26 Oct. 2022 (Wed)

ICSICT-2022

2022 IEEE 16th International Conference on Solid-State and Integrated Circuit Technology

Oct. 25-28, 2022

Nanjing Platinum Hanjue Hotel Nanjing, China.

Session D1:
Analog Circuit

Spatial and Temporal Dynamics of Non-Uniform Active Resistor Networks

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Outline

- Research Objective
- Research Background
- Active Resistive Network
    - - Spatial Impulse Response
    - - Temporal Dynamics
- Uniform Network Dynamics
- Non-Uniform Network Dynamics
- Conclusion

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Research Objective

Our previous theorem:

Spatial and temporal stability conditions are equal
for uniform resistive network including negative resistors

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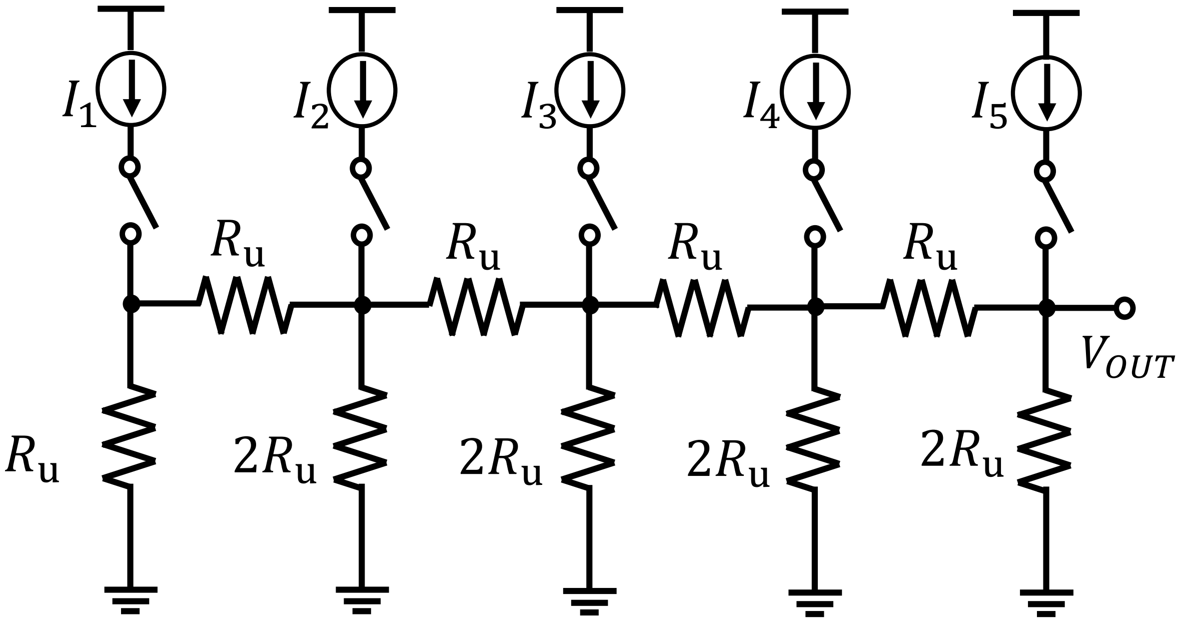
This research:

Investigation of spatial and temporal dynamics
for non-uniform resistive network

Outline

Resistive Network Circuit (1)

R-2R resistive ladder DAC

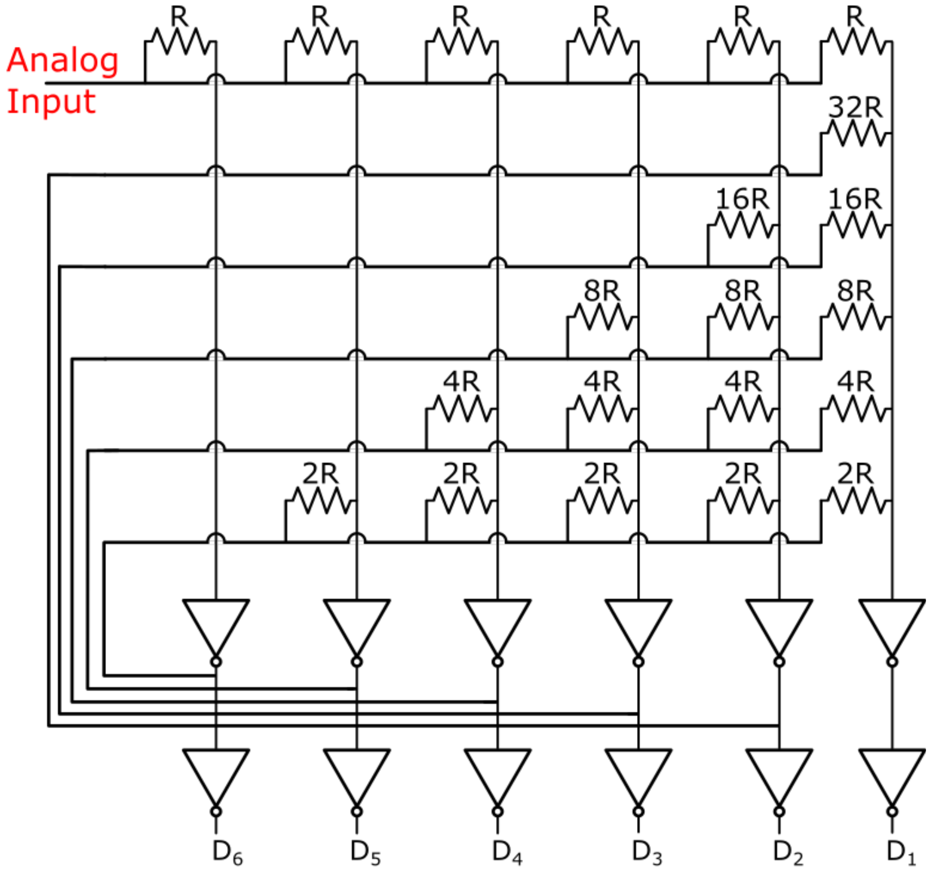


Advantages
- High speed
- No need for decoder

Disadvantages
- Glitch
- Non-monotonicity

Resistive Network Circuit (2)

Asynchronous SAR ADC



Advantages

- High speed
- Low power
- Small circuit

[1] Z . Xu, X. Bai, D. Yao, A. Kuwana, H. Kobayashi,
"Revisit to Hopfield Network for Asynchronous SAR ADC and DAC",
IEEE 3rd International Conference on Circuits and Systems, Chengdu, China (Oct. 2021)

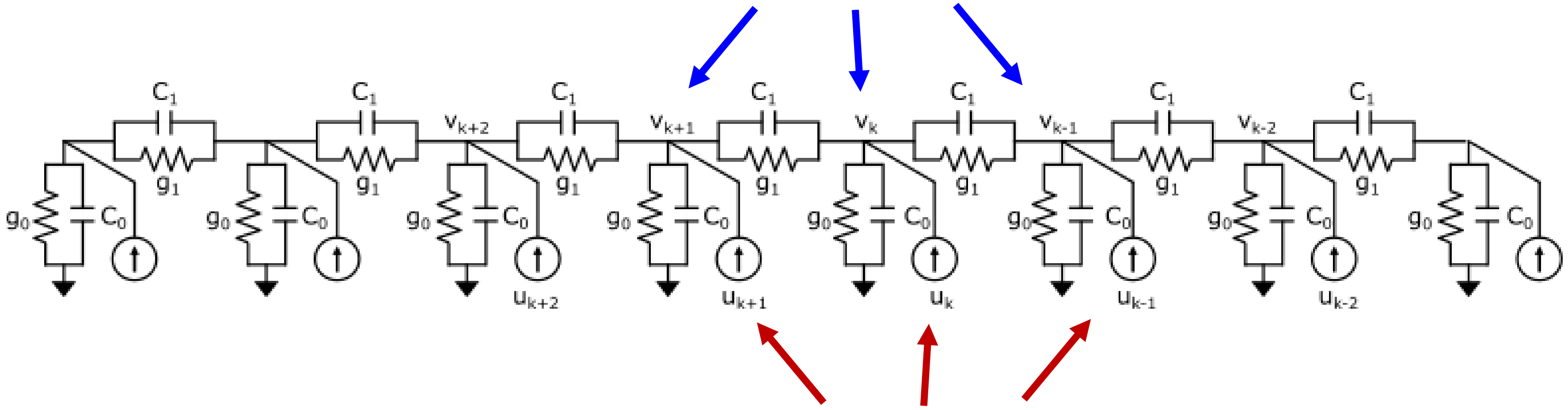
Resistive Network Circuit (3)

High-speed analog image processor

1D image case

$g_0 = 1/R_0$
$g_1 = 1/R_1$

Node voltages: Output image



Injected currents at nodes: Input image

[2] C. A. Mead, Analog VLSI and Neural Systems, Addison Wesley, 1989

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Active Resistive Network:
➡ Including positive and negative resistors

Resistive Network Circuit (4)

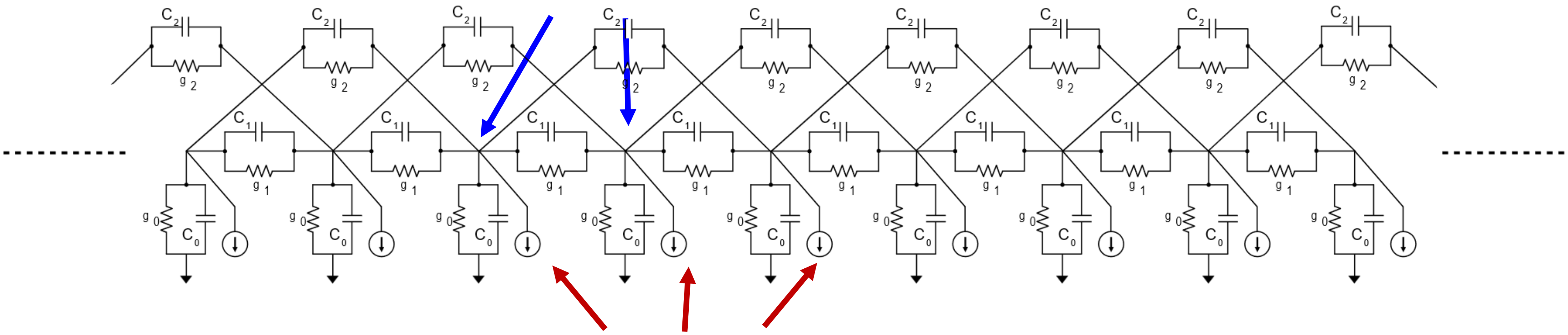
High-speed analog image processor

$g_0 = 1/R_0$
$g_1 = 1/R_1$
$g_2 = 1/R_2$

1D image case

Negative resistor $R_2 = -4R_1 < 0$

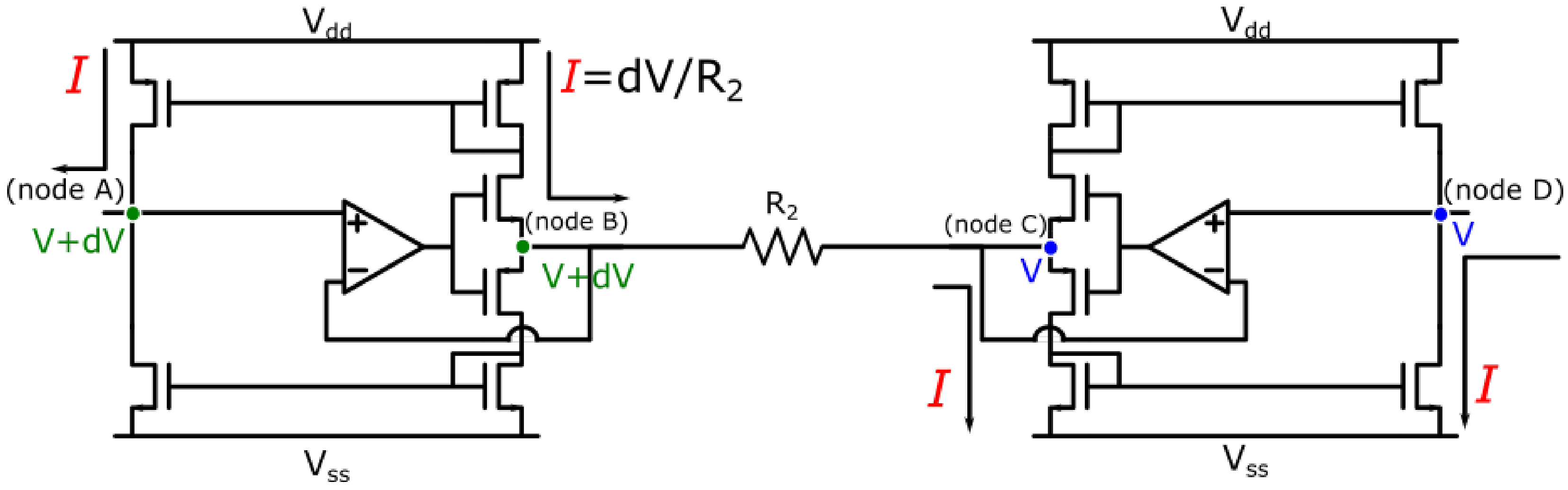
Node voltages：Output image

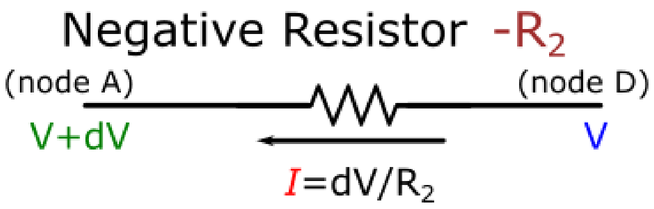


Injected currents at nodes：Input image

[3] H. Kobayashi, J. L. White, A. A. Abidi, "An Active Resistor Network for Gaussian Filtering of Images", IEEE Journal of Solid-State Circuits (May 1991)

Negative Resistor with Standard CMOS





V+dV @ Node A > V @ Node D

Current $I = \frac{dV}{R_2}$

[3] H. Kobayashi, J. L. White, A. A. Abidi, “An Active Resistor Network for Gaussian Filtering of Images", IEEE Journal of Solid-State Circuits (May 1991)

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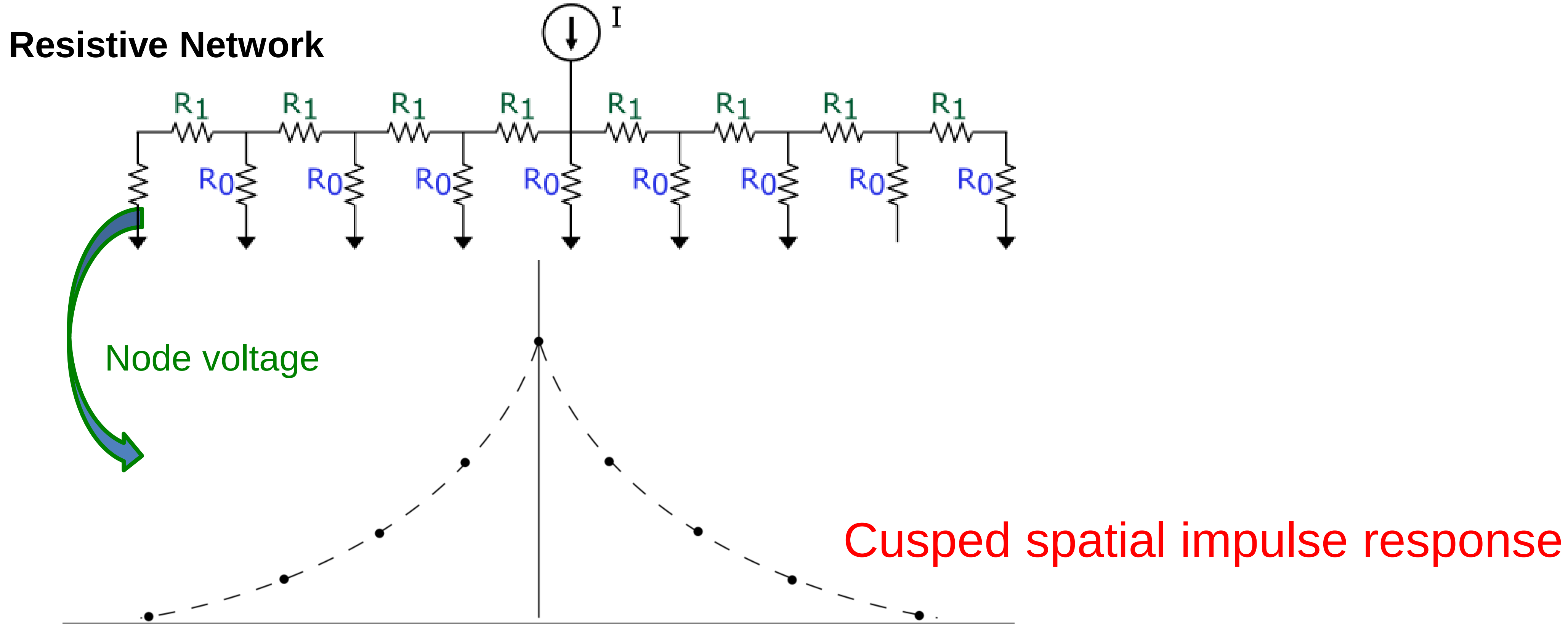
Spatial Impulse Response (1)

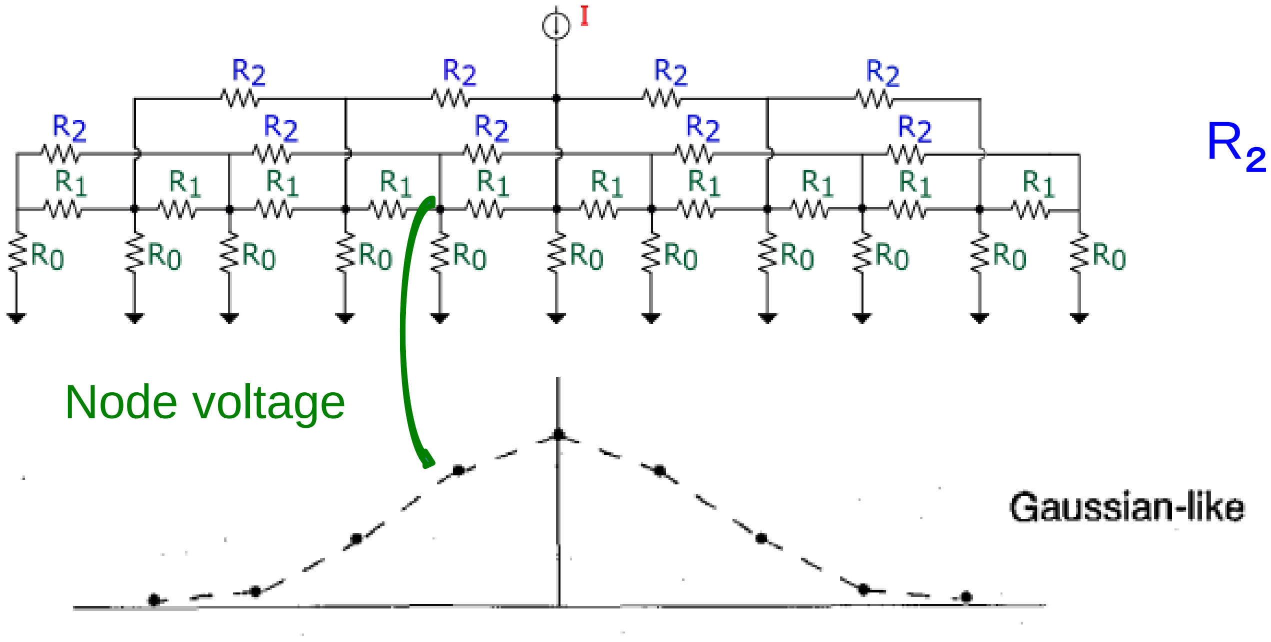
High-speed analog image processor (Retina chip)



[5] C. A. Mead, Analog VLSI and Neural Systems, Addison Wesley, 1989

Spatial Impulse Response (2)

High-speed analog image processor (Gaussian chip)



R_2 : Negative Resistor

Flat-top spatial impulse response

[3] H. Kobayashi, J. L. White, A. A. Abidi, “An Active Resistor Network for Gaussian Filtering of Images", IEEE Journal of Solid-State Circuits (May 1991)

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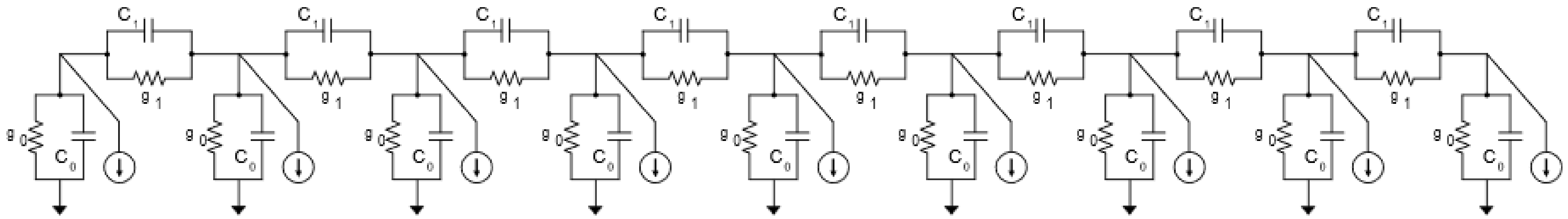
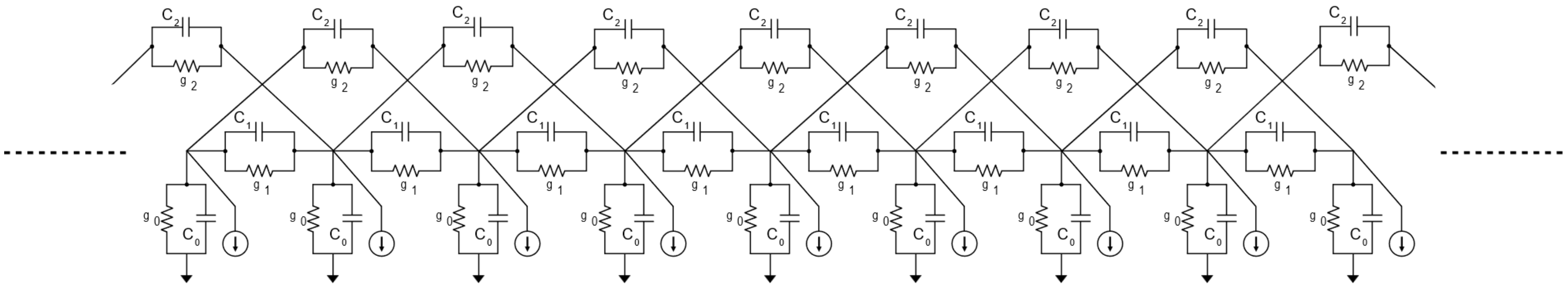
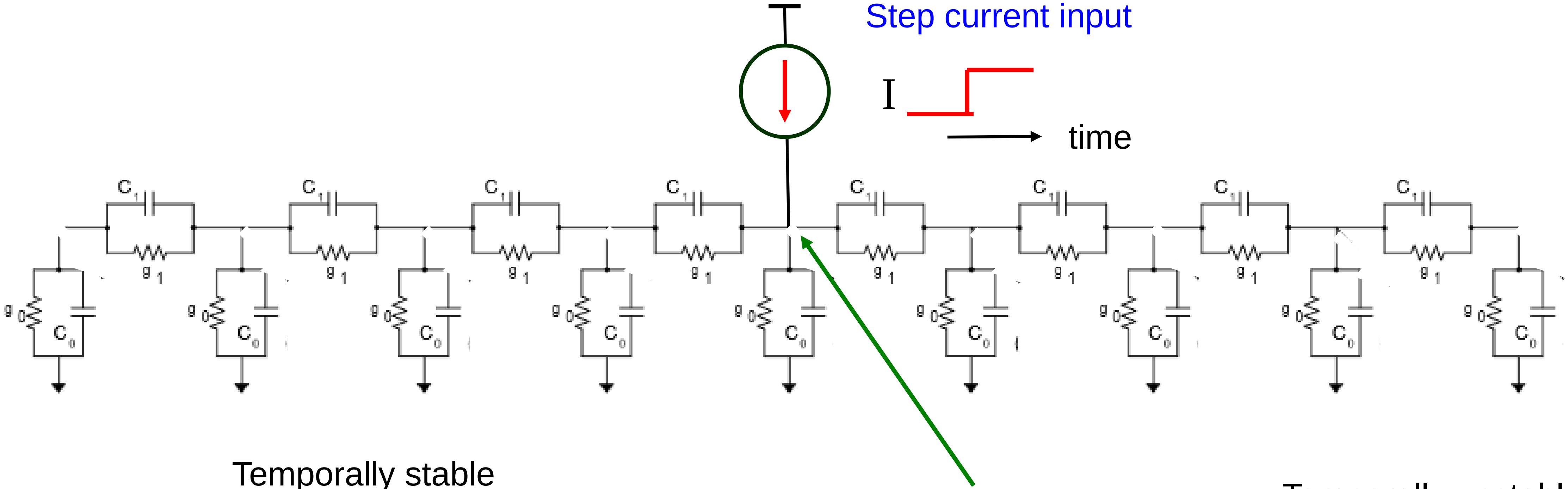
Temporal Dynamics with R, C

Capacitances are considered for temporal dynamics

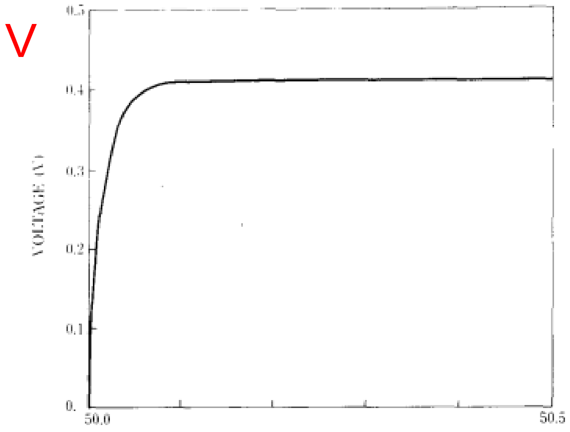




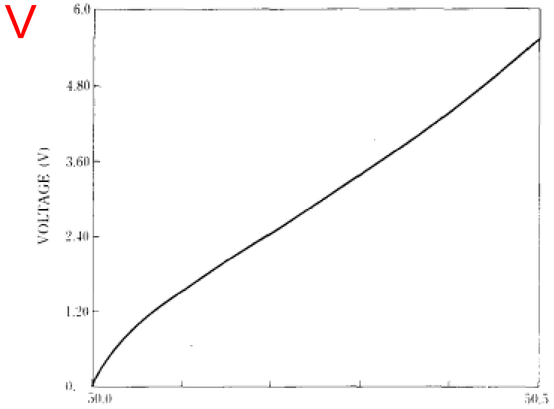
Temporal Step Response



Temporally stable

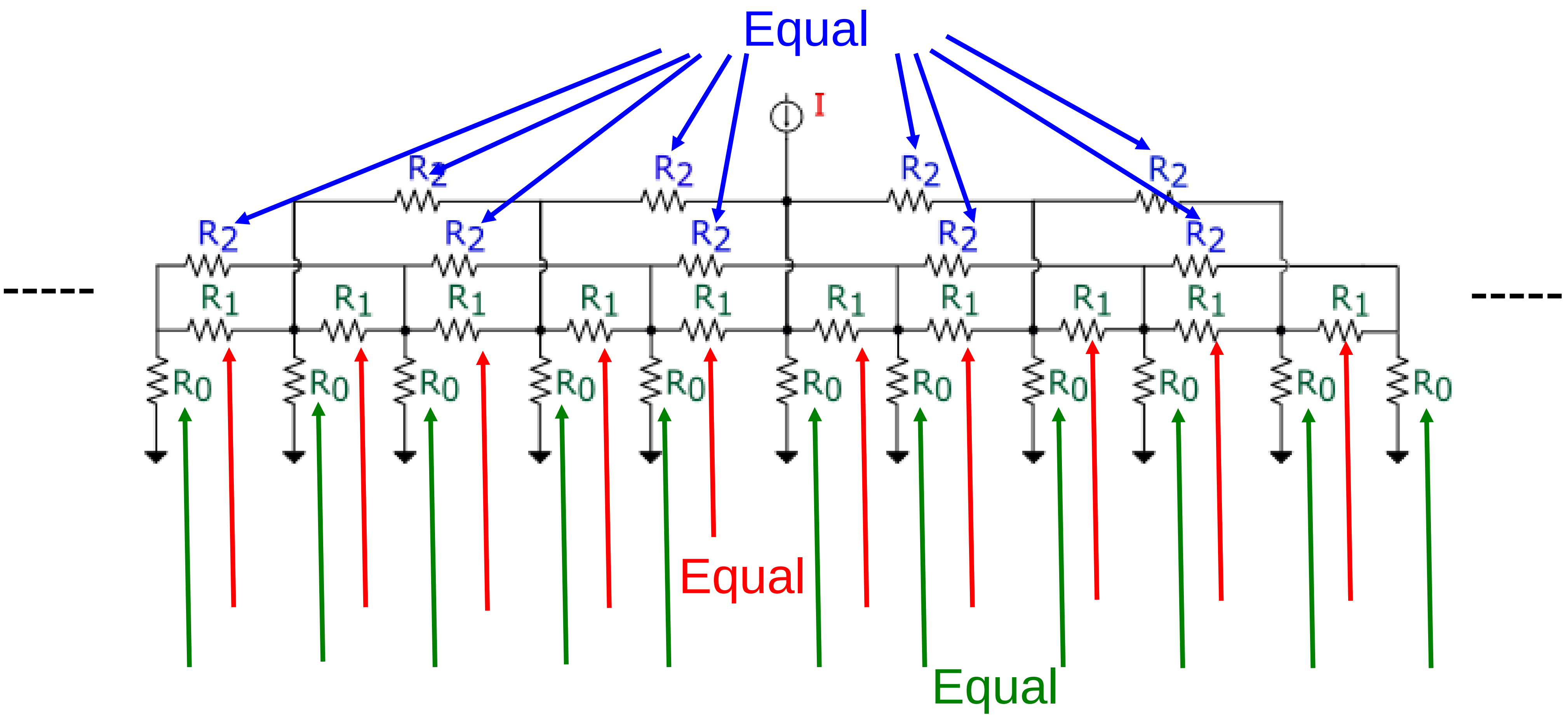


Temporally unstable



Outline

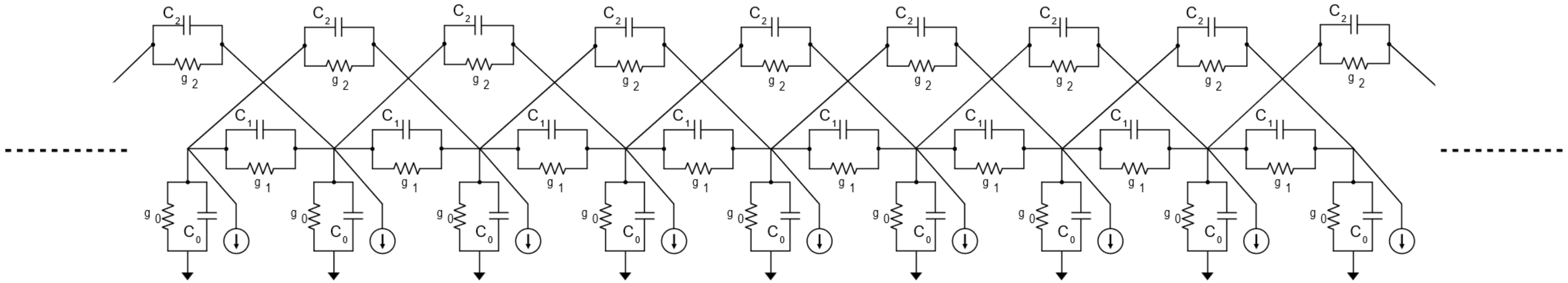
Uniform Resistor Network



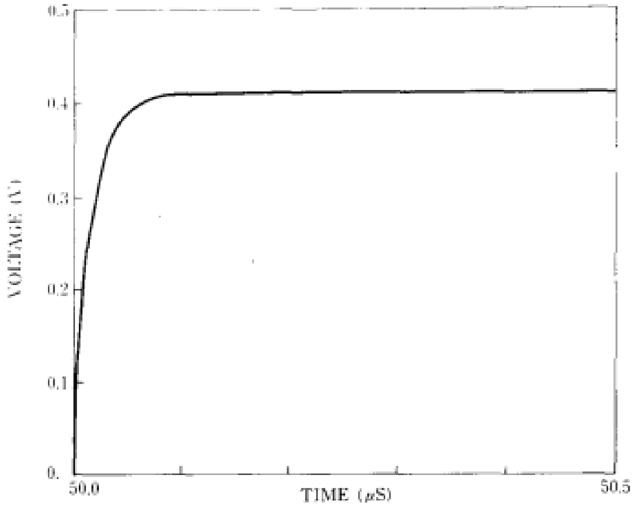
- Shift invariant
- Spatial transfer function

Simulation Results: Spatial Temporal Stabilities

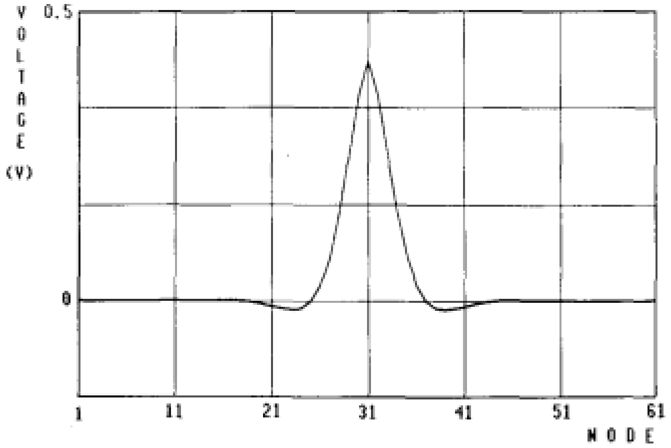
$R_0 = 1/g_0 = 200\text{k}\Omega,\quad R_1 = 1/g_1 = 5\text{k}\Omega,\quad R_2 = 1/g_2 = -20\text{k}\Omega$



Temporally stable

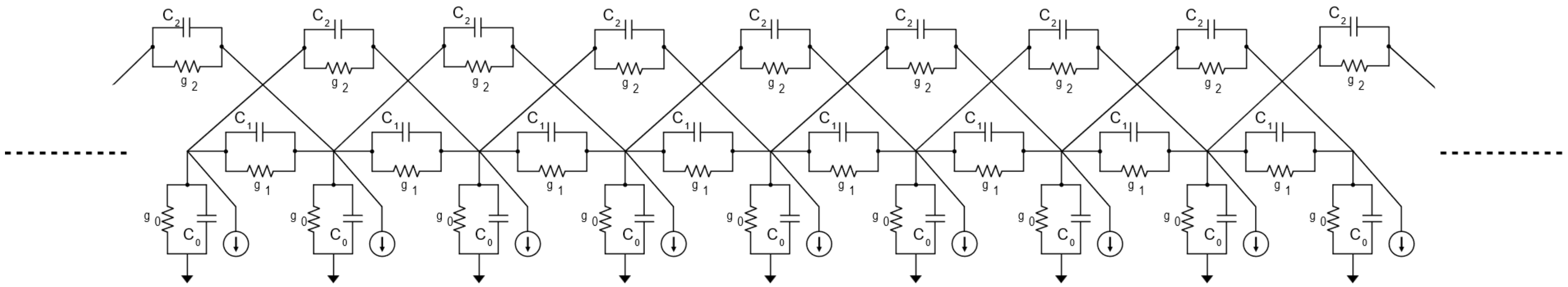


Spatially stable

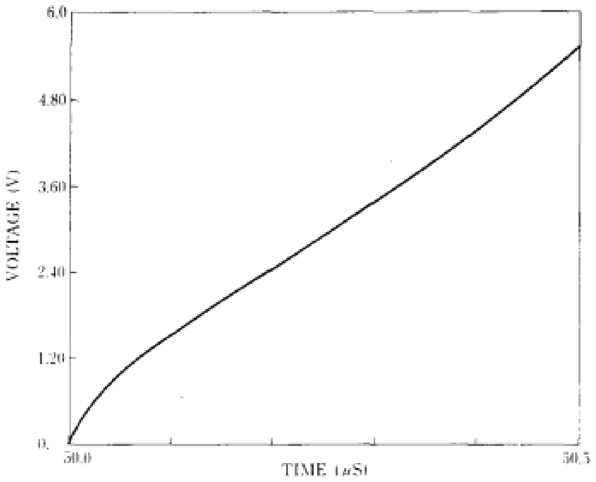


Simulation Results: Spatial Temporal Instabilities

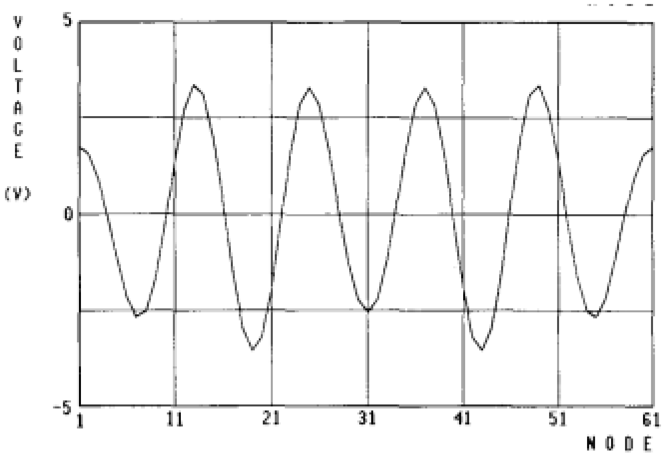
$R_0 = 1/g_0 = 200\text{k}\Omega, \quad R_1 = 1/g_1 = 5\text{k}\Omega, \quad R_2 = 1/g_2 = -17\text{k}\Omega$



Temporally unstable



Spatially unstable



Circuit Network Theorem

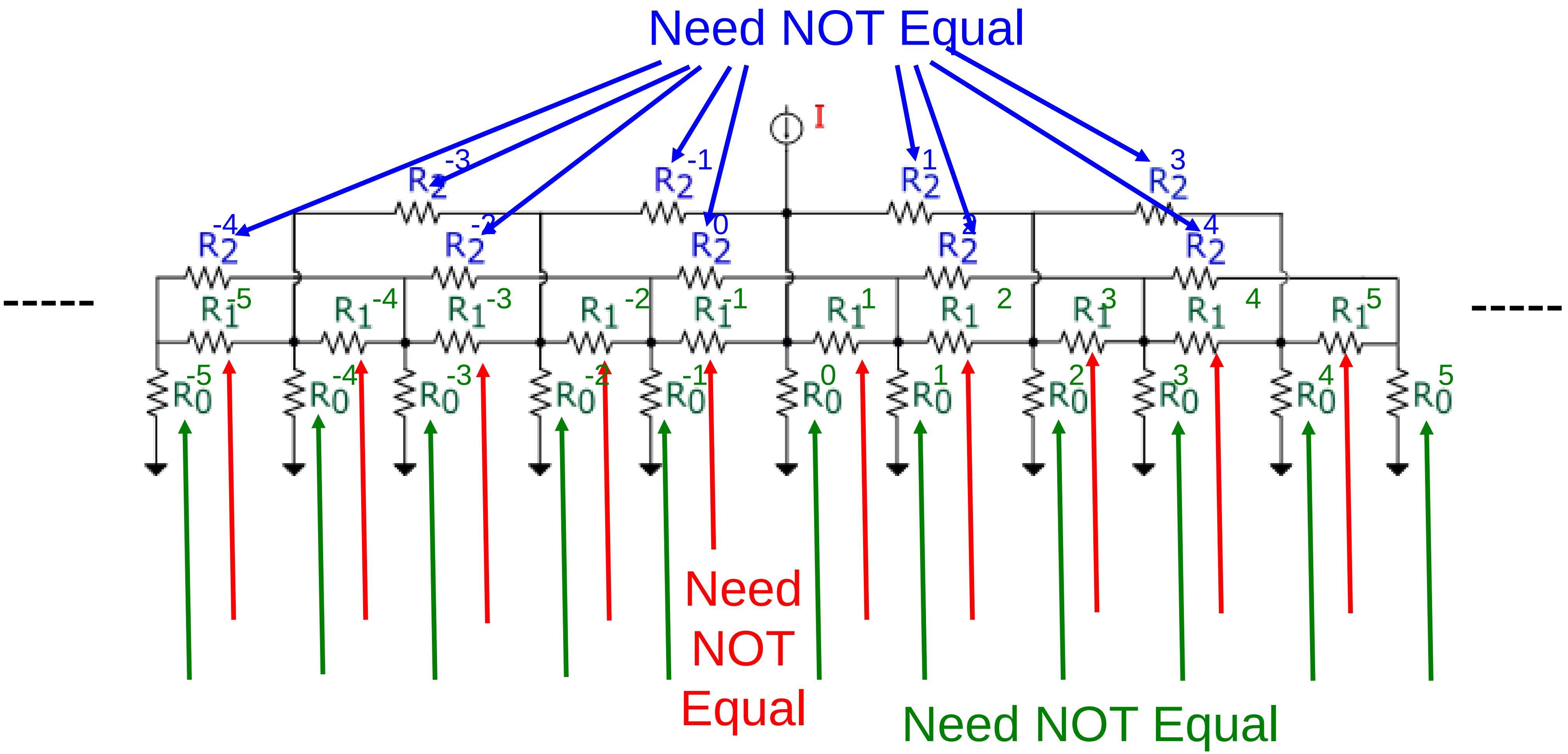
For uniform network with positive and negative resistors, spatial and temporal stability conditions are equivalent.

[4] T. Matsumoto, H. Kobayashi, Y. Togawa,
"Spatial Versus Temporal Stability Issues in Image Processing Neuro Chips",
IEEE Trans. Neural Networks, (July 1992).
[5] H. Kobayashi, T. Matsumoto, J. Sanekata,
"Two-Dimensional Spatio-Temporal Dynamics of Analog Image Processing Neural Networks",
IEEE Trans. Neural Networks (Oct. 1995).

How about non-uniform network ?

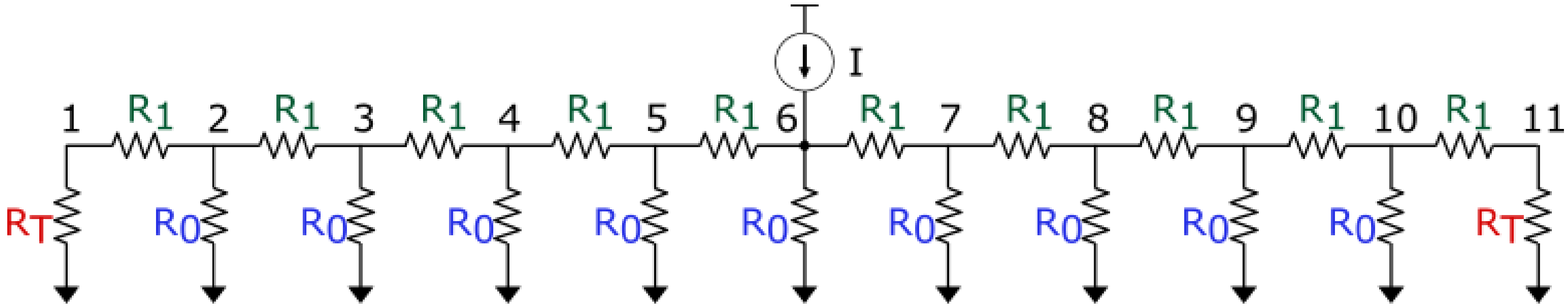
Outline

Non-Uniform Resistor Network



- Shift variant
- Spatial transfer function CANNOT be defined

Spatial Impulse Response of Non-Uniform Network

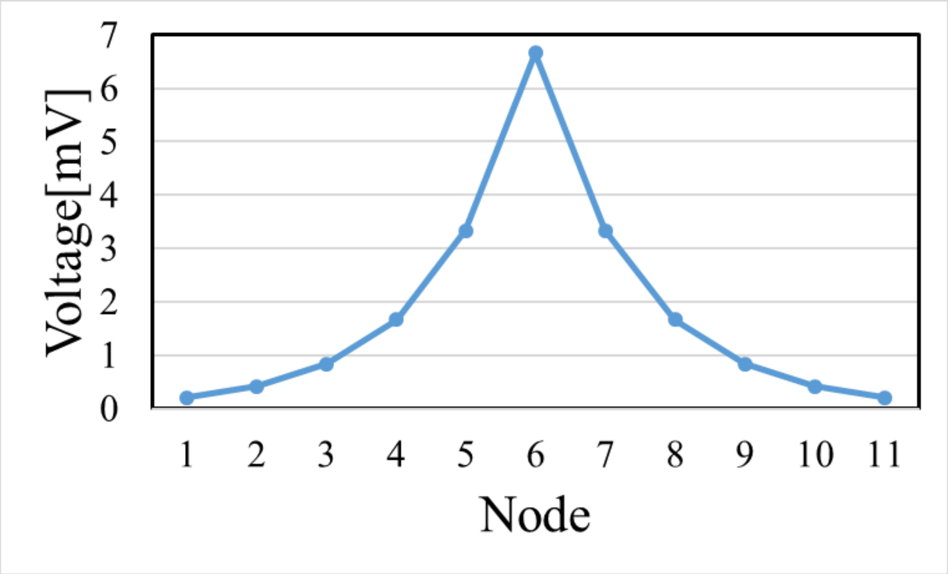


R1, RT may be different

→ Non-uniform network

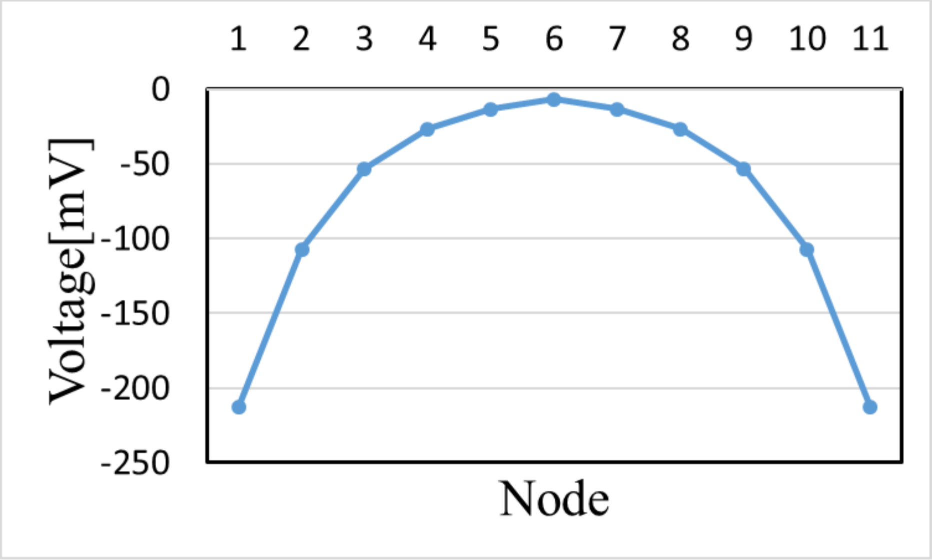
$R_0 = 2\mathrm{k}\Omega, R_1 = 1\mathrm{k}\Omega,$ I =0.01mA

Spatial Impulse Response



$R_T = 1\mathrm{k}\Omega$

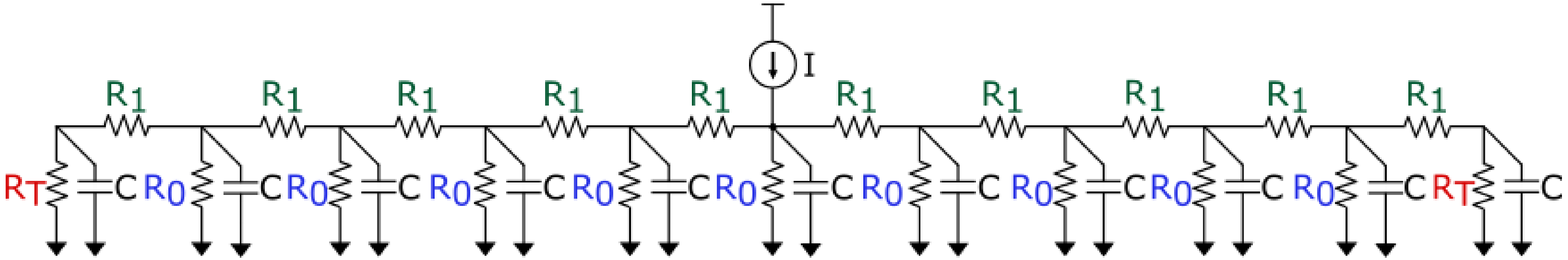
Well behaved



$R_T = -2\mathrm{k}\Omega$

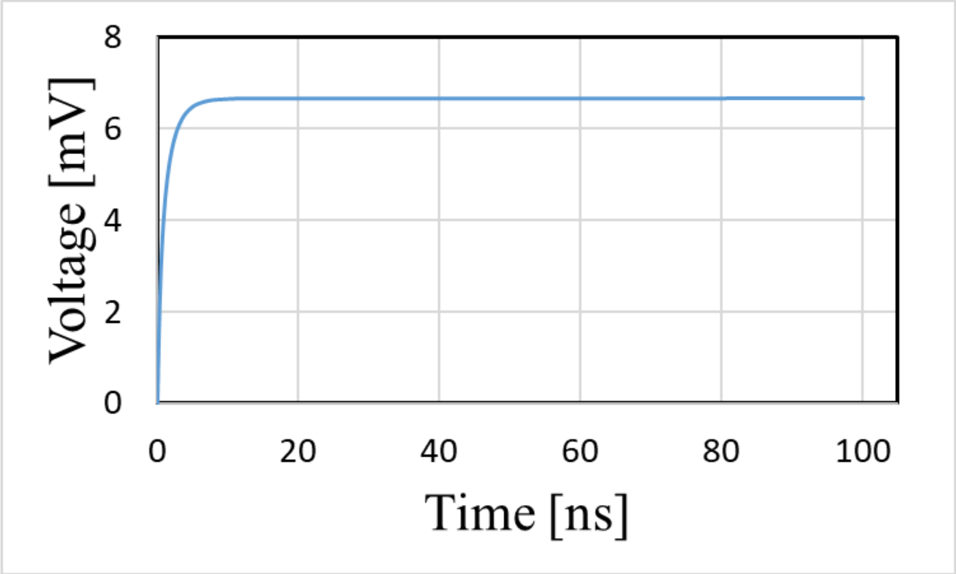
Violently behaved

Temporal Dynamics of Non-Uniform Network



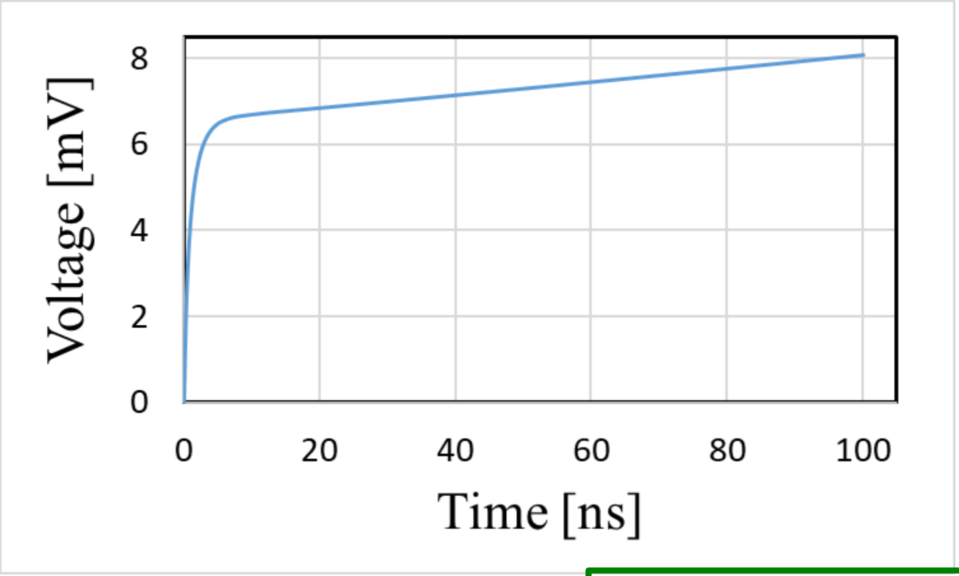
$R_0 = 2\mathrm{k}\Omega$, $R_1 = 1\mathrm{k}\Omega$, $C = 1\mathrm{pF}$, Step I =0.01mA

Step response at the center node.



$R_\mathrm{T} = 1\mathrm{k}\Omega$

Temporally stable



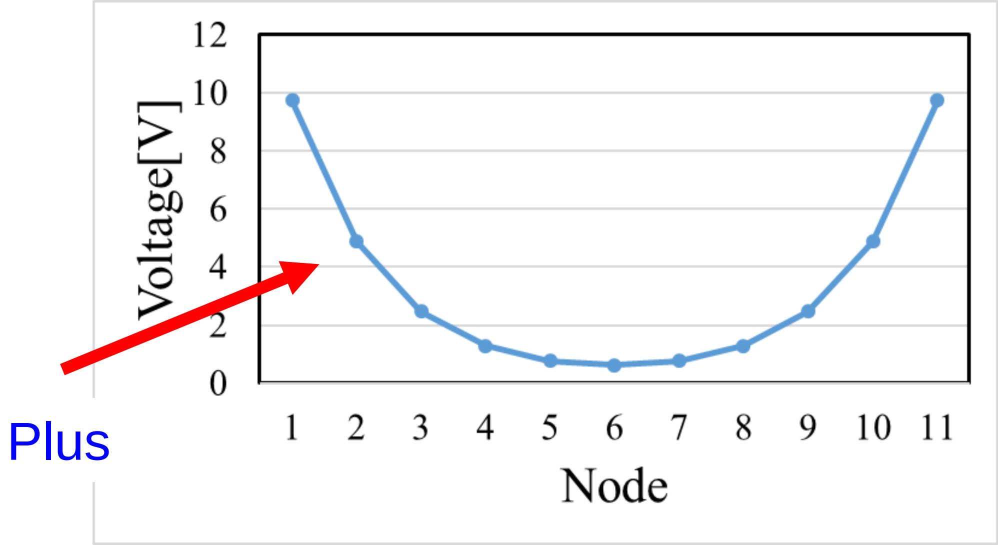
$R_\mathrm{T} = -2\mathrm{k}\Omega$

Temporally unstable

Boundary Condition

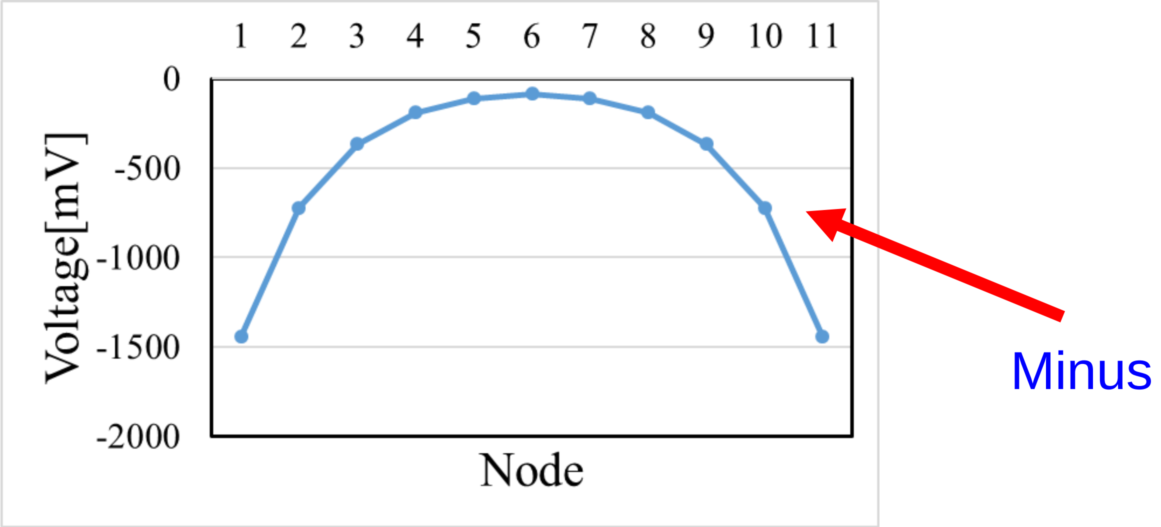
$R_0 = 2\mathrm{k}\Omega$, $R_1 = 1\mathrm{k}\Omega$, $C = 1\mathrm{pF}$, Step I =0.01mA

$R_\mathrm{T} = -2.006\mathrm{k}\Omega$



- Modestly well behaved spatial impulse response
- Temporally stable

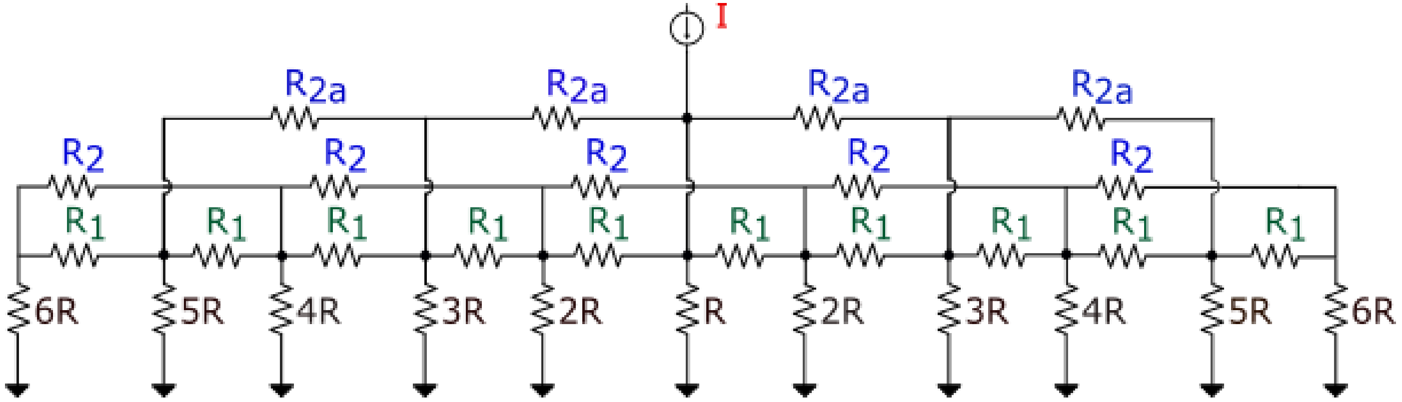
$R_\mathrm{T} = -2.005\mathrm{k}\Omega$



- Violently behaved spatial impulse response
- Temporally unstable

Close relationships between spatial and temporal dynamics

General Non-Uniform Network

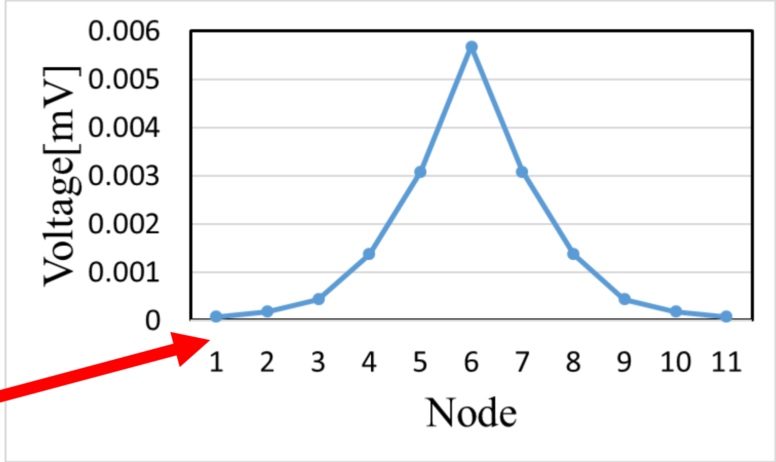


$R_2 = -4\text{k}\Omega$

$R_1 = 1\text{k}\Omega$

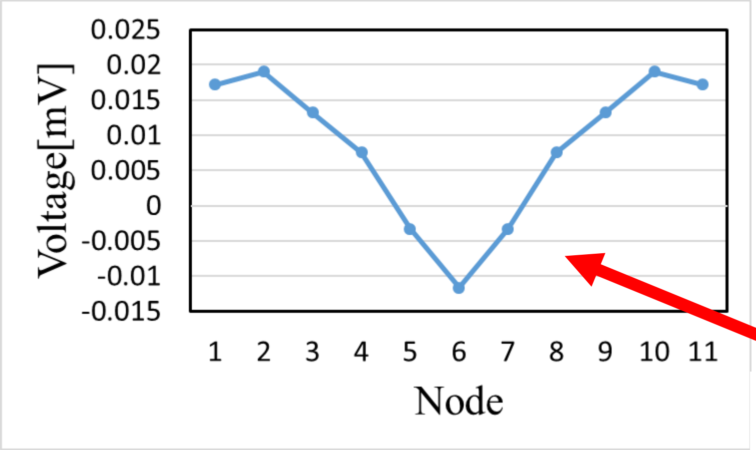
$R = 1\text{k}\Omega$

$R_{2a} = -10\text{k}\Omega$



Plus

$R_{2a} = -1\text{k}\Omega$

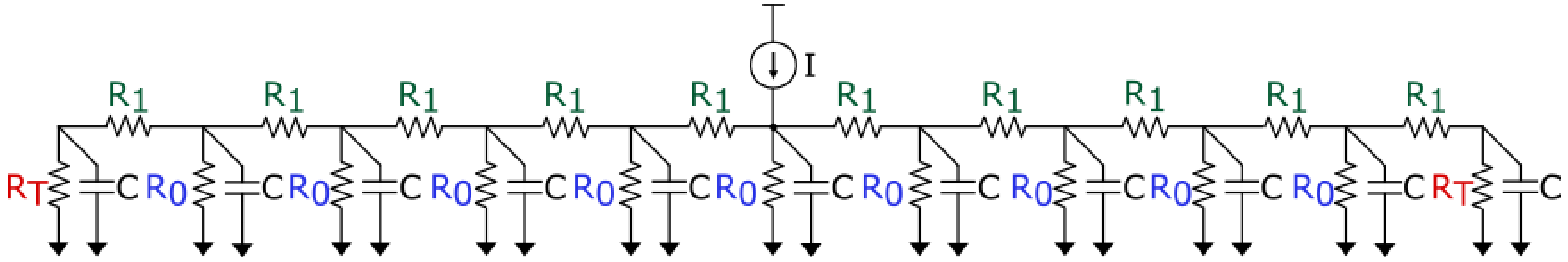


Minus

- Modestly well behaved spatial impulse response
- Temporally stable

- Violently behaved spatial impulse response
- Temporally unstable

Theoretical Analysis of Non-Uniform Network



Kirchhoff Current Law at node *k*

$$C\frac{\mathrm{d}}{\mathrm{d}t}v_k = -g_0 v_k + g_1(v_{k+1} - v_k) + g_1(v_{k-1} - v_k)$$

Here $g_0 = 1/R_0,\quad g_1 = 1/R_1$

State Equation

$$c\frac{d}{dt}\boldsymbol{v} = \boldsymbol{A}\,\boldsymbol{v} + \boldsymbol{i}$$

$$\boldsymbol{v} = (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11})$$

$$\boldsymbol{i} = (0,0,0,0,0,I,0.0,0,0,0)$$

$$\boldsymbol{A} = \begin{pmatrix} y_T & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_1 & y & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_1 & y & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & y & g_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_1 & y & g_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & y & g_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_1 & y & g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_1 & y & g_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1 & y & g_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1 & y & g_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1 & y_T \end{pmatrix}$$

$$y = -g_0 - 2g_1, \qquad y_\mathrm{T} = -g_\mathrm{T} - g_1.$$

Theoretical Analysis of Non-Uniform Network (2)

State Equation $c\frac{d}{dt}\boldsymbol{v} = \boldsymbol{A}\,\boldsymbol{v} + \boldsymbol{i}$

- Spatial impulse response

$$0 = \boldsymbol{A}\,\boldsymbol{v} + \boldsymbol{i}$$

$$\boldsymbol{v} = -\,\boldsymbol{A}^{-1}\boldsymbol{i} = -\boldsymbol{D}\,\boldsymbol{\Lambda}^{-1}\,\boldsymbol{D}^{-1}\,\boldsymbol{I}$$

Here
$\boldsymbol{\Lambda}$ = diag $(\lambda_1, \lambda_2, \lambda_3, .., \lambda_{11})$, $\boldsymbol{\Lambda}^{-1}$ = diag $(\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1}, .., \lambda_{11}^{-1})$
$\boldsymbol{D}$: array of eigenvectors.

- Temporal stability

⇕

$\boldsymbol{A}$ is negative definite

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Conclusion

Circuit network theorem:

Equivalence between spatial and temporal stabilities for uniform network with negative resistors

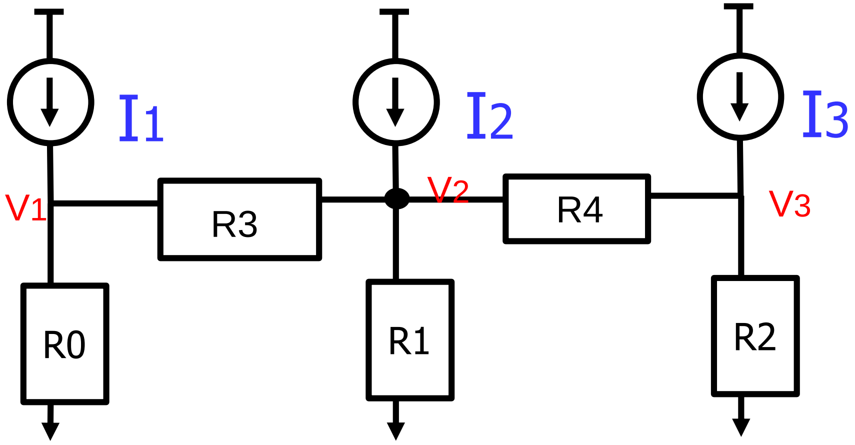
↓ Generalization

This research has shown

Close relationships between spatial and temporal dynamics for non-uniform network with negative resistors

Appendix

State Equation Derivation



$$g_0 = 1/R_0$$

$$g_1 = 1/R_1$$

$$g_2 = 1/R_2$$

$$g_3 = 1/R_3$$

$$g_4 = 1/R_4$$

Kirchhoff Current Law at each node

At node V1

$$I_1 = (V_1 - V_2)g_3 + V_1 g_0$$

At node V2

$$I_2 = (V_2 - V_1)g_3 + V_2 g_1 + (V_2 - V_3)g_4$$

At node V3

$$I_3 = (V_3 - V_2)g_4 + V_3 g_2$$

$$I_1 = (g_0 + g_3)V_1 + (-g_3)V_2 + 0$$
$$I_2 = -g_3 V_1 + (g_1 + g_3 + g_4)V_2 - g_4 V_3$$
$$I_3 = 0 + (-g_4)V_2 + (g_2 + g_4)V_3$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} g_0 + g_3 & -g_3 & 0 \\ -g_3 & g_1 + g_3 + g_4 & -g_4 \\ 0 & -g_4 & g_2 + g_4 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$