

Effect of the Delay Elements Variation on Time-to-Digital Converter Linearity

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Abstract. We investigated the effect of the variation in the delay elements to measure the difference in time between two signals on the nonlinearity of the TDC (time-to-digital converter) in this study. Even if the sum of the variations were the same, the different order of the elements would affect the nonlinearity. A sequence of delay elements with positive or negative deviations results in a larger nonlinearity.

1. Introduction

In electronic instrumentation and signal processing, a time-to-digital converter (abbreviated as TDC) is used as a device for recognizing timing events and providing a digital representation of the time they occurred [1][2]. For example, the TDC might output the time of arrival for each incoming pulse. Some applications wish to measure the time interval between two timing events rather than some notion of an absolute time.

The TDC is used in many different applications, where the time interval between two signal pulses (start and stop pulses) should be determined. Measurement is started and stopped, when either the rising or the falling edge of a signal pulse crosses a set threshold. These requirements are fulfilled in many physical experiments, like time-of-flight and lifetime measurements in atomic and high energy physics, experiments that involve laser ranging and electronic research involving the testing of integrated circuits and high-speed data transfer [3].

The accuracy (linearity) of the input time vs. output code is very important for TDC obviously. Our research group has proposed a self-calibration method [4][5]. In particular, we have recently been proposing a self-calibration method using metallic ratio sampling technique [6][7]. In this study, we investigated the effect of the variation in the delay elements on the nonlinearity of the TDC.

2. Time-to-Digital Converter (TDC)

2.1 Basic Flash-Type TDC

Figure 1 shows the configuration of a basic flash-type TDC; the reference CLK passes through a buffer delay line, which consists of a chain of inverters, and the delayed reference CLK signals are used as the data input for some flip-flop (DFF) circuits. The measured signal is used as the clock signal of the flip-flops. We obtain the outputs of the flip-flops as a thermometer code, according to the rise-edge-timing interval between the reference “Start” edge and the “Stop” edge, and the encoder transforms it into a binary code. The time resolution is determined by the gate delay τ .

Consider an example where $\tau = 1.0$ and the time between “Start” and “Stop” (Input Time) is 2.7. As shown in Fig. 1, a “Stop” edge is entered in the middle of the third delay element. A thermometer-code is obtained where D0, D1 and D2 are 1 and D3 onwards are 0. This is 10 (2 in decimal number) when converted to binary code, and the “Input Time” is accurately output as a digital value.

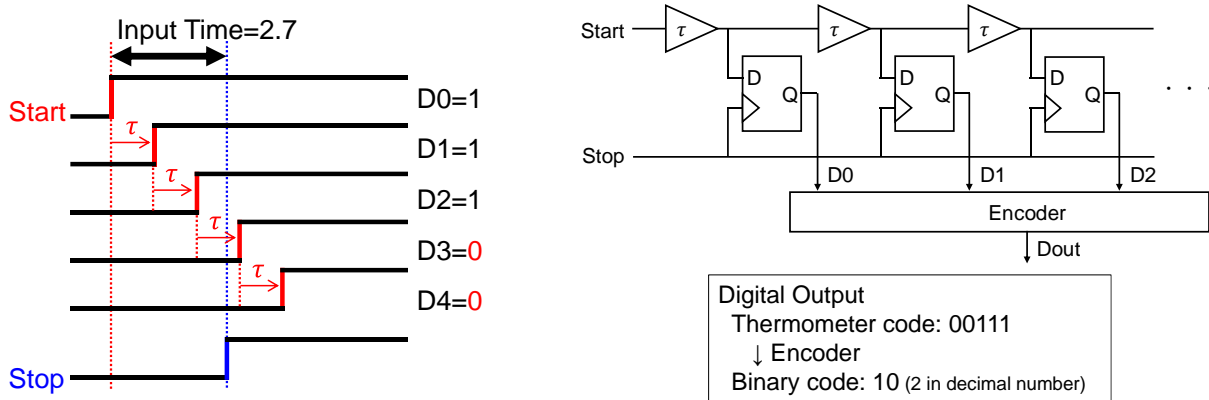


Fig. 1. A flash-type TDC architecture and an example of operation.

2.2 TDC with Delay Elements Variation

Consider the case where the delay element described in Section 2.2 has a variation ($\Delta\tau_i$). As shown in Fig. 2, the “Start” edge is shifted by the amount of the variation. Due to the shift, a “Stop” edge is introduced in the middle of the second delay element. A thermometer-code is obtained where D0 and D1 are 1 and D2 onwards are 0. This is 11 (1 in decimal number) when converted to binary code, and the “Input Time” cannot be output accurately as a digital value.

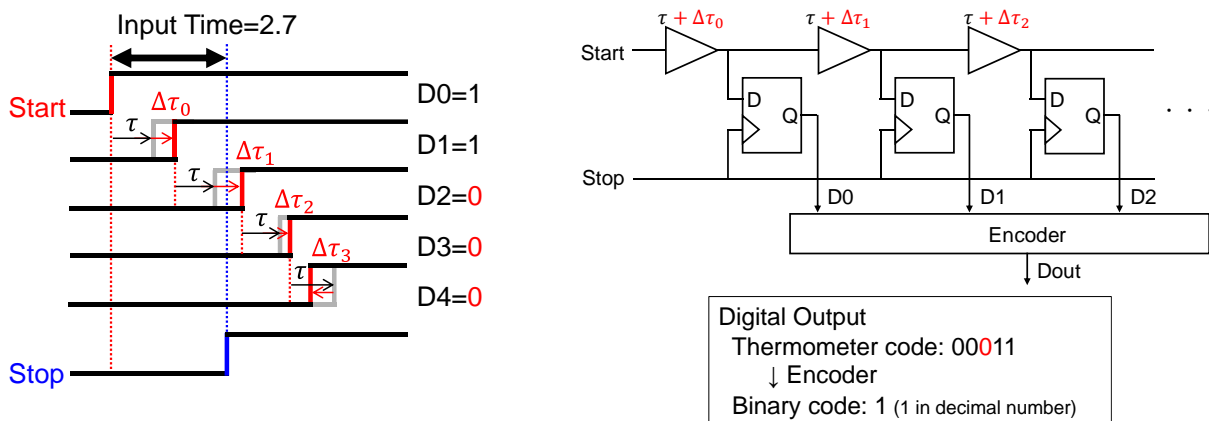
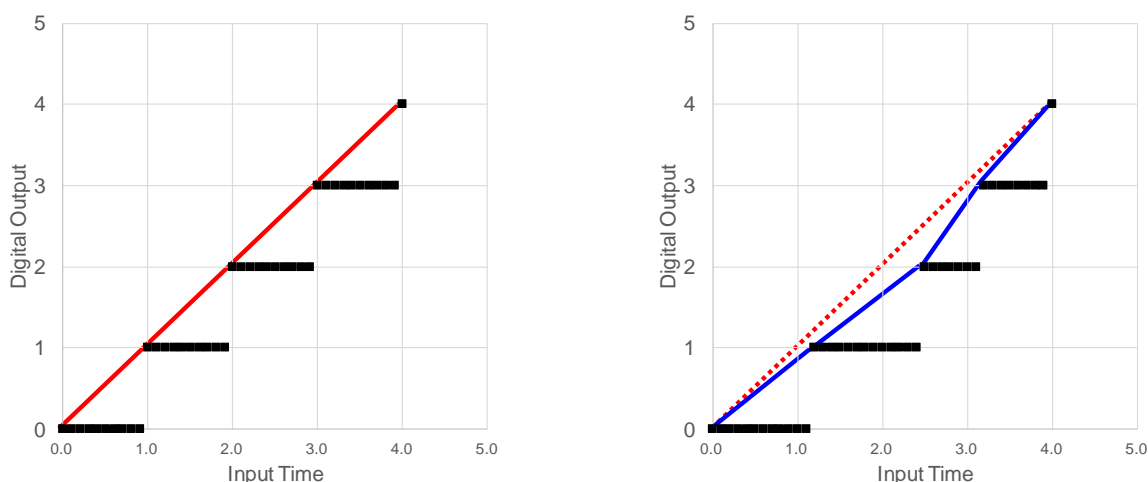


Fig. 2. An example of operation with delay elements variation.
As in $\Delta\tau_3$, the variation could be negative.

The inputs and outputs of the TDC are shown in Fig. 3. The “Digital Output” obtained by changing the “Input Time” from 0.0 to 4.0 in 0.1 increments in the system shown in Fig. 1 are plotted by black squares in Fig. 3(a). As shown by the red line, Input and Output have perfect linearity.

An example with variation is shown in Fig. 3(b) (Note: For clarity of the figure, a larger variation than the image in Fig. 2 was used as an example input). The blue line shows the relationship between input and output. The red dotted line is the same as in Fig.3(a). The variation gives a large non-linearity to the blue line.



(a) Without delay elements variation. (b) With delay elements variation.

Fig. 3. TDC inputs and outputs.

The digital output is indicated by a decimal number binary code.

3. Simulation

3.1 Problem Settings

The RMS, defined in Eq. (1), is used as a measure of non-linearity. It is the root-mean-square of the difference diff_i between no variation (no nonlinearity). diff_i is defined in Eq. (2). It is also shown in green lines in Fig. 4 which is created by adding information to Fig. 3(b).

$$\text{RMS} = \sqrt{\frac{\sum_{i=0}^N (\text{diff}_i)^2}{N}} \quad (1)$$

$$\text{diff}_i = \text{Minimum}(\text{Input time} |_{\text{Digital Output}=i}) - i \quad (2)$$

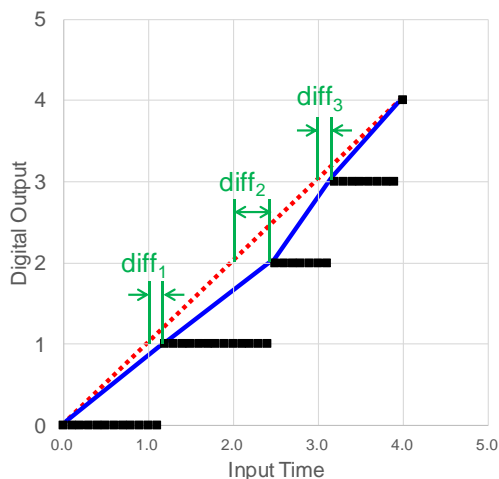


Fig. 4. Definition of “ diff_i ” in Eq. (2).

The total delay elements variation is set to zero as follows:

$$\sum_{i=0}^N \Delta\tau_i = 0 \tag{3}$$

The parameters in this study were set as follows: $N = 7$ (that is, number of delay elements is 8.), $\tau = 1.0$, $|\Delta\tau_i| = 0.01$. All 70 possibilities were examined for patterns of variation. Some examples are given in Table 1.

Table 1. Delay elements variation patterns (-0.01 is painted gray for clarity).

	$\Delta\tau_0$	$\Delta\tau_1$	$\Delta\tau_2$	$\Delta\tau_3$	$\Delta\tau_4$	$\Delta\tau_5$	$\Delta\tau_6$	$\Delta\tau_7$
Patten 0	-0.01	-0.01	-0.01	-0.01	0.01	0.01	0.01	0.01
Patten 1	-0.01	-0.01	-0.01	0.01	-0.01	0.01	0.01	0.01
Patten 2	-0.01	-0.01	-0.01	0.01	0.01	-0.01	0.01	0.01
Patten 3	-0.01	-0.01	-0.01	0.01	0.01	0.01	-0.01	0.01
Patten 4	-0.01	-0.01	-0.01	0.01	0.01	0.01	0.01	-0.01
Patten 5	-0.01	-0.01	0.01	-0.01	-0.01	0.01	0.01	0.01
Patten 6	-0.01	-0.01	0.01	-0.01	0.01	-0.01	0.01	0.01
Patten 7	-0.01	-0.01	0.01	-0.01	0.01	0.01	-0.01	0.01
Patten 8	-0.01	-0.01	0.01	-0.01	0.01	0.01	0.01	-0.01
...								
Patten 68	0.01	0.01	0.01	-0.01	0.01	-0.01	-0.01	-0.01
Patten 69	0.01	0.01	0.01	0.01	-0.01	-0.01	-0.01	-0.01

3.2 Results

The RMS defined in Eq. (1) has been calculated for the 70 different patterns introduced in Table 1 and the results are shown in Fig. 5.

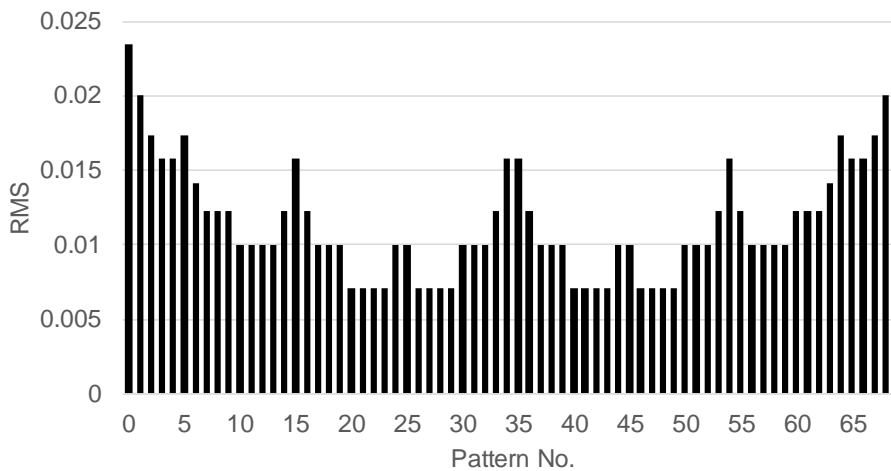
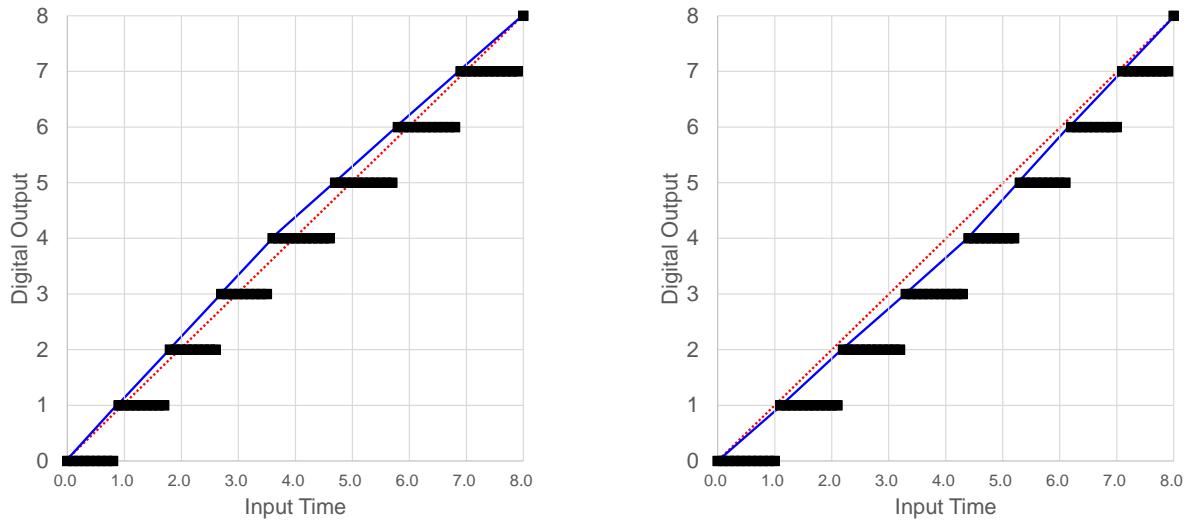


Fig. 5. Calculated RMS.

The shape of the graph in Fig. 5 is symmetrical. For example, pattern 0 and pattern 69 have the same RMS. Figure 6 shows an imaginary diagram. It is targeted across the red broken line of linearity without delay elements variation. Therefore, the following discussion is only for patterns 0 to 34.



(a) Patten 0.

(b) Patten 69.

Fig. 6. Example of TDC nonlinearity that results in the same RMS.

(Note: The red and blue lines almost overlap each other when $|\Delta\tau_i| = 0.01$, making it difficult to understand, so these images were created with $|\Delta\tau_i| = 0.1$.)

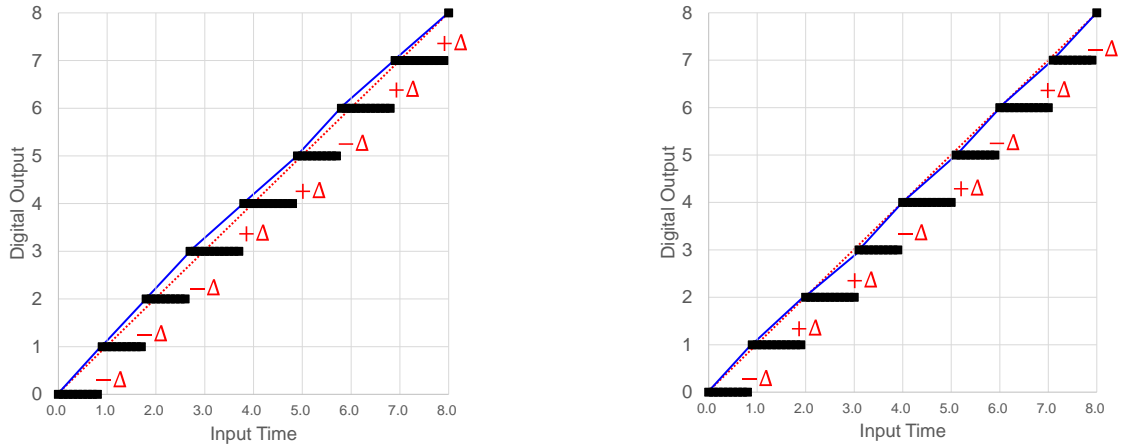
The pattern of cases with large RMS (>0.015) is shown in Table 2. A sequence of 3 or 4 $\Delta\tau_i = 0.01$ or $\Delta\tau_i = -0.01$ causes larger RMS. On the other hand, RMS becomes smaller when $\Delta\tau_i = 0.01$ and $\Delta\tau_i = -0.01$ almost alternate, as shown in Table 3. Figure 7 shows one example each of the TDC nonlinearity cases with large and small RMS. Alternating $\Delta\tau_i = 0.01$ and $\Delta\tau_i = -0.01$ cancels out the deviation from the red line, resulting in smaller RMS.

Table 2. Delay elements variation patterns with large RMS.

	$\Delta\tau_0$	$\Delta\tau_1$	$\Delta\tau_2$	$\Delta\tau_3$	$\Delta\tau_4$	$\Delta\tau_5$	$\Delta\tau_6$	$\Delta\tau_7$
Patten 0	-0.01	-0.01	-0.01	-0.01	0.01	0.01	0.01	0.01
Patten 1	-0.01	-0.01	-0.01	0.01	-0.01	0.01	0.01	0.01
Patten 2	-0.01	-0.01	-0.01	0.01	0.01	-0.01	0.01	0.01
Patten 3	-0.01	-0.01	-0.01	0.01	0.01	0.01	-0.01	0.01
Patten 4	-0.01	-0.01	-0.01	0.01	0.01	0.01	0.01	-0.01
Patten 5	-0.01	-0.01	0.01	-0.01	-0.01	0.01	0.01	0.01
Patten 15	-0.01	0.01	-0.01	-0.01	-0.01	0.01	0.01	0.01
Patten 34	-0.01	0.01	0.01	0.01	0.01	-0.01	-0.01	-0.01

Table 3. Delay elements variation patterns with small RMS.

	$\Delta\tau_0$	$\Delta\tau_1$	$\Delta\tau_2$	$\Delta\tau_3$	$\Delta\tau_4$	$\Delta\tau_5$	$\Delta\tau_6$	$\Delta\tau_7$
Patten 20	-0.01	0.01	-0.01	0.01	-0.01	0.01	-0.01	0.01
Patten 21	-0.01	0.01	-0.01	0.01	-0.01	0.01	0.01	-0.01
Patten 22	-0.01	0.01	-0.01	0.01	0.01	-0.01	-0.01	0.01
Patten 23	-0.01	0.01	-0.01	0.01	0.01	-0.01	0.01	-0.01
Patten 26	-0.01	0.01	0.01	-0.01	-0.01	0.01	-0.01	0.01
Patten 27	-0.01	0.01	0.01	-0.01	-0.01	0.01	0.01	-0.01
Patten 28	-0.01	0.01	0.01	-0.01	0.01	-0.01	-0.01	0.01
Patten 29	-0.01	0.01	0.01	-0.01	0.01	-0.01	0.01	-0.01



(a) Large RMS example (Patten 2). (b) Small RMS example (Patten 29).
Fig. 7. Examples of TDC nonlinearity that results of large and small RMS.

Table 3 lists the patterns with the smallest RMS; Table 4 lists the patterns with slightly larger RMS (RMS=0.01). Comparing Table 3 and Table 4, consider the factors that increase RMS: In all of the patterns listed in Table 3, Eq. (4) is satisfied. The RMS becomes smaller as a result of cancelation local variations.

$$\Delta\tau_0 + \Delta\tau_1 = \Delta\tau_2 + \Delta\tau_3 = \Delta\tau_4 + \Delta\tau_5 = \Delta\tau_6 + \Delta\tau_7 = 0 \quad (4)$$

In Table 4, Eq. (4) is not satisfied. For example, in Pattern 10, the variation relationship is as in Eq. (5), resulting in increased linearity and larger RMS.

$$\Delta\tau_0 + \Delta\tau_1 < 0, \quad \Delta\tau_2 + \Delta\tau_3 > 0, \quad \Delta\tau_4 + \Delta\tau_5 = \Delta\tau_6 + \Delta\tau_7 = 0 \quad (5)$$

Table 4. Delay elements variation patterns with slightly larger RMS (=0.01).

	$\Delta\tau_0$	$\Delta\tau_1$	$\Delta\tau_2$	$\Delta\tau_3$	$\Delta\tau_4$	$\Delta\tau_5$	$\Delta\tau_6$	$\Delta\tau_7$
Patten 10	-0.01	-0.01	0.01	0.01	-0.01	0.01	-0.01	0.01
Patten 11	-0.01	-0.01	0.01	0.01	-0.01	0.01	0.01	-0.01
Patten 12	-0.01	-0.01	0.01	0.01	0.01	-0.01	-0.01	0.01
Patten 13	-0.01	-0.01	0.01	0.01	0.01	-0.01	0.01	-0.01
Patten 17	-0.01	0.01	-0.01	-0.01	0.01	0.01	-0.01	0.01
Patten 18	-0.01	0.01	-0.01	-0.01	0.01	0.01	0.01	-0.01
Patten 19	-0.01	0.01	-0.01	0.01	-0.01	-0.01	0.01	0.01
Patten 24	-0.01	0.01	-0.01	0.01	0.01	0.01	-0.01	-0.01
Patten 25	-0.01	0.01	0.01	-0.01	-0.01	-0.01	0.01	0.01
Patten 30	-0.01	0.01	0.01	-0.01	0.01	0.01	-0.01	-0.01
Patten 31	-0.01	0.01	0.01	0.01	-0.01	-0.01	-0.01	0.01
Patten 32	-0.01	0.01	0.01	0.01	-0.01	-0.01	0.01	-0.01

4. Conclusion

We investigated the effect of the variation in the delay elements of the TDC on the nonlinearity of the TDC in this study. It was found that even if the sum of the variations is the same, the deviation from the ideal TDC (RMS) becomes larger when the order of the variations is different. In particular, a sequence of delay elements with positive or negative deviations results in a larger RMS. On the other hand, if the positive and negative deviations of the delay elements locally cancel each other out, the RMS becomes smaller.

The following are planned for future works:

- To investigate various patterns of variation, such as $\Delta\tau_0 = 0.04$, $\Delta\tau_1 = \Delta\tau_2 = \Delta\tau_3 = 0.0$, $\Delta\tau_4 = \Delta\tau_5 = \Delta\tau_6 = \Delta\tau_7 = -0.01$, for example. although this time we only investigated $|\Delta\tau_i| = 0.01$.
- We attempt to self-calibration with the golden ratio proposed by our research group. We will investigate the patterns of the delay element variation that necessitate calibration of the TDC.

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