A new method to apply the synthetic exponential technique to infection model AID with COVID-19

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Keywords: analogy method, COVID-19, modeling, estimation, number of infected patients

Abstract. The AID model can interpret the time series peak figure of newly infected COVID-19 patients. The model employs a simple governing equation. The citizens can easily find the resultant trigger input. However, the reason why the shape of the increase occurs was not apparent. This paper clarifies why the increase occurs due to the linear sum of multiple exponential peaks. As a result, we can estimate the continuation of the trigger input that caused the infected peak.

1. Introduction

In Japan, the eighth wave of novel coronavirus infections is approaching, as shown in Fig. 1. One of the authors has already modeled the peak figure of new infections by an engineering analogy, termed the AID model. There are two significant points in the model. First, there is some cause, and as a result, the number of infected people peaks. Second, the number of infected people will proceed in the order of AID (Attack, Increase, and Decay). This AID model will give us interpretation without knowing the medical details. Since citizens can predict the end of the infectious disease peak, ordinary citizens

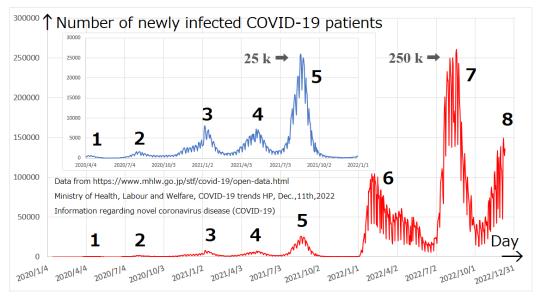


Fig.1 Number of new infections COVID-19 in Japan.

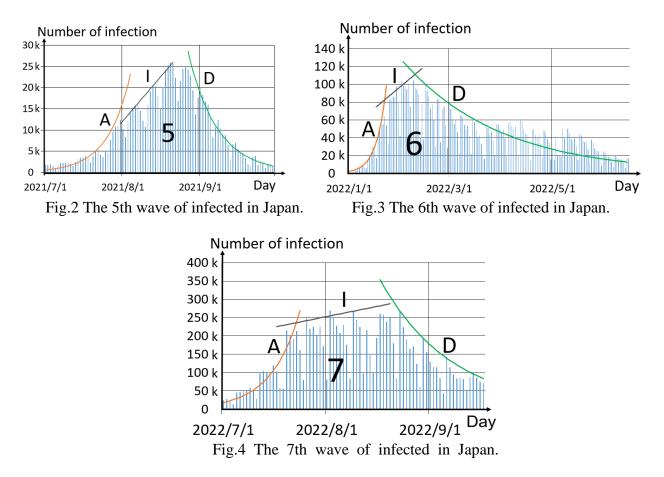
who do not know medicine will not be worried. Moreover, we can write a simple governing equation, and if we trace the phenomenon further, we can presume the cause of the infection outbreak. The governing equations are simple, and we can interpret them with a simple theory based on analogies.

However, it was unclear what mechanism caused the linear increase in the AID model. This paper presents one way of understanding the mechanism.

2. Current situation in Japan

Fig.1 shows a time series graph of the number of new PCR-positive cases in Japan. Compared to the peaks up to the 5th wave, we can find that the 6th and 7th waves are more significant than the 5th. Furthermore, we can see the extended period from spreading the infection to attenuation.

Figs.2-4 are analyzed using the AID model for the 5th to 7th waves. Table 1 shows the parameters obtained by least-squares analysis of these tendencies.



In every wave of COVID-19 infections, the initial shape of the wave increases *exponentially* in the early stage. Therefore, we can explain the increase as the population growth model by analogy. The governing equation of the model is x'(t) = cx(t), where x(t) is the number of infections, and the solution is x(t) = A exp(ct). The constant c's are calculated by the least squares method as shown in the Table 1, with n of c_n indicates n-th wave. Let us call this exponential increase "Attack."

The following increase comes in every wave of COVID-19 infections, the middle shape of the wave increases *linearly* in the middle stage. In the AID model, we interpret the cling inertia., with the governing equation of the model is x'(t) = a, and the solution is x(t) = at + b. The constant a's

are calculated by the least squares method as shown in the Table 1, with n of a_n indicates n-th wave. We call this linear increase "Increase."

In the last stage, in every wave of COVID-19 infections, the final shape of the wave decreases *exponentially*. Therefore, we can explain the decrease as the RC dissipater in the electrical circuit. The governing equation of the model is x'(t) = -kx(t), where x(t) is the number of infections, and the solution is $x(t) = A \exp(-kt)$. The constant k's are calculated by the least squares method as shown in the Table 1, with n of k_n indicates n-th wave. We call this exponential decrease "Decay."

Table 1 The calculated AD constants of 5th through 7th waves			
	5th wave	6th wave	7th wave
Attack constant	$c_5 = 0.1098 /\mathrm{d}$	$c_6 = 0.1728 /\mathrm{d}$	$c_7 = 0.1176 /d$
Increase constant	$a_5 = 622.9 / \mathrm{d}$	$a_6 = 1592.8 / \mathrm{d}$	$a_7 = 1837.1 / d$
Decay constant	$k_5 = 0.0862 /\mathrm{d}$	$k_6 = 0.0174 / \mathrm{d}$	$k_7 = 0.0467 / \mathrm{d}$

Table 1 The calculated AID constants of 5th through 7th waves

* All the time constants are expressed in reciprocal numbers per day. For example, the time constant of 2 days is expressed as 0.5/d according to the SI notation of unit.

The above sequence, we call the AID transition. In this paper, we propose a more straightforward approach. Using an AD model with an exponential increase (Attack) and an exponential decay (Decay) is appropriate. Their linear sum may be able to express the shape of the peak. Let us try the linear sum approximation of the AD model from this point of view.

3. Linear Sum of AD Models

The waveform of Fig. 5 is the primary AD model figure considered in this paper. For example, Figs.6-8 show the composition of several waves. We can create an AID shape in those figures by adding AD waves together at different delayed times. As shown in the figure, we made the AID figure with increasing characteristics. Furthermore, we can offer a longer raised part or a nearly flat shape.

This idea is a convolution because we perform the linear sum with time shifting. If the input is x(t) and the impulse response is $h(\tau)$, the output is y(t). Here, the two AD impulse responses have a constant original time constant even if we obtain a sum of multiple AD partial of only the Attack parts. That is if the attack expressed as

$$h(\tau) = A \cdot e^{c\tau},\tag{1}$$

the shifted add

$$h(\tau) + a \cdot h(\tau - t_0) = A \cdot e^{c\tau} + aA \cdot e^{c(\tau - t_0)} = (1 + ae^{-ct_0}) \cdot h(\tau)$$

= $A(1 + ae^{-ct_0}) \cdot e^{c\tau}$ (2)

holds the same time constant. For the decaying part, the time constant is also the same.

So even if the amplitude is different, the time constant does not change. That is why we can solve the impulse response AD from the peak waveform.

To generalize

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau.$$
(3)

Upon encountering and solving this inverse problem, we will find the peak shape to deduce the triggering event that caused the peak.

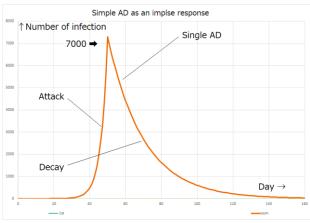


Fig.5 The AD model impulse response.

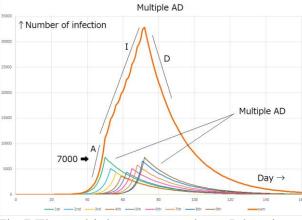


Fig.7 The multiple sum can enlarge I duration.

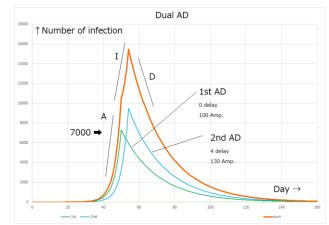


Fig.6 The sum of 2 distinct AD yields AID.

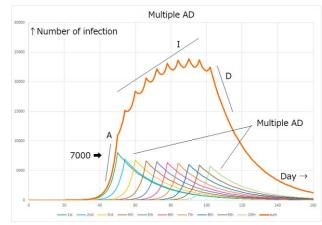


Fig.8 The multiple sum also yiels calm I figure.

4. Conclusion

First of all, let us remark on a discussion here. The multiple sums of AD correspond to different modes of epidemics, but each is minor. Some epidemics may happen in different regions in Japan or may repeatedly happen in the same places. The Increase duration holds much information in a pandemic. Not knowing medical issues, we can analyze the infection trends, even if the citizens.

In conclusion, let us summarize the significance of what this paper informs us about the COVID-19 pandemic. We traditionally interpret the figure of the Increase mode in the AID model as a function of inertia, such as residual heat after the loss of input. Our hypothetical results are a good representation of reality and make sense. The interpretation of the Increase mode is caused by convolution, apart from actual physical phenomena. It should be possible to explain each of these physical phenomena, and the method presented here can be expected to develop in the future.

References

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