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Signal Estimation by Prony's Method for Application to ADC Testing

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Abstract. The Prony's method is an algorithm that can estimate the parameters of the original wave from a minimum of four sampling points. With the aim of applying the Prony's method to ADC testing, especially for on-line/field testing which requires simple hardware and software, this study investigated the effect of noise on the estimation accuracy of the Prony's method. When the noise is large, meaningful values cannot be obtained because the root in the middle of the Prony's method algorithm cannot be calculated. The limits of the noise magnitude for Prony's method were found by simulation. In the range of the noise magnitude that the root can be calculated, the error in the estimate value increased exponentially as the noise magnitude increased.

1. Introduction

Testing techniques are becoming very difficult and important for high-performance analog integrated circuits [1,2]. One technique for estimating the frequency of signals is the FFT, which is particularly often used when strict coherent conditions cannot be created, as the Hanning window allows accurate spectrum estimation with a single FFT.

FFT has good spectral measurement accuracy, but requires a large number of sample values. An algorithm that can estimate frequencies from a small number of sample values is the Prony's method, which originally arose from an attempt to express experimental data as a linear sum of several exponential functions in order to calculate the rate of expansion of a gas [3]. The Prony's method can be utilized to measure the velocity, noise power and time of passage of a constant velocity moving source and can be applied to the location estimation of stationary noise sources [4]. Although the Prony's method is a beautiful algorithm, it is said to be unable to estimate frequencies accurately when the signal contains noise [5]. In this study, the effect of noise on estimation accuracy is investigated for application to ADC on-line/field testing.

2. Principle of Prony's Method

The amplitude, frequency, initial phase and DC bias of a sinusoidal signal are $\boldsymbol{A}$, $\boldsymbol{f}$, $\boldsymbol{\theta_0}$ and $\boldsymbol{d}$ respectively. If the sampling frequency is $\boldsymbol{f_s}$, the sampling value of the n-th point is as follows:

$$x(n) = A\cos\left\{\frac{2\pi f n}{f_s} + \theta_{\mathbf{0}}\right\} + d \tag{1}$$

Equation (1) can be rewritten using conjugate complex numbers as follows

$$x(n) = mZ_1^n + m^* Z_1^{*n} + d \qquad (n = 0, 1, 2, \ldots) \tag{2}$$

where the conjugate complex number of Z is written as Z^*. m and Z_1 are respectively as follows:

$$m = \frac{A}{2}\exp j\theta_{\mathbf{0}}$$
$$Z_1 = \exp j\delta \qquad (\delta = \arg Z_1 = \frac{2\pi f}{f_s}) \tag{3}$$

In other words, the parameters determining the sinusoidal signal (A, f, and θ) can be determined by m and the complex number Z_1 on the unit circle.

Here, consider the following polynomial $P(n)$:

$$P(n) = (Z - Z_1)(Z - Z_1^*)(Z - 1) \tag{4}$$
$$= Z^3 - aZ^2 + aZ - 1 \tag{5}$$

where

$$a = 2\,\mathrm{Re}[Z_1] + 1 \tag{6}$$

The real part of the complex number Z is denoted by $\mathrm{Re}[Z]$ and the imaginary part by $\mathrm{Im}[Z]$.

Using the coefficients a of the polynomial (5), consider the following equation:

$$x(n+3) - ax(n+2) + ax(n+1) - x(n)$$

The polynomial (5) has Z_1 and Z_1^* as solutions as defined in Eq. (4). In other words, Eq. (7) holds. Transforming Eq. (7) yields Eq. (8). Equation (8) implies that a can be calculated from the values of four sampled points.

$$x(n+3) - ax(n+2) + ax(n+1) - x(n) = 0 \tag{7}$$
$$a = \frac{x(n) - x(n+3)}{x(n+1) - x(n+2)} \tag{8}$$

From a, Eq. (3) and (6), the following can be obtained:

$$\mathrm{Re}[Z_1] = \frac{a-1}{2} \tag{9}$$
$$\mathrm{Im}[Z_1] = \sqrt{1 - (\mathrm{Re}[Z_1])^2} \tag{10}$$
$$Z_1 = \mathrm{Re}[Z_1] + j\,\mathrm{Im}[Z_1] \tag{11}$$
$$f = f_s \arg\frac{Z_1}{2\pi} \tag{12}$$

In summary, the frequency f can be estimated by substituting the four sampling points $x(n)$, $x(n+1)$, $x(n+2)$, $x(n+3)$ into Eq. (8) and calculating Eqs. (9), (10), (11) and (12) in turn.

Note five or more sample values can be obtained, a in Eq. (8) is obtained by the least-squares method. Other parameters (amplitude A, initial phase θ_0) can also be obtained. As this paper is concerned with the estimation of only the frequency f using the lowest number of sample values, the details are omitted.

3. Problem Setting

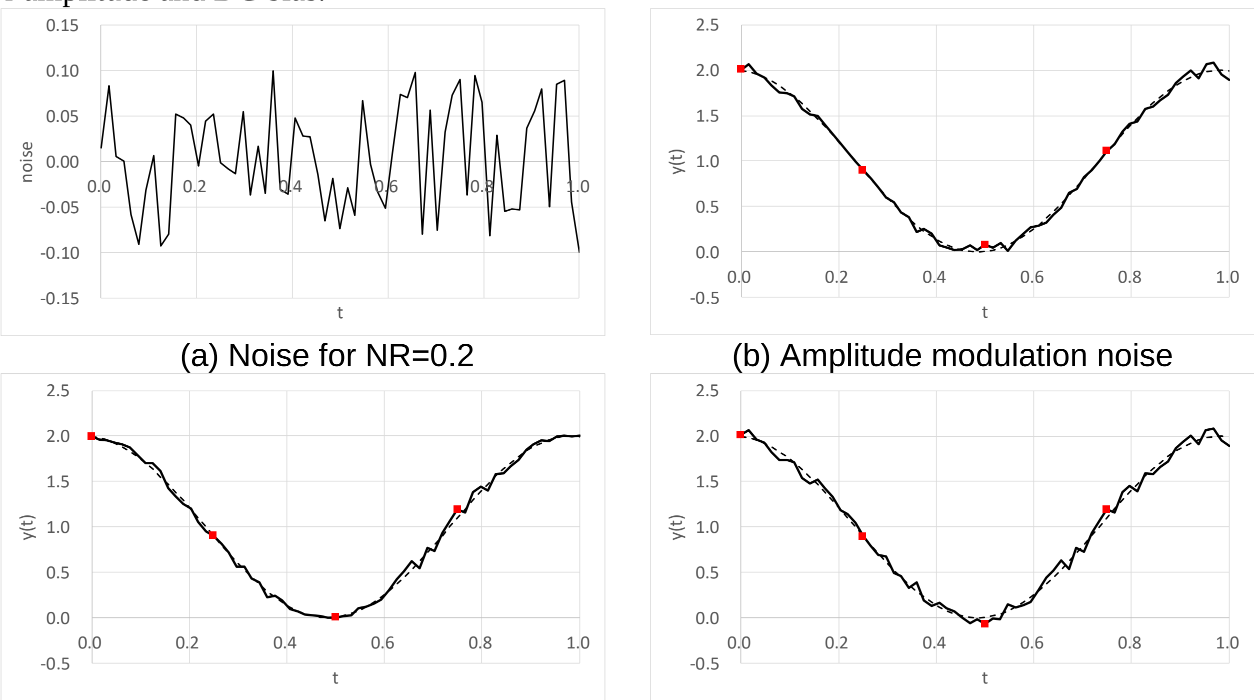
When estimating waveform parameters using the Prony's method, the estimation accuracy deteriorates if the sampling values are mixed with noise. There are three types of noise: $a(t)$ in equation (13) is amplitude modulation noise, $\theta(t)$ is phase noise and $c(t)$ is additive noise. This study investigates the effect of the type and magnitude of noise on the estimation accuracy.

$$y(t) = A\{1 + a(t)\}\cos\left\{\frac{2\pi f t}{f_s} + \theta_0 + \theta(t)\right\} + d\{1 + c(t)\} \tag{13}$$

The magnitude of the noise is defined as the range over which the sample values vary. It is referred to as the "Noise Ratio (NR)" in this paper. For example, "additive noise, NR=0.2" means that sample values vary within the range of Eq. (14).

$$y(t) = A\{1 + a(t)\}\cos\left\{\frac{2\pi f t}{f_s} + \theta_0 + \theta(t)\right\} + d\{1 \pm 0.1\} \tag{13}$$

A sine wave with amplitude 1.0, initial phase 0.1, DC bias 1.0 and frequency 1.0 was used as the "original waveform". A random number sequence generated using the standard C rand function is superimposed as noise. Simulations were carried out in the range NR=0.00, 0.01, ⋯ and 1.00. An example of noise for NR=0.2 and its superimposition on the "original wave" in the three different ways shown in equation (13) is shown in Fig. 1. The units are radian for phase, and dimensionless values for amplitude and DC bias.



(a) Noise for NR=0.2 (b) Amplitude modulation noise

(c) Phase noise (d) Additive noise

Fig. 1. An example of noise and sine waves with three kinds of superimposed noises. In (b)(c)(d), the broken line is the "original wave", the solid line is the noise-superimposed wave and the four red squares are sample points.

4. Results

1000 different patterns of noise were created (Fig. 1(a) shows an example). The process of estimating f from four sample values using Eqs. (8)-(12) was repeated 1000 times.

If $|\mathrm{Re}[Z_1]| > 1$ in Eq.(10), the root cannot be calculated (In C language, "NaN (Not a Number)" is returned). The number of times of NaN in 1000 times is shown as a percentage in the Fig. 2. Naturally, the larger the noise magnitude, the higher the probability of a NaN being returned.

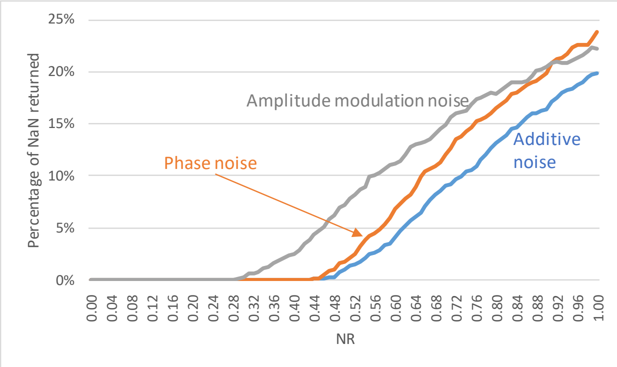
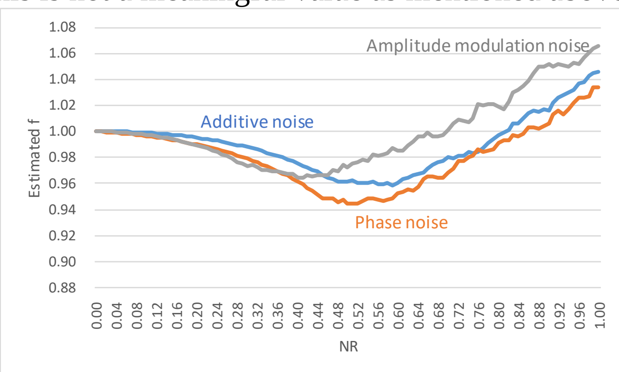


Fig. 2. Simulation results (the number of times of NaN in the process of estimating f 1000 times). The horizontal axis is the magnitude of the noise and the vertical axis is the percentage of times a NaN was returned.

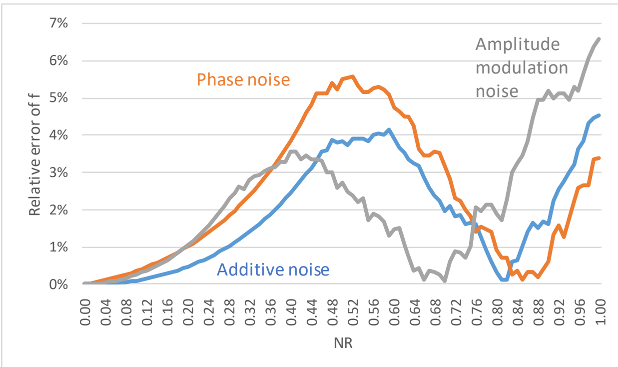
The estimated f and the deviation from the original wave ($f = 1.0$) are shown in Fig. 3, which is the arithmetic mean of 1000 estimates.

Note the case of NaN is excluded. For example, in the case of additive noise, according to Fig. 2, when NR=0.60, NaN is returned about 41 times (4.1% of 1000 times), that is the estimated f when NR=0.60 in Fig. 3(a) is the arithmetic mean of 959 estimation. If there are more NaN, i.e. NR > 0.30 for amplitude modulation noise, NR > 0.45 for phase noise and NR > 0.48 for additive noise, there is less confidence in the estimation of f.

In the range of NR where NaN do not appear, according to Fig. 3(a), f gradually decreases from the original value ($f = 1.0$). According to Fig. 3(b), the relative error increases exponentially, with Amplitude modulation noise, phase noise and additive noise having larger errors in that order; when NaN starts to appear, f becomes larger. There is a point of NR where the relative error is zero, but this is not a meaningful value as mentioned above.



(a) Estimated f



(b) Absolute value of the relative error of estimated f and original f (=1.0)

Fig. 3. Simulation results (estimated f).
The horizontal axis is the magnitude of the noise.

5. Conclusion

This study focused on the Prony's method as a method for estimating the original waveform using the sample values obtained by sampling, and investigates the effects of additive noise, phase noise and amplitude modulation noise on the measured values. When the noise is large, meaningful values cannot be obtained because the root in the middle of the Prony's method algorithm cannot be calculated. The limits of the noise magnitude for Prony's method were found. In the range of the noise magnitude that the root can be calculated, the error in the estimate value increased exponentially as the noise magnitude increased. As what could be carried out in this paper is only a qualitative study, quantitative evaluation and theoretical analysis should be the subject of future work.

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