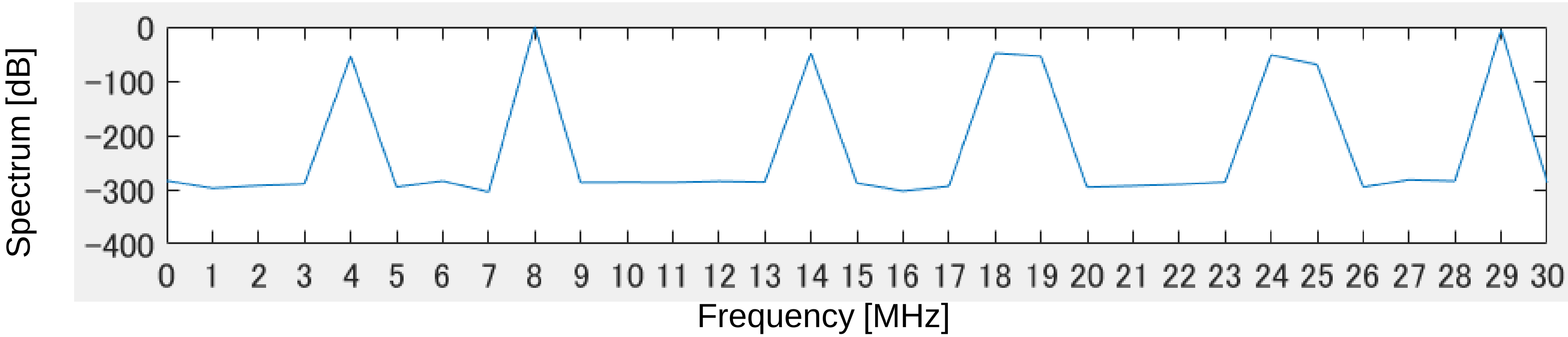


Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	12	0.00
f_2	60	-6.07	2	-6.07
$3f_1$	210	-51.9	7	-51.9
$3f_2$	180	-70.0	6	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	22	-48.4
$2f_2 - f_1$	50	-54.4	21	-54.4
$2f_1 + f_2$	200	-48.4	26	-48.4
$2f_2 + f_1$	190	-54.4	16	-54.4

Two-Tone Test Simulation ($f_{s5} = 31$ MHz)

Sampling Frequency: $f_{s5} = 31$ MHz



Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
f_1	70	0.00	8	0.00
f_2	60	-6.07	29	-6.07
$3f_1$	210	-51.9	24	-51.9
$3f_2$	180	-70.0	25	-70.0

Theory			Simulation	
	Freq. [MHz]	Power [dBc]	Residue freq. [MHz]	Power [dBc]
$2f_1 - f_2$	80	-48.4	18	-48.4
$2f_2 - f_1$	50	-54.4	19	-54.4
$2f_1 + f_2$	200	-48.4	14	-48.4
$2f_2 + f_1$	190	-54.4	4	-54.4

Residue HD, IMD power are the same as theorical HD, IMD power
Residue sampling is applicable to two-tone test

Summary

- Proposed a method to estimate high-frequency signal using multiple low-frequency sampling circuits.
- Confirmed its operation by theory and simulation.
- Measurable range can be wide: proportional to multiplication of multiple sampling frequencies.
- Measurable frequency resolution can be fine: proportional to number of FFT points

Possible Applications:

- Two-tone signal device testing
- Bluetooth device testing

OUTLINE

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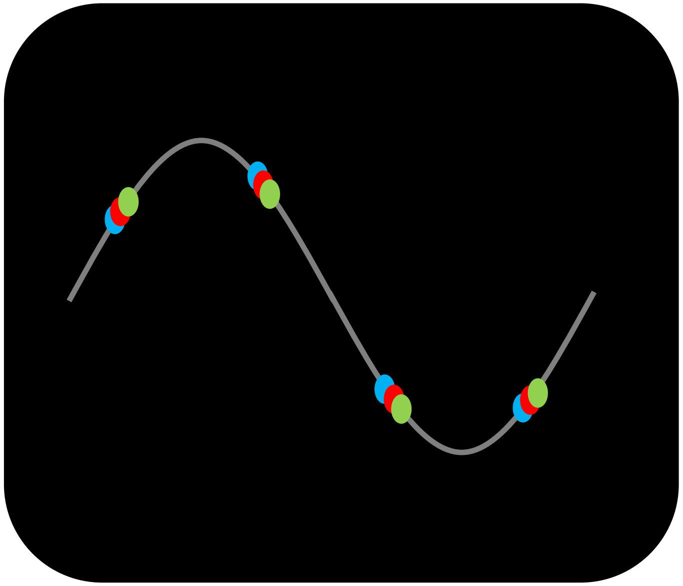
- <u>Metallic Ratio Sampling</u>
- Conclusion

[1] S.Yamamoto, H. Kobayashi, et. al., "Metallic Ratio Equivalent-Time Sampling and Application to TDC Linearity Calibration" IEEE Trans. Device and Materials Reliability（Mar. 2022)

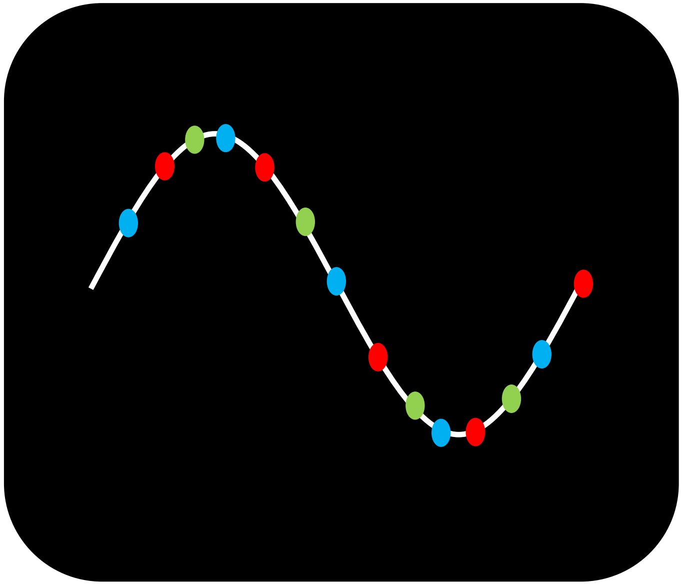
[2] Y. Sasaki, H. Kobayashi, et. al., "Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System", 27th IEEE Asian Test Symposium (Oct. 2018)

Research Objective

Objective: For efficient IC testing, high efficiency waveform acquisition with equivalent-time sampling.



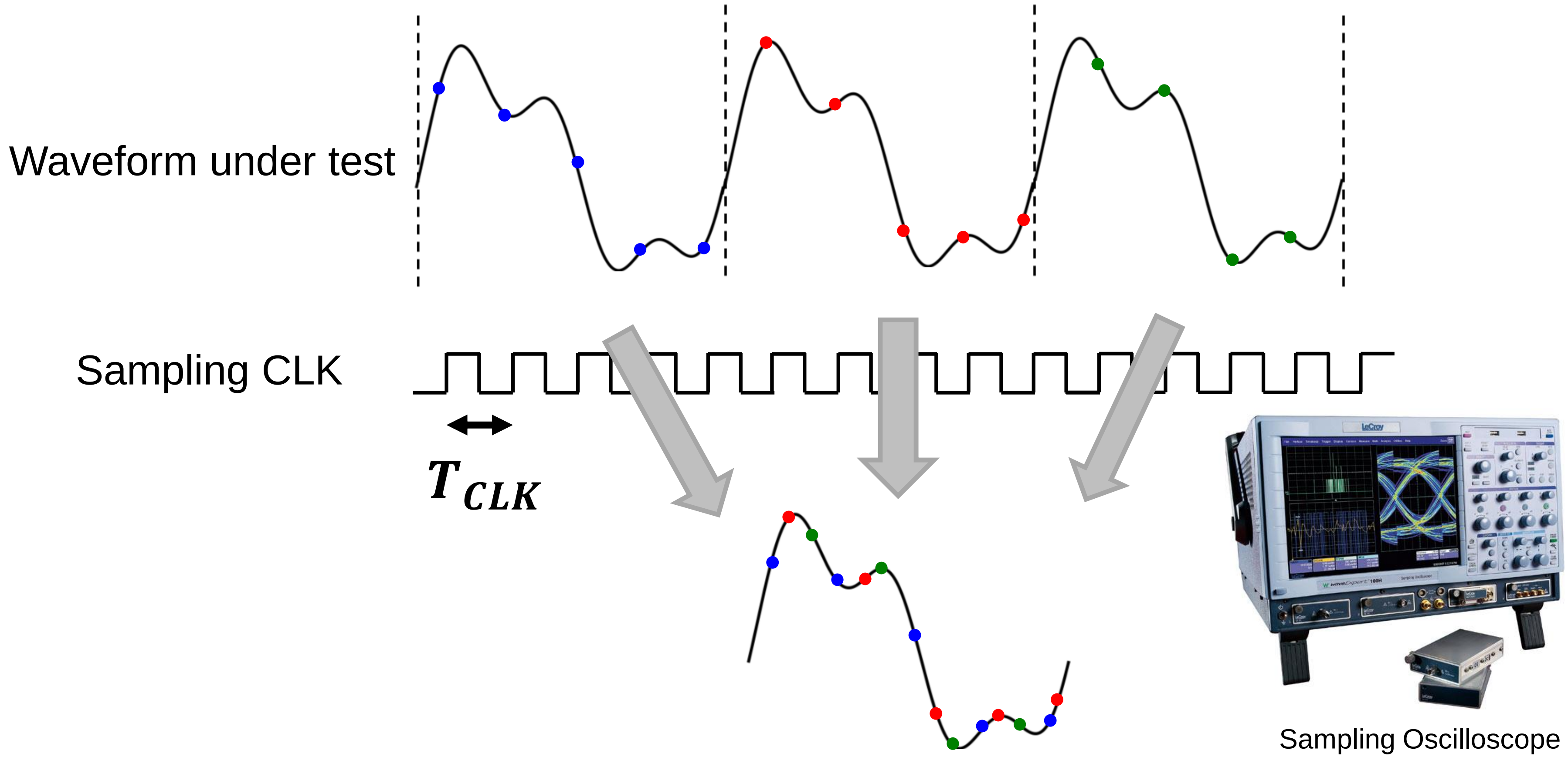
Sampling points: localized



Sampling points: distributed

Equivalent-Time Sampling

- Technique for sampling repetitive waveform
- Used in sampling oscilloscope and ATE

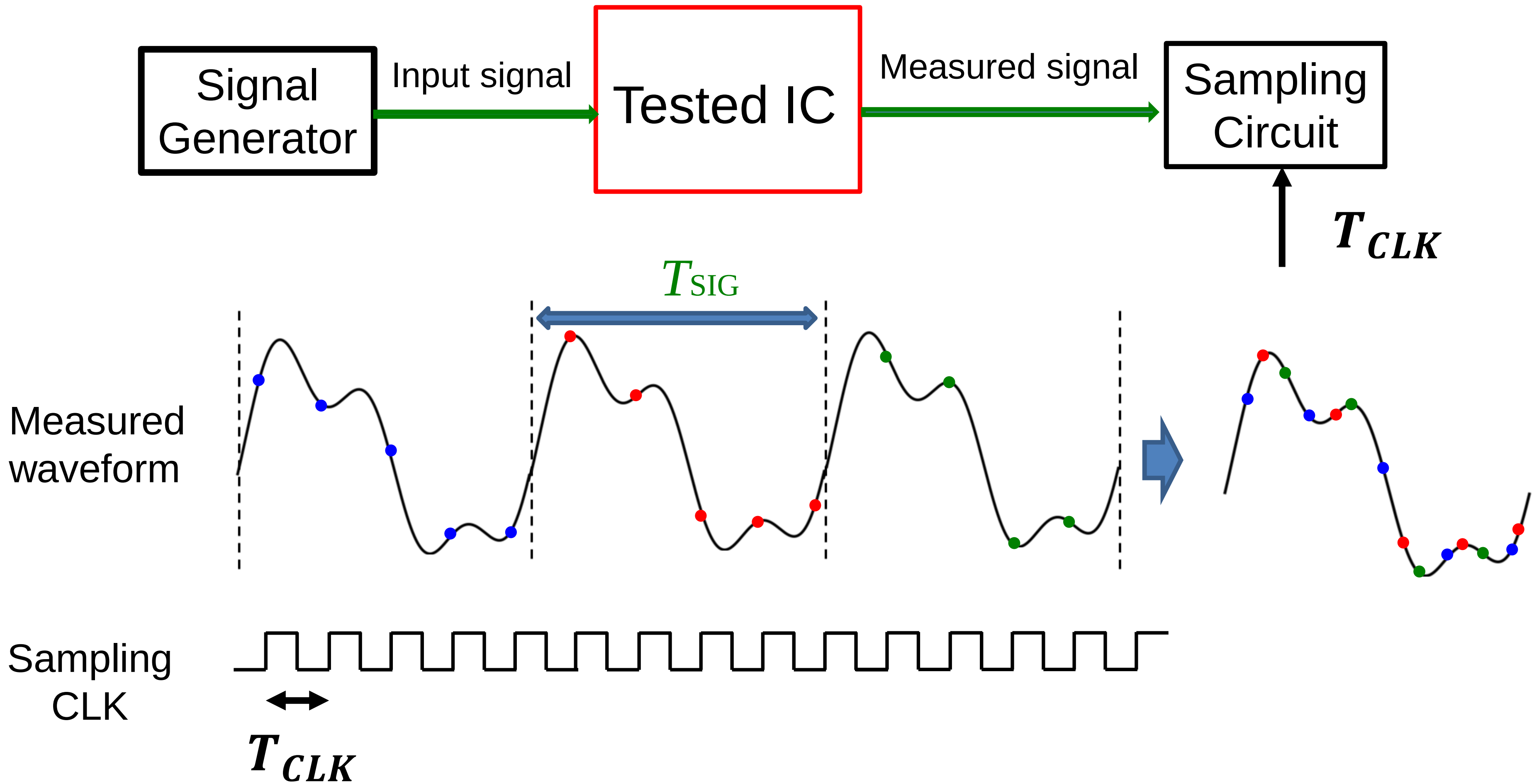


Sampling Oscilloscope

https://teledynelecroy.com/japan/products/scopes/we/default.asp

IC Testing and Equivalent-Time Sampling

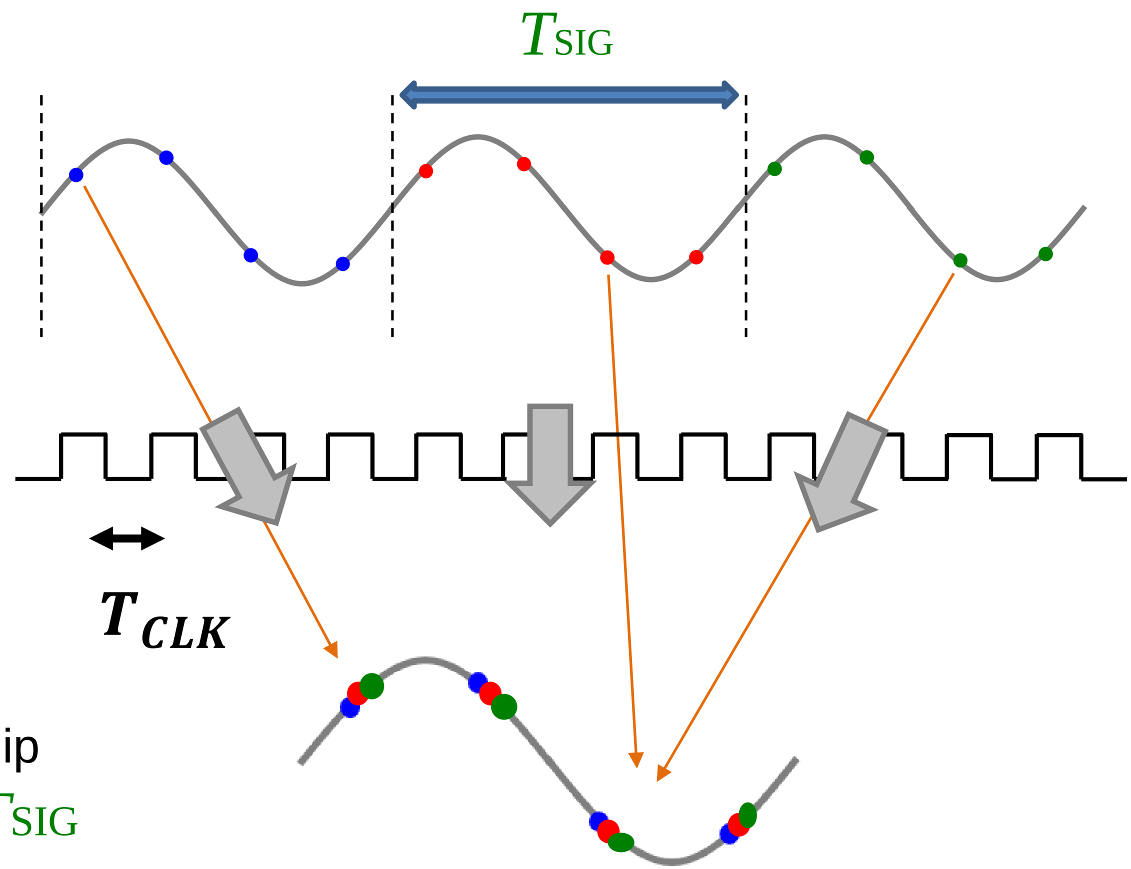
- Input signal → Controlled during IC testing
  Input signal period T_{SIG} → Output signal period T_{SIG}



Waveform Missing Phenomena

Waveform under test

Sampling CLK



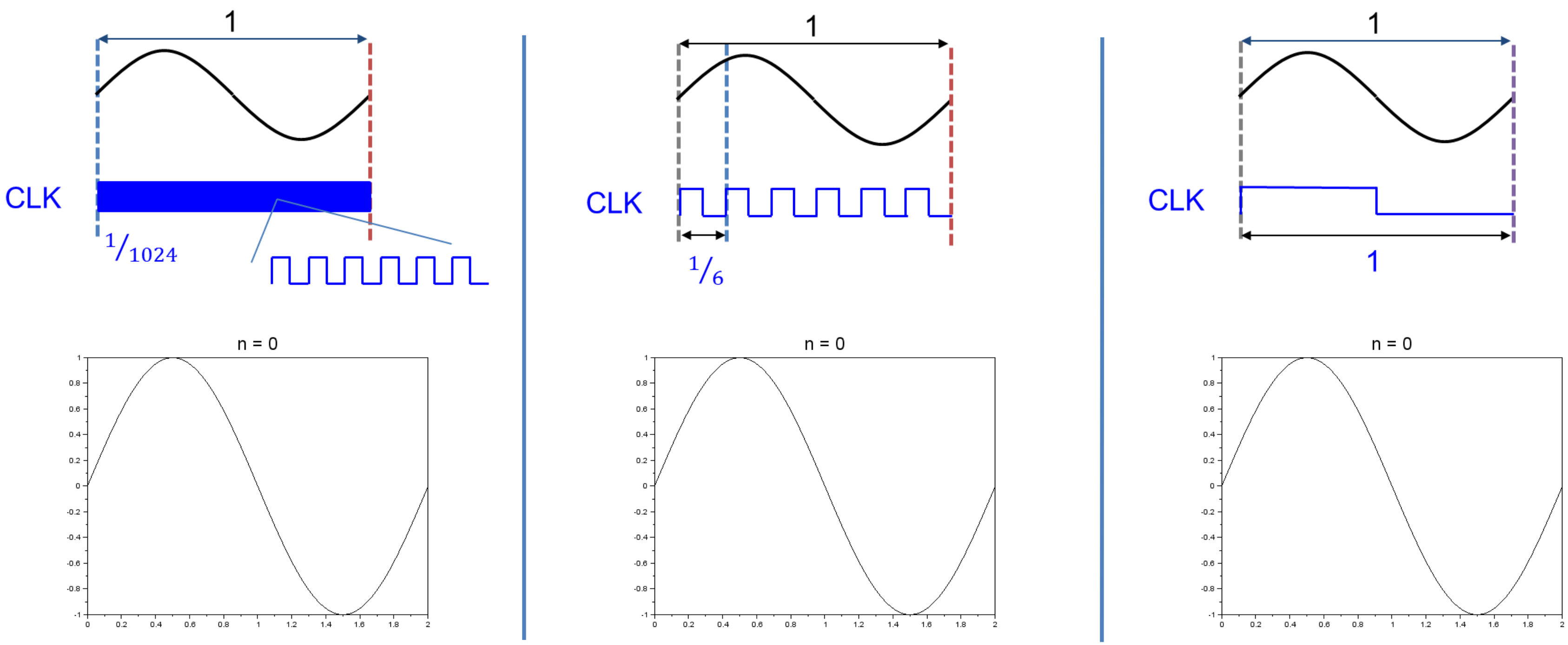
Caused by relationship
between T_{CLK} and T_{SIG}

A lot of data → reconstruct one period

Test time : LONG

Waveform Missing Conditions

$f_{CLK} \gg f_{sin}$ $\qquad$ $f_{CLK} \approx \frac{1}{\alpha} f_{sin} \left(\alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \cdots, \frac{1}{6}, \cdots\right)$ $\qquad$ $f_{CLK} \approx f_{sin}$



Sampling points: Localized

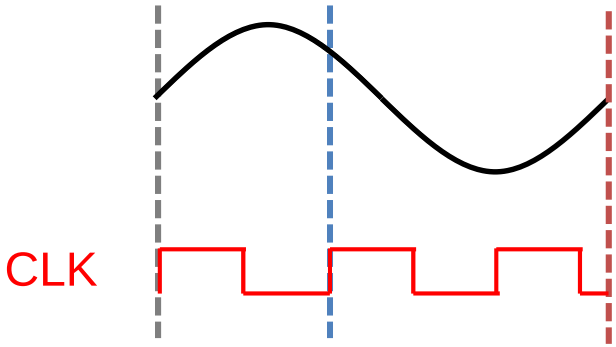
➡ One-period reconstruction time : Long

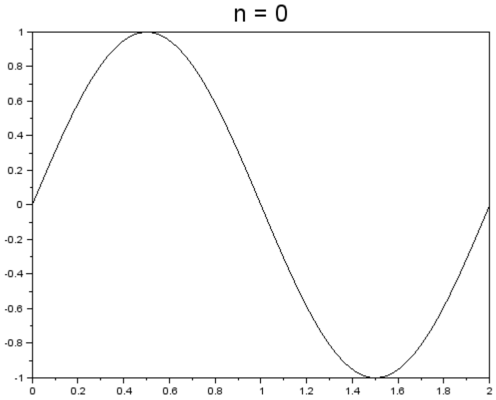
Efficient Waveform Acquisition Condition

Proper CLK

Sampling points: distributed

High efficiency waveform acquisition





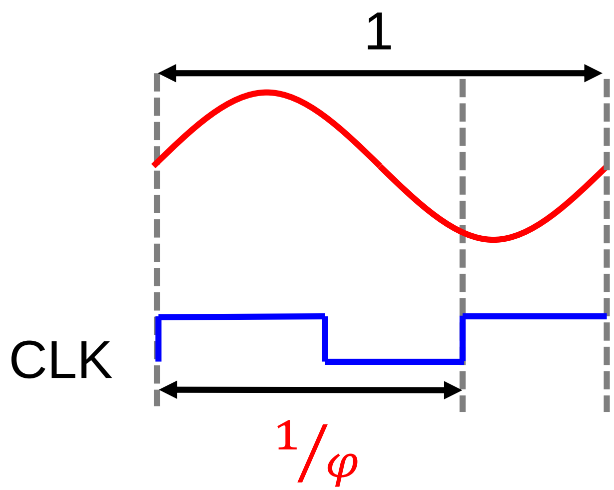
Sampling points: Distributed

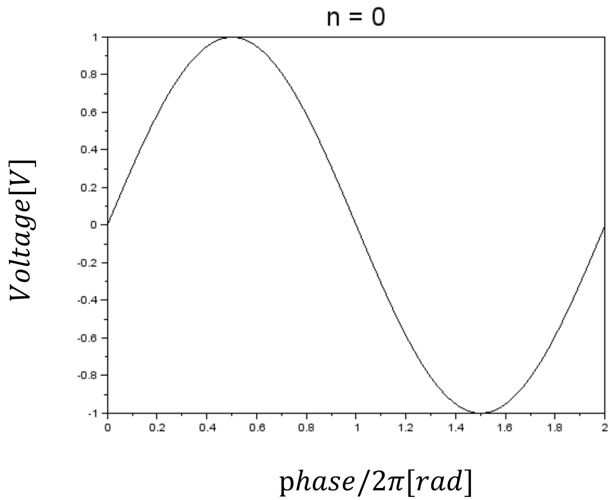
➡ One-period reconstruction time Short

Golden Ratio Sampling

$$f_{CLK} = \varphi \times f_{sig}$$

φ : golden ratio (= 1.6180339887…)

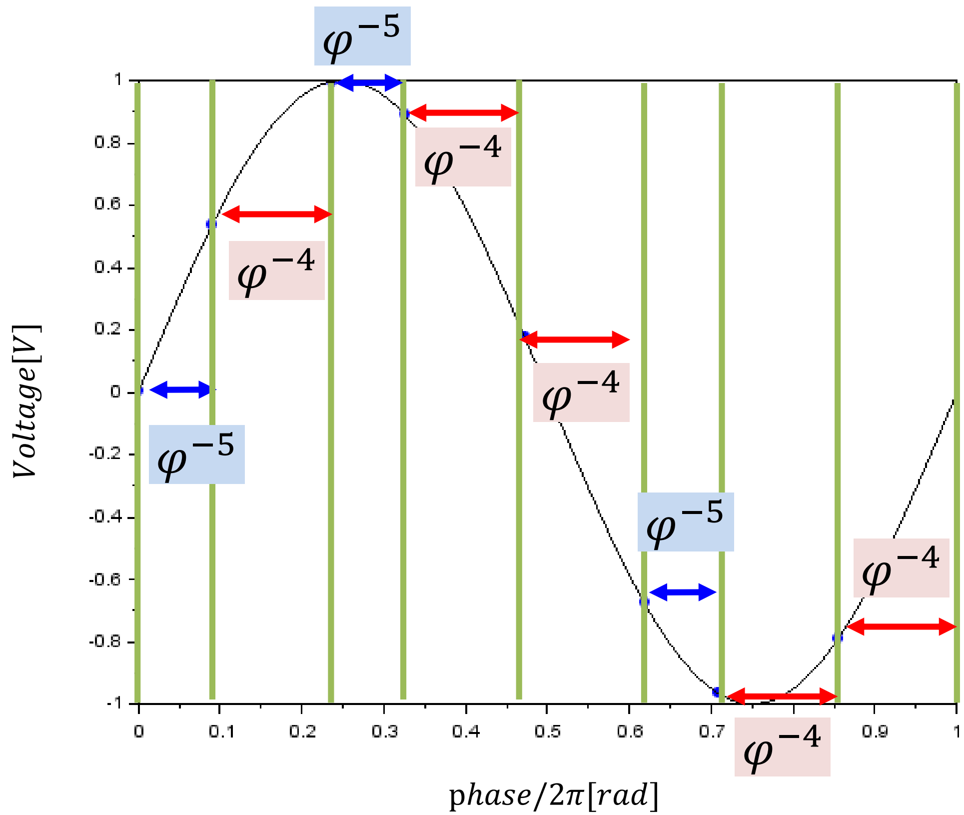




Sampling points ➡ Uniformly distributed

[2] Y. Sasaki, H. Kobayashi, et. al., "Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System", 27th IEEE Asian Test Symposium (Oct. 2018)

Distance of Adjacent Sampling Points



$\boldsymbol{\varphi}$: golden ratio (= 1.6180339887…)

Maximum distance／Minimum distance $= \varphi$ or φ^2

➡ Sampling points : Not too close & Not too far

Metallic Ratio

Metallic ratio

$$1 : \frac{n + \sqrt{n^2 + 4}}{2} \qquad (n = 1, 2, 3 \cdots)$$

M : Metallic number

$n = 1$: Golden ratio (M = 1.6180…)

$n = 2$: Silver ratio (M = 2.4142…)

$n = 3$: Bronze ratio (M = 3.3027…)

⋮

$n = m$: 1 : M

Difference from reciprocal

$M - \frac{1}{M}$ = Natural Number

Continued fraction

$$M = n + \frac{1}{n + \frac{1}{n + M}}$$

Limit of adjacent term ratio

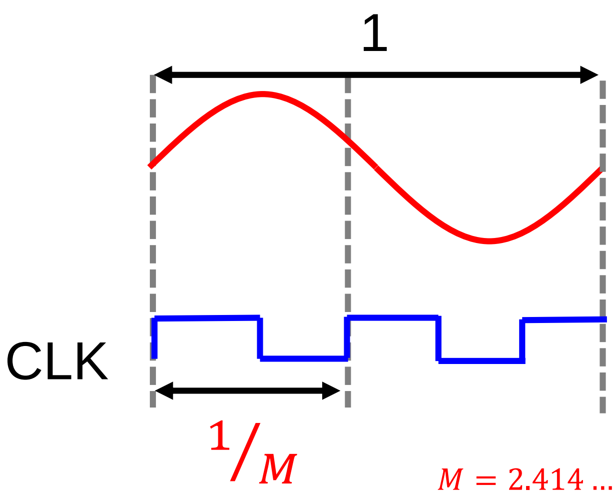
$$F_0 = 0, F_1 = 1, F_{n+2} = nF_{n+1} + F_n$$

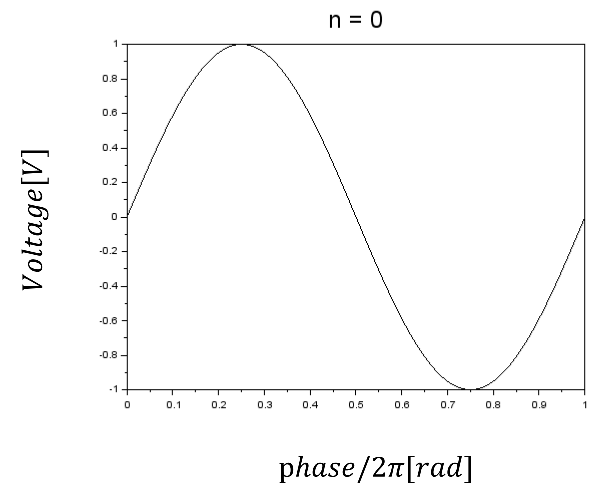
Metallic Ratio Sampling

Fixed f_{CLK} → Test ADC with various f_{sig}

$$f_{CLK} = M \times f_{sig}$$

M : Metallic ratio





In the case of silver ratio

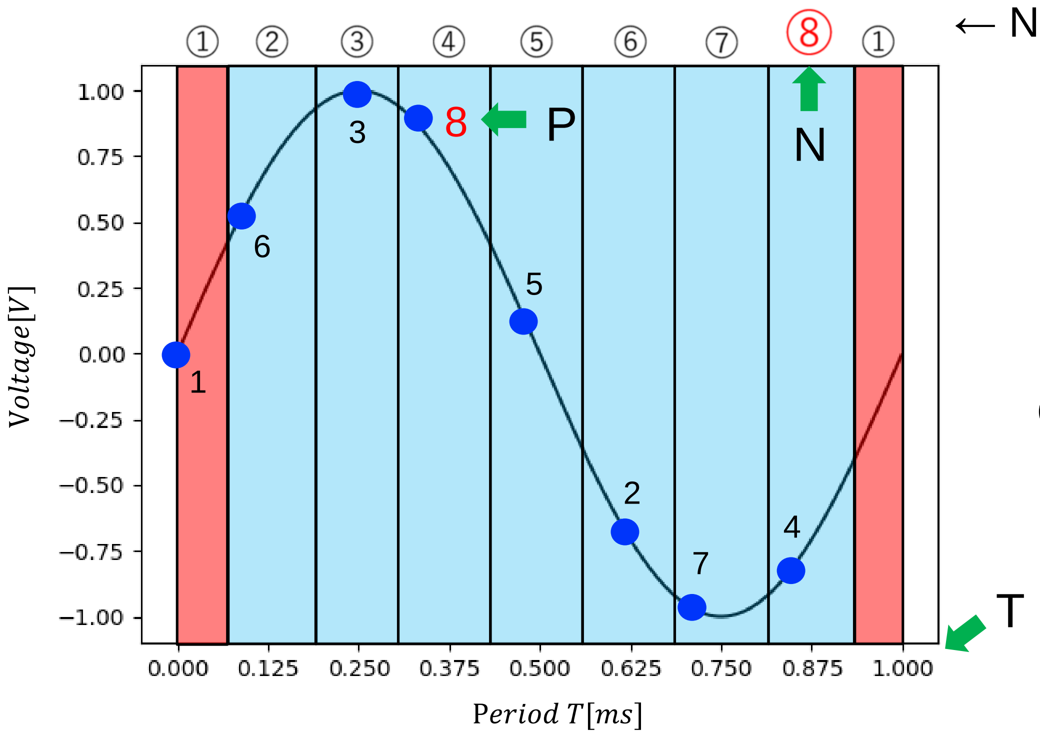
Sampling points ➡ Always distributed evenly in phase

Sampling Efficiency Definition

N : Number of divisions in period T E : Sampling efficiency

P : Number of points → All divisions have at east one point in them.

$$E = \frac{N}{P}$$



← Number to identify segmented area

● Sampling points and order

Difference between adjacent sampling points < $\frac{2T}{N}$

Golden ratio sampling 8 divisions.

$P = 8, N = 8, \ T = 1.0$

$\therefore E = \frac{8}{8} = 1.0$

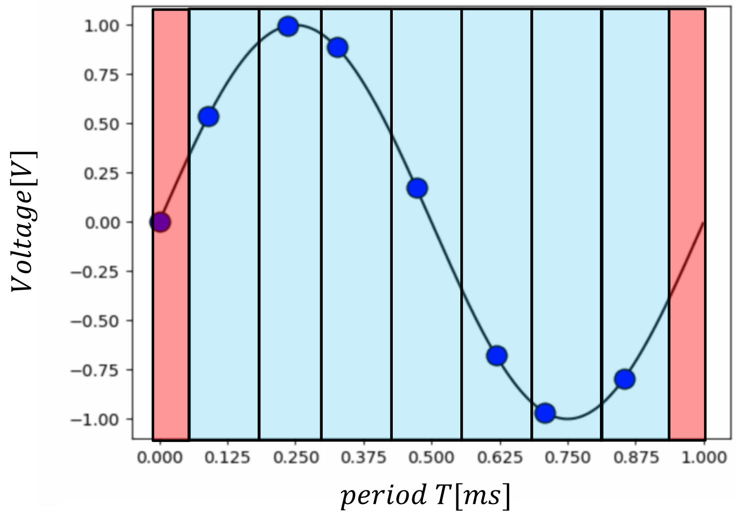
Difference between adjacent sampling points → < $\frac{2}{8}$

In case of golden ratio sampling 8 divisions.

Sampling Efficiency by Metallic Ratios

In case of 8 divided sections

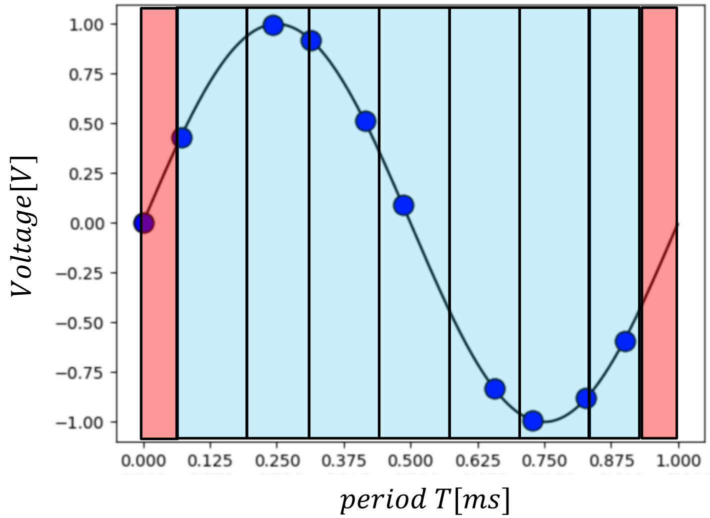
$n = 1$ ($M = 1.6180..$) Golden ratio



Need 8 points

$$P = 8, N = 8, T = 1.0$$
$$\therefore E = \frac{8}{8} = 1.0$$

$n = 2$ ($M = 2.4142..$) Silver ratio



Need 10 points

$$P = 10, N = 8, T = 1.0$$
$$\therefore E = \frac{8}{10} = 0.8$$

Efficiency → varies by metallic ratio

Summary

- We have found
  highly efficient waveform acquisition conditions

  - Golden ratio sampling

    $$f_{CLK} = \varphi \times f_{sig}$$

    φ : Metallic ratio (=1.618…)

  - Metallic ratio sampling

    $$f_{CLK} = M \times f_{sig}$$

    M : Metallic ratio

- Applicable to RF/analog IC testing

  Input signal, Sampling clock ⇨ Controllable

- They have found some rules
  in the viewpoint of number theory

Conclusion

- Waveform sampling is a key for RF/analog device testing.

- New sampling technology can be developed based on number theory

  - Residue Sampling

  - Metallic Ratio Sampling

“ Number theory is queen of mathematics.”

Carl Friedrich Gauss

OUTLINE

Appendix

Proactive Use of Finite Aperture Time
in Sampling Circuit for Sensor Interface

[1] Y. Yan, H. Kobayashi, et. al., "Proactive Use of Finite Aperture Time
in Sampling Circuit for Sensor Interface“
5th International Conference on Technology and Social Science (Dec. 2021)

Research Background

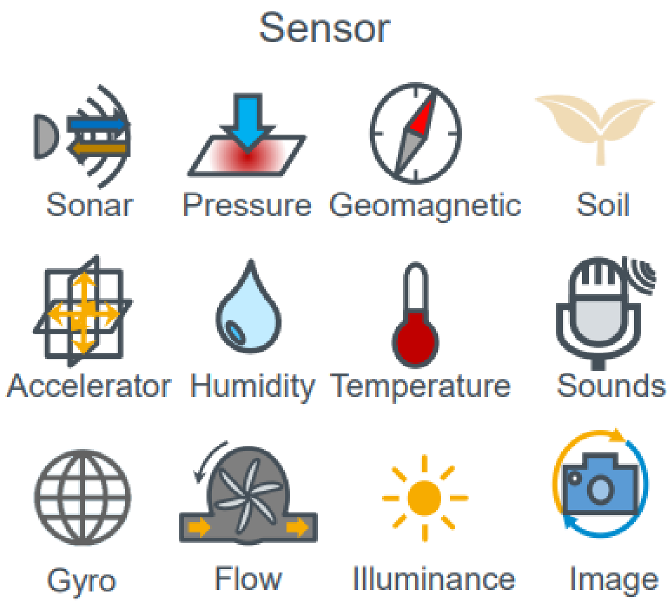
IoT systems prevail

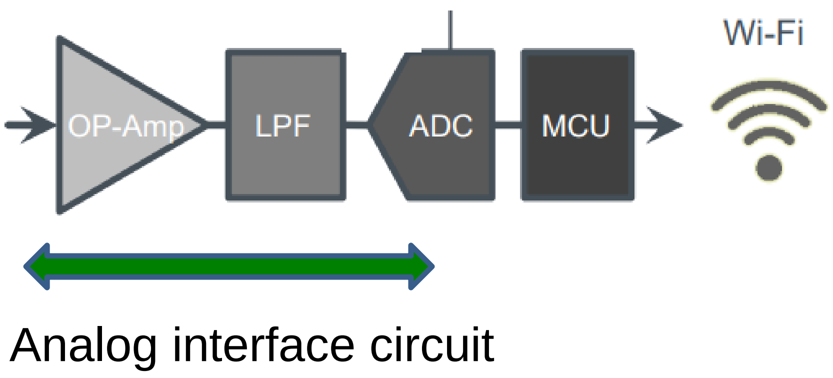
A lot of sensors

low frequency signals

Analog interface circuit

low power, miniature size



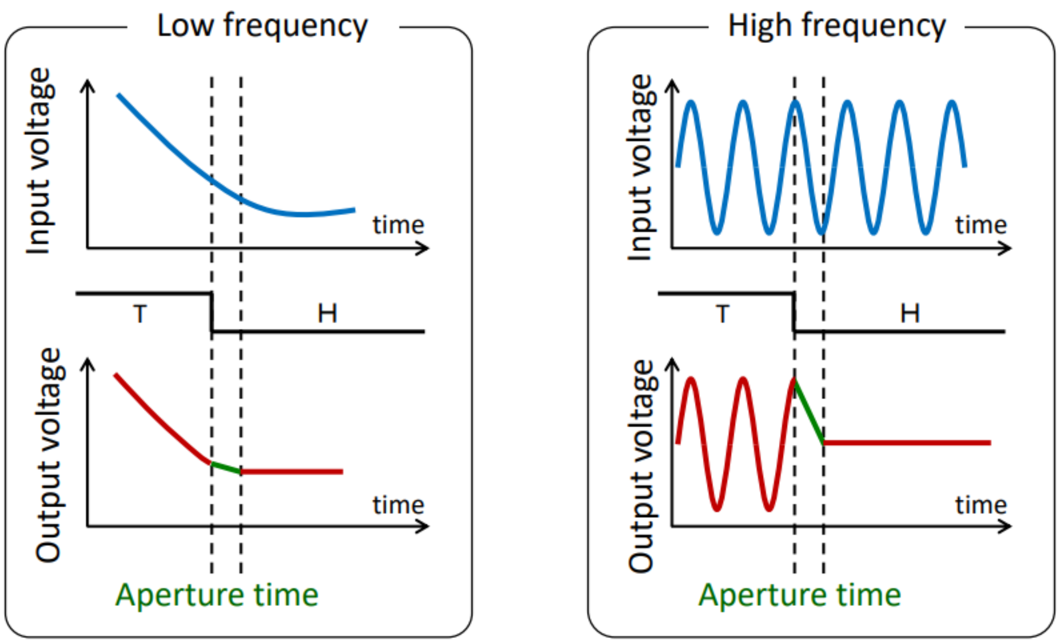


Research Objective

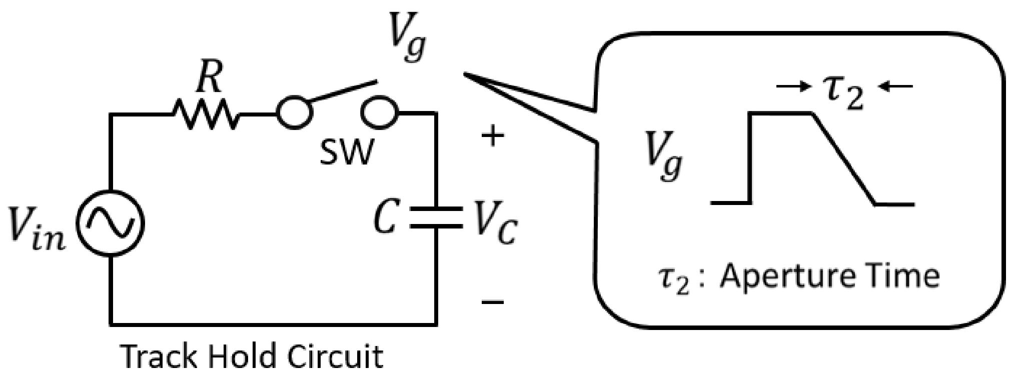
Clarification of
proactive use of finite aperture time in sampling circuit

- Low-pass filter chip area reduction

- Low frequency signal quality improvement



Low Pass Filtering Effect of Aperture Time



Explicit transfer function

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

Here τ_1= RC.

Finite aperture time τ_2 ➡ Lowpass filter action

- Harmful for high frequency signal sampling
- Good for low frequency signal sampling

➡ Lowpass filter simplification

Summary

- Finite aperture time in sampling circuit

  Low frequency signal acquisition:

  Proactive use for lowpass filtering

- One more comment

  Pedestal error:
  caused by charge injection and clock feedthrough.

  Pedestal error reduction by long aperture time.