

Conjecture on Spatial-Temporal Response Relationship for Spatially Shift-Variant Networks with Positive and Negative Resistors

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Abstract. This paper discusses the spatial and temporal response of *spatially shift-variant* (non-uniform) networks with positive and negative resistors. Response of general spatially shift-variant networks whose resistor components are different from each other is investigated with extensive simulation and the following conjecture is obtained: The network is temporally unstable if there is a node where the input current is injected and its node voltage as the spatial impulse response is negative. Also, its reason is discussed.

1. Introduction

Resistor networks are widely used in analog and mixed-signal circuits, such as DACs, ADCs and vision chips [1-6]. This study is motivated by the vision chip in [6] which the network consists of positive and negative resistors (Fig. 1); we found there that when the negative resistance effect becomes large, the resistor network in the chip becomes unstable. It was shown in [7-10] that there are two concepts of the network stability: spatial and temporal, and both conditions are equivalent. However there only the shift-invariant network with infinite size [7, 9] or circulant configuration [8] is considered, and its termination resistor effect is not handled. Previous works in [7-10] considered that the spatial stability condition is “all poles Z_p of the non-causal spatial transfer function of the network satisfy $|Z_p| \neq 1$ ” [11] and showed that it is equivalent to the temporal stability condition.

In this paper, we consider a shift-variant resistive network where each node has resistors connection to the k nearest neighbor nodes ($k=1, 2, 3, \dots$) and also to the ground [7, 8], and their values can be different in the network. In our previous paper [12], it was shown that both spatial and temporal network behaviors have some relationships even for the shift-variant network; when it is temporally unstable, its spatial response behaves in a violent manner. In this paper, we further investigate their relationships and consider a conjecture through simulations.

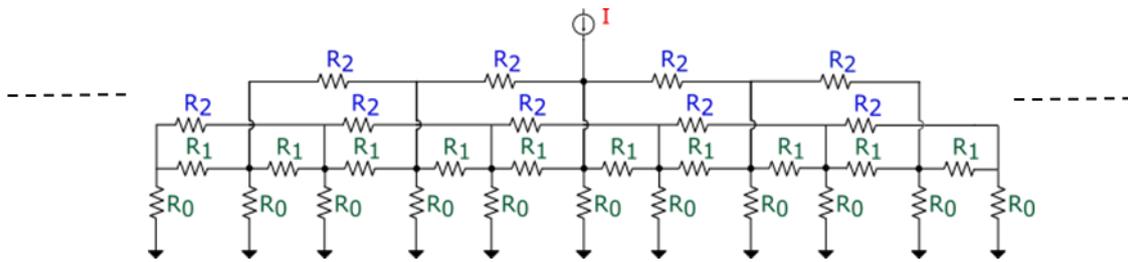


Fig.1. Active resistor network composed of R_0 , R_1 , R_2 , where R_2 is a negative resistor with $R_2 = -4R_1$ [6].

2. Conjecture on Spatial and Temporal Response of Active Resistor Network

Spatial Response

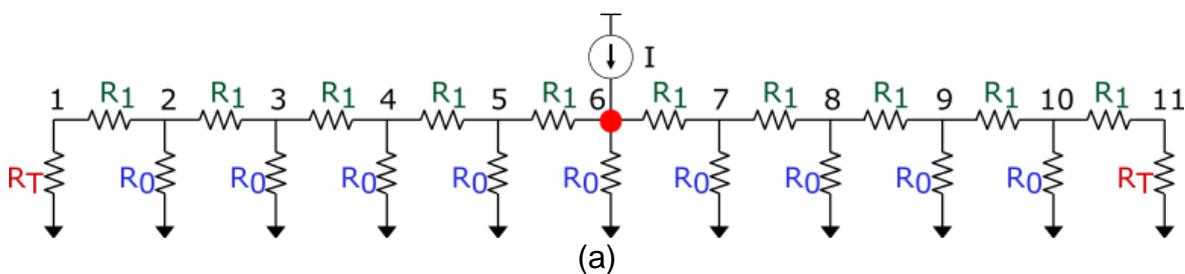
Let $R_0 = 2R$, $R_1 = R$, $R_T = R$ in Fig. 2 (a). The current I is injected at the center node (Fig. 2 (a)) and Fig. 2 (b) shows the spatial impulse response. We see that the node voltage decays to zero as the node approaches the network ends and all node voltages are positive.

Next letting $R_0 = 2R$, $R_1 = R$, $R_T = -2R$ (negative resistor) in Fig. 2 (a). Fig. 2 (c) is its spatial impulse response, and we see that the node voltage explodes as the node approaches to the network ends (nodes 1, 11) and all node voltages are negative.

Temporal Response

Next, we consider the temporal response by considering existence of capacitors C from each node to ground (Fig. 3 (a)). When the step input current $I(t)$ is injected at the center node, each node voltage converges to the stable equilibrium as time goes on for $R_0 = 2R$, $R_1 = R$, $R_T = R$ (temporally stable, Fig. 3 (b)) whereas it explodes as time goes for $R_0 = 2R$, $R_1 = R$, $R_T = -2R$ (temporally unstable, Fig. 3 (c)).

We found in SPICE simulation that for $R_0 = 4 \text{ k}\Omega$, $R_1 = 2 \text{ k}\Omega$, $R_T = -2.006 \text{ k}\Omega$ in Fig. 2, the spatial impulse response in Fig. 2 (a) is shown in Fig. 4 (a) and we checked with simulation that the network in Fig. 2 (b) is temporally stable. Notice that all the node voltages are positive in Fig. 4 (a). On the other hand, for $R_0 = 4 \text{ k}\Omega$, $R_1 = 2 \text{ k}\Omega$, $R_T = -2.005 \text{ k}\Omega$, the spatial impulse response is shown in Fig. 4 (b) and we checked that the network is temporally unstable. Notice that all the node voltages are negative in Fig. 5 (b). We also see that R_T of the temporal stability boundary is between $-2.005 \text{ k}\Omega$ and $-2.006 \text{ k}\Omega$.



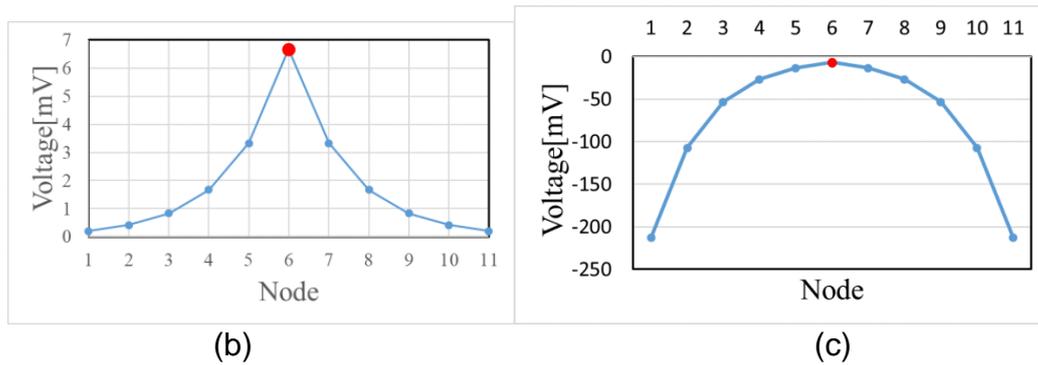


Fig. 2. $R_0 = 2 \text{ k}\Omega$, $R_1 = 1 \text{ k}\Omega$, current $I = 10 \text{ }\mu\text{A}$. (b) Node voltages (spatial impulse response) for $R_T = 1 \text{ k}\Omega$. (c) Node voltages for $R_T = -2 \text{ k}\Omega$.

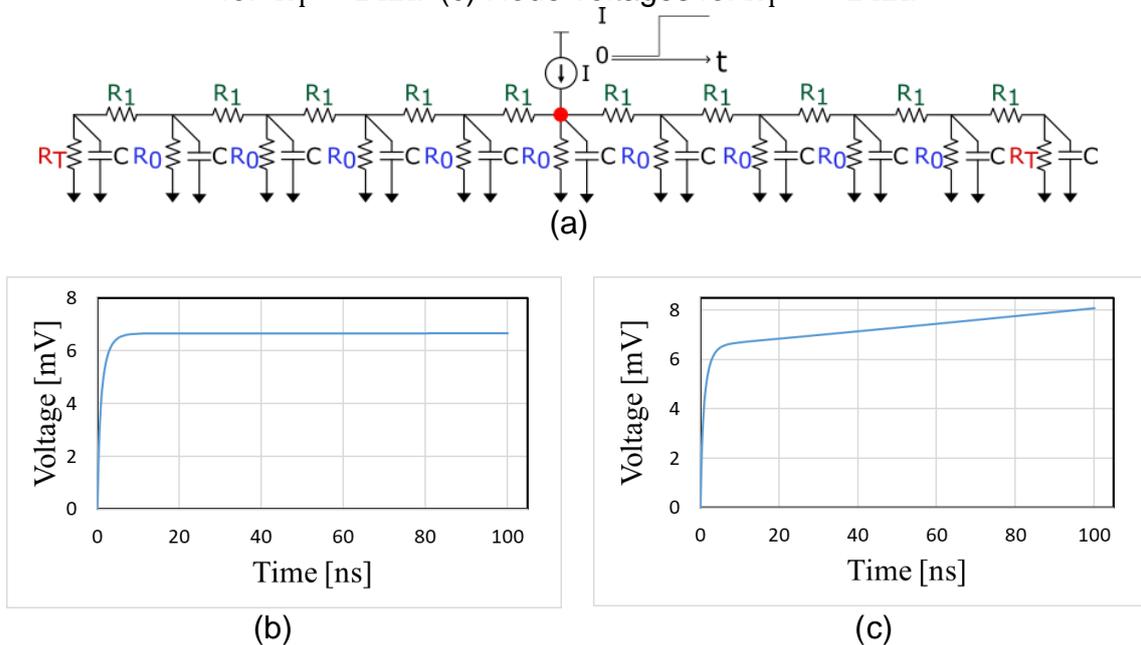


Fig. 3. Temporal response for the step input current $I (=10 \text{ }\mu\text{A})$ with $C = 1 \text{ pF}$. (a) Circuit with $R_0 = 2 \text{ k}\Omega$, $R_1 = 1 \text{ k}\Omega$. (b) Center node voltage for $R_T = 1 \text{ k}\Omega$ (temporally stable). (c) Center node voltage for $R_T = -2 \text{ k}\Omega$ (temporally unstable).

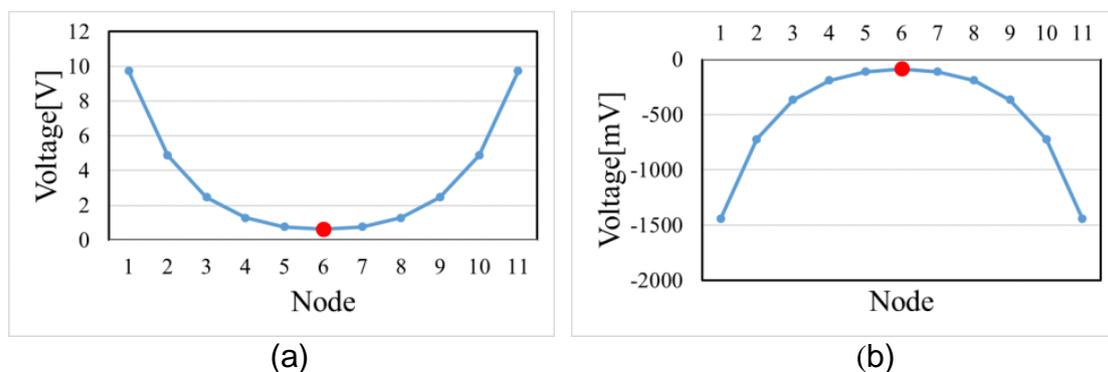


Fig. 4. Spatial impulse response for the network in Fig. 2 (a) with $R_0 = 2 \text{ k}\Omega$, $R_1 = 1 \text{ k}\Omega$, $I = 10 \text{ }\mu\text{A}$. (a) $R_T = -2.006 \text{ k}\Omega$ (temporally stable). (b) $R_T = -2.005 \text{ k}\Omega$ (temporally unstable).

Our conjecture is as follows:

Conjecture:

The network is temporally unstable if there is a node where the input current is injected and its node voltage as the spatial impulse response is negative.

Remarks:

(i) The above conjecture can be rephrased as follows:

The network is temporary unstable if there exist one or more negative nodes in the network, where a negative node means the resistance between the node and the ground is negative.

(ii) The network can be temporally unstable even if there is no node where the input current is injected and its node voltage as the spatial impulse response is negative. In other words, the network can be temporary unstable even if there exists no negative voltage node in the network,

(iii) If the input current is injected and its node voltage as the spatial impulse response is negative, the conductance seen from the node to the network is negative, whereas the node voltage is positive, the conductance is positive.

(iv) The network can be temporally stable or unstable if a node voltage where the spatial impulse current injected is positive.

(v) The network can be temporally unstable even if all the node voltages of the spatial impulse response for the input current injected at a node are positive.

These remarks (ii) (iii) (iv) (v) do not contradict the above conjecture. In the following sections, we investigate the conjecture and remarks with simulations.

3. Simulation for Various Active Resistor Networks

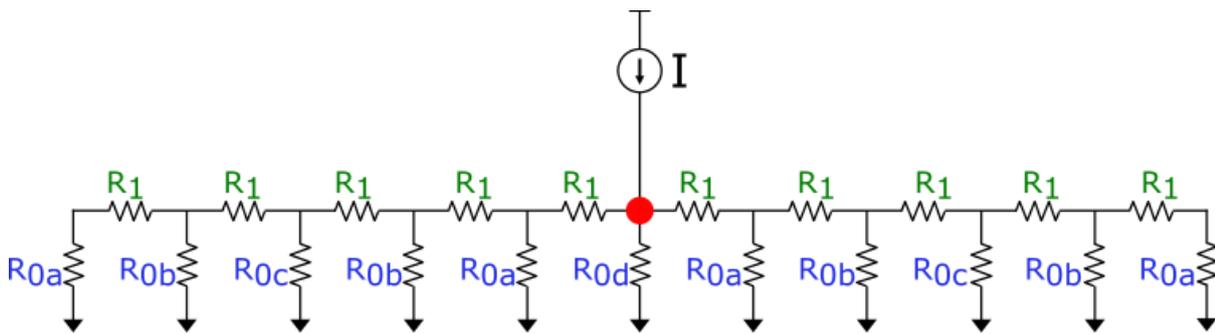
Now we consider the resistive networks in Figs. 5 (a), (b).

Figs. 5 (c), (d) show the spatial impulse response and temporal response for Circuit A in Fig. 5 (a). We see that in both cases, the network is temporally stable and the center node voltage is positive where the input current is injected to obtain the spatial impulse response; this is consistent with the conjecture. Notice that when the temporal response is considered, a capacitor ($C=1$ pF) connection from each node to the ground is assumed.

Fig. 5 (e) shows the spatial impulse response and temporal response for Circuit A in Fig. 5 (a). We see that in both cases, the network is temporally unstable and the center node voltage is negative where the input current is injected; this is consistent with the conjecture.

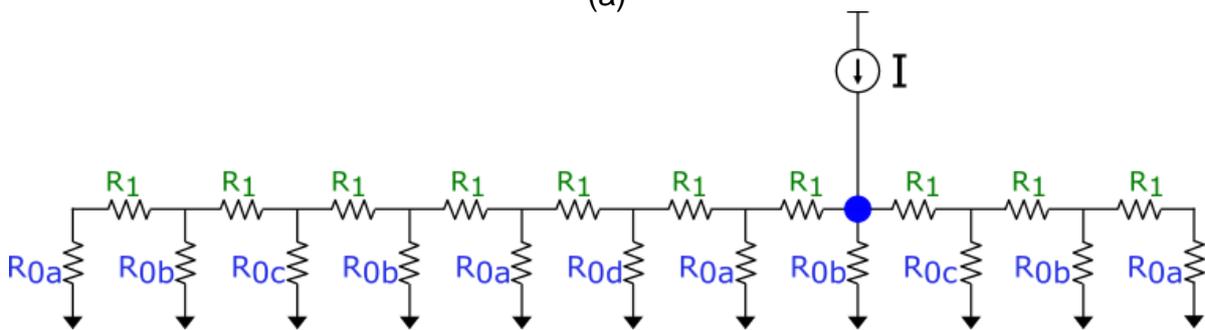
Fig. 5 (f) shows the spatial impulse response and temporal response for Circuit A in Fig. 5 (a) and Circuit B in Fig. 5 (b). We see that the network is temporally unstable. For Circuit A, the center node voltage is positive when the input current is injected there, whereas for Circuit B, the right node voltage is negative when the input current is injected there. Notice that Circuit A case (upper) does not violate the conjecture, and Circuit B case (lower) shows the consistency with the conjecture. Notice that when the temporal response is considered, a capacitor ($C=1$ pF) connection from each node to the ground is assumed.

In the case of Fig. 5 (g), the same argument holds as Fig. 5 (f).



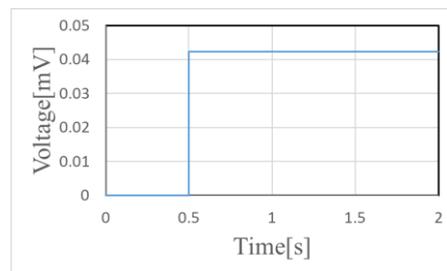
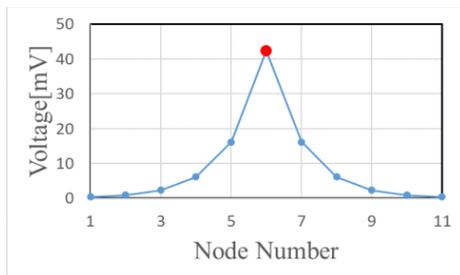
Circuit A: Current is injected at the center node.

(a)



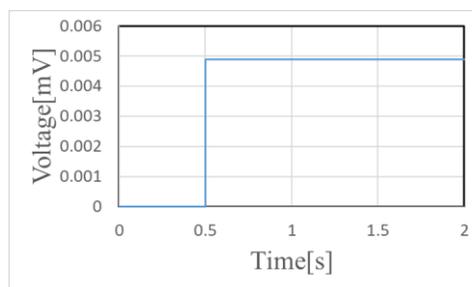
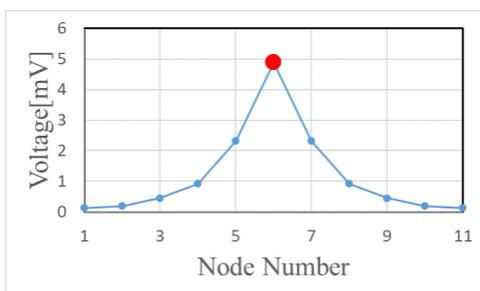
Circuit B: Current is injected at the right node.

(b)



Circuit A: Positive voltage at the current injected node Temporally stable

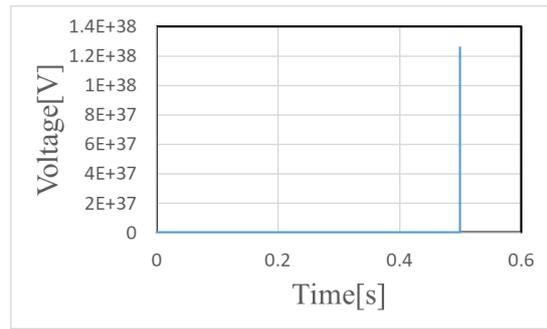
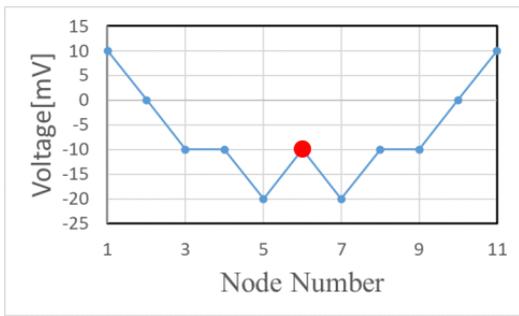
(c) $R_{0a} = R_{0b} = R_{0c} = 1k\Omega, R_{0d} = -1k\Omega, R_1 = 1k\Omega, I = 10\mu A.$



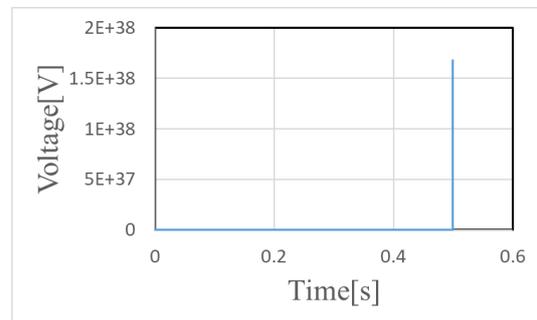
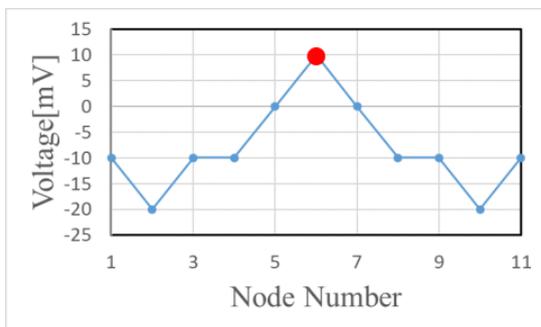
Circuit A: Positive voltage at current injected node Temporally stable

(d) $R_{0a} = R_{0c} = 2k\Omega, R_{0b} = R_{0d} = 1k\Omega, R_1 = 1k\Omega, I = 10\mu A.$

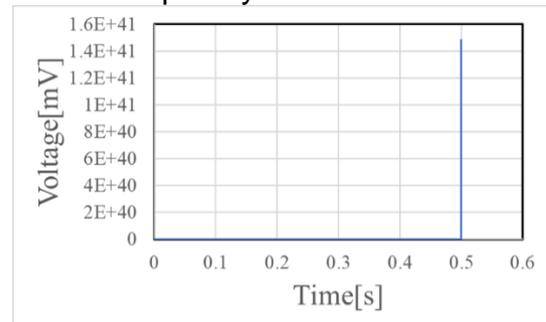
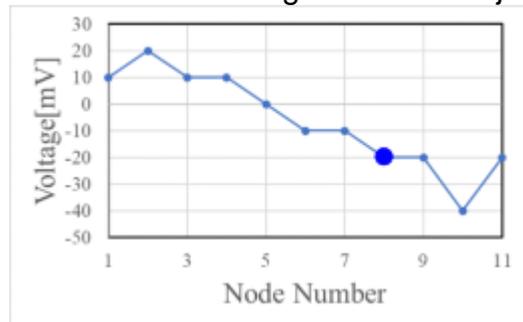
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6th International Conference on Technology and Social Science 2022 (ICTSS 2022)**



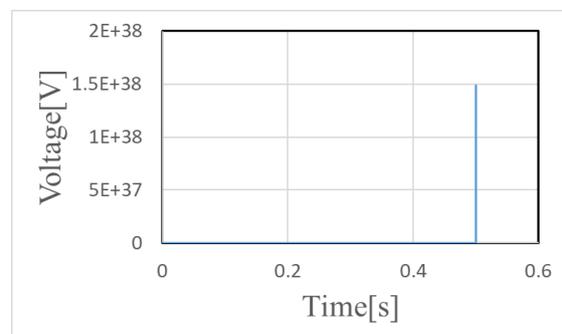
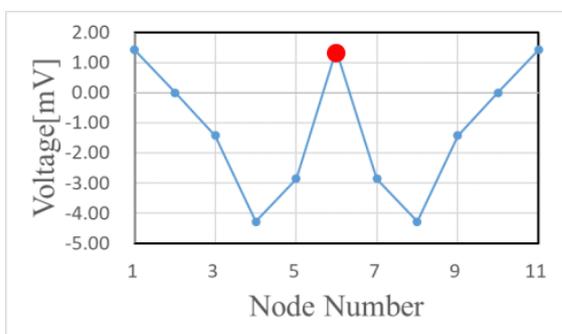
Circuit A: Negative voltage at current injected node Temporally unstable
(e) $R_{0a} = R_{0c} = -1k\Omega, R_{0b} = R_{0d} = 1k\Omega, R_1 = 1k\Omega, I = 10\mu A$.



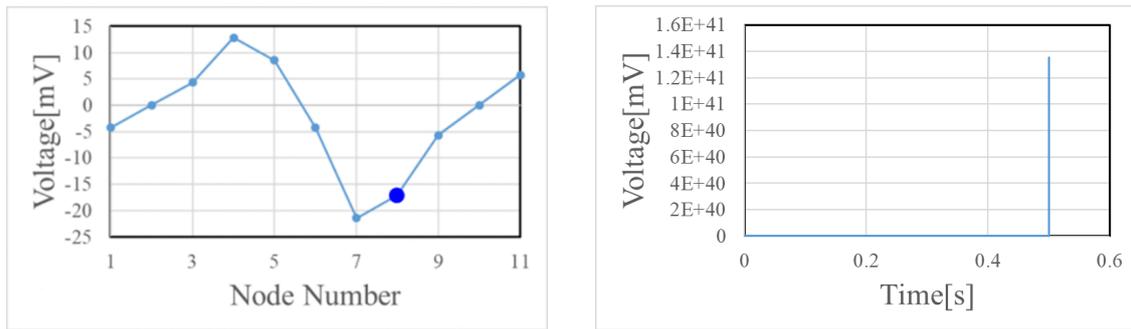
Circuit A: Positive voltage at current injected node Temporally unstable



Circuit B: Negative voltage at current injected node Temporally unstable
(f) $R_{0a} = R_{0c} = 1k\Omega, R_{0b} = R_{0d} = -1k\Omega, R_1 = 1k\Omega, I = 10\mu A$.



Circuit A: Positive voltage at current injected node Temporally unstable



Circuit B: Negative voltage at current injected node Temporally unstable
(g) $R_{0a} = R_{0b} = -1\text{k}\Omega, R_{0c} = R_{0d} = 1\text{k}\Omega, R_1 = 1\text{k}\Omega, I = 10\mu\text{A}$.

Fig. 5. Spatially shift-variant active resistive networks with the first nearest node resistor connection.

Let us move to another example in Fig. 6. Fig. 6 (a) shows the circuit with $R = 1\text{ k}\Omega, R_1 = 1\text{ k}\Omega, R_2 = -4\text{ k}\Omega, I = 10\text{ }\mu\text{A}$. For $R_{2a} = -10\text{ k}\Omega$, Fig. 6 (b) shows its spatial impulse response where all the node voltages are positive and we checked that the network is temporally stable when C is connected from each node to ground. On the other hand, for $R_{2a} = -1\text{ k}\Omega$, Fig. 6 (c) shows the spatial impulse response where the center node voltage is negative, and we checked that it is temporally unstable. These results are consistent with the conjecture.

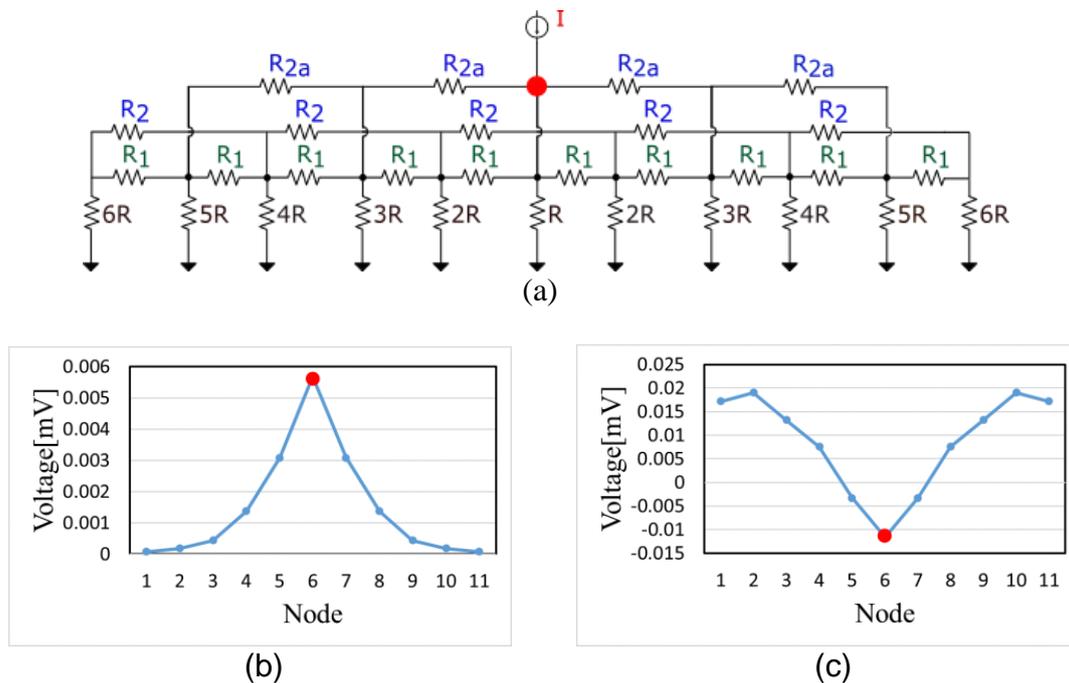


Fig. 6. A shift-variant resistor network with $R = 1\text{ k}\Omega, R_1 = 1\text{ k}\Omega, R_2 = -4\text{ k}\Omega, I = 10\text{ }\mu\text{A}$.

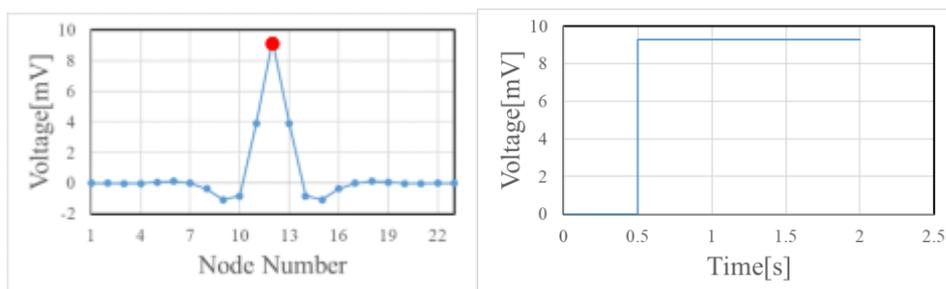
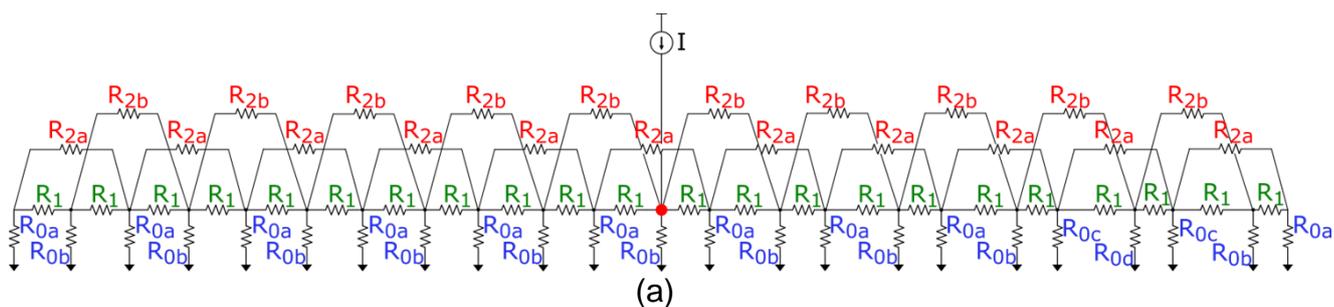
(a) Circuit. (b) Spatial impulse response for $R_{2a} = -10\text{ k}\Omega$. (c) The one for $R_{2a} = -1\text{ k}\Omega$.

Fig. 7 shows another example, and the network with the injected current at the center node is shown in Fig. 7 (a). Fig. 7 (b) shows the spatial impulse response (left), where the current-intected node voltage is positive, and the temporal step response (right); we see that the network is temporally stable.

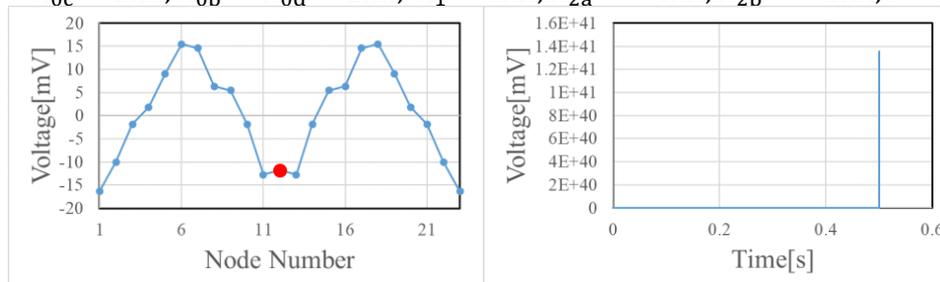
Fig. 7 (c) shows the spatial impulse response (left), where the current-injected node voltage is negative, and the temporal step response (right); we see that the network is temporally stable.

On the other hand, Fig. 7 (d) shows the spatial impulse response (left), where the current-injected node voltage is *positive*, and the temporal step response (right); we see that the network is temporally *unstable*. However, Fig. 8 (a) shows the network with the same parameter values as Fig. 7 (d) but with the injected current at the right node. We see in Fig. 8 (b) that the current-injected node voltage is *negative* for the spatial impulse response (left), and it is temporally *unstable* (right).

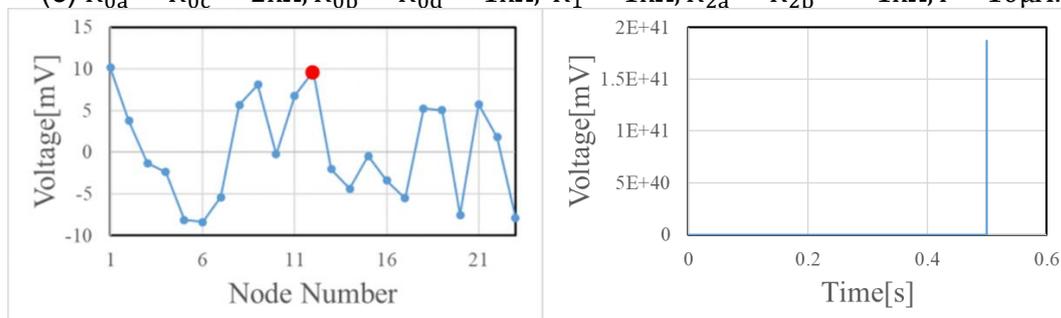
These are consistent with the conjecture.



(b) $R_{0a} = R_{0c} = 2k\Omega, R_{0b} = R_{0d} = 1k\Omega, R_1 = 1k\Omega, R_{2a} = -4k\Omega, R_{2b} = -2k\Omega, I = 10\mu A$.



(c) $R_{0a} = R_{0c} = 2k\Omega, R_{0b} = R_{0d} = 1k\Omega, R_1 = 1k\Omega, R_{2a} = R_{2b} = -1k\Omega, I = 10\mu A$.



(d) $R_{0a} = 2k\Omega, R_{0b} = 0.5k\Omega, R_{0c} = R_{0d} = -2k\Omega, R_1 = 1k\Omega, R_{2a} = R_{2b} = -1k\Omega, I = 10\mu A$.

Fig. 7. A shift-variant resistive network with the first and second nearest node connection.

(a) Circuit. (b) (c) (d) Spatial impulse response and temporal response.

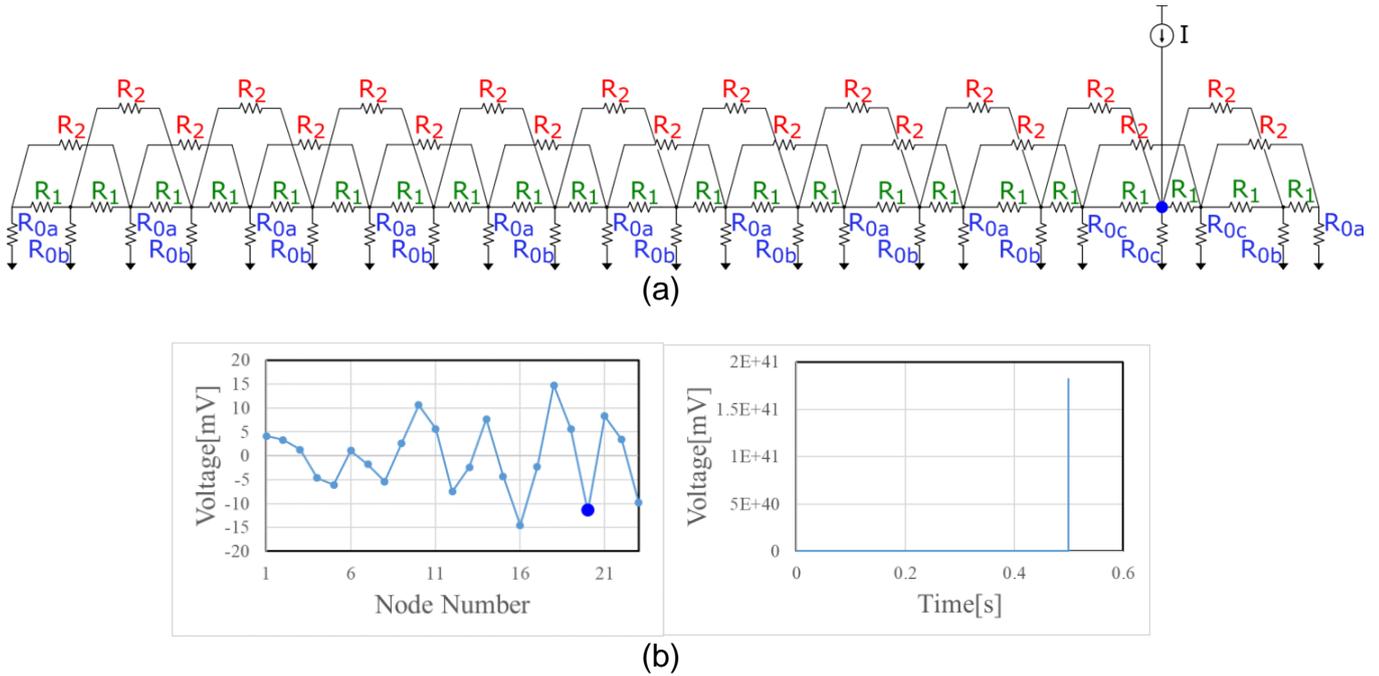


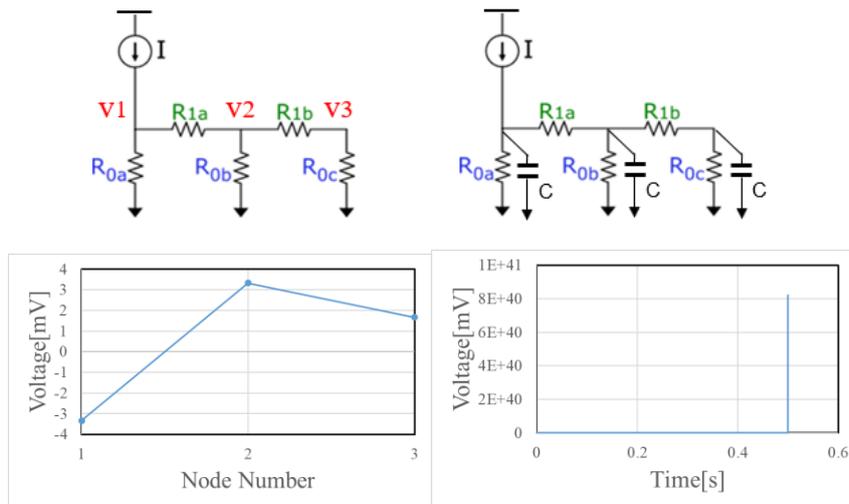
Fig. 8. A shift-variant resistive network with the first and second nearest node connections where the current is injected at the right node. $R_{0a} = 2\text{k}\Omega$, $R_{0b} = 0.5\text{k}\Omega$, $R_{0c} = -2\text{k}\Omega$, $R_1 = 1\text{k}\Omega$, $R_2 = -1\text{k}\Omega$, $I = 10\mu\text{A}$.
(a) Circuit. (b) Spatial impulse response and temporal response.

4. Discussion

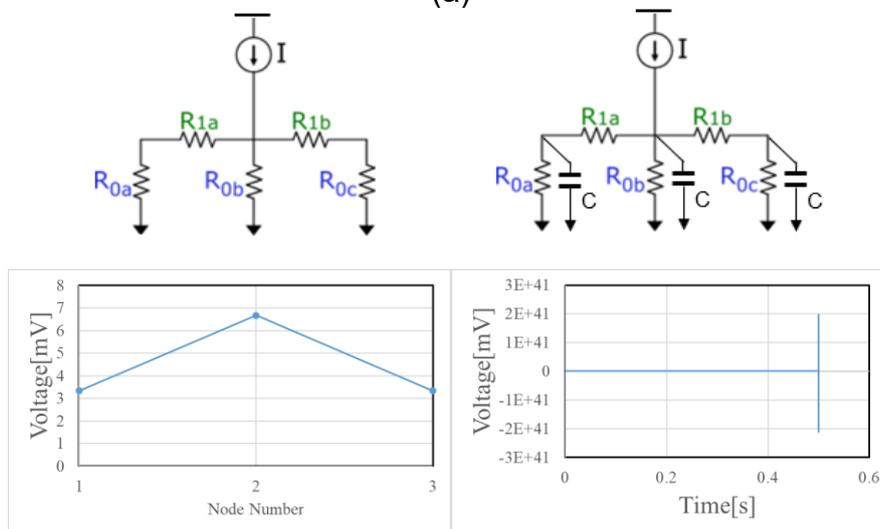
Fig. 9 shows the networks with 3 nodes. Fig. 9 (a) shows the network with the current injected at the left node; v_1 is negative and it is temporally unstable. Fig. 9 (b) shows the network with the current injected at the center node and its spatial impulse response as well as temporal response. What is interesting there is that all of node voltages v_1 , v_2 , v_3 are positive in the spatial impulse response, but it is temporally unstable. Fig. 9 (c) shows the one at the right node; its node voltage is positive but it is temporally unstable. These results are consistent with the conjecture.

These can be interpreted as shown in Fig. 10. As Fig. 10 (a) shows, the resistance seen from the center node is negative, while as Fig. 10 (b) shows, the one from the left node is positive. Also, as Fig. 10 (b) shows, the one from the right node is positive. Fig. 11 shows the temporal response for each equivalent network. We conjecture that the network is temporally unstable in Fig. 11 (a) and when its equivalent resistance seen from one node is negative, it becomes temporally unstable.

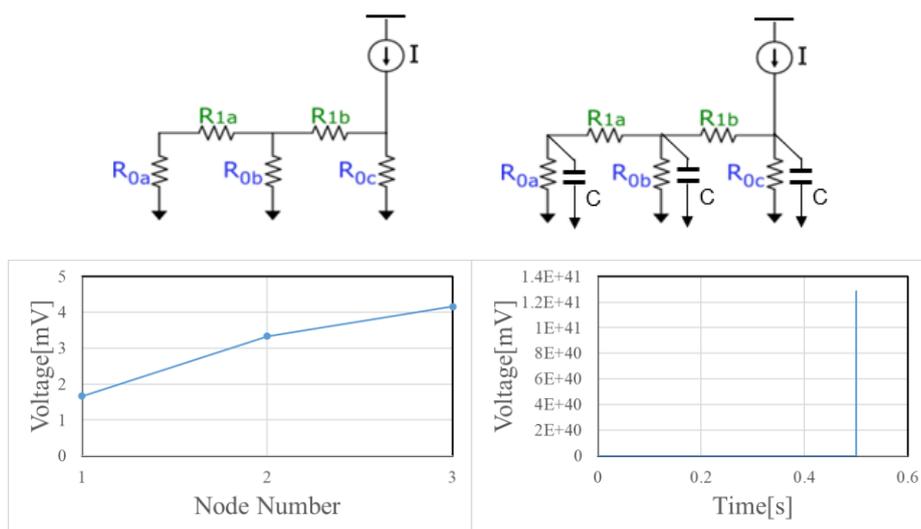
We have checked the eigenvalues of the 3×3 conductance matrix (or system matrix) [11] of the network in Fig. 9 and they are -0.0022749 , 0.001 and 0.0052749 . Since -0.0022749 is negative and the conductance matrix is not positive definite, the network is temporally unstable from the viewpoint of the state equation analysis.



(a)



(b)



(c)

Fig. 9 Network with 3 nodes. $R_{0a} = -1 \text{ k}\Omega$, $R_{0b} = 1 \text{ k}\Omega$, $R_{0c} = 0.5 \text{ k}\Omega$, $R_{1a} = -1 \text{ k}\Omega$, $R_{1b} = 0.5 \text{ k}\Omega$, $I = 10 \text{ }\mu\text{A}$, $C = 1 \text{ pF}$. (a) Current is injected at v2. (b) It is at v1. (c) It is at v3.

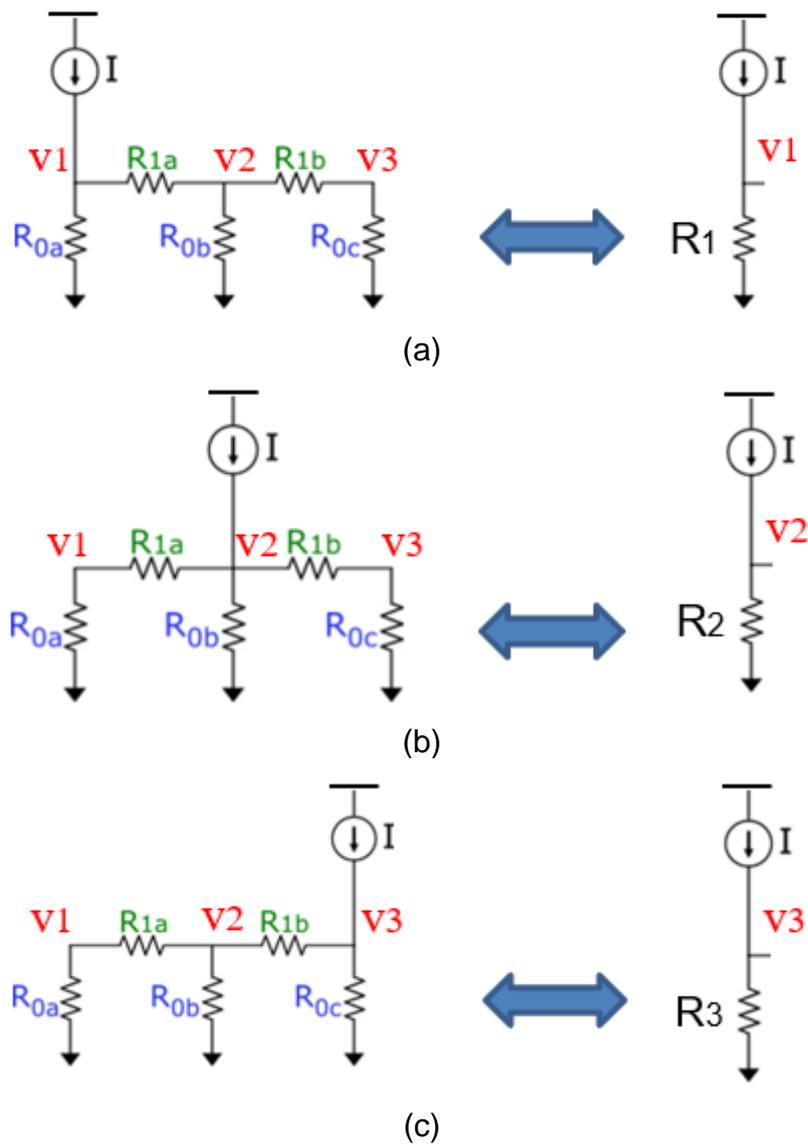


Fig. 10. Network with 3 nodes and equivalent one. $R_{0a} = -1 \text{ k}\Omega$, $R_{0b} = 1 \text{ k}\Omega$, $R_{0c} = 0.5 \text{ k}\Omega$, $R_{1a} = -1 \text{ k}\Omega$, $R_{1b} = 0.5 \text{ k}\Omega$, $I = 10 \text{ }\mu\text{A}$. (a) It is at $v1$. $R_1 = -0.333 \text{ k}\Omega$. (b) Current is injected at $v2$. $R_2 = 1.5 \text{ k}\Omega$. (c) It is at $v3$. $R_3 = 0.417 \text{ k}\Omega$.

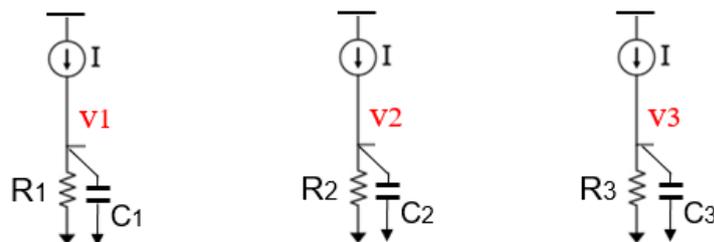


Fig. 11. Equivalent network temporal response. (a) $R_1 = -0.333 \text{ k}\Omega$ (temporally unstable mode). (b) $R_2 = 1.5 \text{ k}\Omega$. (c) $R_3 = 0.417 \text{ k}\Omega$.

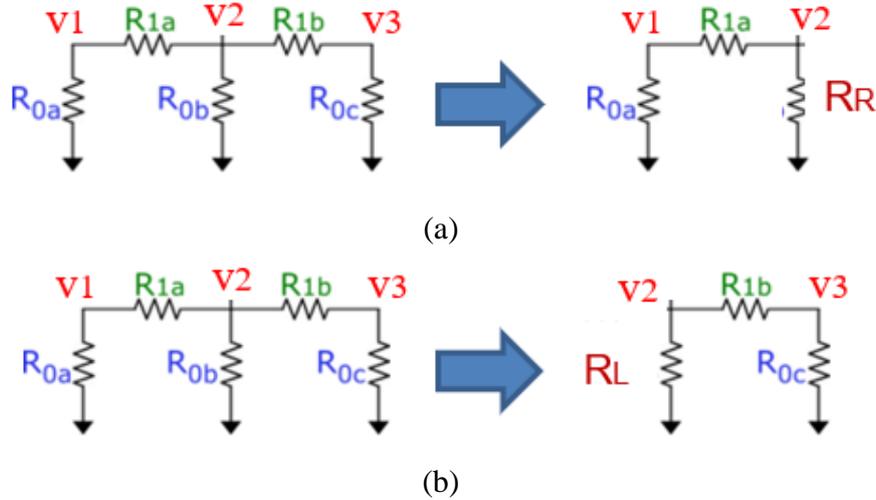


Fig. 12. Network with 3 nodes and its equivalent ones with 2 nodes.

(a) $R_R = R_{0b} || (R_{1b} + R_{0c})$. (b) $R_L = R_{0b} || (R_{1a} + R_{0a})$.

Now let us consider their temporal response as shown in Fig. 13.

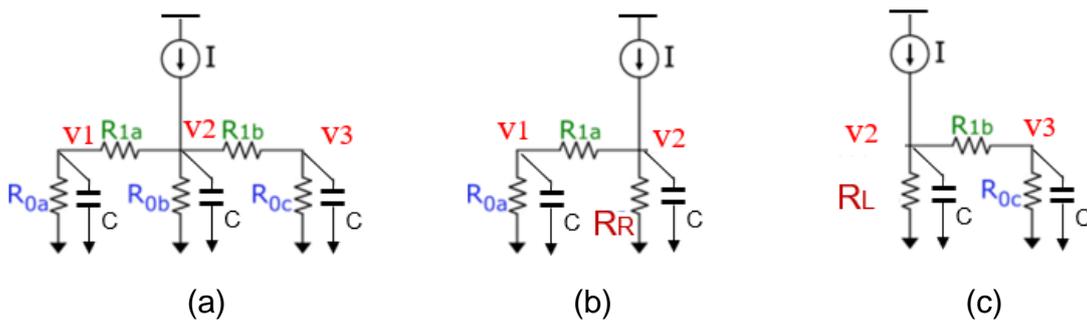


Fig. 13. Temporal response of network with 3 nodes and its equivalent ones with 2 nodes.

(a) 3 nodes. (b) 2 nodes. $R_R = R_{0b} || (R_{1b} + R_{0c})$. (c) 2 nodes. $R_L = R_{0b} || (R_{1a} + R_{0a})$.

The following parameter values are used:

$$R_{0a} = -1 \text{ k}\Omega, R_{0b} = 1 \text{ k}\Omega, R_{0c} = 0.5 \text{ k}\Omega, R_{1a} = -1 \text{ k}\Omega, R_{1b} = 0.5 \text{ k}\Omega, I = 10 \text{ }\mu\text{A}, C = 1 \text{ pF}.$$

Then we have the following:

$$R_R = R_{0b} || (R_{1b} + R_{0c}) = 0.5 \text{ k}\Omega. \quad R_L = R_{0b} || (R_{1a} + R_{0a}) = 1 \text{ k}\Omega.$$

Letting $g_2 = 1/R_{0a}$, $g_1 = 1/R_{1a}$, $g_0 = 1/R_R$ in Fig. 13 (b) and the circuit in Fig. 14 (a) is obtained. Also letting $g_3 = 1/R_L$, $g_4 = 1/R_{1b}$, $g_5 = 1/R_{0c}$ in Fig. 13 (c) and Fig. 14 (b) is obtained.

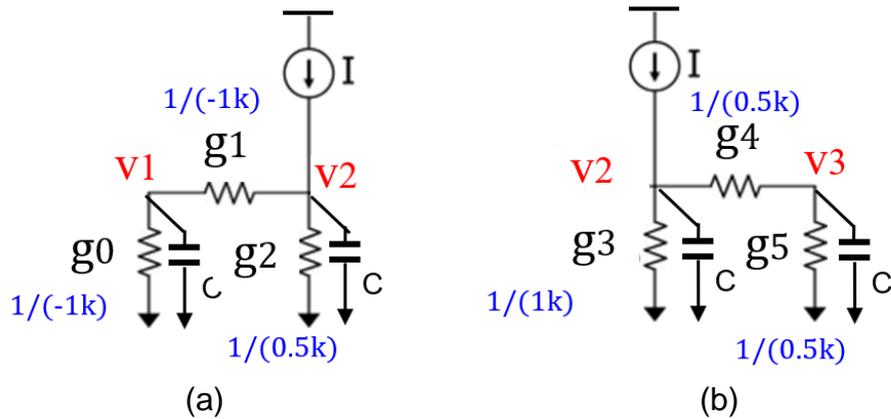


Fig. 14. (a) Network in Fig. 13 (b) with given parameter values. (b) The one in Fig. 13 (c).

The state equation for the network in Fig. 14 (a) is given by

$$C \frac{d}{dt} v_1 = -g_0 v_1 + g_1 (v_2 - v_1) = -(g_0 + g_1) v_1 + g_1 v_2$$

$$C \frac{d}{dt} v_2 = -g_2 v_2 + g_1 (v_1 - v_2) + I = g_1 v_1 - (g_0 + g_1) v_2 + I$$

$$C \frac{d}{dt} \mathbf{v} = \mathbf{A} \mathbf{v} + \mathbf{i}$$

$$\mathbf{v} = (v_1, v_2)^T, \mathbf{i} = (0, I)^T$$

$$\mathbf{A} = \begin{bmatrix} -(g_1 + g_2) & g_1 \\ g_1 & -(g_0 + g_1) \end{bmatrix}$$

We obtain the eigenvalues of \mathbf{A} :

$$|\mathbf{A} - \lambda \mathbf{E}| = \begin{vmatrix} -(g_1 + g_2) - \lambda & g_1 \\ g_1 & -(g_0 + g_1) - \lambda \end{vmatrix} = 0$$

$$\lambda^2 + (g_0 + 2g_1 + g_2)\lambda + g_0 g_1 + g_0 g_2 + g_1 g_2 = 0$$

$$\lambda^2 - 10^{-3} \times \lambda - 3 \times 10^{-6} = 0 \rightarrow \lambda = 0.0005 (1 \pm \sqrt{13}).$$

Since one eigenvalue is negative, the network in Fig. 13 (b) with given parameter values is temporally unstable. On the other hand, the network in Fig. 14 (b) consists of only positive conductors without negative ones and hence it is temporally stable. This means that the network in Fig. 15 (a), (c) are temporally unstable whereas the one in Fig. 15 (b) is temporally stable. This is worth pursuing.

We consider only a capacitor C from each node to ground. If capacitors from each node to other nodes are considered, a capacitance matrix should be taken care of. We just remark here that if the capacitance matrix is positive definite, the same conjecture as the above mentioned one would be valid.

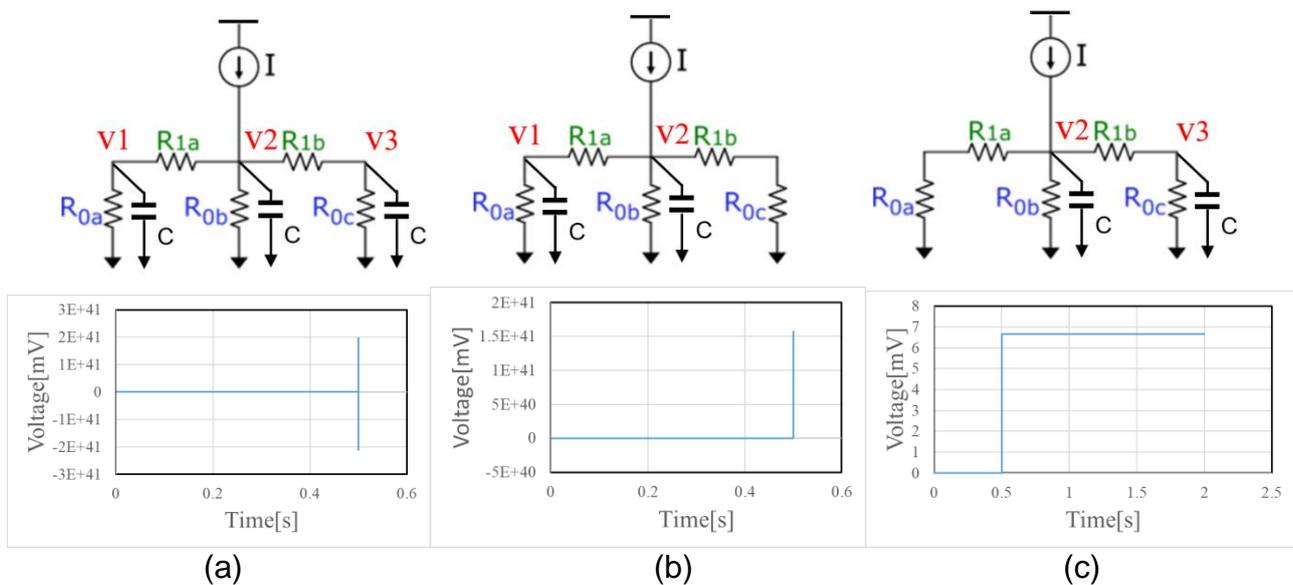


Fig. 15. $R_{0a} = -1 \text{ k}\Omega$, $R_{0b} = 1 \text{ k}\Omega$, $R_{0c} = 0.5 \text{ k}\Omega$, $R_{1a} = -1 \text{ k}\Omega$, $R_{1b} = 0.5 \text{ k}\Omega$, $I = 10 \mu\text{A}$, $C = 1 \text{ pF}$. (a) Temporally unstable. (b) Temporally unstable. (c) Temporally stable.

Fig. 16 shows an interesting network example of Remark (ii) in Section 2 where “the node voltage at which the current is injected is positive” is valid for all nodes but the network is temporally unstable.

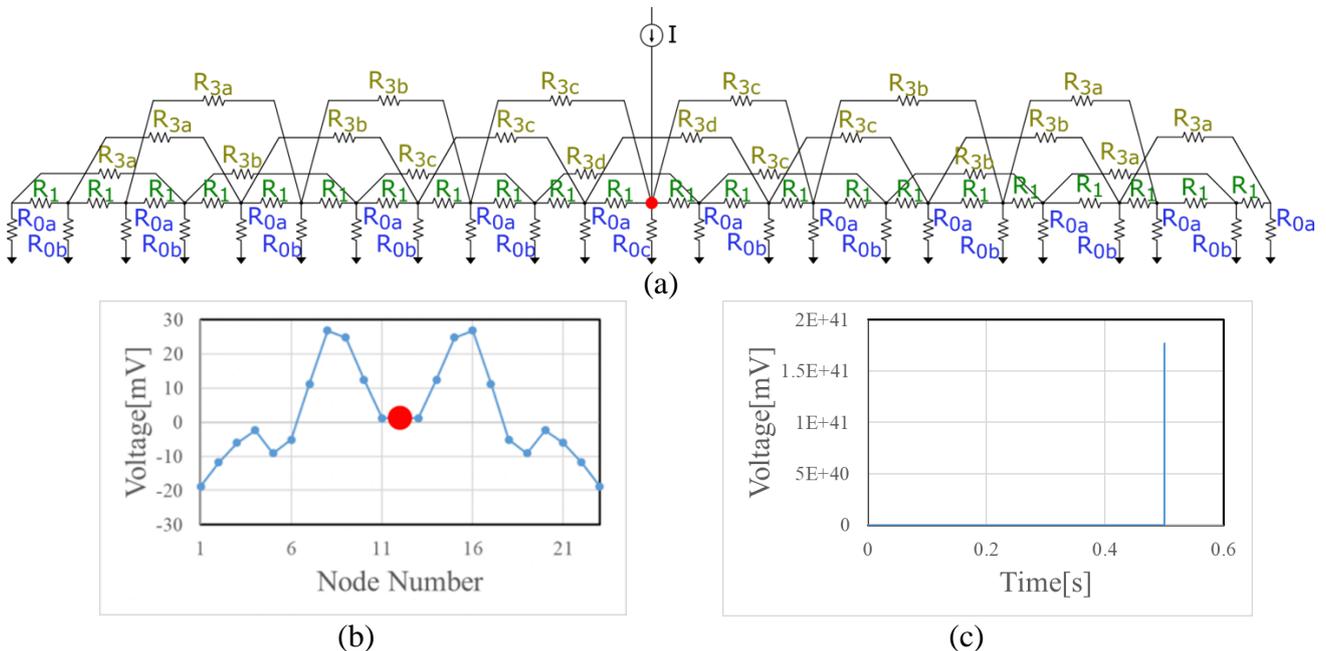


Fig. 16 Network example where “the node voltage at which the current is injected is positive” is valid for all nodes and the network is temporally unstable. (a) $R_{0a} = 2 \text{ k}\Omega$, $R_{0b} = 3 \text{ k}\Omega$, $R_{0c} = -0.25 \text{ k}\Omega$, $R_1 = 1 \text{ k}\Omega$, $R_{3a} = -1 \text{ k}\Omega$, $R_{3b} = -2 \text{ k}\Omega$, $R_{3c} = -3 \text{ k}\Omega$, $R_{3d} = -4 \text{ k}\Omega$, $I = 10 \mu\text{A}$
(b) Spatial impulse response where the current is injected at the center node.
(c) Temporally unstable response.

4. Conclusion

We have shown that the spatial and temporal response of the spatially shift-variant network composed of positive and negative resistors has close relationships, and investigated one conjecture based on theoretical analysis and simulations; “*The network is temporally **unstable** if there is a node where the input current is injected and its node voltage as the spatial impulse response is negative. Even If there is not such a node, it can be temporally unstable*”. Here for the spatial response, no capacitors are included in the network, whereas for the temporal response, a capacitor from each node to ground is considered in the network. Our simulation results have supported this conjecture; in other words, we have NOT found a network example in simulation that *the network is temporally **stable** where there is a node where the input current is injected and its node voltage as the spatial impulse response is negative*.

We close this paper by remarking that the RC linear circuit theory is considered as a mature research area, but still there are challenging problems.

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References

- [1] M. Hirai, S. Yamamoto, H. Arai, A. Kuwana, H. Tanimoto, Y. Gendai, H. Kobayashi, "Systematic Construction of Resistor Ladder Network for N-ary DACs", *IEEE International Conference on ASIC*, Oct. 2019.
- [2] H. Pan, A. A. Abidi, "Spatial Filtering in Flash A/D Converters", *IEEE Trans. Circuits and Systems II*, Aug. 2003.
- [3] O. Carnu, A. Leuciuc, "Design Issues for Low Voltage, High Speed Folding and Interpolating A/D Converters", *IEEE Midwest Symposium on Circuits and Systems*, 2002.
- [4] C. Mead, Analog VLSI and Neural Systems, *Addison-Wesley*, 1989.
- [5] A. Moini, Vision Chips, *Springer*, 1999.
- [6] H. Kobayashi, J. L. White, A. A. Abidi, "An Active Resistor Network for Gaussian Filtering of Images", *IEEE Journal of Solid-State Circuits*, May 1991.
- [7] T. Matsumoto, H. Kobayashi, Y. Togawa, "Spatial Versus Temporal Stability Issues in Image Processing Neuro Chips", *IEEE Trans. Neural Networks*, July 1992.
- [8] H. Kobayashi, T. Matsumoto, J. Sanekata, "Two-Dimensional Spatio-Temporal Response of Analog Image Processing Neural Networks", *IEEE Trans. Neural Networks*, May 1995.
- [9] H. Kobayashi, T. Matsumoto, "Spatial and Temporal Response of Vision Chips Including Parasitic Inductances and Capacitances", *IEICE Trans. Fundamentals*, March 1999.
- [10] J. L. White, A. N. Willson Jr., "On the Equivalence of Spatial and Temporal Stability for Translation Invariant Linear Resistive Networks", *IEEE Trans. Circuits and Systems I*, Sept. 1992.
- [11] D. E. Dudgeon, R. M. Mersereau, Multidimensional Digital Signal Processing, Prentice-Hall (1983).

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11th International Science, Social Sciences, Engineering and Energy Conference
(I-SEEC 2022) and,
6th International Conference on Technology and Social Science 2022 (ICTSS 2022)**

- [12] M. Chiba, K. Otomo, S. Katayama, K. Yoshihiro, A. Kuwana, H. Kobayashi, H. Tanimoto
"Spatial and Temporal Dynamics of Non-Uniform Active Resistor Networks", **IEEE 16th
International Conference on Solid-State and Integrated Circuit Technology**, Oct. 2022.