Signal Estimation by Prony's Method for Application to ADC Testing

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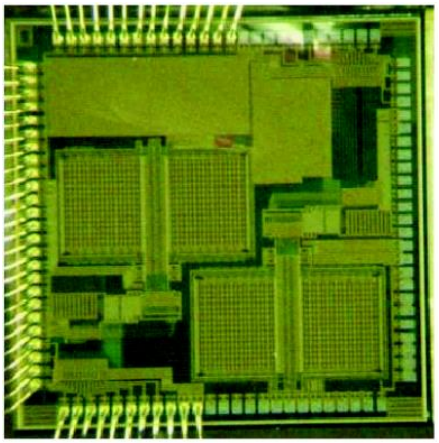
Research Background: ADC Testing

For high performance analog-to-digital converters (ADCs), their low-cost, high-quality testing techniques are becoming more important.



ADC Chip

LSI Tester
for mass production at shipping stage

FFT and ADC Testing

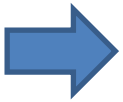
For ADC testing, Fast Fourier transform (FFT) is used

for signal frequency estimation.

-Advantage: Accurate estimation of output spectrum.

Window function can be used even if not coherent sampling

- Disadvantage: A large number of sample values are required.



Suitable for mass production testing at shipping stage

Not suitable for on-line/field testing

Prony Method

- Prony method [1]

  -Advantage: Use only a small number of sampled data for original signal estimation.

  -Disadvantage: Not very accurate estimation

[1] C. Prony, F.M. Gaspard and R. De Baron, “Essai Experimental et Analytique”. ***Journal de l'école Polytechnique de Paris***, Vol. 1, pp. 24-76 1795.

Research Objective

- With the aim of applying the Prony's method to ADC on-line/field testing, this study investigates the noise effect on its estimation accuracy.

※ On-line testing system for ADC must be simple.

Our Approach

- Find the limit of noise magnitude
  to obtain meaningful estimation.

- Clarify the relationship between noise magnitude and estimation accuracy when the noise is smaller than limit.

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Waveform Under Estimation

Cosine wave input

- Input signal frequency: f
- Sampling frequency: fs
- Sine wave parameters:

  A: Amplitude, θ: Initial phase, d: DC offset

- Specimen value of point at n

$$x(n) = A\cos\left(\frac{2\pi f n}{f_s} + \theta\right) + d$$

Principle of Prony's Method

- Replace 4 points; x (n), x (n+1), x (n+2), x (n+3)

$$a = \frac{\{x(n) - x(n+3)\}}{\{x(n+1) - x(n+2)\}}$$

$$z_r = \frac{(a-1)}{2}$$

$$f = f_s \frac{\left\{ \arg\left(z_r + j\sqrt{(1 - {z_r}^2)} \right) \right\}}{2\pi}$$

Estimate *f* from 4 points of sampled values

Principle of Prony's Method

Calculation from 4 points; x (n), x (n+1), x (n+2), x (n+3)

$$a = \frac{\{x(n) - x(n+3)\}}{\{x(n+1) - x(n+2)\}}$$

$$z_r = \frac{(a-1)}{2}$$

$$f = f_s \frac{\left\{ \arg\left(z_r + j\sqrt{(1 - {z_r}^2)} \right) \right\}}{2\pi}$$

Must be positive or zero

If negative, fetal error: NaN (Not a Number)

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Problem Setting

- “Original waveform”

  Sine wave with

  Frequency: 1.0

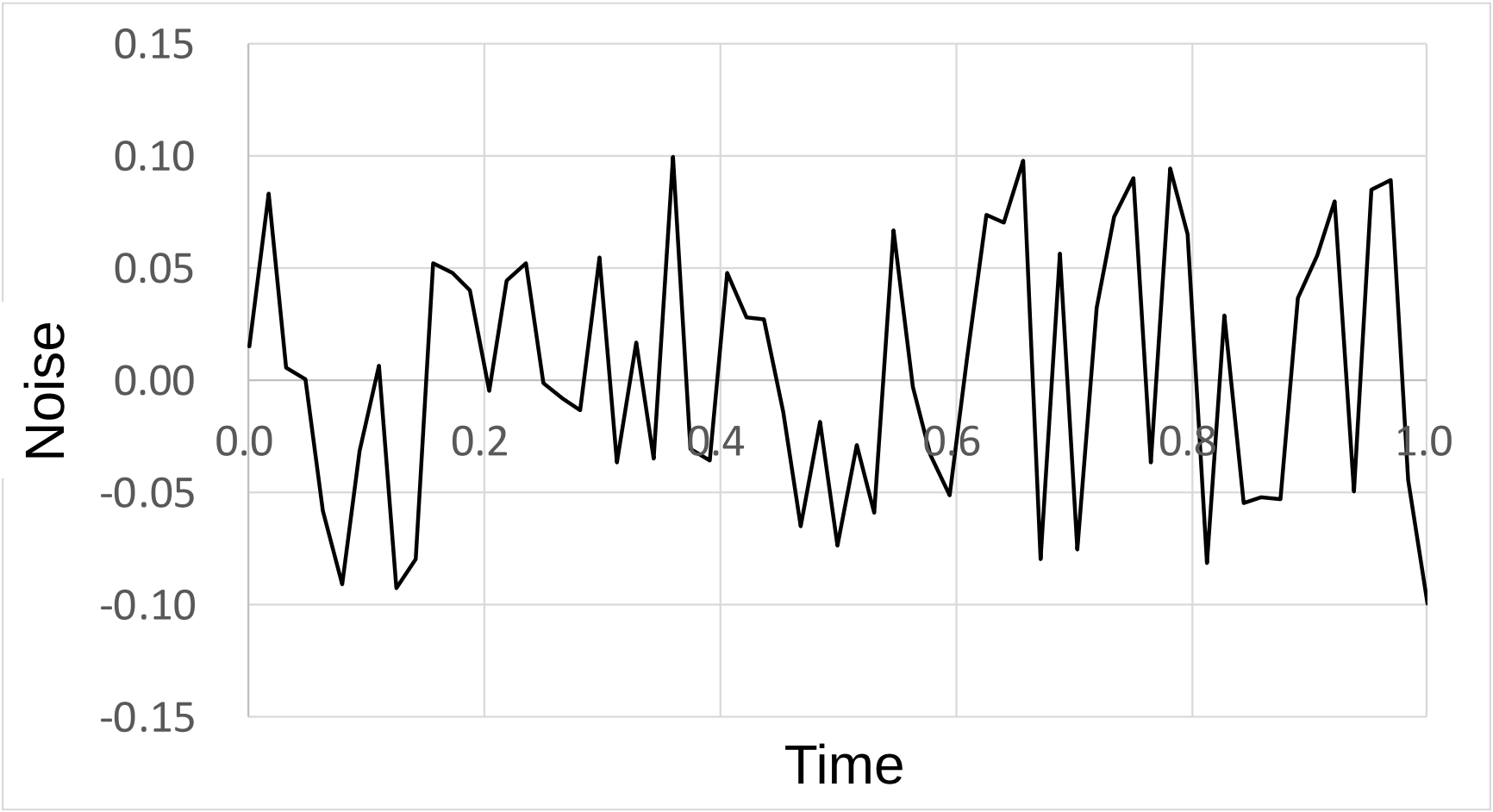
  Amplitude: 1.0

  Initial phase: 0.1

  DC component: 1.0
- Number of Samples: 4
- Superimposed “Noise”

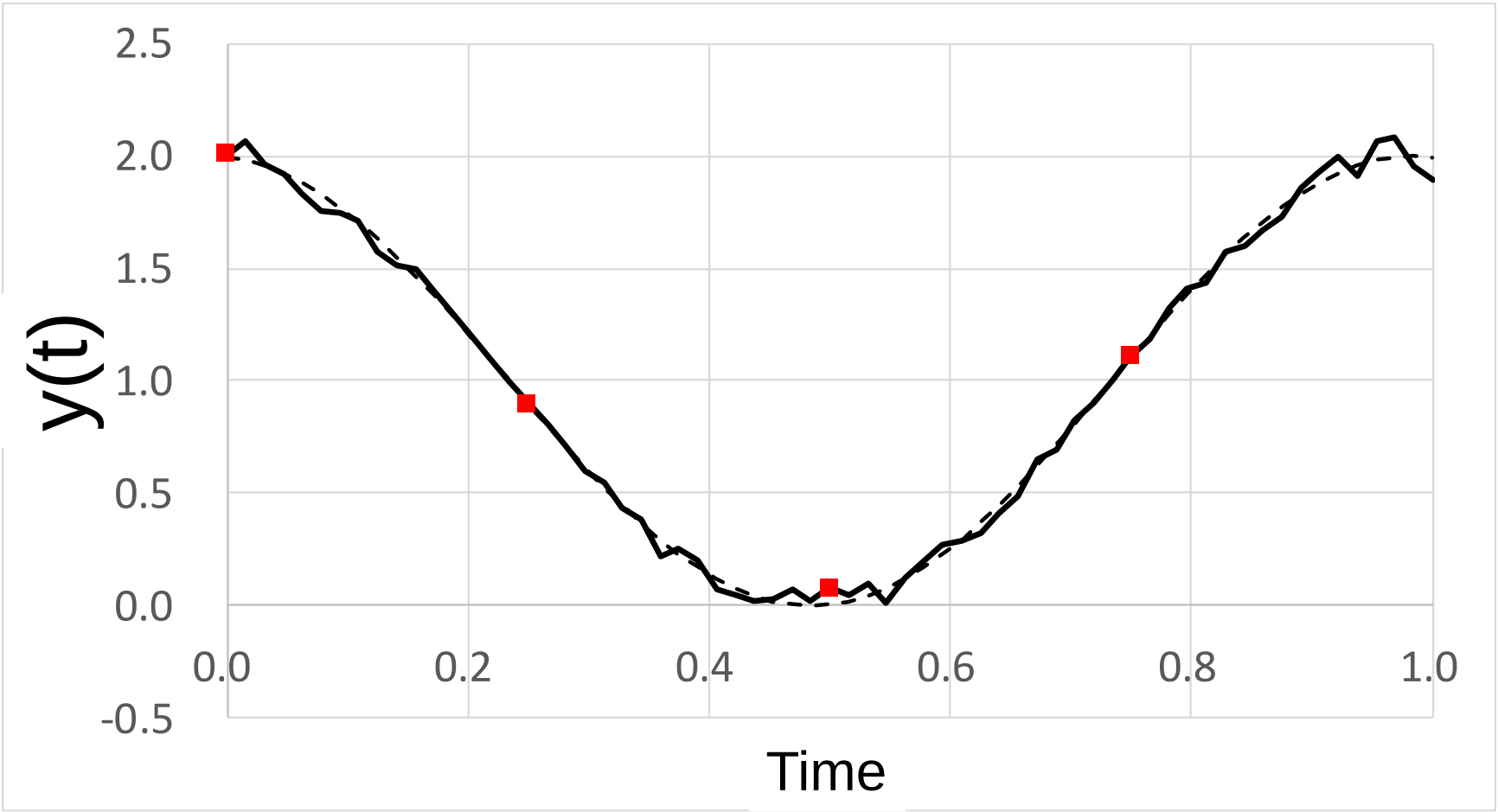
  A random number sequence generated by standard C rand function

Noise Superimposed to Original



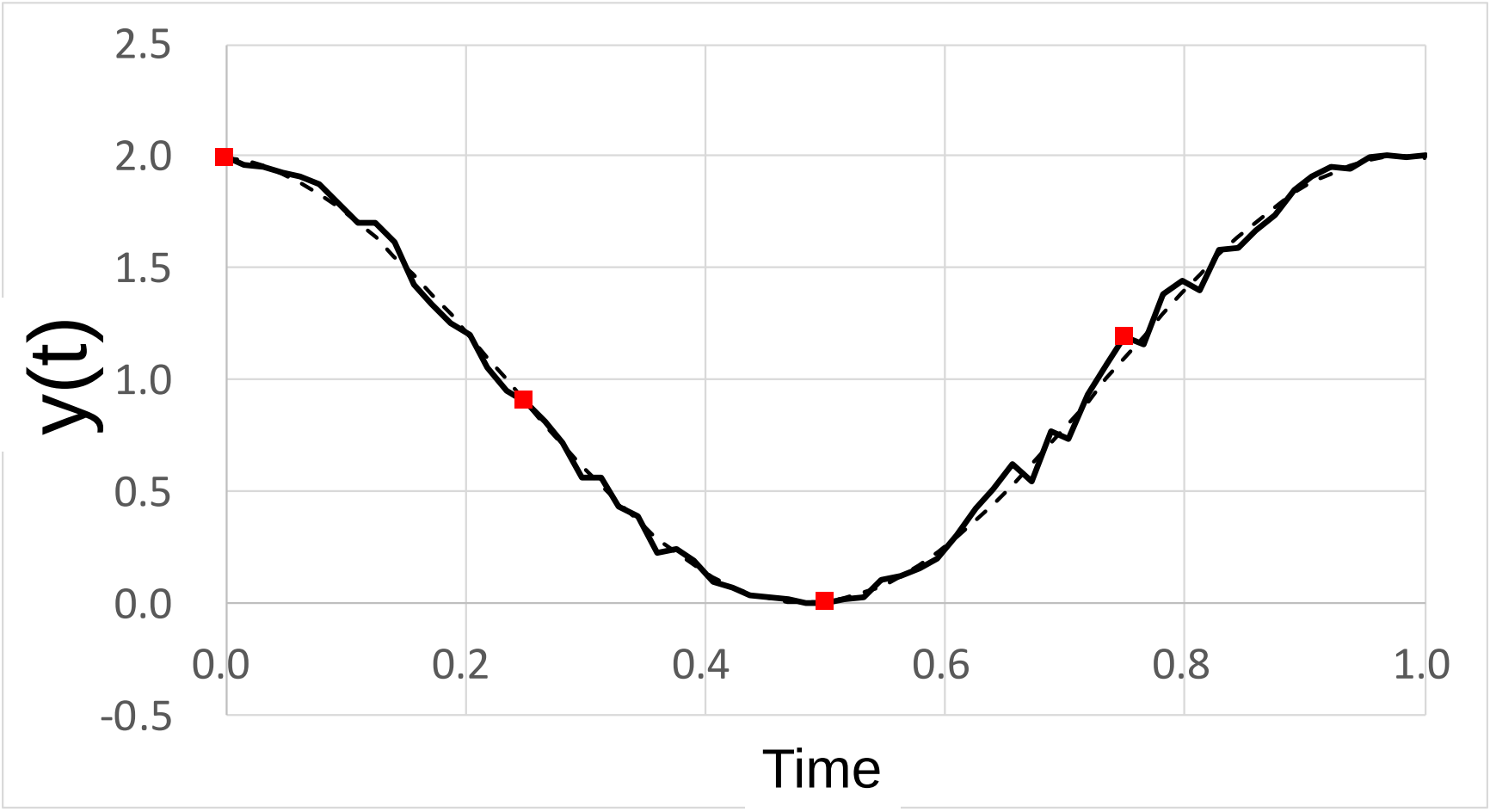
Noise for NR (Noise Ratio)=0.2

Amplitude Modulation Noise



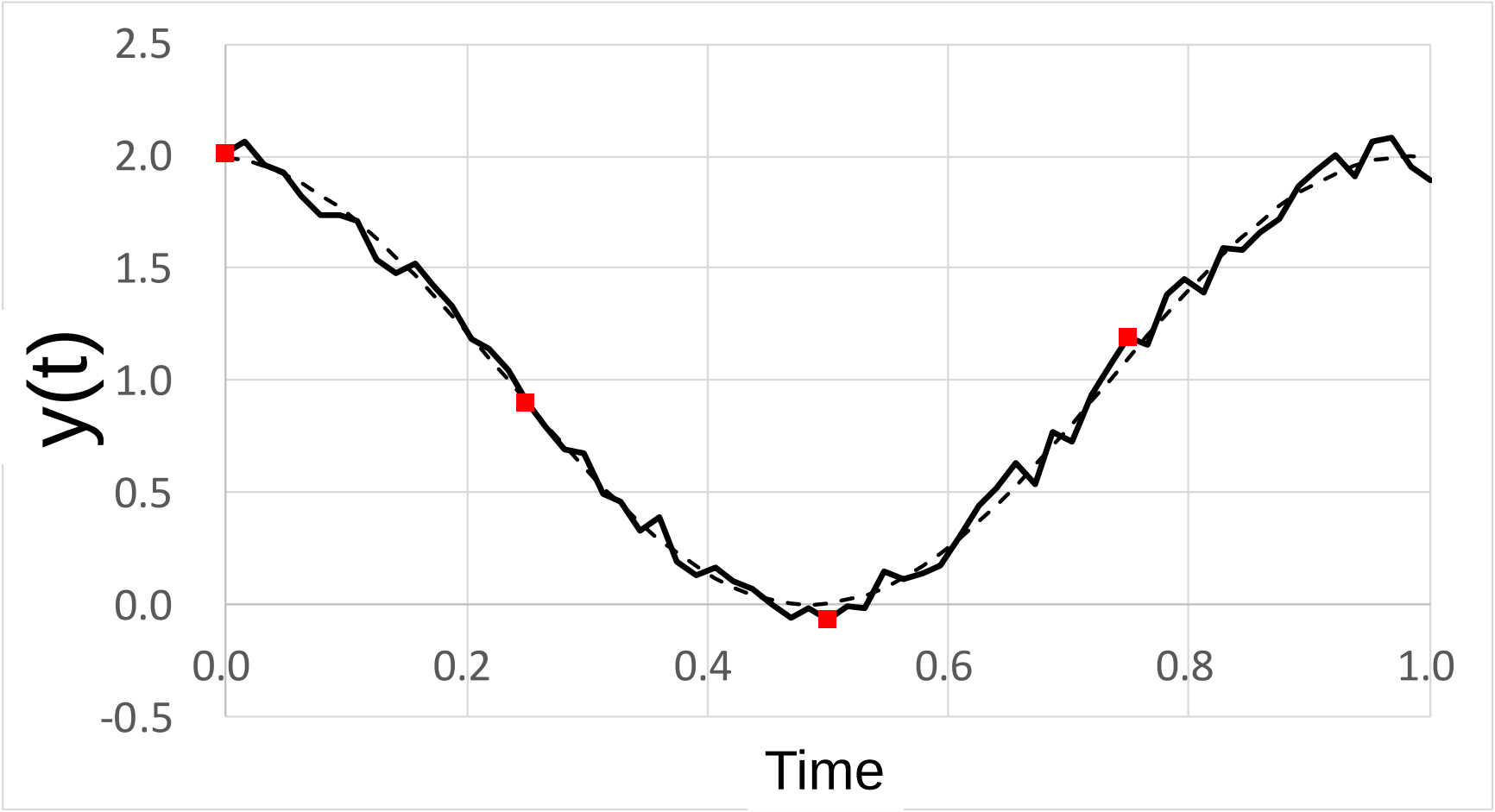
$$y(t) = A\{1 + a(t)\}\cos\left\{\frac{2\pi f t}{f_s} + \theta_0\right\} + d$$

Phase Noise



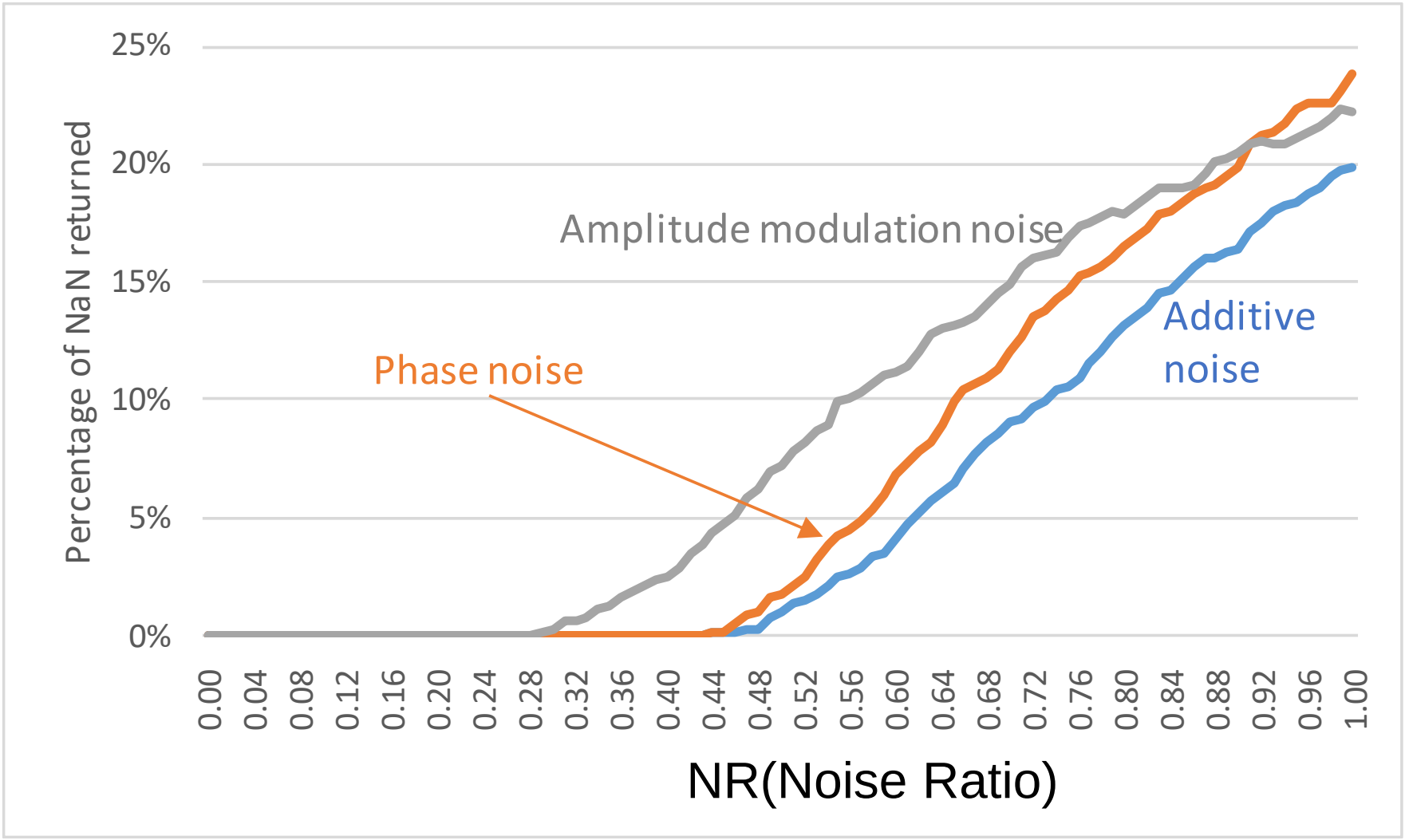
$$y(t) = A\cos\left\{\frac{2\pi f t}{f_s} + \theta_0 + \theta(t)\right\} + d$$

Additive Noise



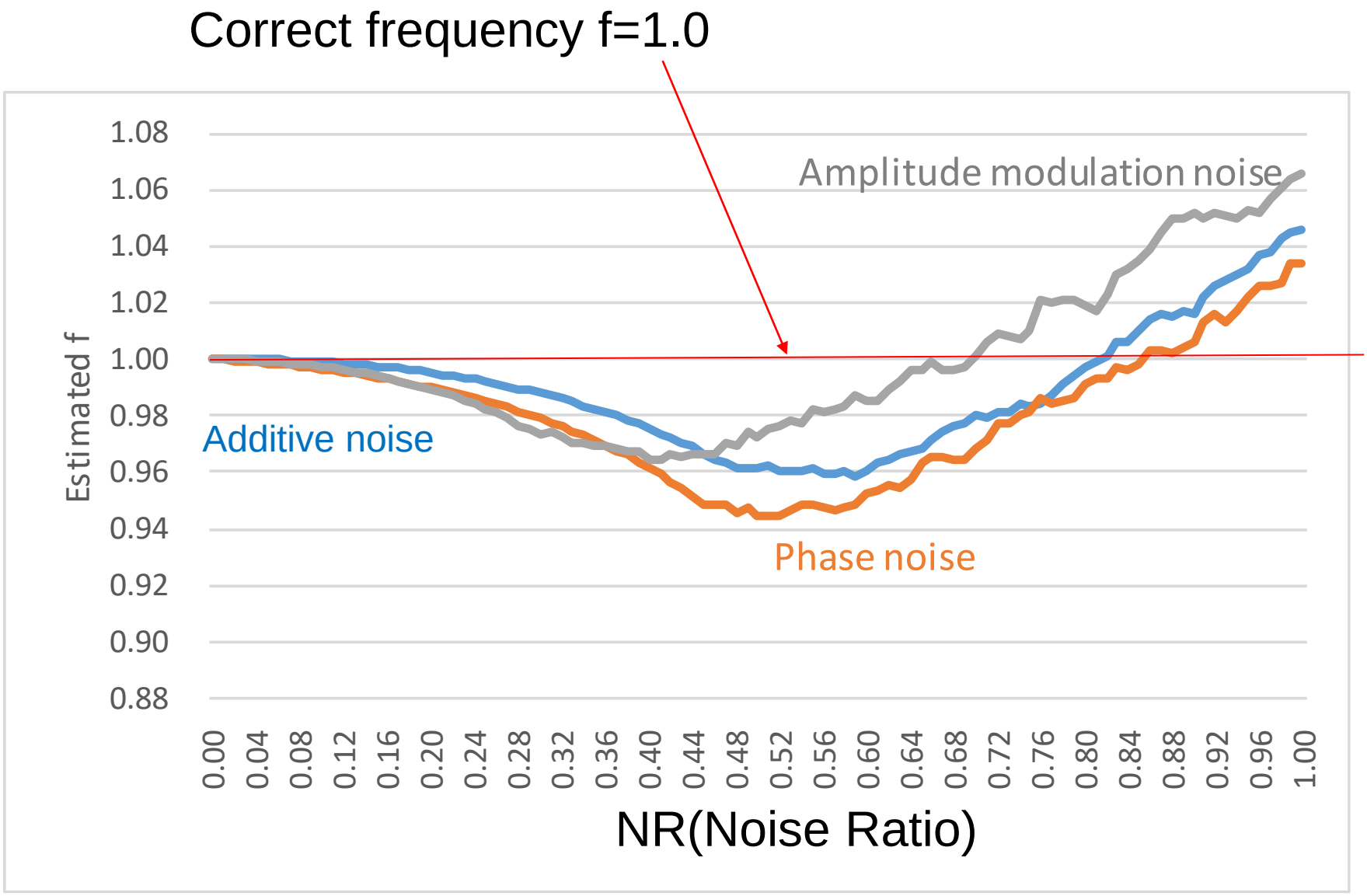
$$y(t) = A\cos\left\{\frac{2\pi f t}{f_s} + \theta_0\right\} + d\{1 + c(t)\}$$

Fatal Estimation Error

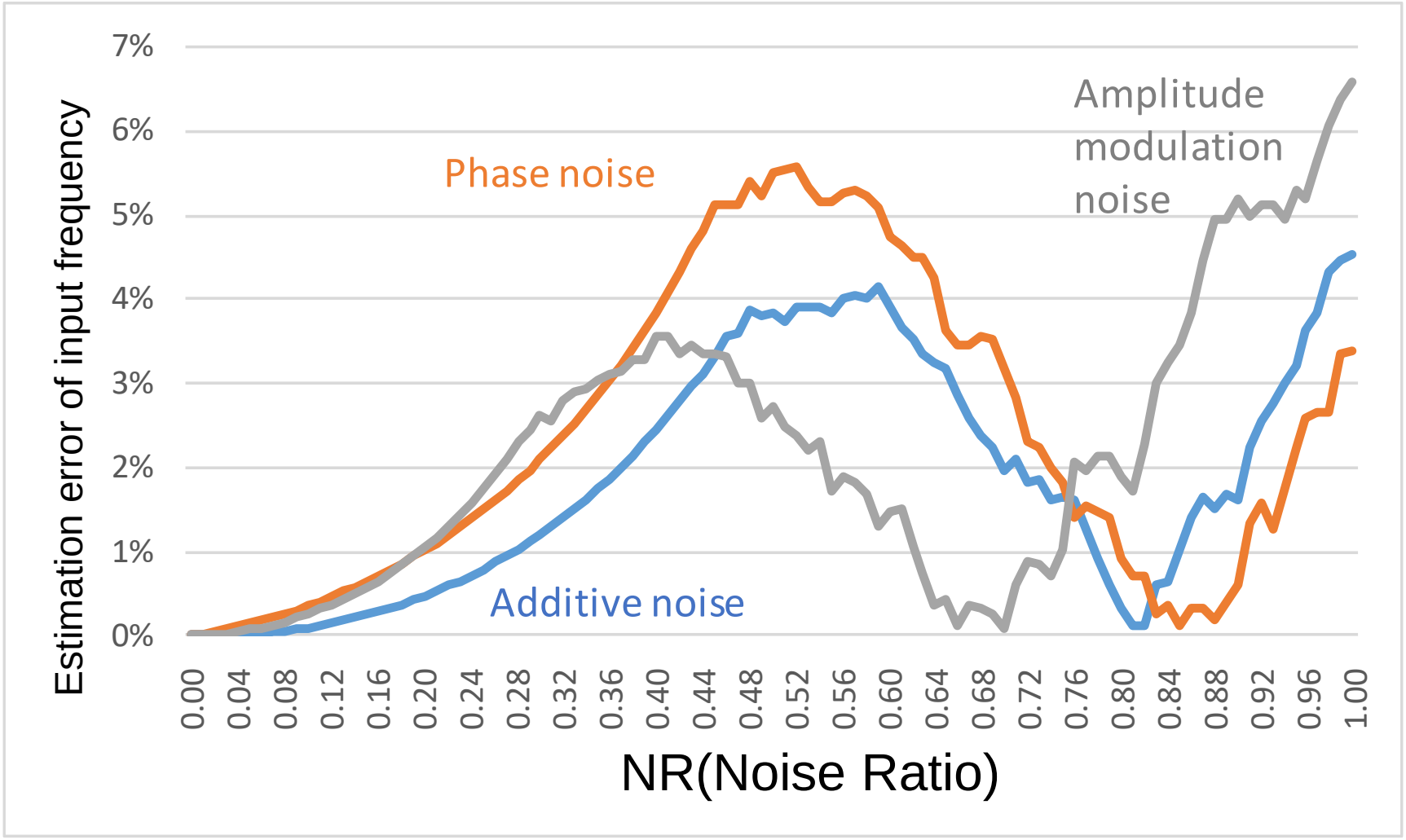


Number of times of NaN (Not a Number) for estimating 1000 times

Estimation of Input Frequency



Estimation Error of Input Frequency



Absolute value of error between estimated and original frequency

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Conclusion

- Focused on Prony's method for estimating original waveform using a few sampled values, which is very simple.
- Investigated effects of additive noise, phase noise and amplitude modulation noise
  - When noise is large, meaningful values cannot be obtained because the root in the algorithm cannot be calculated.
  - Limits of noise magnitude for Prony's method were found.
  - Below the limits, estimation error increases exponentially as the noise magnitude increases.
- Next work:

We will show quantitatively that Prony's method is applicable for on-line/field ADC testing because it requires only a small number of sampled data.

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Thank you for lattention

謝謝

Merci de votre attention