***Proceedings of Joint Conference of***
***11th International Science, Social Sciences, Engineering and Energy Conference (I-SEEC 2022) and,***
***6th International Conference on Technology and Social Science 2022 (ICTSS 2022)***

# Time-to-Digital Converter Linearity Calibration with Metallic Ratio Sampling

Shuhei Yamamoto[1, a, *], Kentaroh Katoh[1,b], Zheming Zhao[1,c], Yujie Zhao[1,d], Shogo Katayama[1,e], Anna Kuwana[1,f], Jianglin Wei[2,g], Keno Sato[3,h], Takashi Ishida[3,i] , Toshiyuki Okamoto[3,j], Tamotsu Ichikawa[3,k], Takayuki Nakatani[1,l], Kazumi Hatayama[1,m], Haruo Kobayashi[1,n]

[1] Division of Electronics and Informatics, Faculty of Science and Technology, Gunma University

1-5-1 Tenjin-cho Kiryu Gunma 376-8515, Japan

[2] Faculty of Intelligence Manufacturing, Yibin University, Yibin Sichuan, 64400 China

[3] ROHM Semiconductor, 2-4-8 Shin-Yokohama, Kouhoku-Ku, Yokohama 222-8575, Japan

[a]<t170d123@gunma-u.ac.jp>, [b]< kentarohkkkatoh@yahoo.co.jp>, [c]< t201d605@gunma-u.ac.jp>, [d]< t202d002@gunma-u.ac.jp >, [e]< t15304906@gunma-u.ac.jp>, [f]< kuwana.anna@gunma-u.ac.jp>,

[g]< weijianglin9@163.com>, [h]< keno.sato@dsn.rohm.co.jp>, [i]< takashi.ishida@lsi.rohm.co.jp>, [j]< toshiyuki.okamoto@lsi.rohm.co.jp>, [k]< tamotsu.ichikawa@lsi.rohm.co.jp>, [l]< takayuki.nakatani1017@gmail.com>, [m]< hatayama@oak.gunma-u.ac.jp>, [n]<koba@gunma-u.ac.jp>

**Keywords:** time-to-digital converter**,** linearity calibration, histogram method, golden ratio, metallic ratio, equivalent-time sampling

**Abstract.** This paper describes an efficient linearity calibration technique for the flash-type time-to-digital converter (TDC) with the histogram method using the metallic ratio sampling. Its principle and simulation results for validation are presented.

## 1. Introduction

The TDC measures the timing difference between two digital signals with a digital value. Thanks to the recent miniaturization of CMOS LSIs, the TDC can perform it with fine time resolution [1, 2]. However, due to the process, temperature and supply voltage (PVT) variations, its linearity may be degraded and its effective remedy is required. In this paper, we show that the metallic sampling method is useful for deciding the frequency ratio of the upper and lower ring oscillators for the TDC linearity calibration with the histogram method. We previously proposed the metallic sampling method for efficient waveform acquisition with equivalent-time sampling in LSI testing [3]. We show here that it can be applied to the TDC linearity calibration with the histogram method; when the input signal frequency and sampling frequency are set to a metallic ratio, its effective linearity calibration can be achieved. We show several simulation results for its validation.

This paper is organized as follows: Section 2 explains the TDC configuration and operation. Section 3 shows the TDC nonlinearity due to the buffer delay mismatches and its calibration method with the histogram method. Section 4 explains the metallic ratio sampling while Section 5 discusses its application to the TDC linearity calibration and demonstrates its effectiveness with simulation results. Section 6 concludes the paper.

## 2. Time-to-Digital Converter (TDC)

Flash TDC measures the time interval between two edges [1, 2]. Fig. 1 shows the configuration of a flash TDC: the reference clock (START) passes through a buffer delay line, which consists of a chain of inverters, and the delayed reference clock (START) signals are used as the data input for some flip-flop (DFF) circuits. The measured signal (STOP) is used as the clock signal of the flip-flops. We obtain the outputs of the flip-flops as a thermometer code, according to the rise-edge-timing interval between the reference START edge and STOP edge, and the encoder transforms the results into a binary code. The time resolution is determined by gate delay $\tau$.

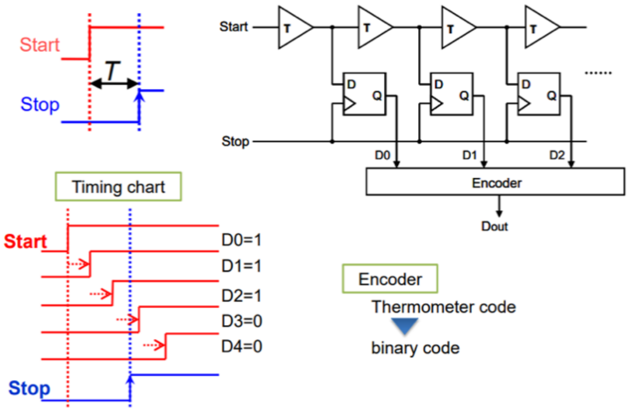


Fig. 1. A flash TDC architecture and its operation.

## 3. Linearity Calibration of TDC

Ideally, all of the buffer delays τ in Fig. 1 are identical. However, in reality they may be mismatched due to PVT variations, which causes the TDC nonlinearity. The delay variation $\Delta\tau 1$, $\Delta\tau 2$, …. can be measured with the histogram data when the frequencies of START and STOP clocks are asynchronous and the large number of the TDC output data are collected (Fig.2) [2, 4]. The Differential Non-Linearity (DNL) and Integral Non-Linearity (INL) at digital code k are respectively given by

$$\mathrm{DNL}\ (k) = \Delta\tau_k/\tau \tag{1}$$

$$\mathrm{INL}\ (k) = \mathrm{DNL}(1) + \mathrm{DNL}(2) + \ldots + \mathrm{DNL}\ (k) \tag{2}$$

Here τ is the average delay value of gate delays. We assume here that the START and STOP are asynchronous with each other and the distribution of the timing differences between START and STOP is statistically uniform and also a large number of data are collected. When the buffers have uniform delay, the distribution of the bins of the constructed histogram is also uniform. On the other hand, if there are some mismatches ($\Delta\tau 1$, $\Delta\tau 2$, ….) among gate delay values, the histogram for the k-th bin is proportional to $\tau+\Delta\tau_k$ or 1+DNL(k). In other words, DLN(k) can be measured from the histogram data. With the obtained histogram, the TDC nonlinearity is compensated during normal operation by calculation of the inverse transfer function obtained by INL in digital domain [2].

For self-calibration, we proposed the configuration of two ring oscillators to generate START and STOP signals which are not synchronized and whose frequencies are $f_1$ and $f_2$ respectively (Fig. 3). However, we found that when the ratio of $f_1$ and $f_2$ is a proper value, the accurate measurement of the buffer delays with the histogram method can be achieved with small number of data; otherwise it cannot be achieved.

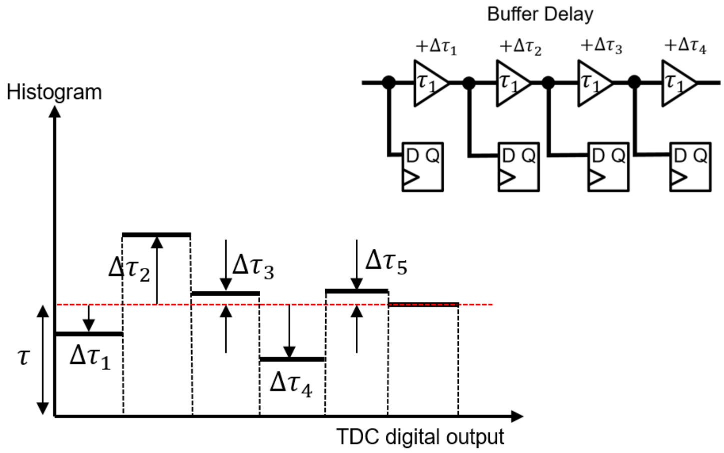


Fig. 2. Delay mismatches and their measurements with the histogram data.

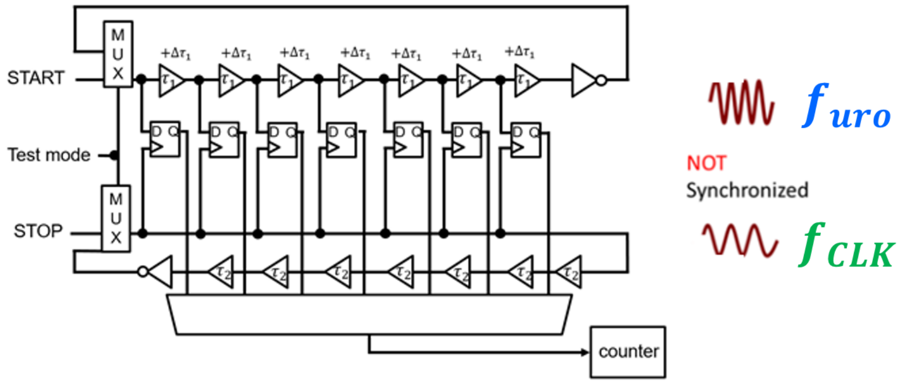


Fig. 3. TDC configuration in linearity self-calibration mode.

## 4. Equivalent-Time Sampling Technique

There are two sampling techniques for waveform acquisition: real-time sampling and equivalent-time sampling [5, 6, 7, 8]. Real-time sampling can sample any input signal of a single event if the sampling theorem condition is satisfied. Equivalent-time sampling is only applicable to repetitive waveform input, but sampling can be performed with a time resolution finer than the period of the sampling clock. Equivalent-time sampling can be classified into random sampling, sequence sampling and coherent sampling, and our previously proposed metallic ratio sampling in [3] is in the case of the random sampling. We consider here to apply the equivalent-time sampling technique with metallic ratio sampling to the effective self-calibration technique of the TDC linearity.

### 4.1 Random Sampling

The random sampling employs an asynchronous sampling clock for the measured waveform signal, and we consider the case that it is periodic and each sampling time from the phase zero of the measured waveform is measured and known. Then after many sampling data acquisitions, a waveform with one period can be reconstructed. Fig. 4 shows the random sampling of the repetitive waveform. When the sampling points appear in random phase over one cycle of the measured signal, it is possible to sample efficiently with a small number of points as shown in Fig. 4.

If the frequency ratio of the sampling clock and the signal under measurement are irrational or asynchronous, all phases can be sampled uniformly over time. However, if the frequency ratio causes bias in the sampling points, which is called as the waveform missing phenomena, the overall waveform cannot be acquired with a small number of sampling points [3, 5]. Thus we investigated the frequency ratio that enables the entire waveform acquisition with a small number of sampling points, where the entire waveform can be sampled with uniform phase distribution throughout the entire measurement.

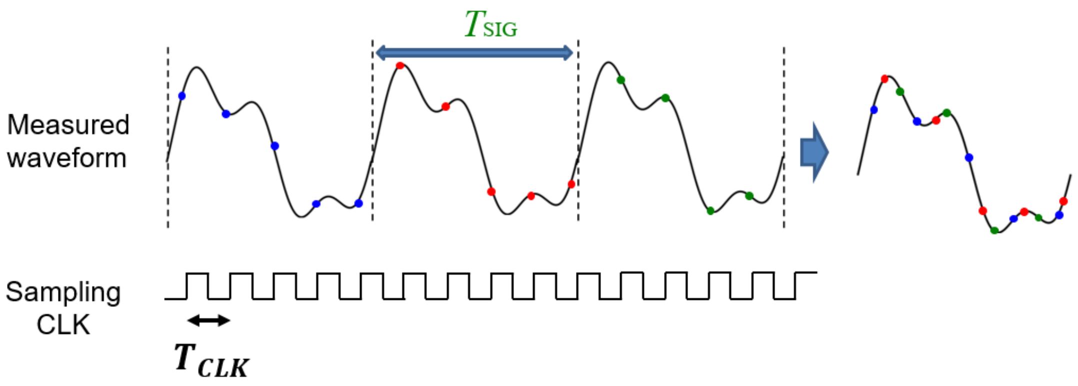


Fig. 4. Principle of random sampling and waveform reconstruction.

**4.2 Metallic Ratio Sampling**

This subsection explains our investigated metallic ratio sampling technique. The metallic ratio (1 : $M_n$) is defined as follows [9]:

$$1 : ( n + \sqrt{(n^2+4)} ) / 2 \qquad (n = 1, 2, 3,\ldots) \tag{3}$$

For example, when n = 1, it is called the golden ratio and $M_1$ =1.6180... . Here $M_n$ is called the n-th metallic number. Setting the ratio of the signal frequency $f_{sig}$ to the sampling frequency $f_{CLK}$ to the metallic ratio is called as the metallic ratio sampling [3]. We call it as the golden ratio sampling for n = 1.

$$f_{CLK} = M_n \times f_{sig} \tag{4}$$

Related to the n-th metallic number $M_n$ , there is a number sequence in which the ratio of adjacent terms ($F_{m+1}$ / $F_m$ ) is converges to the n-th metallic ratio as m goes to infinity [9].

$$F_0 = 0,\ F_1 = 1,\ F_{m+2} = nF_{m+1} + F_m \qquad (n = 1,2,3\ldots,\ m = 0,1,2\ldots) \tag{5}$$

Notice that the metallic ratio frequency clocks can be generated based on Eq. (5).

We found that for the metallic ratio sampling, the ratio of the minimum and maximum distances between sampling points of consecutive numbers is always as follows [3]:

$$1 / M_n \quad \text{or} \quad ( n - 1 ) / M_n + 1/M_n^2$$

In other words, the ratio of the maximum to minimum distance between adjacent sampling points is constant, allowing stable sampling while maintaining mathematically guaranteed efficiency. In addition, the sampling time resolution improves in proportion to the number of data.

When n=1, the ratio of the maximum distance to the minimum distance is the smallest, and this ratio increases as the metallic number n increases. This means that the smaller the metallic number n, the higher the efficiency but the slower the sampling rate, and the larger the metallic number n, the lower the efficiency but the faster the sampling rate.

**5. Application of Metallic Sampling to Linearity Calibration of Flash-type TDC**

This section shows that the metallic ratio sampling is a powerful tool for getting accurate calibration of the flash TDC.

Here $f_{uro}$, $f_{CLK}$ are defined as the oscillation frequencies of the upper and lower ring oscillators, respectively in Fig. 3. Also $T_{uro} = 1/f_{uro}$, $T_{CLK} = 1/f_{CLK}$. With zero initial phase difference between the upper and lower oscillations, we can express the n-th timing difference with the two ring oscillators $\Delta n$ as follows.

$$\Delta n = \text{remainder of } nT_{CLK} \text{ divided by } T_{uro}. \tag{6}$$

If the data sequence distributes randomly, accurate calibration result can be obtained with relatively small number of histogram data (in other words, in short calibration time). Accordingly, for accurate calibration, the ratio between $f_{CLK}$ and $f_{uro}$ is important; this was inspired by our another research result that the ratio between $f_{CLK}$ and $f_{sig}$ is important for accurate waveform acquisition described in the previous subsections. Here Ratio is defined as $f_{CLK}=\text{Ratio}\times f_{uro}$. We found that for Ratio = $M_n$, good randomness of sampling phase is obtained.

The pseudo-randomness in the case of the golden ratio is evaluated compared with several ratios. The calibration is performed with some non-linear TDC with calibration for Ratio = $M_1$, $M_2$, $M_3$, $\pi$, e, $e^{\pi}$, $\tau$, 4.321, and $\sqrt{5}$. Then the constructed histograms are compared. The higher the randomness is, the more uniform the histogram is. In this evaluation, $T_{uro}$ is 1024, the length of the delay buffers in the TDC is 128.

Fig. 5 shows the simulation results, while Fig. 6 shows the ones in Fig. 5 with the vertical axis enlarged to 230~270 for the ratio of $M_1$, $M_2$, $M_3$, e, $e^{\pi}$, $\tau$, 4.321, $\sqrt{5}$ while 180~300 for the ratio = $\pi$. When Ratio = $M_1$, $M_2$, $M_3$, the distribution of the constructed histogram is almost uniform; this means that sufficient randomness is obtained.

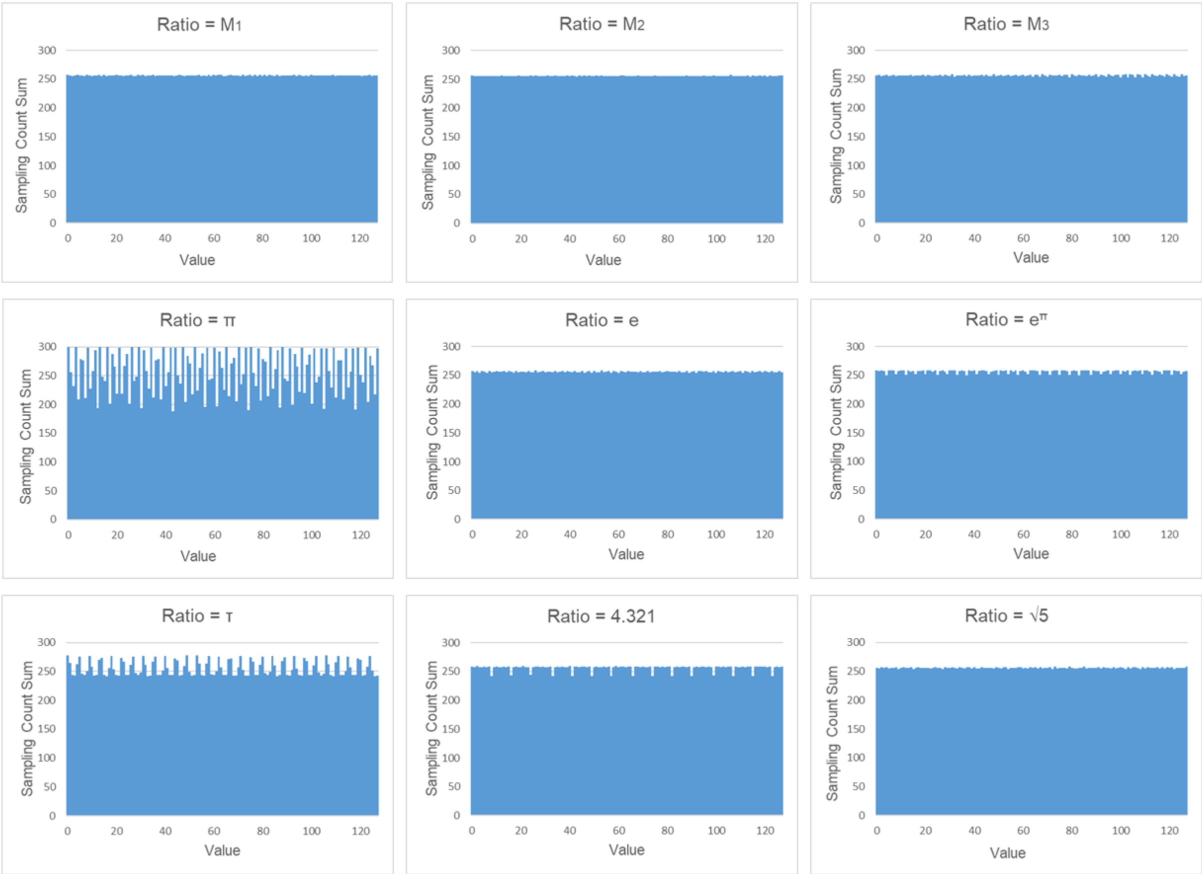


Fig. 5. Constructed histogram with various two-frequency ratios.

The INL after compensation is evaluated with numerical simulation. The calibration is performed with the nonlinear TDC when Ratio = $M_1$, $M_2$, and 4.321, and the INL is calculated from the obtained histogram. There, $T_{uro}$ is 1024, the number of the delay buffers is 128, while Gaussian noise of 10 % standard deviation is added to the buffer delays. Fig. 7 shows the simulation results.

We see from the above results that accuracy of the linearity calibration largely depends on the frequency ratio between the upper and lower ring oscillator frequencies. The golden ratio and other metallic ratios realizes accurate calibration results according to the metallic ratio sampling theory. On the other hand, when $\pi$ is used as the frequency ratio, the calibration accuracy is poor. Accurate calibration can also be achieved by using e, $\sqrt{5}$, etc. However, since no law is found for calibration using different ratios, we have to find a new value each time. We see that the metal ratio sampling method provides a good guideline for accurate calibration of the flash TDC linearity with the histogram method.

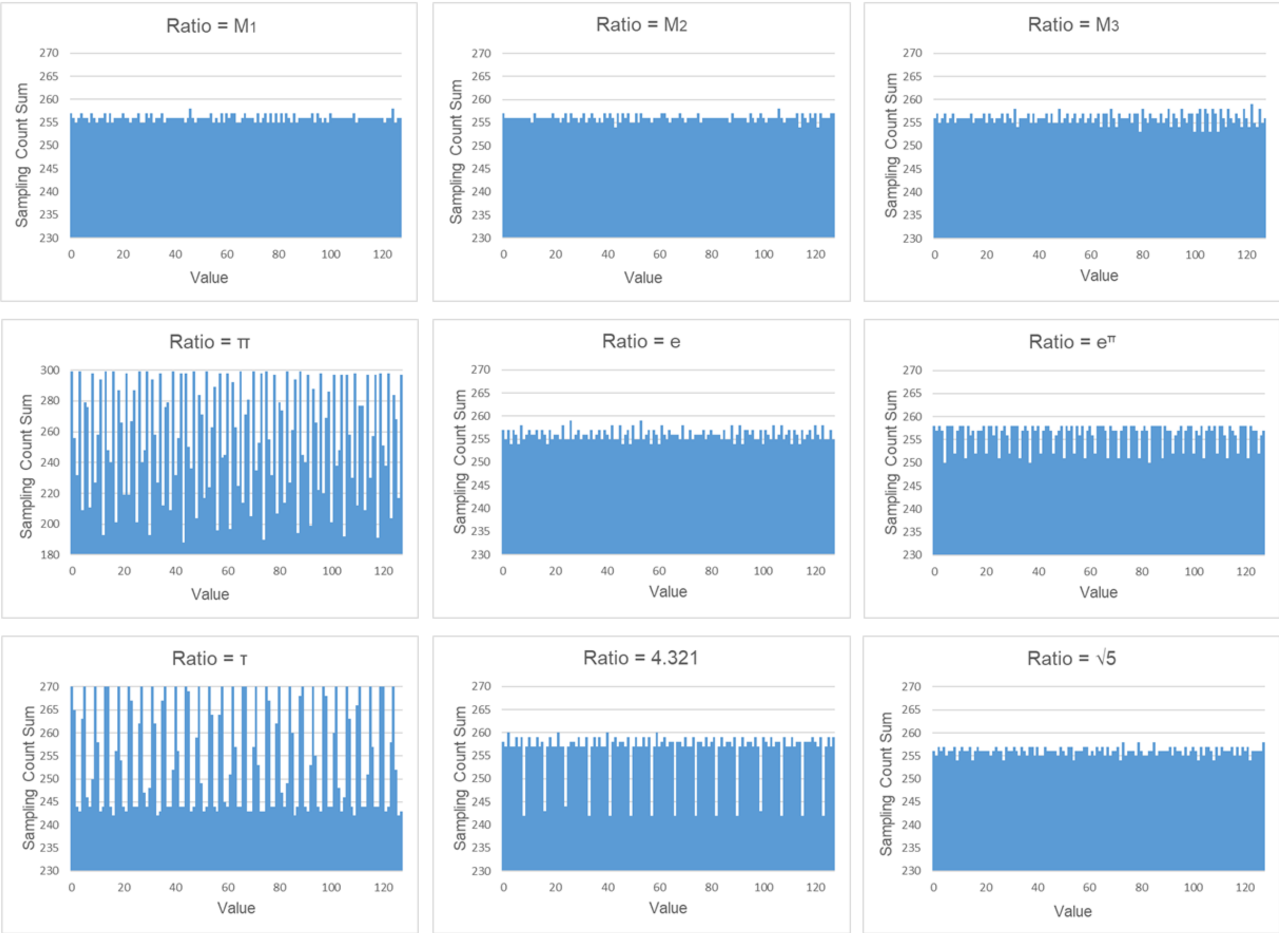


Fig. 6. Enlarged constructed histogram with various ratios.

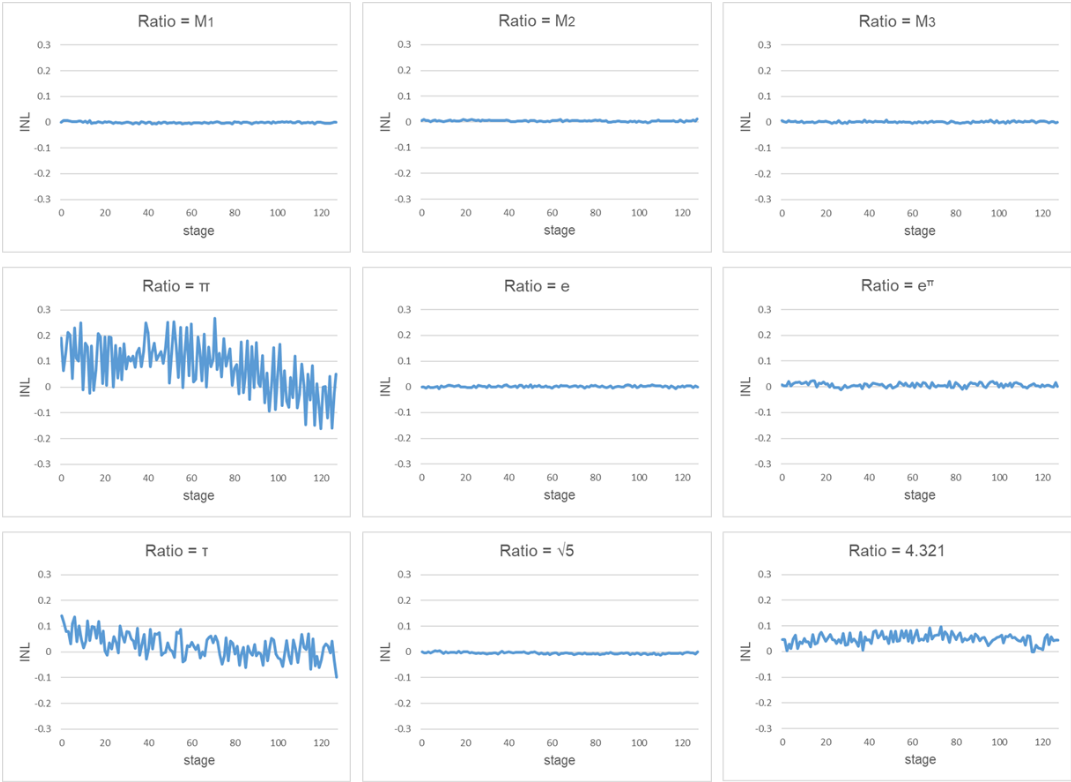


Fig. 7. INLs for various two-frequency ratios.

## 5. Conclusion

In this paper, we have shown that the metallic ratio sampling is efficient for the flash TDC linearity calibration with the histogram method; the TDC nonlinearity can be measured and compensated with a small number of data. We found that the accuracy of calibration is largely dependent on the two-frequency ratio of the upper and lower ring oscillators, and when the frequency ratio is the metallic ratio, high accuracy calibration can be achieved with a small number of histogram data; otherwise, the calibration may not be accurate. These were verified with simulations. The results show that the use of metallic ratio sampling can provide stable and highly accurate corrections.

We close this paper by remarking the following: we have shown here that the metallic ratio sampling technique is effective for the TDC linearity calibration, but additionally it is also effective for the ADC linearity testing with the histogram method using a ramp input as shown in [10].

## References

[1] Y. Arai, T. Baba: "A CMOS Time to Digital Converter VLSI for High-Energy Physics", ***IEEE Symposium on VLSI Circuits***, (Tokyo, Japan) 1988.

[2] S. Ito, S. Nishimura, H. Kobayashi, S. Uemori, Y. Tan, N. Takai, T. J. Yamaguchi, K. Niitsu: “Stochastic TDC Architecture with Self-Calibration”, ***IEEE Asia Pacific Conference on Circuits and Systems*** (Kuala Lumpur, Malaysia) Dec. 2010.

[3] S. Yamamoto, Y. Sasaki, Y. Zhao, J. Wei, A. Kuwana, K. Sato, T. Ishida, T. Okamoto, T. Ichikawa, T. Nakatani, T. Tran, S. Katayama, K. Hatayama, H. Kobayashi: "Metallic Ratio Equivalent-Time Sampling: A Highly Efficient Waveform Acquisition Method", ***27th IEEE International Symposium on On-Line Testing and Robust System Design***, June 2021.

[4] K. Katoh, Y. Doi, S. Ito, H. Kobayashi, E. Li, N. Takai, O. Kobayashi: "An Analysis of Stochastic Calibration of TDC Using Two Ring Oscillators", ***23th IEEE Asian Test Symposium,*** (Yilan,Taiwan) Nov. 2013.

[5] D. E. Toeppen: "Acquisition Clock Dithering in a Digital Oscilloscope", ***Hewlett-Packard Journal,*** Vol.48, No.2, pp.26-28, Feb. 1997.

[6] K. Rush and D. J. Oldfield: "A Data Acquisition System for 1-GHz Digitizing Oscilloscope", ***Hewlett-Packard Journal,*** Vol.37, No.4 pp.4-11, April 1986.

[7] M. Kimura, A. Minegishi, K. Kobayashi, H. Kobayashi: "A New Coherent Sampling System with a Triggered Time Interpolation", ***IEICE Trans. Fundamentals***, Vol.E84-A, No.3, pp.713-719 March 2001.

[8] M. Kimura, K. Kobayashi, H. Kobayashi: "A Quasi-Coherent Sampling Method for Wideband Data Acquisition", ***IEICE Trans. Fundamentals,*** Vol. E85-A, No. 4, pp.757-763, April 2002.

[9] M. Leonard Eugene Dickson: History of the Theory of Numbers, vol. 2, Diophantine Analysis, ***Dover***, 2005.

[10] Y. Zhao, K. Katoh, A. Kuwana, S. Katayama, J. Wei, H. Kobayashi, T. Nakatani, K. Hatayama, K. Sato, T. Ishida, T. Okamoto, T. Ichikawa, "Revisit to Histogram Method for ADC Linearity Test: Examination of Input Signal and Ratio of Input and Sampling Frequencies", ***Journal of Electronic Testing: Theory and Applications, Springer,*** vol. 38, pp. 21-38, March 2022.