Conjecture on Spatial-Temporal Response Relationship for Spatially Shift-Variant Networks with Positive and Negative Resistors

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Outline

- Research Objective
- Research Background
- Active Resistive Network
    - Spatial Impulse Response
    - Temporal Dynamics
- Uniform Network Dynamics
- Non-Uniform Network Dynamics
- Conclusion

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Research Objective

Our previous theorem:

Spatial and temporal stability conditions are equal for uniform resistive network including negative resistors

↓

This research:

Investigation of spatial and temporal dynamics for non-uniform resistive network

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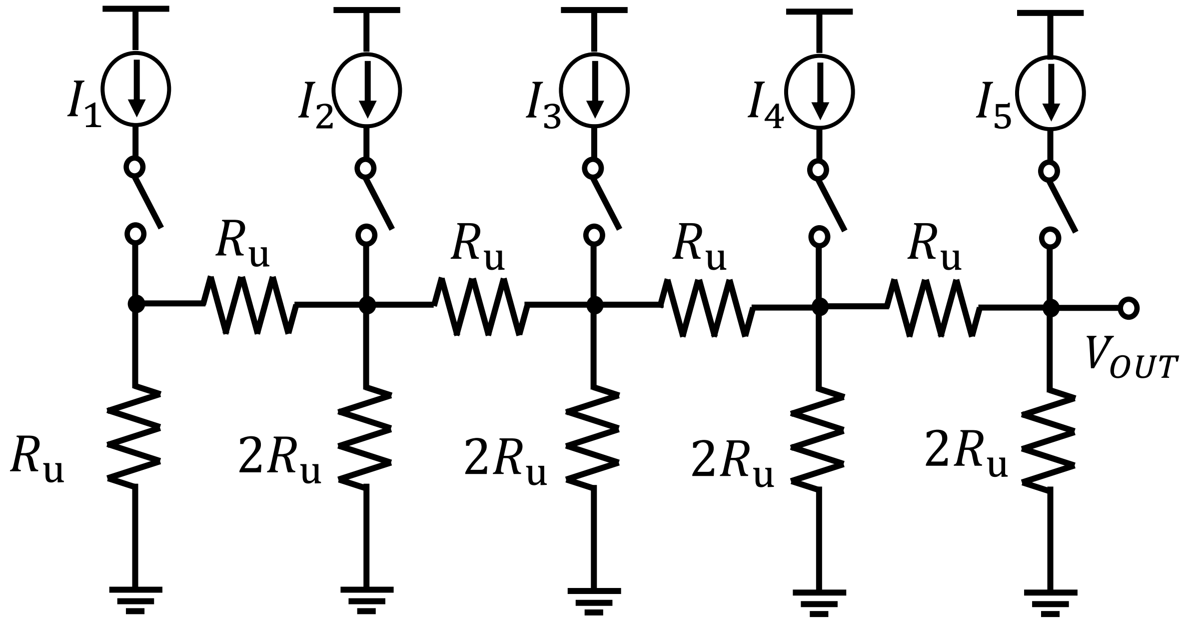
Resistive Network Circuit (1)

R-2R resistive ladder DAC



Advantages

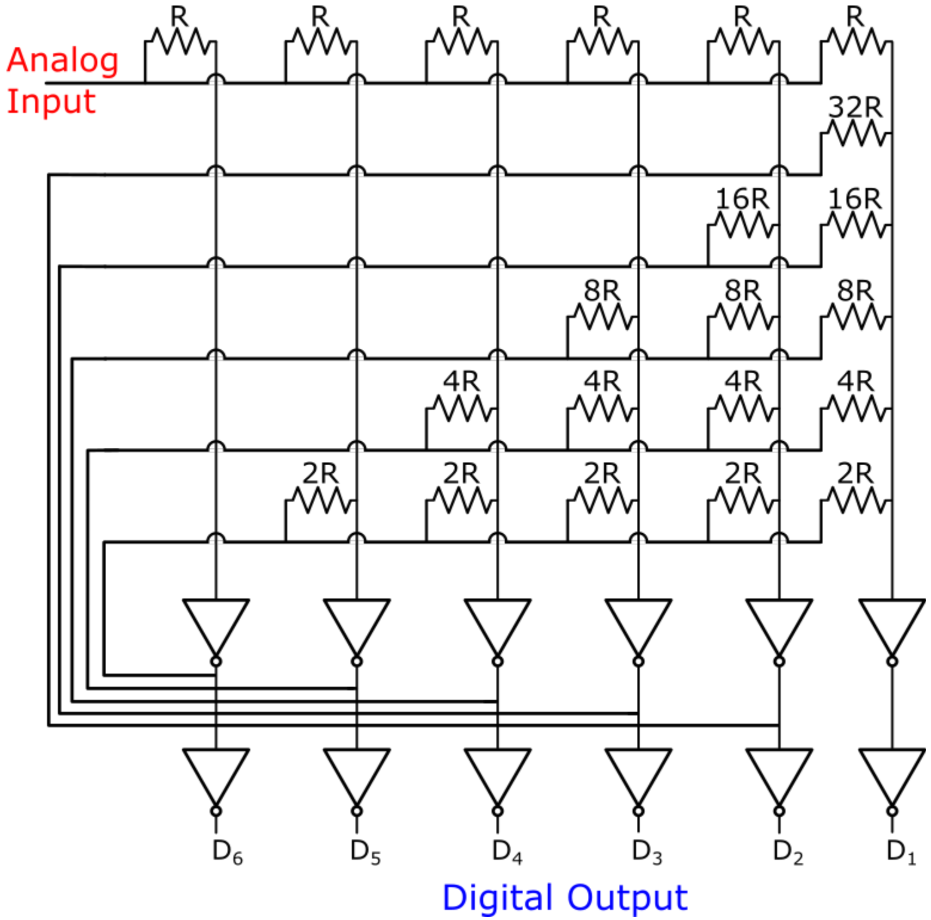
- High speed
- No need for decoder

Disadvantages

- Glitch
- Non-monotonicity

Resistive Network Circuit (2)

Asynchronous SAR ADC



Advantages

- High speed
- Low power
- Small circuit

[1] Z . Xu, X. Bai, D. Yao, A. Kuwana, H. Kobayashi,
"Revisit to Hopfield Network for Asynchronous SAR ADC and DAC",
IEEE 3rd International Conference on Circuits and Systems, Chengdu, China (Oct. 2021)

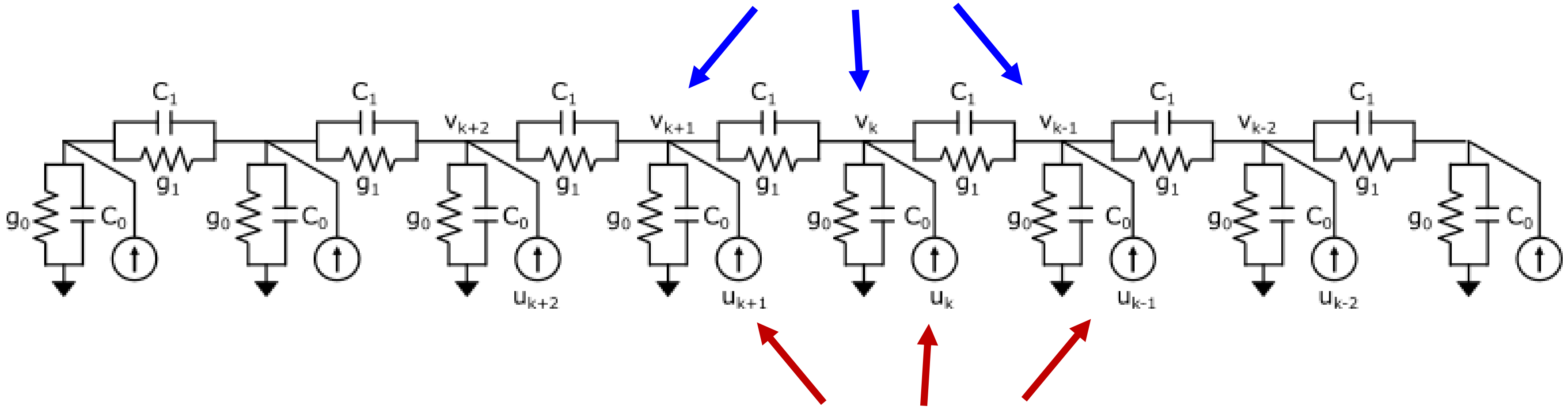
Resistive Network Circuit (3)

High-speed analog image processor

1D image case

$g_0 = 1/R_0$
$g_1 = 1/R_1$

Node voltages： Output image



Injected currents at nodes： Input image

[2] C. A. Mead, Analog VLSI and Neural Systems, Addison Wesley, 1989

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Active Resistive Network:
➡ Including positive and negative resistors

Resistive Network Circuit (4)

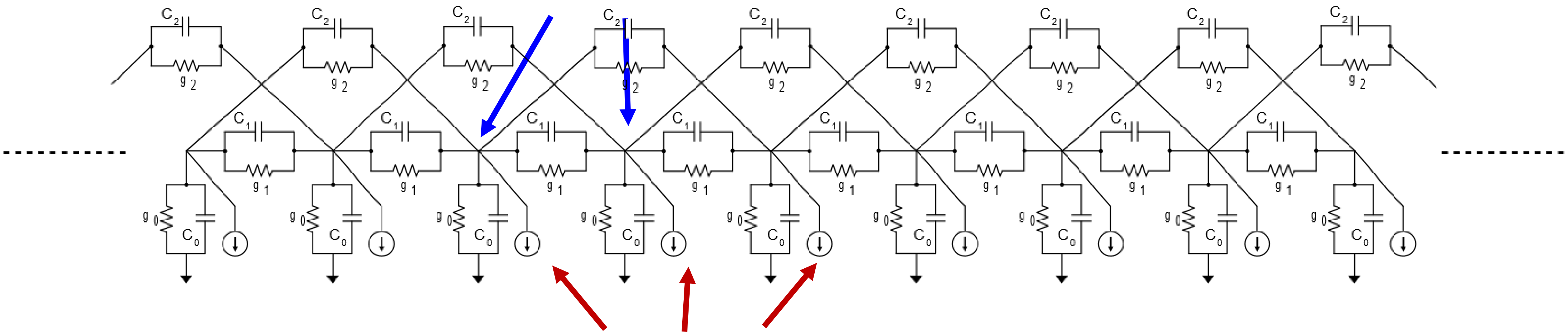
High-speed analog image processor

$g_0 = 1/R_0$
$g_1 = 1/R_1$
$g_2 = 1/R_2$

1D image case

Negative resistor $R_2 = -4R_1 < 0$

Node voltages： Output image



Injected currents at nodes： Input image

[3] H. Kobayashi, J. L. White, A. A. Abidi, “An Active Resistor Network for Gaussian Filtering of Images",
IEEE Journal of Solid-State Circuits (May 1991)

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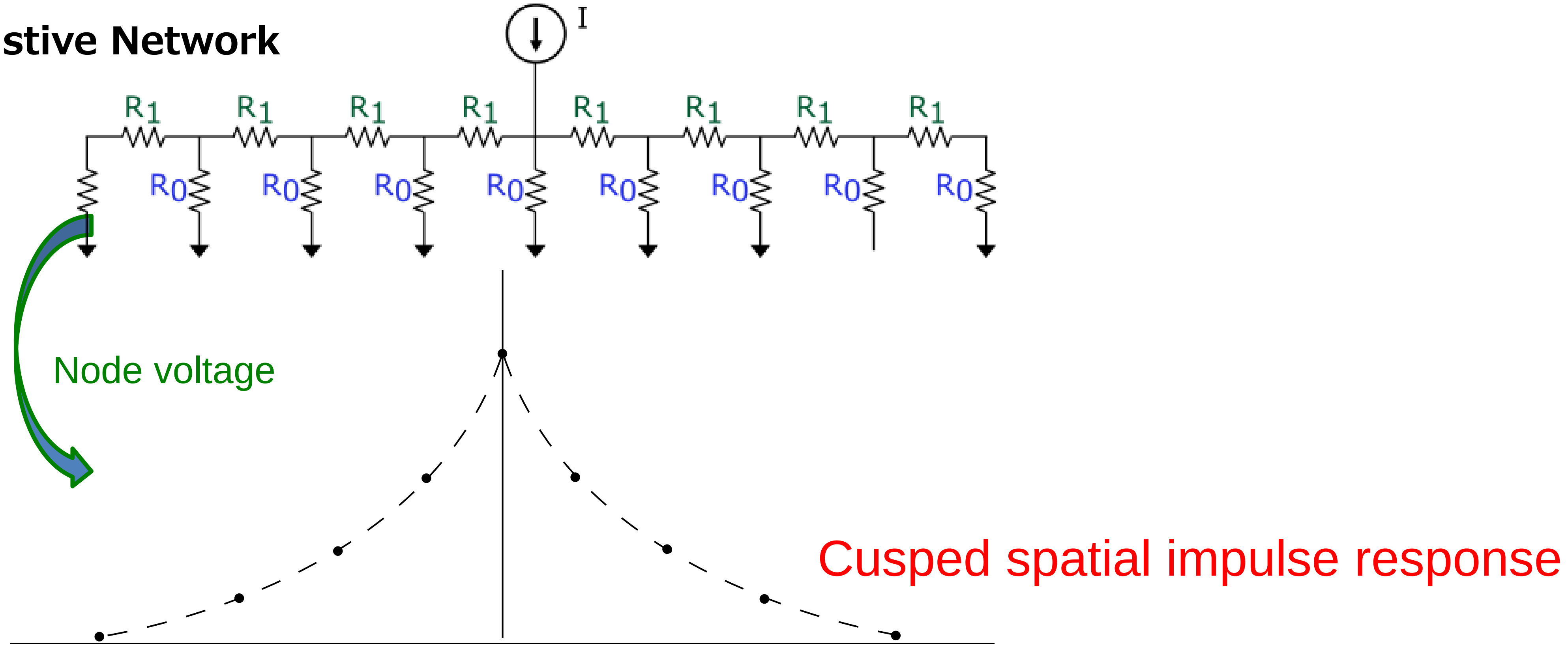
Spatial Impulse Response (1)

High-speed analog image processor (Retina chip)

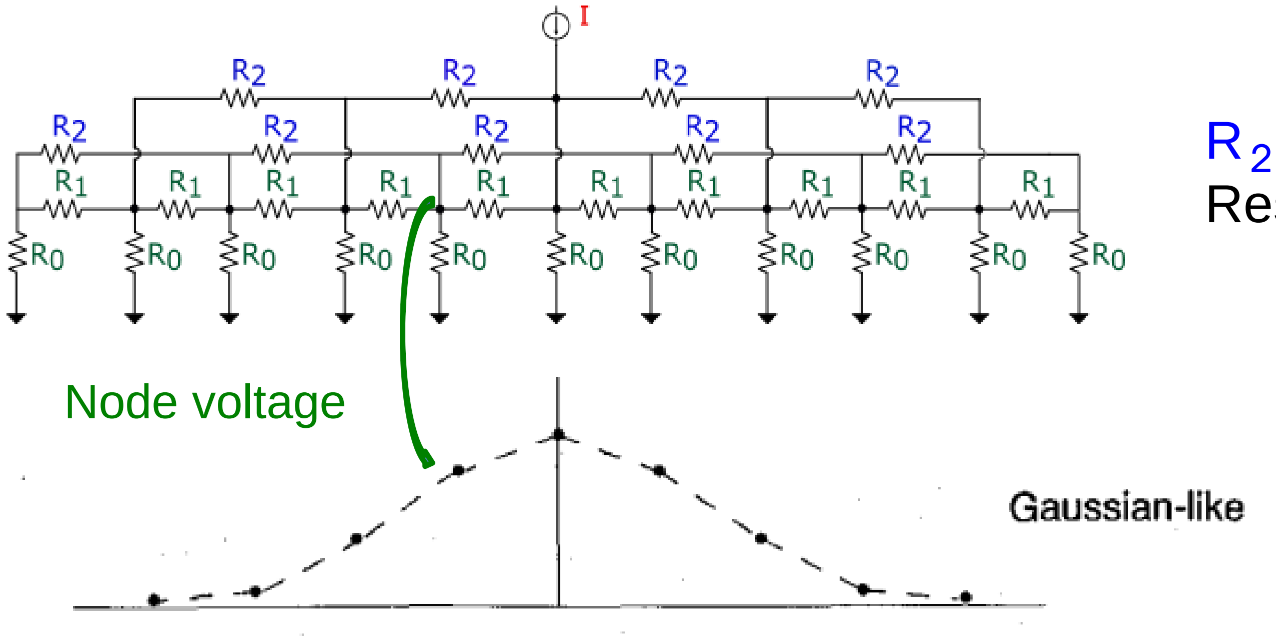
Resistive Network



[5] C. A. Mead, Analog VLSI and Neural Systems, Addison Wesley, 1989

Spatial Impulse Response (2)

High-speed analog image processor (Gaussian chip)



R_2 : Negative Resistor

Flat-top spatial impulse response

[3] H. Kobayashi, J. L. White, A. A. Abidi, "An Active Resistor Network for Gaussian Filtering of Images", IEEE Journal of Solid-State Circuits (May 1991)

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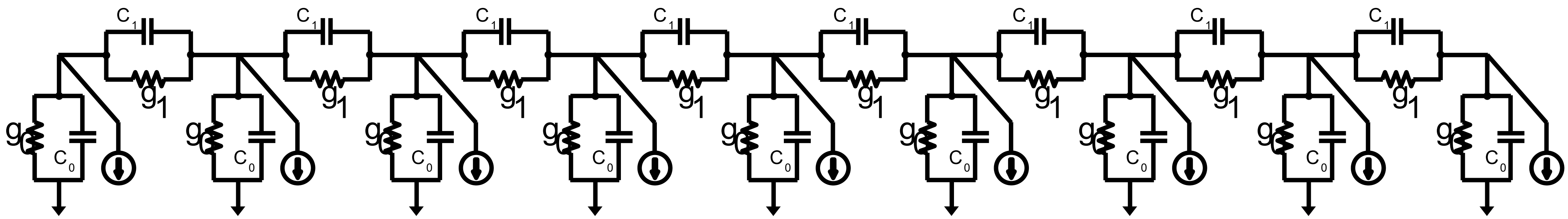
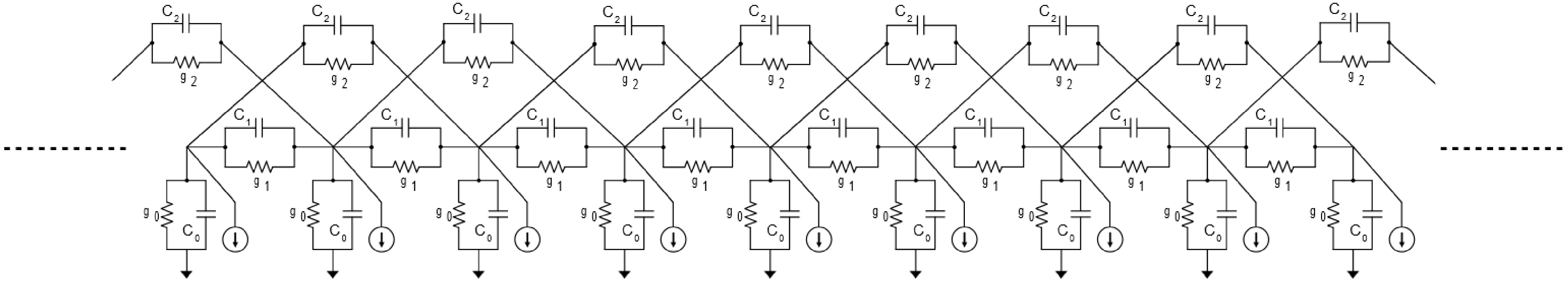
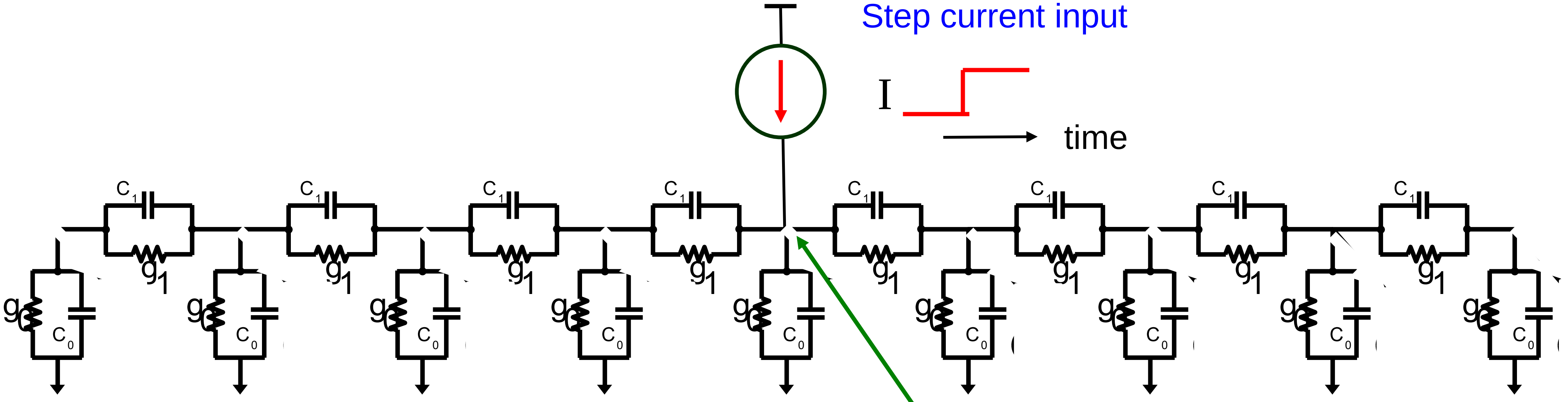
Temporal Dynamics with R, C

Capacitances are considered for temporal dynamics



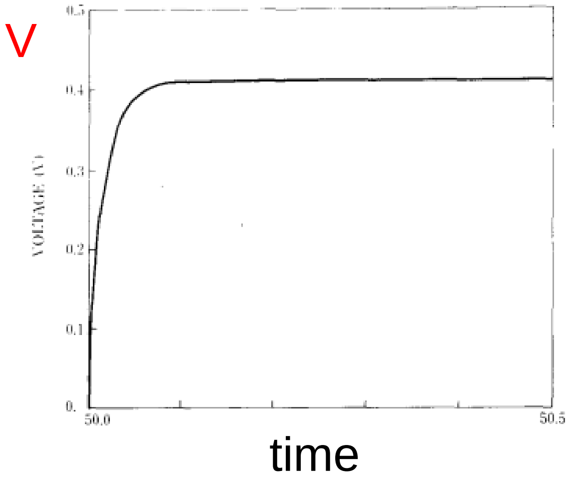


Temporal Step Response

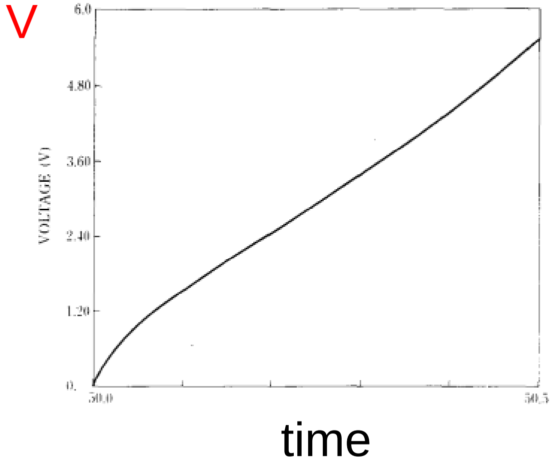


Observe this node voltage V

Temporally stable



Temporally unstable



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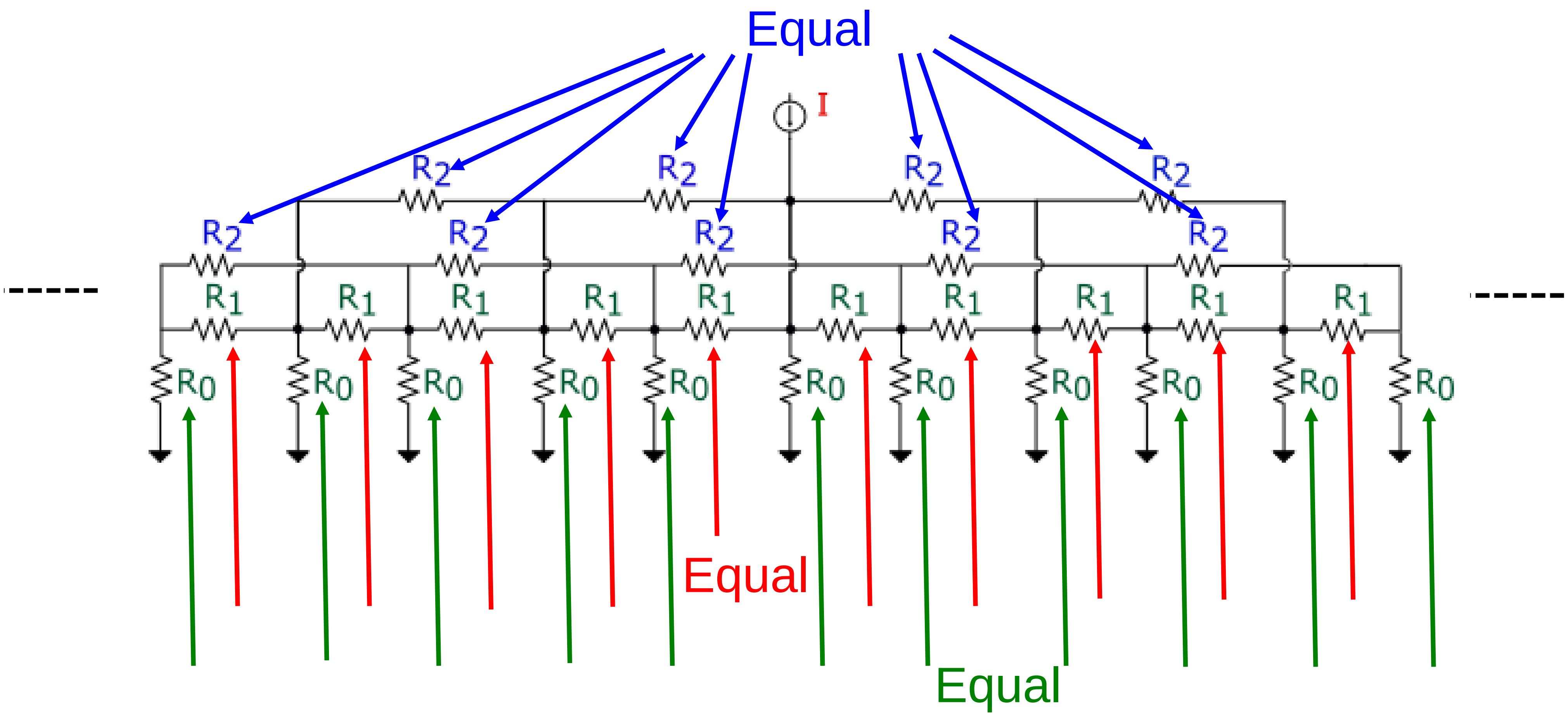
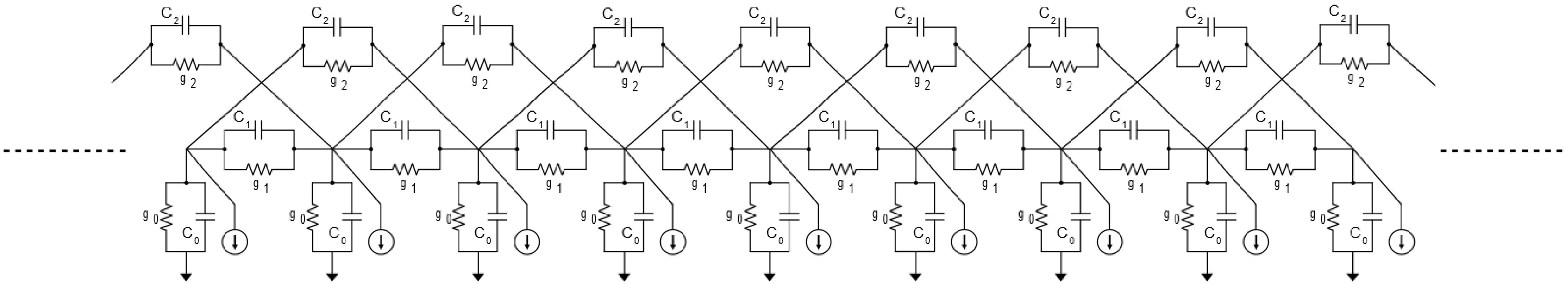
Uniform Resistor Network



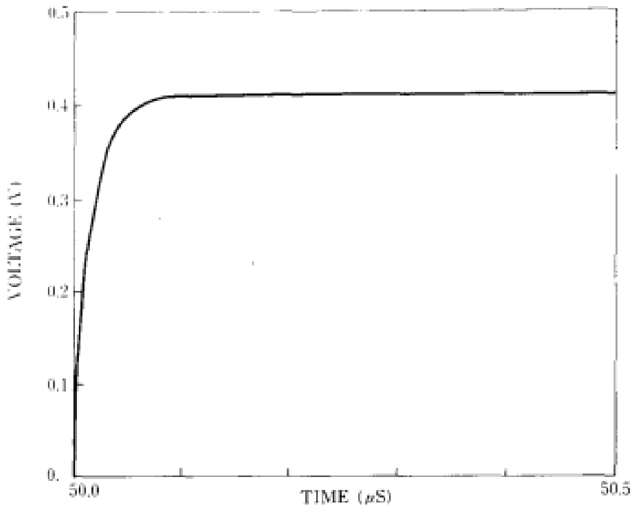
- Shift invariant
- Spatial transfer function

Simulation Results: Spatial Temporal Stabilities

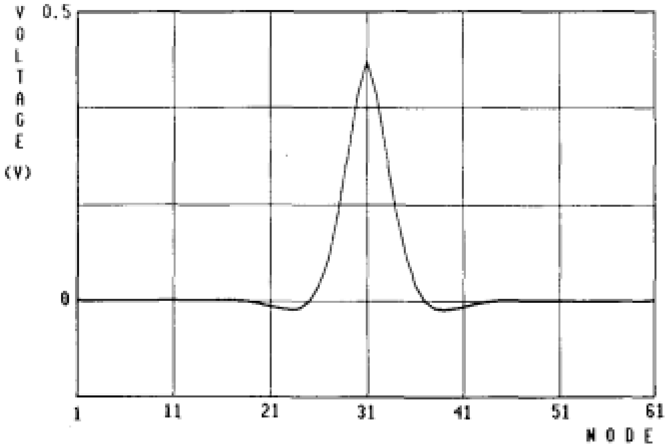
$R_0 = 1/g_0 = 200\text{k}\Omega, \quad R_1 = 1/g_1 = 5\text{k}\Omega, \quad R_2 = 1/g_2 = -20\text{k}\Omega$



Temporally stable

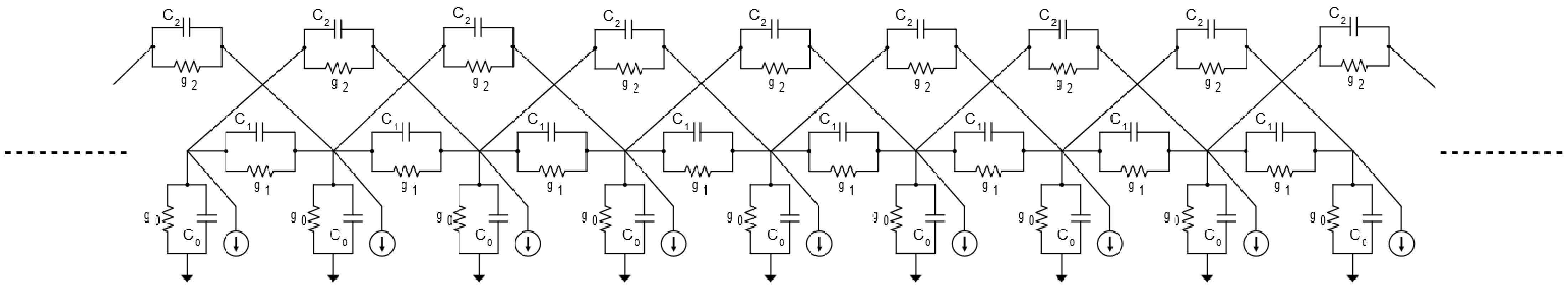


Spatially stable

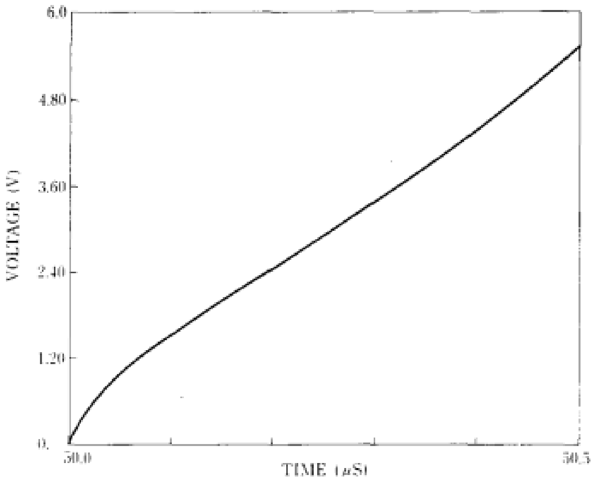


Simulation Results: Spatial Temporal Instabilities

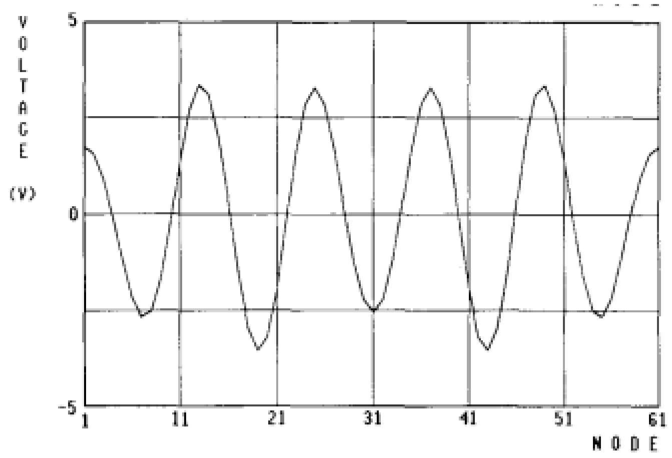
$$R_0 = 1/g_0 = 200\text{k}\Omega, \quad R_1 = 1/g_1 = 5\text{k}\Omega, \quad R_2 = 1/g_2 = -17\text{k}\Omega$$



Temporally unstable



Spatially unstable



Circuit Network Theorem

For uniform network with positive and negative resistors, spatial and temporal stability conditions are equivalent.

[4] T. Matsumoto, H. Kobayashi, Y. Togawa,
"Spatial Versus Temporal Stability Issues in Image Processing Neuro Chips",
IEEE Trans. Neural Networks, (July 1992).
[5] H. Kobayashi, T. Matsumoto, J. Sanekata,
"Two-Dimensional Spatio-Temporal Dynamics of Analog Image Processing Neural Networks",
IEEE Trans. Neural Networks (Oct. 1995).

How about non-uniform network ?

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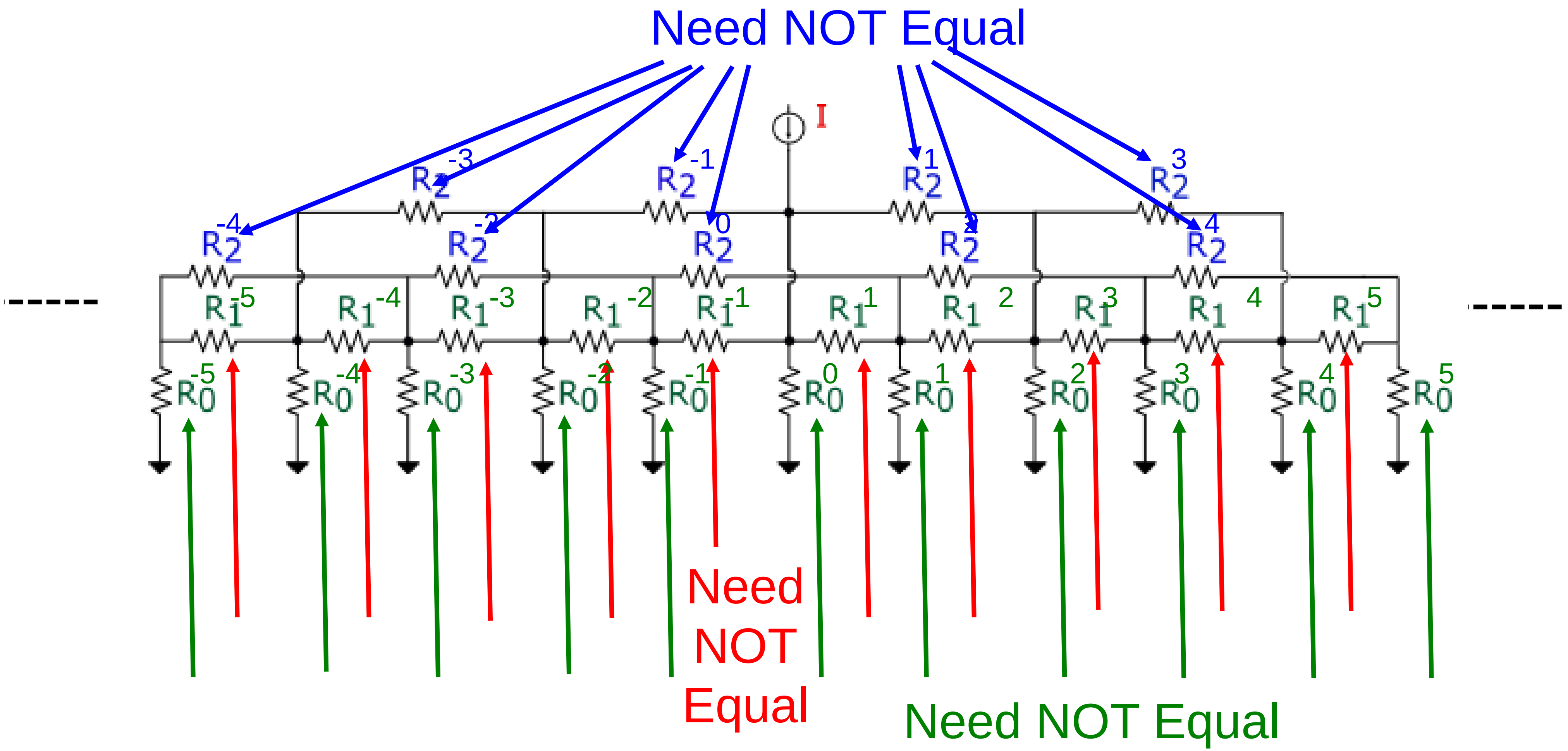
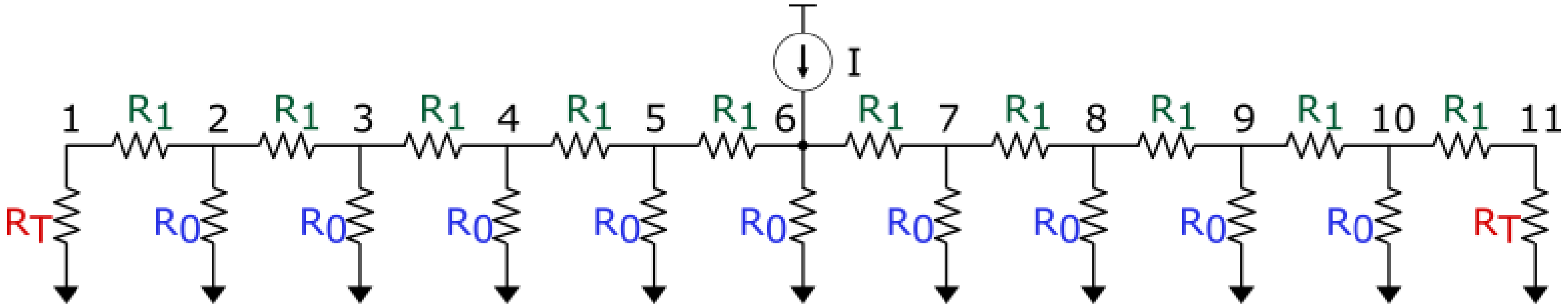
Non-Uniform Resistor Network



- Shift variant
- Spatial transfer function CANNOT be defined

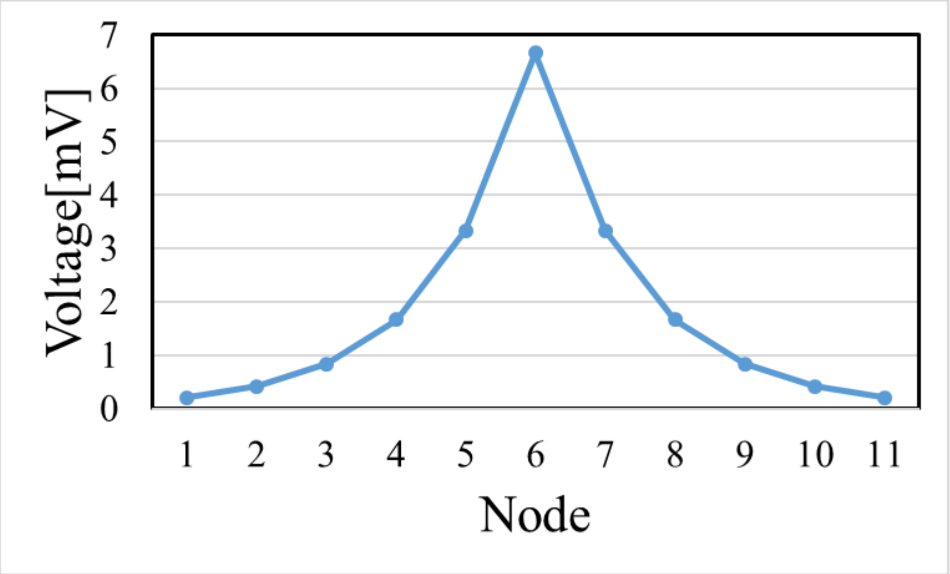
Spatial Impulse Response of Non-Uniform Network



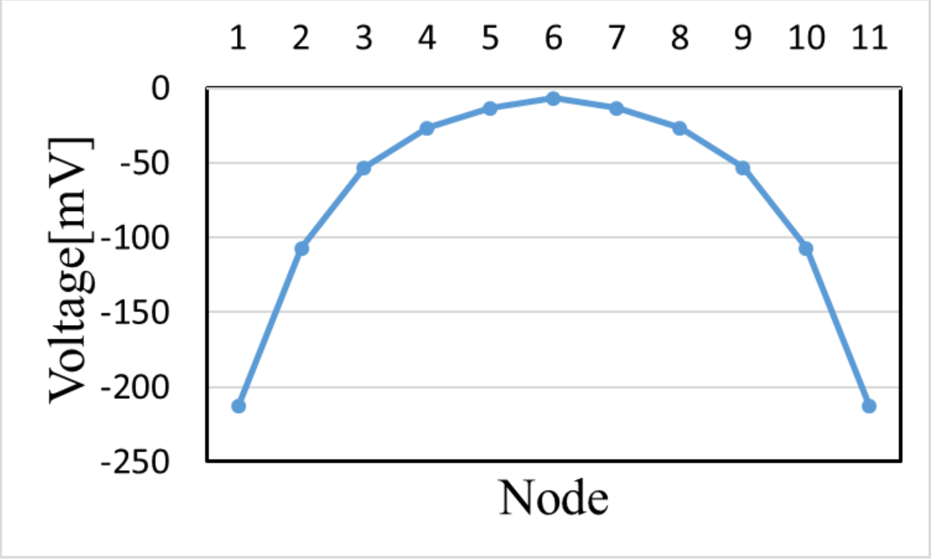
R1, RT may be different → Non-uniform network

$R_0 = 2\mathrm{k}\Omega, R_1 = 1\mathrm{k}\Omega$, I =0.01mA

Spatial Impulse Response

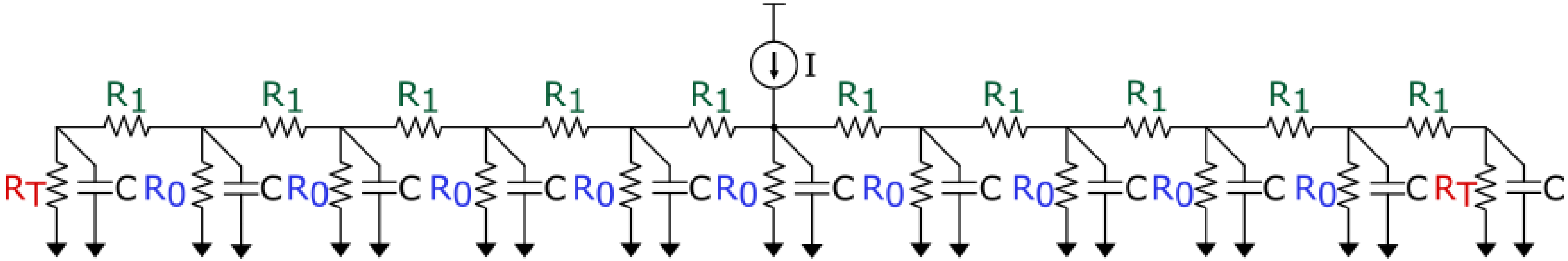


$R_T = 1\mathrm{k}\Omega$ Well behaved



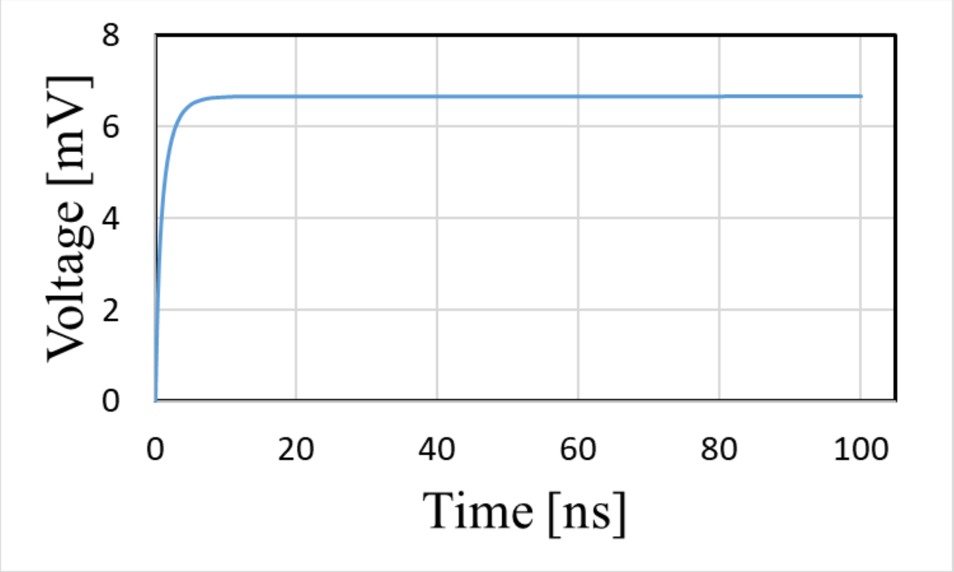
$R_T = -2\mathrm{k}\Omega$ Violently behaved

Temporal Dynamics of Non-Uniform Network



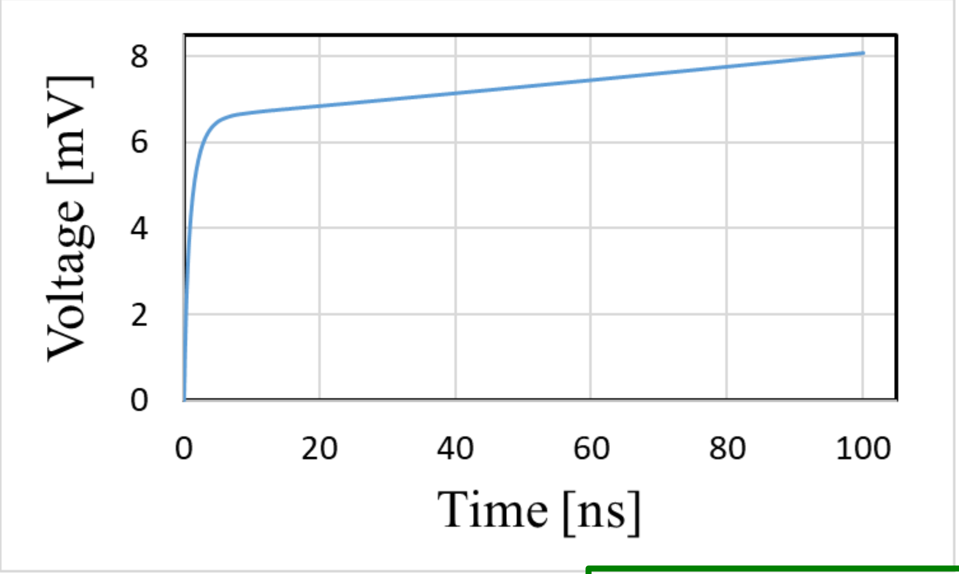
$R_0 = 2\text{k}\Omega, R_1 = 1\text{k}\Omega,\ C = 1\text{pF},$ Step I =0.01mA

Step response at the center node.



$R_T = 1\text{k}\Omega$

Temporally stable



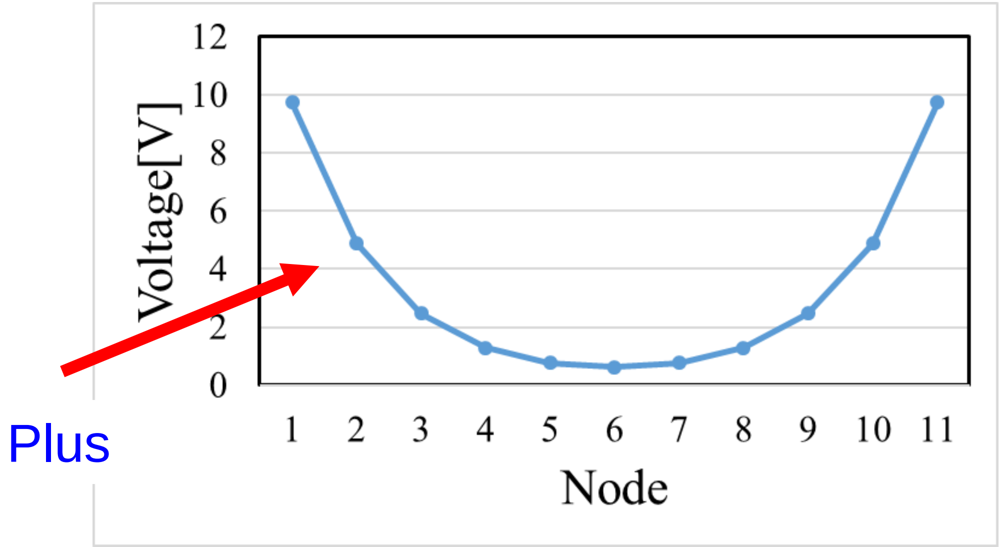
$R_T = -2\text{k}\Omega$

Temporally unstable

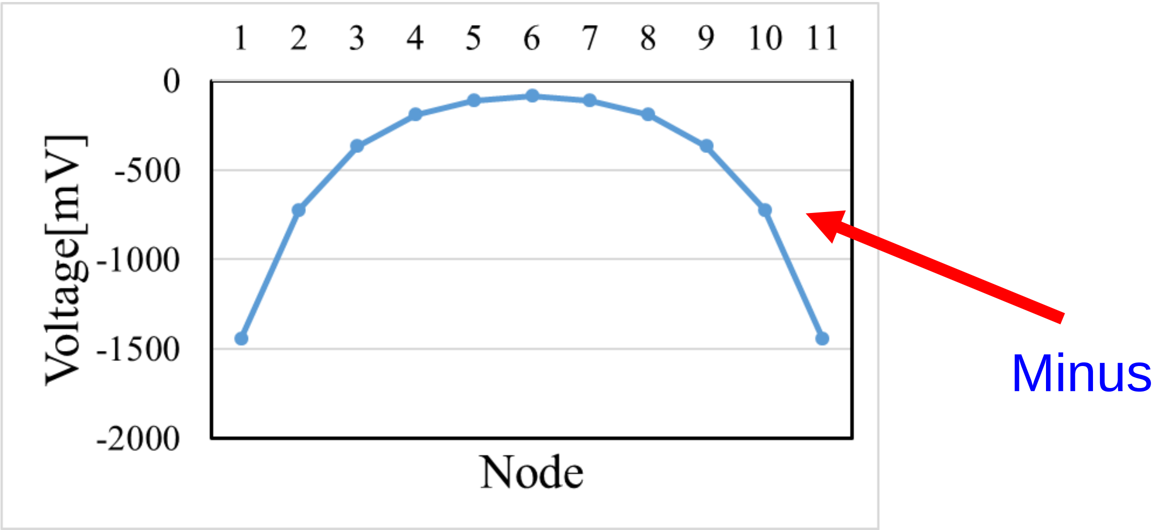
Boundary Condition

$R_0 = 2\mathrm{k}\Omega$, $R_1 = 1\mathrm{k}\Omega$, $C = 1\mathrm{pF}$, Step I =0.01mA

$R_\mathrm{T} = -2.006\mathrm{k}\Omega$



$R_\mathrm{T} = -2.005\mathrm{k}\Omega$

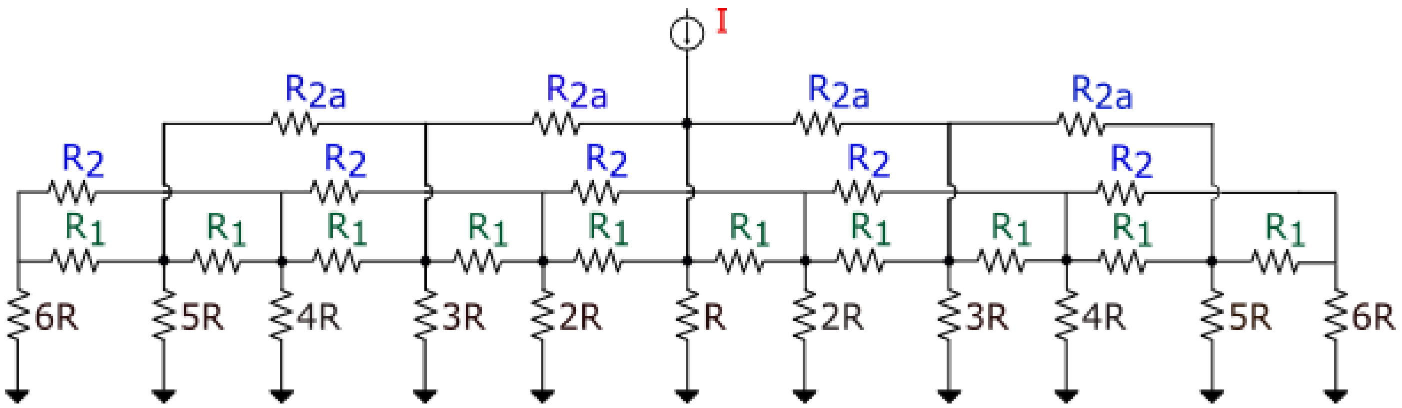


- Modestly well behaved spatial impulse response
- Temporally stable

- Violently behaved spatial impulse response
- Temporally unstable

Close relationships between spatial and temporal dynamics

General Non-Uniform Network

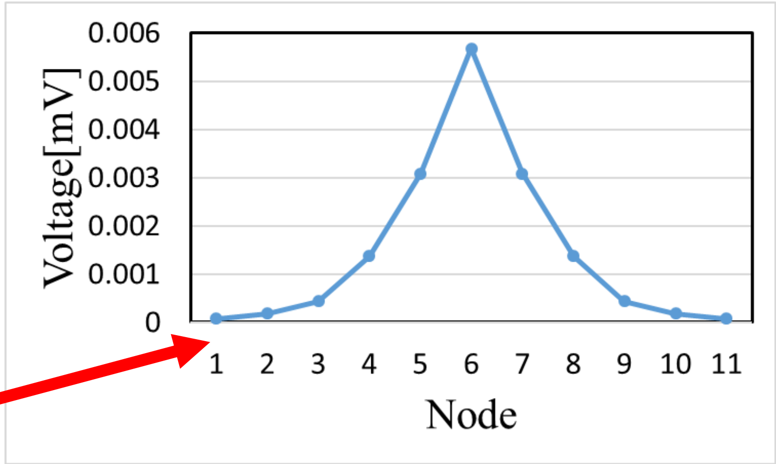


$R_2 = -4\text{k}\Omega$

$R_1 = 1\text{k}\Omega$

$R = 1\text{k}\Omega$

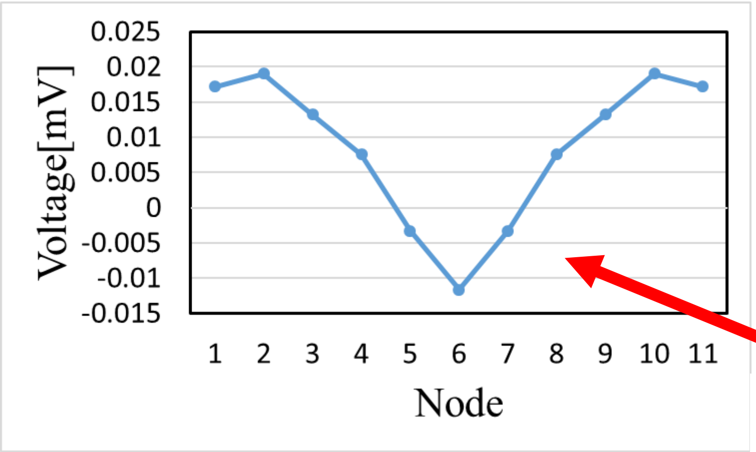
$R_{2a} = -10\text{k}\Omega$



Plus

- Modestly well behaved spatial impulse response
- Temporally stable

$R_{2a} = -1\text{k}\Omega$



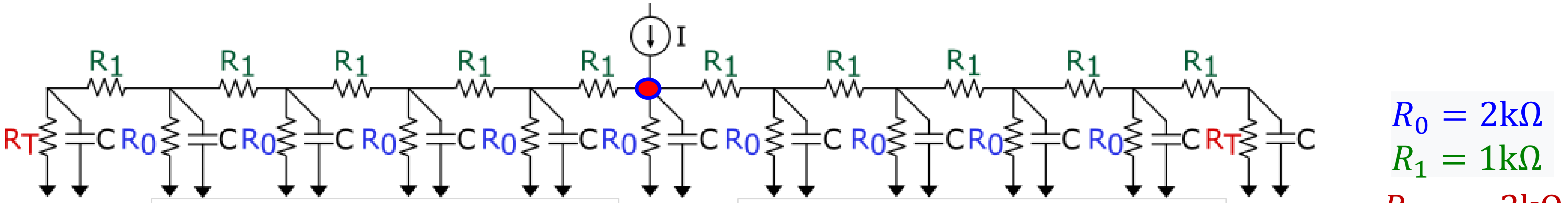
Minus

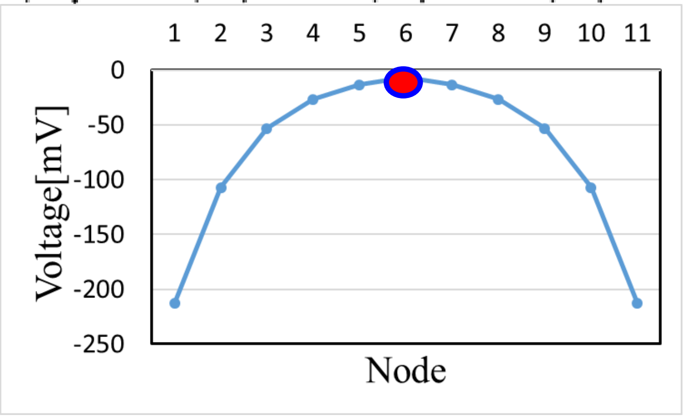
- Violently behaved spatial impulse response
- Temporally unstable

Our Conjecture

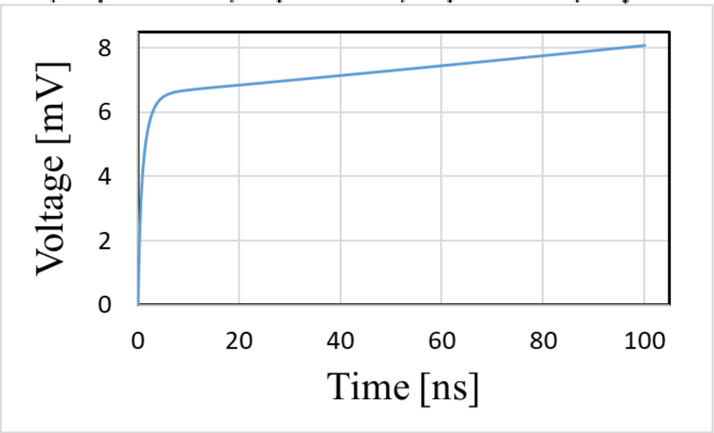
If there is a node where the input current is injected and its node voltage as the spatial impulse response is negative,

➡ *the network is temporally unstable*



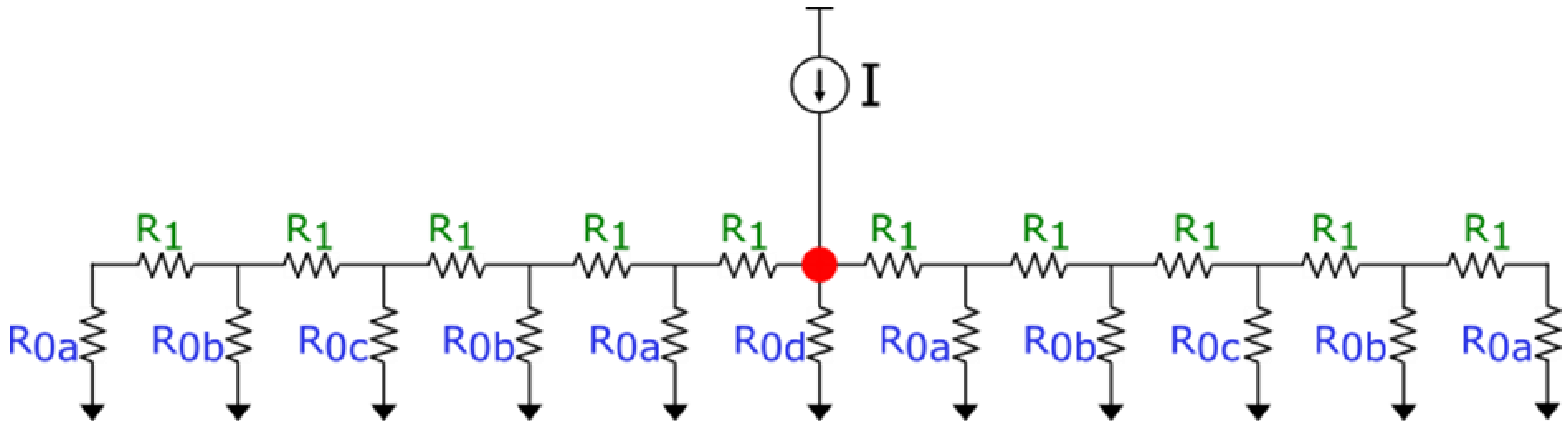


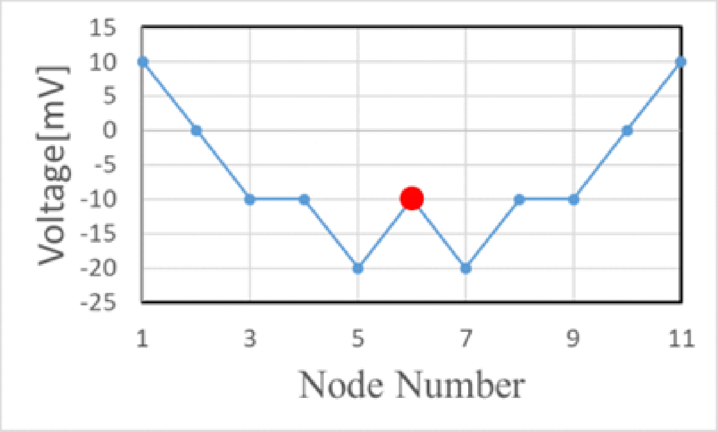
Spatial Impulse Response

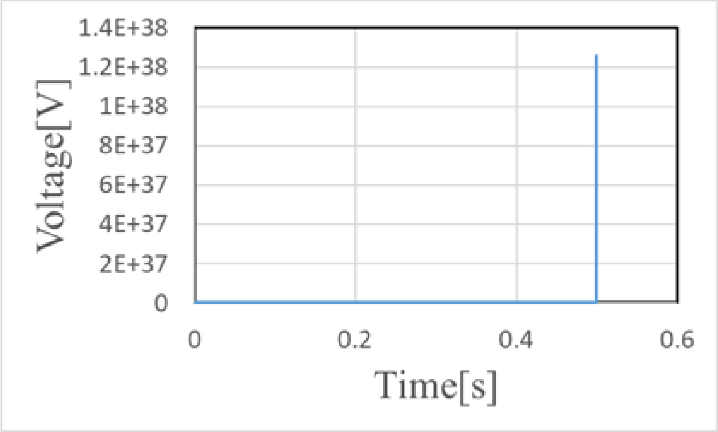


Temporally unstable

Example 1: 1st nearest connection

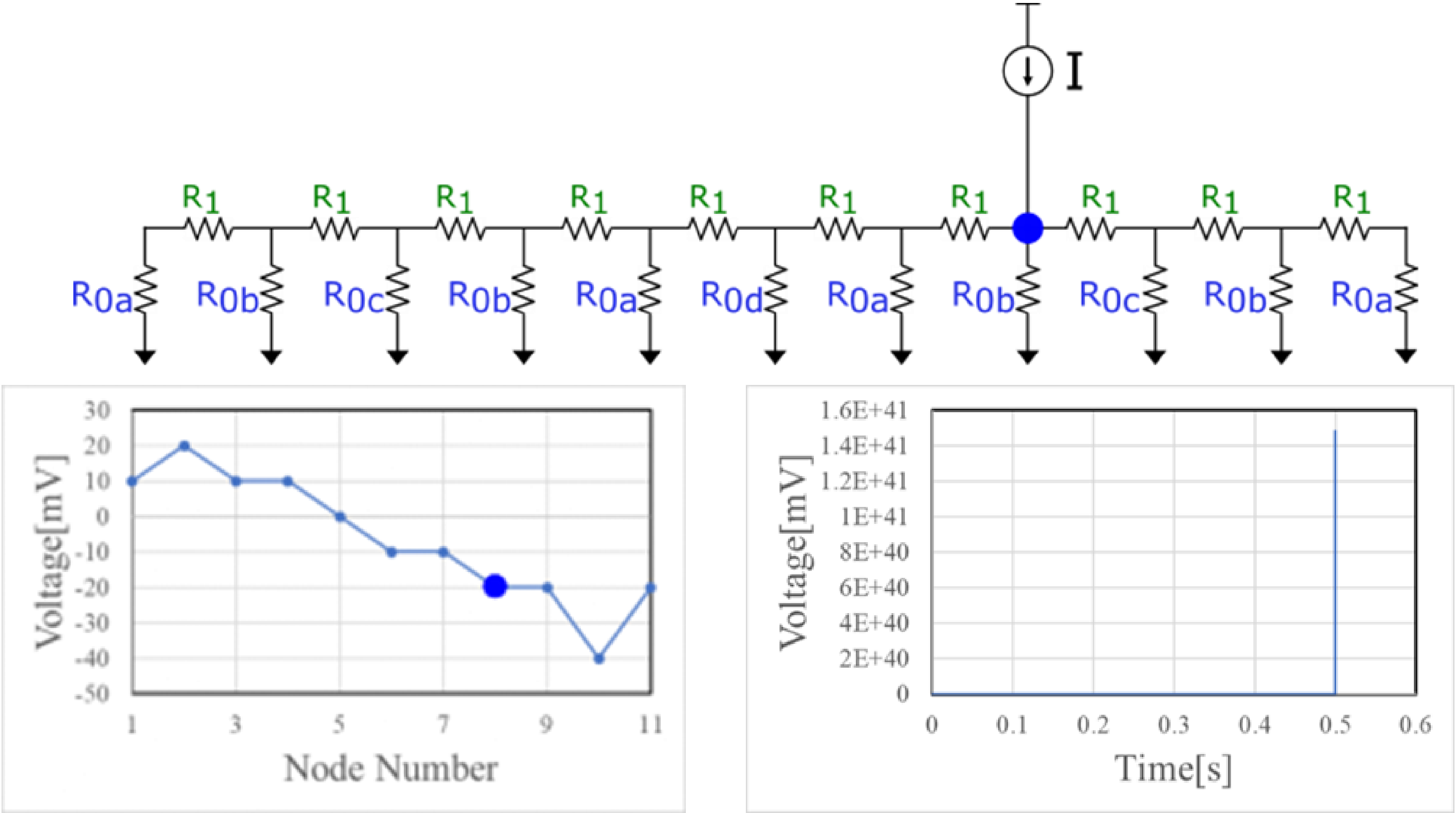






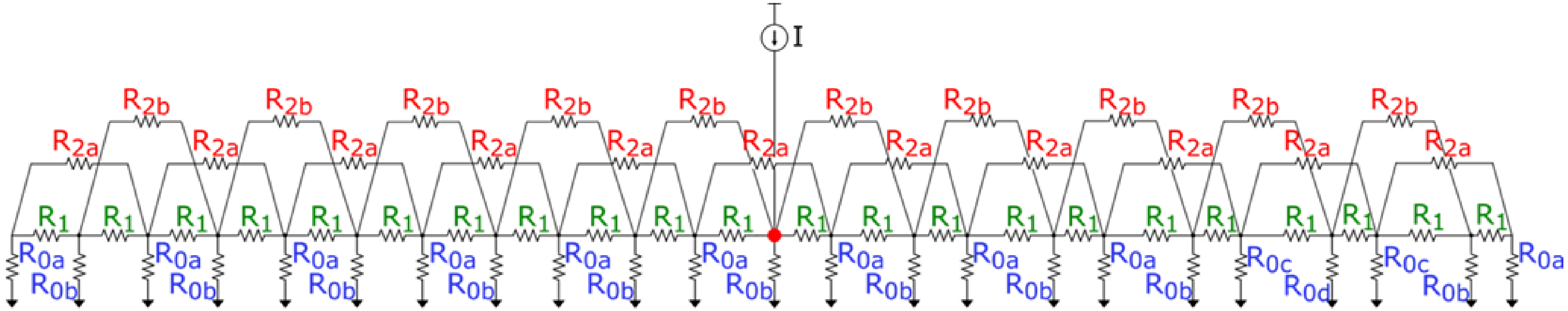
$$R_{0a} = R_{0c} = -1k\Omega, R_{0b} = R_{0d} = 1k\Omega, R_1 = 1k\Omega, I = 10\mu A.$$

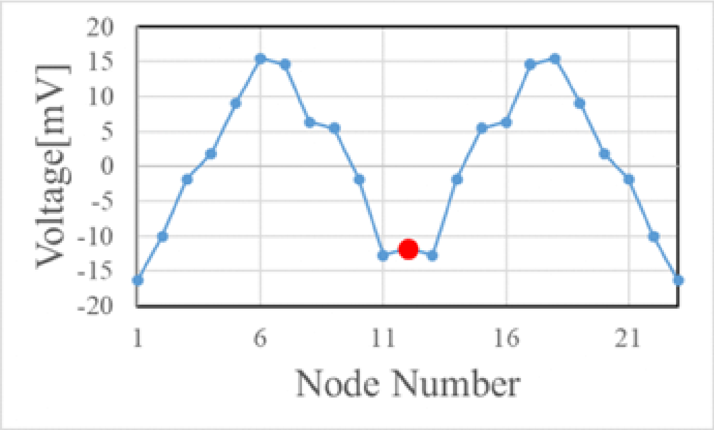
Example 2 : 1st nearest connection

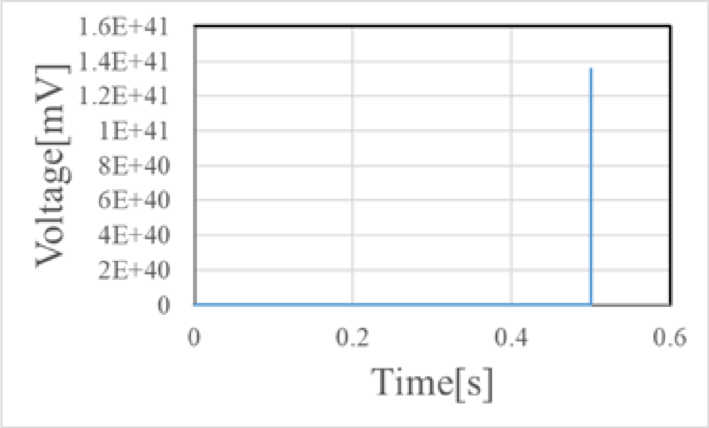


$$R_{0a} = R_{0c} = 1k\Omega, R_{0b} = R_{0d} = -1k\Omega, R_1 = 1k\Omega, I = 10\mu A.$$

Example 3 : 2nd nearest connection

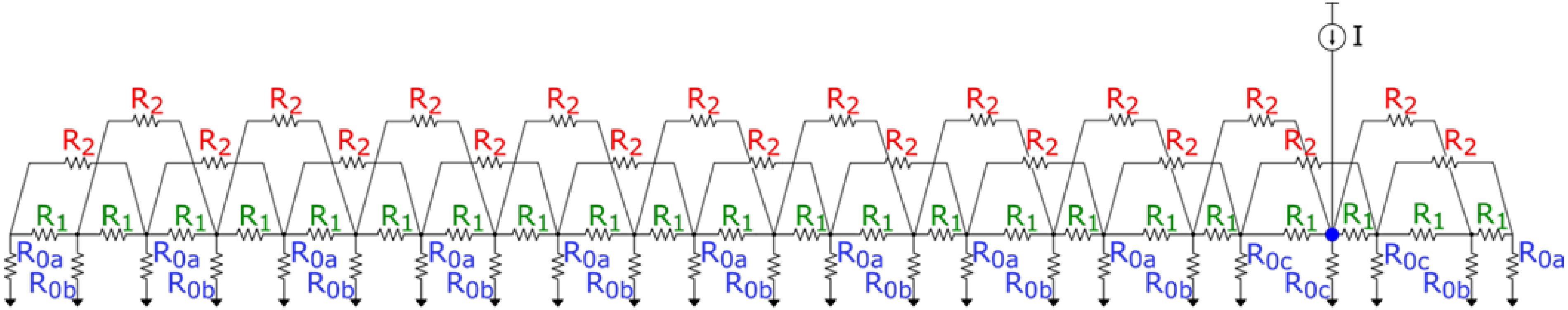


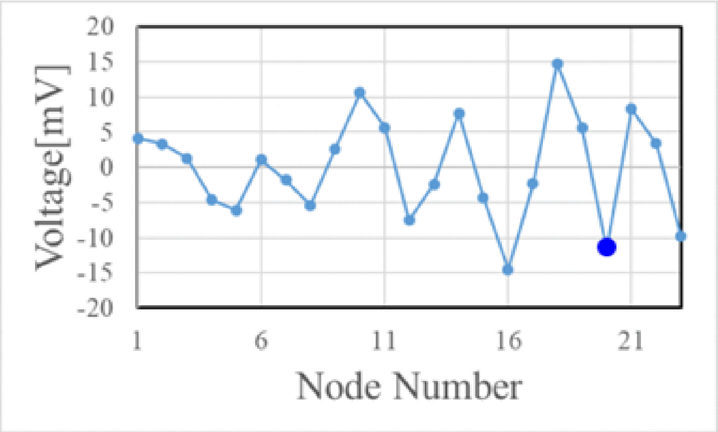


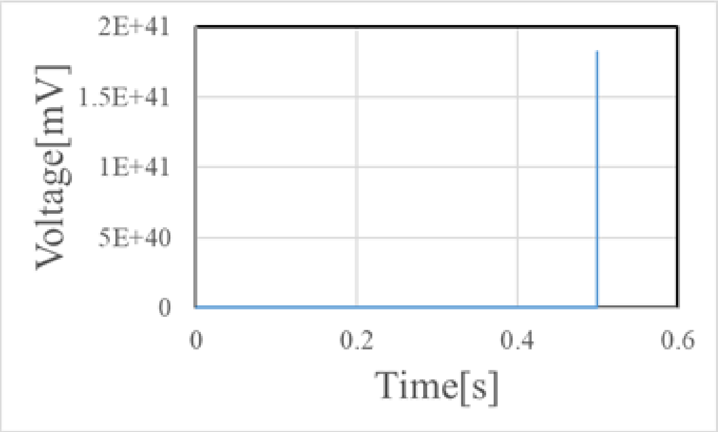


$$R_{0a} = R_{0c} = 2k\Omega, R_{0b} = R_{0d} = 1k\Omega, R_1 = 1k\Omega, R_{2a} = R_{2b} = -1k\Omega, I = 10\mu A.$$

Example 4 : 2nd nearest connection

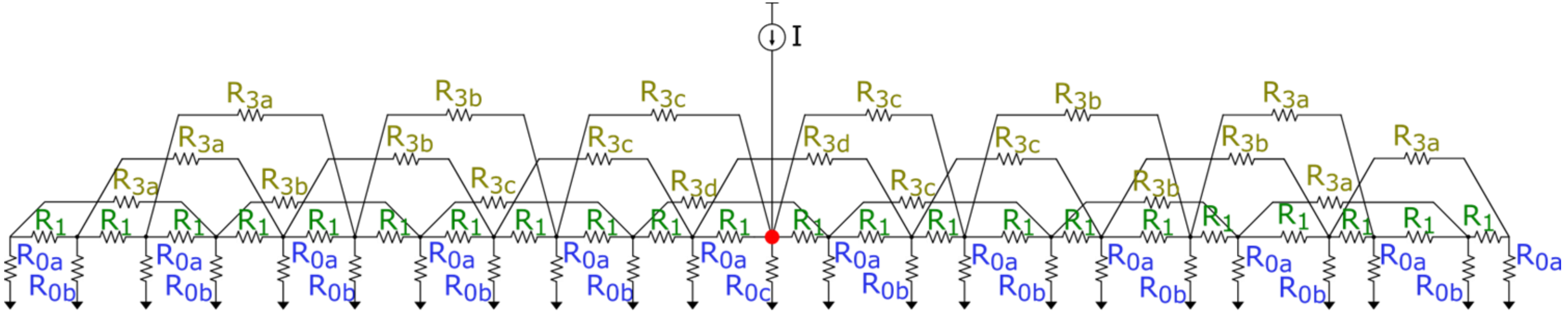


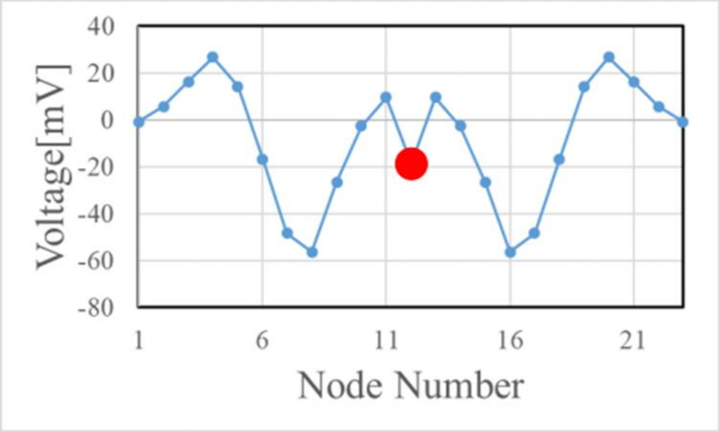


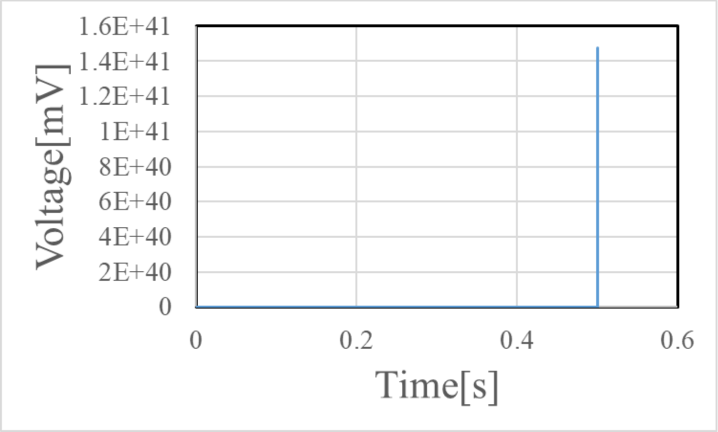


$$R_{0a} = 2k\Omega, R_{0b} = 0.5k\Omega, R_{0c} = -2k\Omega, R_1 = 1k\Omega, R_2 = -1k\Omega, I = 10\mu A.$$

Example 5: 3rd nearest connection







$$R_{0a} = 2k\Omega, R_{0b} = 3k\Omega, R_{0c} = -0.25k, R_1 = 1k\Omega, R_{3a} = -4k,$$
$$R_{3b} = -3k, R_{3c} = -2k, R_{3d} = -1k\Omega, I = 10\mu A$$

Outline

Conclusion

Circuit network theorem:

Equivalence between spatial and temporal stabilities for uniform network with negative resistors

Generalization

This research has shown in simulation for non-uniform network:

if there is a node where the input current is injected and its node voltage as the spatial impulse response is negative,

→ *the network is temporally unstable*