

Time-to-Digital Converter Linearity Calibration with Metallic Ratio Sampling

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OUTLINE

- Background
- Time-to-Digital Converter (TDC)
 - Linearity Calibration of TDC
- Equivalent-Time Sampling
 - Metallic Ratio Sampling Efficiency
- Time-to-Digital Converter Linearity Calibration
 - What is DNL, INL
 - Simulation Result
- Conclusion

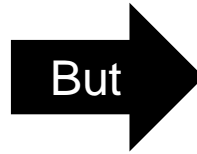
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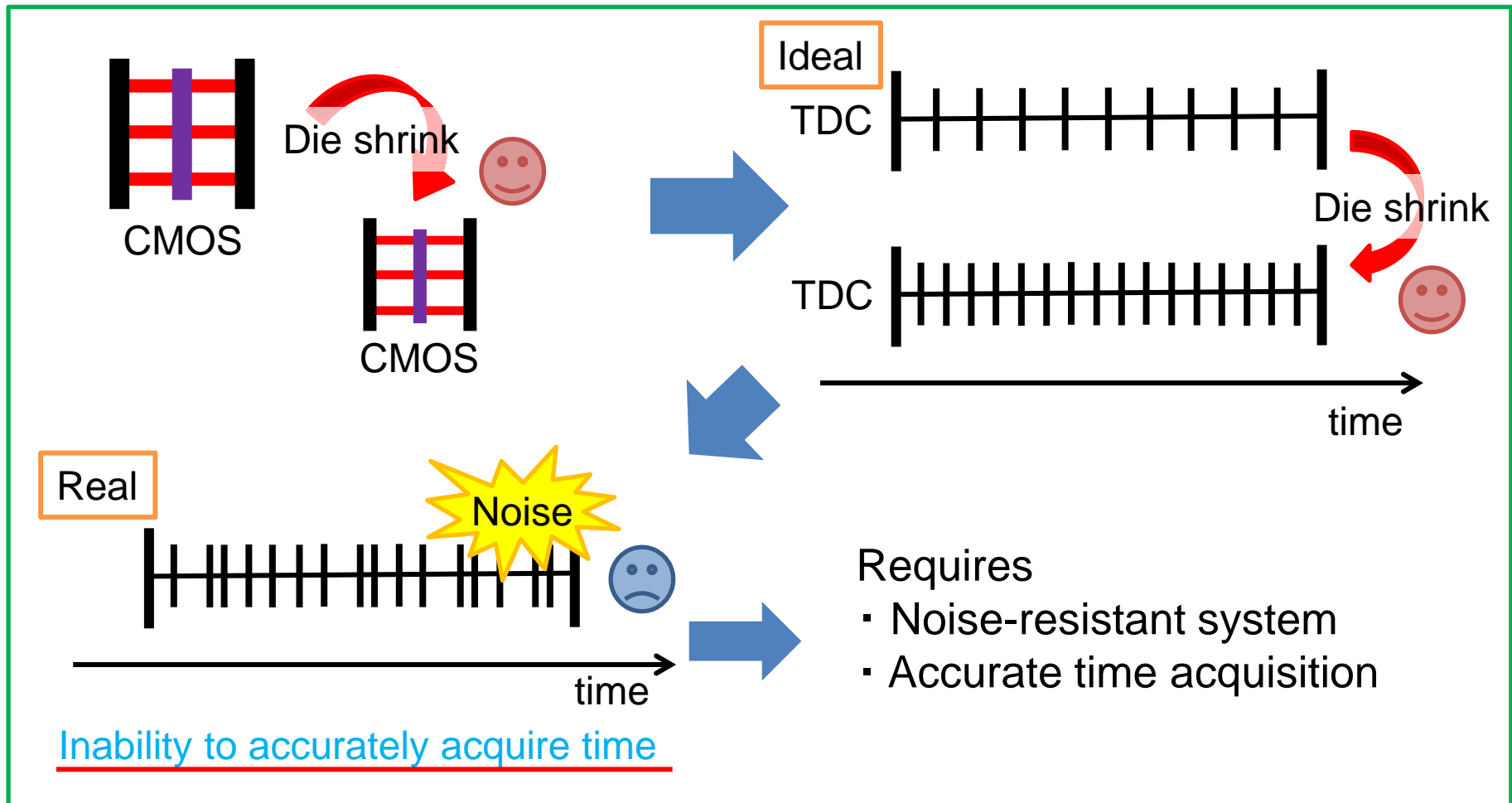
Background

CMOS LSI miniaturization

- **Lower** supply voltage
- **Increased** operating speed



- Requires measurement accuracy
- Noise has a large impact



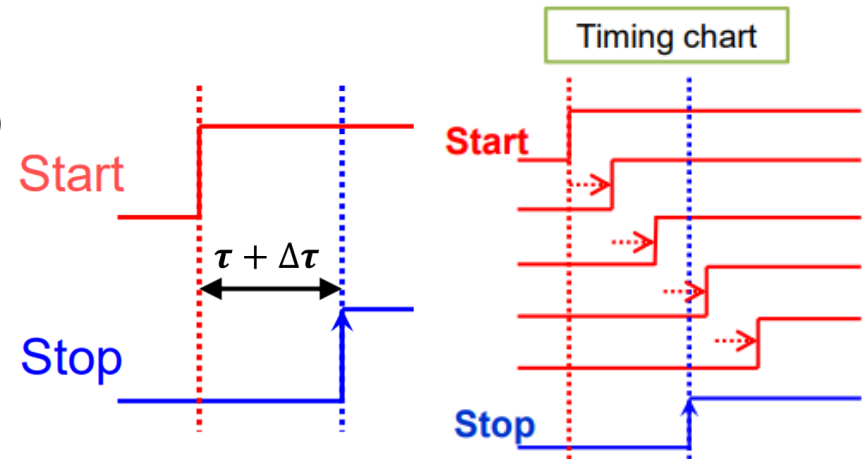
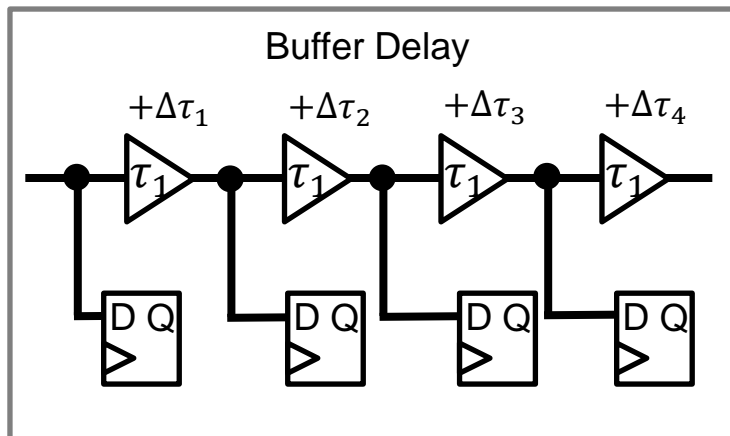
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Time-to-Digital Converter (TDC)

TDC (Time-to-Digital Converter)

A circuit that measures the time difference between the edges of two timing signals to obtain a digital output. Delay is achieved by using an buffer.



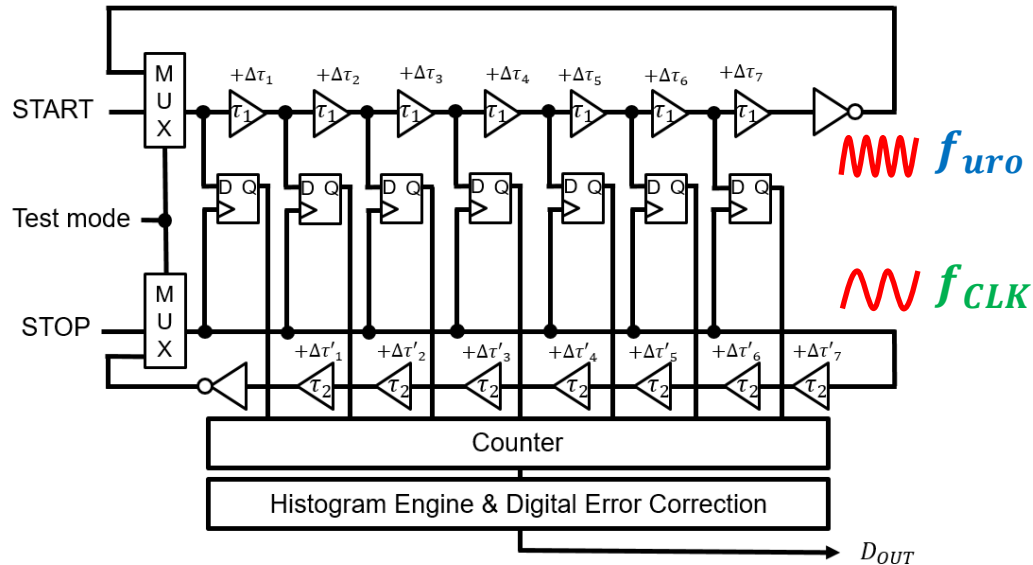
Variability : $\Delta\tau$

→ Incorrect time acquisition



Linearity Calibration of TDC

Sampling with upper and lower ring oscillators and DFF

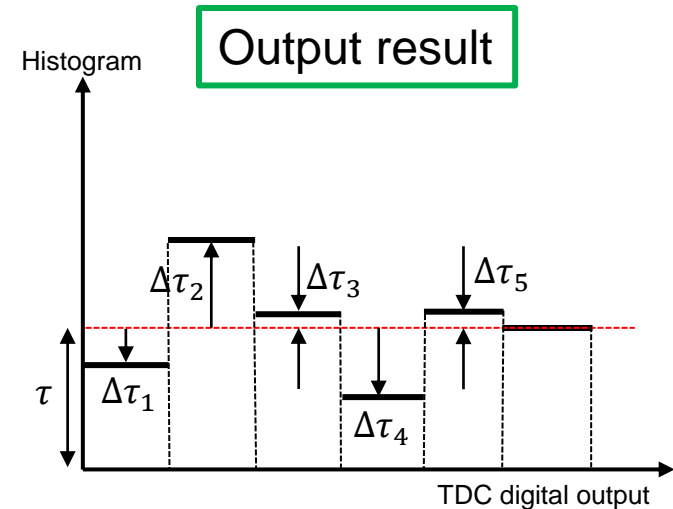


Self-calibration Circuit

Ring Oscillator

- Upper frequency : f_{uro}
- Lower frequency : f_{CLK}

Sampling accuracy depends on
the ratio of f_{uro} to f_{CLK}



Output is Relative delays
Non-linearity is digitally corrected



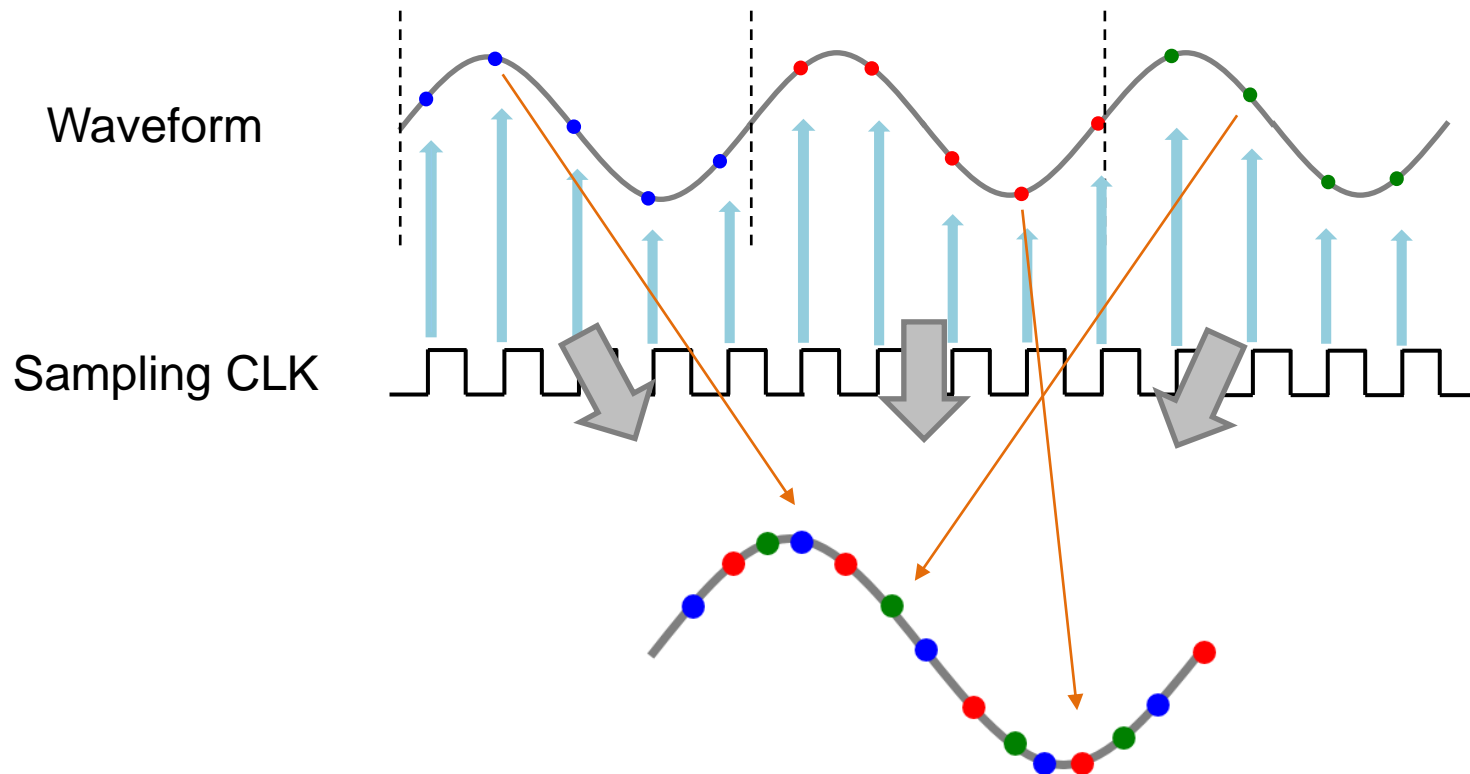
Sampling accuracy : **High**
→ **Increased effectiveness**
of digital correction



OUTLINE

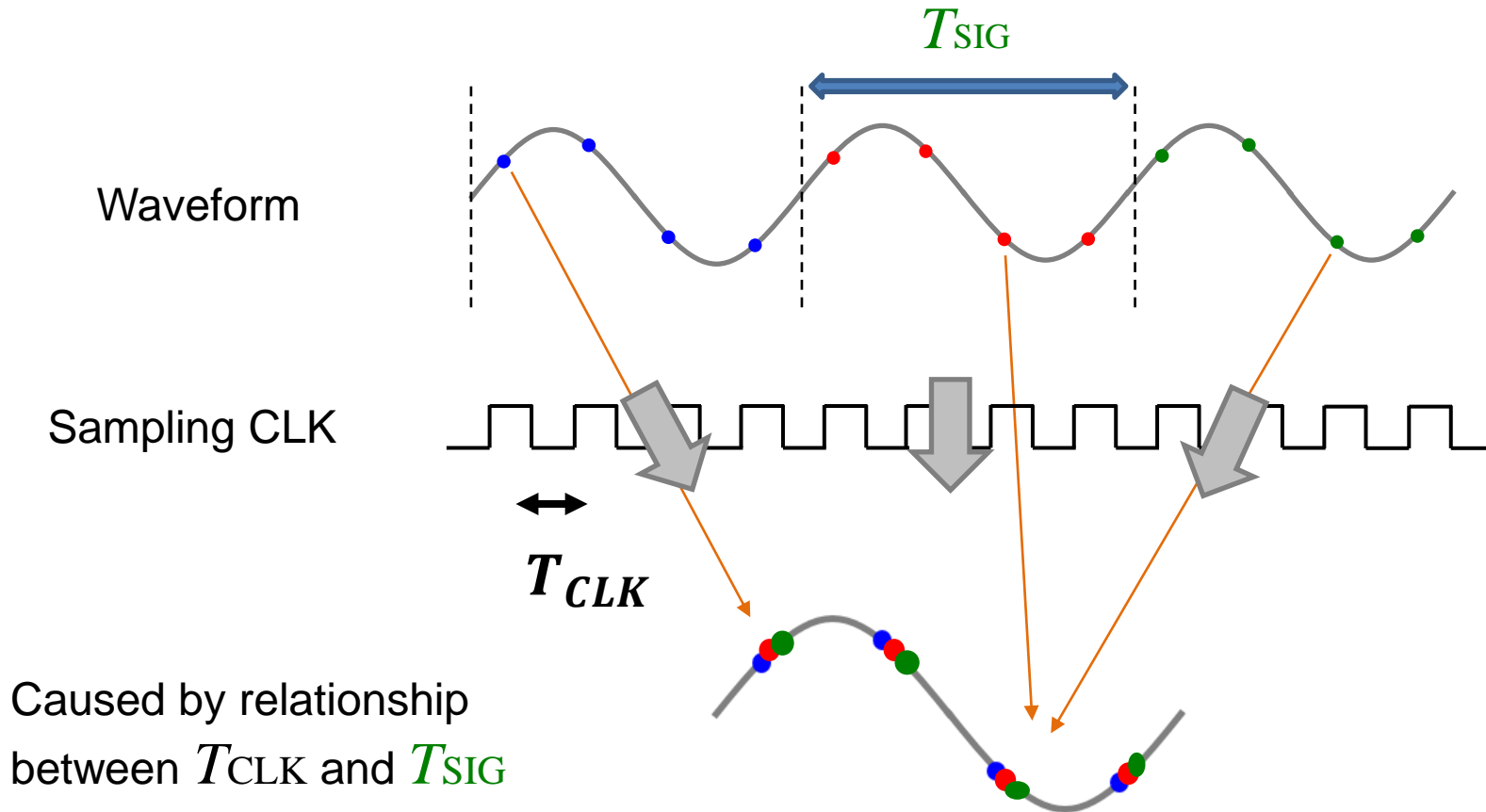
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Equivalent-Time Sampling



Sampling repetitive waveform with asynchronous CLK
➡ Construct one-period waveform

Waveform Missing Phenomena



A lot of data → reconstruct one period

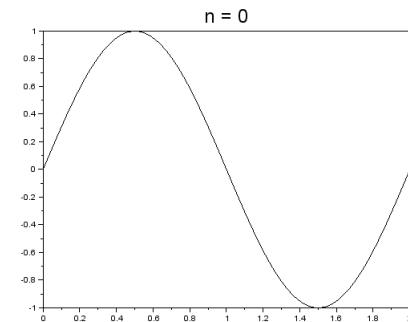
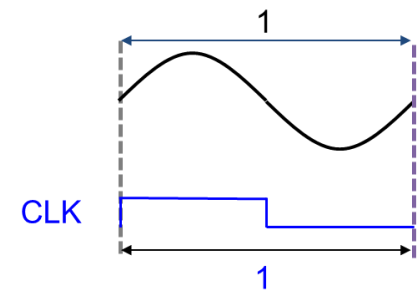
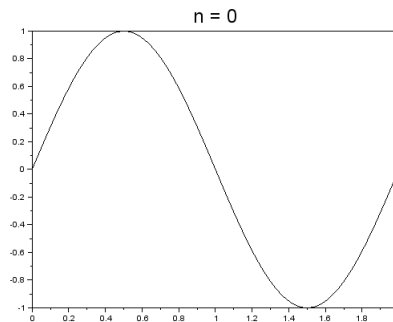
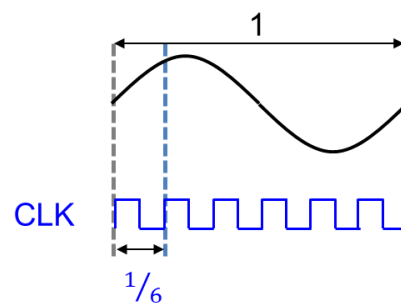
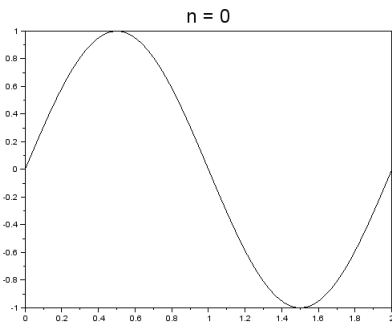
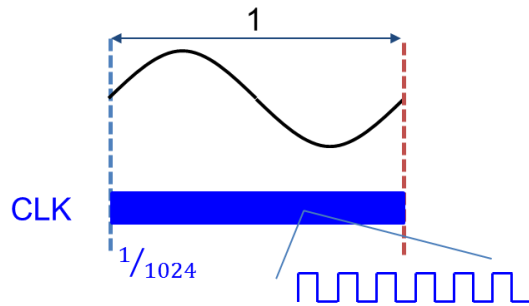


Randomness : Low



Waveform Missing Condition

$$f_{CLK} \gg f_{sin} \quad f_{CLK} \approx \frac{1}{\alpha} f_{sin} \left(\alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \dots, \frac{1}{6}, \dots \right) \quad f_{CLK} \approx f_{sin}$$



Sampling points: **Localized**



Distance ratio between adjacent sampling points: **Large**



Metallic Ratios

Metallic ratios

$$1 : \frac{n + \sqrt{n^2 + 4}}{2} \quad (n = 1, 2, 3 \dots)$$



M_n : Metallic number

$n=1$: Golden ratio ($M_1 = 1.6180\dots$)

$n=2$: Silver ratio ($M_2 = 2.4142\dots$)

$n=3$: Bronze ratio ($M_3 = 3.3027\dots$)

⋮

$n=m$: $1 : M_n$

Difference from reciprocal

$$M_n - \frac{1}{M_n} = \text{Natural Number}$$

Continued fraction

$$M_n = n + \frac{1}{n + \frac{1}{n + M_n}}$$

Limit of adjacent term ratio

$$F_0 = 0, F_1 = 1, F_{k+2} = nF_{k+1} + F_k$$

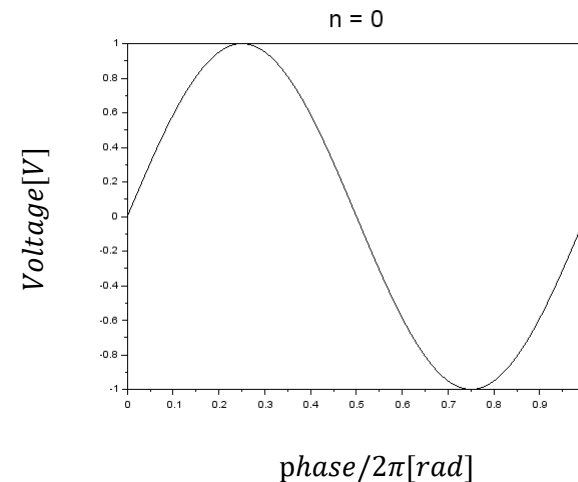
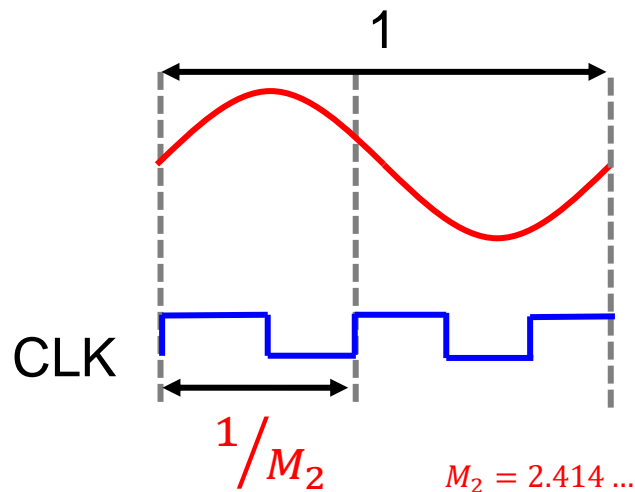
$$k = 0, 1, 2, 3, 4, \dots$$

Metallic Ratio Sampling

Fixed f_{CLK} \rightarrow Test ADC with various f_{sig}

$$f_{CLK} = M_n \times f_{sig}$$

M_n : Metallic ratio



In the case of silver ratio

Sampling points \rightarrow Always distributed evenly in phase

OUTLINE

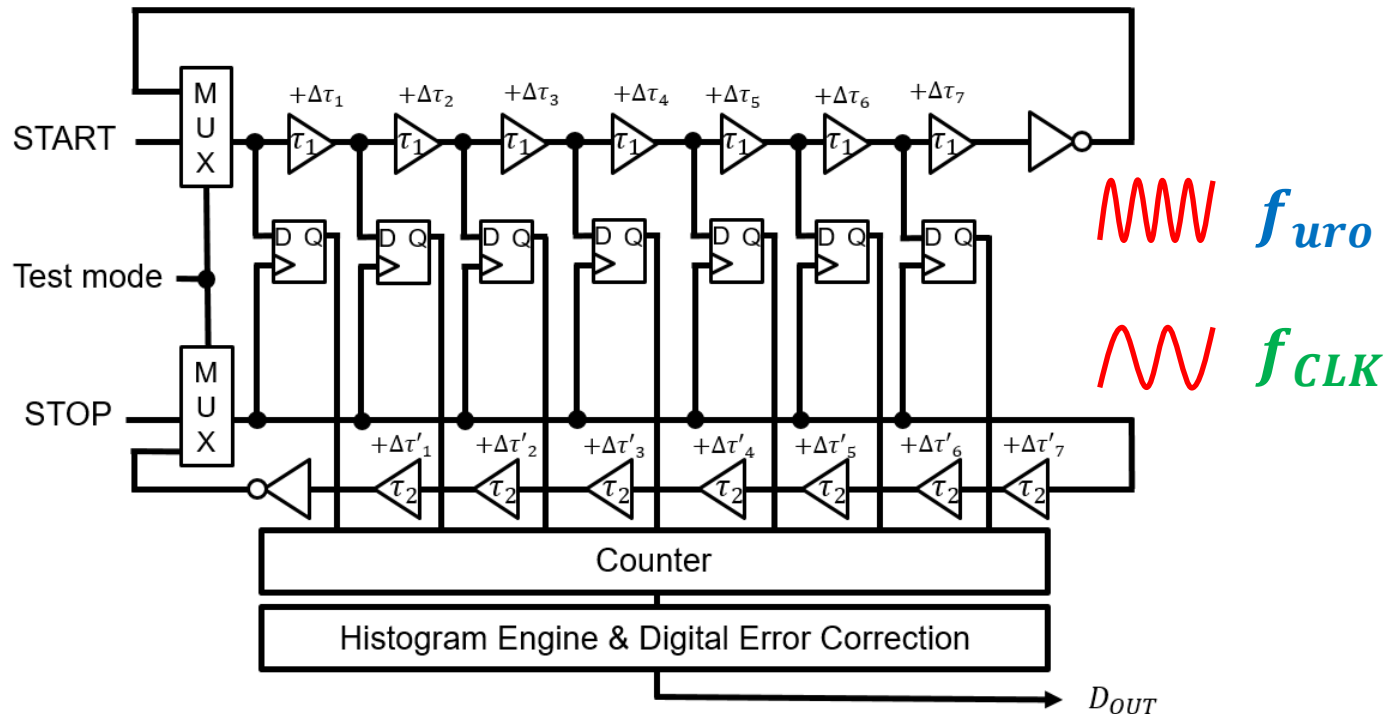
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Time-to-Digital Converter Linearity Calibration

Sampling accuracy is determined by frequency ratio

→ Metallic Ratio Sampling Technology Applied

$$f_{uro} : f_{CLK} = 1 : M_n$$



Metallic Ratio Sampling Technology Improves Sampling Accuracy

→ Improved accuracy of digital correction

DNL & INL

Important ADC testing items

DNL(differential non-linearity)

- Difference between actual step width and ideal value

$$\text{DNL}(k) = \Delta\tau_k / \tau$$

DNL : close to zero 

DNL : away from zero 

INL(integral non-linearity)

- Deviation from ideal conversion line

$$\text{INL}(k) = \text{DNL}(1) + \text{DNL}(2) + \dots + \text{DNL}(k)$$

INL : close to zero 

INL : away from zero 

Histogram with two-frequency ratios

Histogram Simulation results

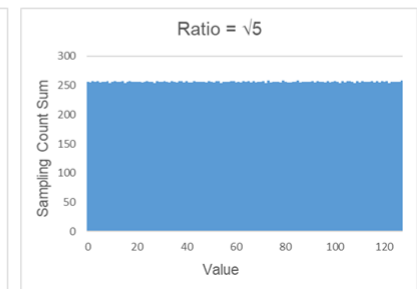
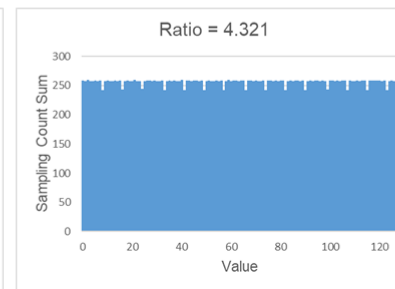
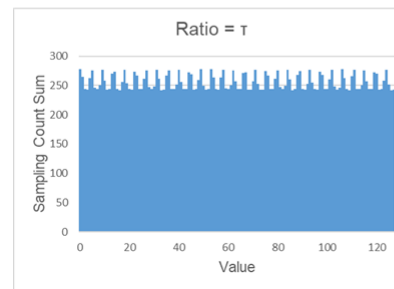
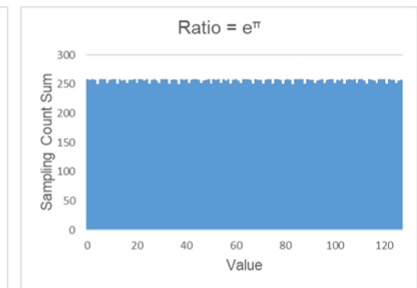
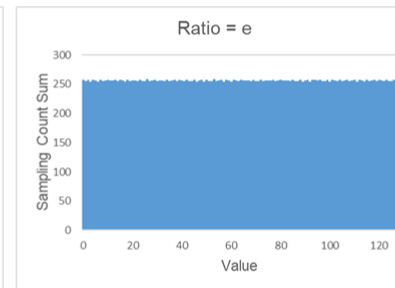
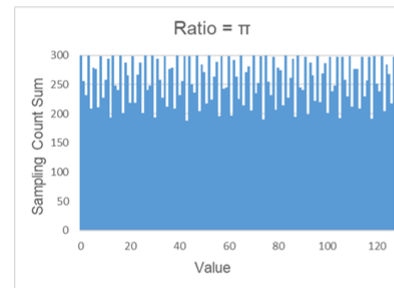
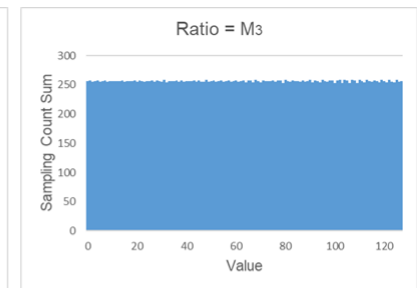
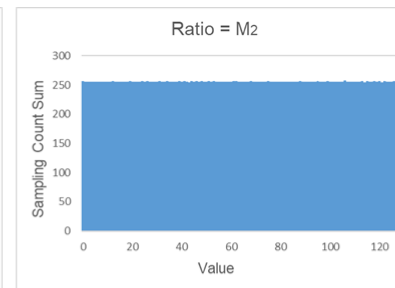
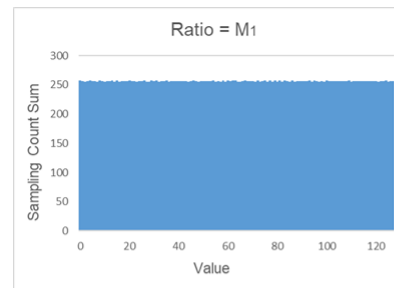
- Metallic Ratio example : M_1, M_2, M_3
- Any Ratio example : $\pi, e, e^\pi, \tau, 4.321, \sqrt{5}$

Condition

- Points : 2^{16}
- TDCs : 2^8

Metallic Ratio

- Accuracy : **High**
- Efficiency : **High**
- Stability : **High**



Any Ratio

- Accuracy :
Depends on Ratio
- Efficiency :
Depends on Ratio
- Stability : **Low**

Histogram : close to Avg. 😊

INL with two-frequency ratios

INL Simulation results

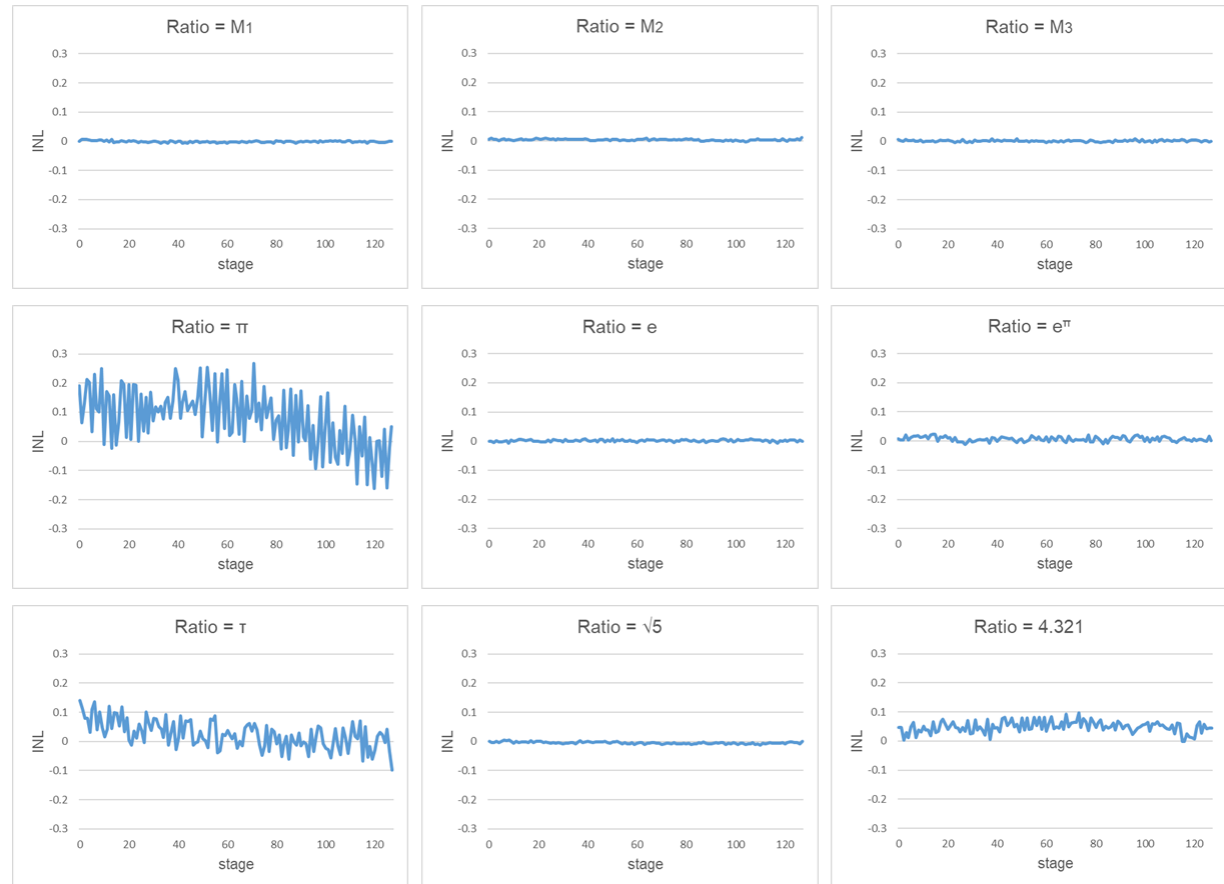
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Condition

- Points : 2^{16}
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Metallic Ratio

- Accuracy : **High**
- Efficiency : **High**
- Stability : **High**



Any Ratio

- Accuracy :
Depends on Ratio
- Efficiency :
Depends on Ratio
- Stability : **Low**

INL : close to zero



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Conclusion

TDC linearity calibration

Metallic Ratio method 😊

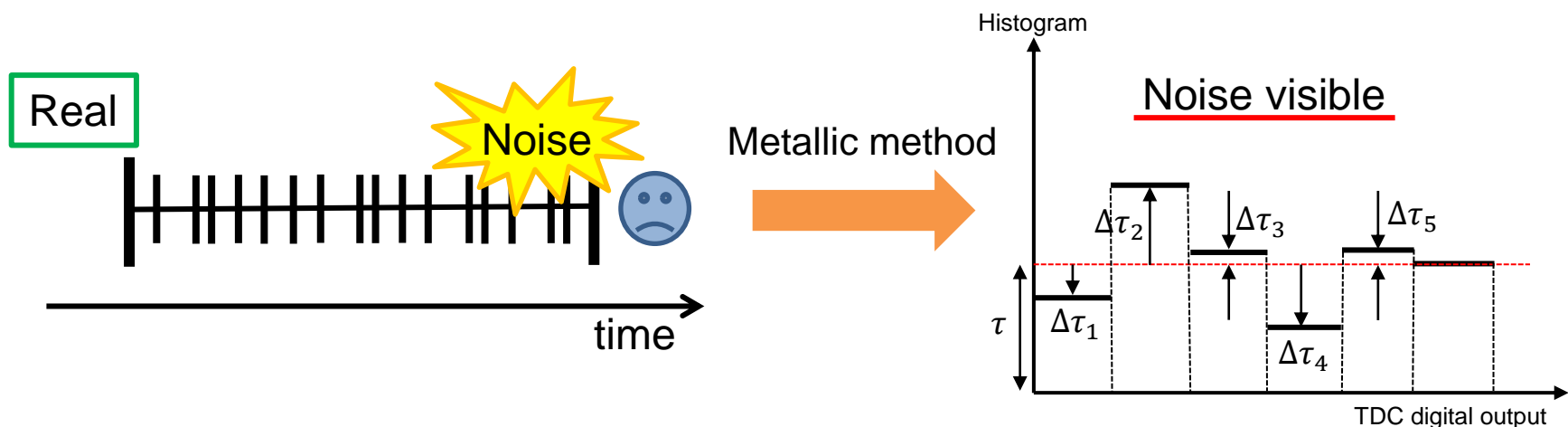
- Sampling accuracy : **High**
- Sampling efficiency : **High**
- Stability : **High**

Conventional method 😞

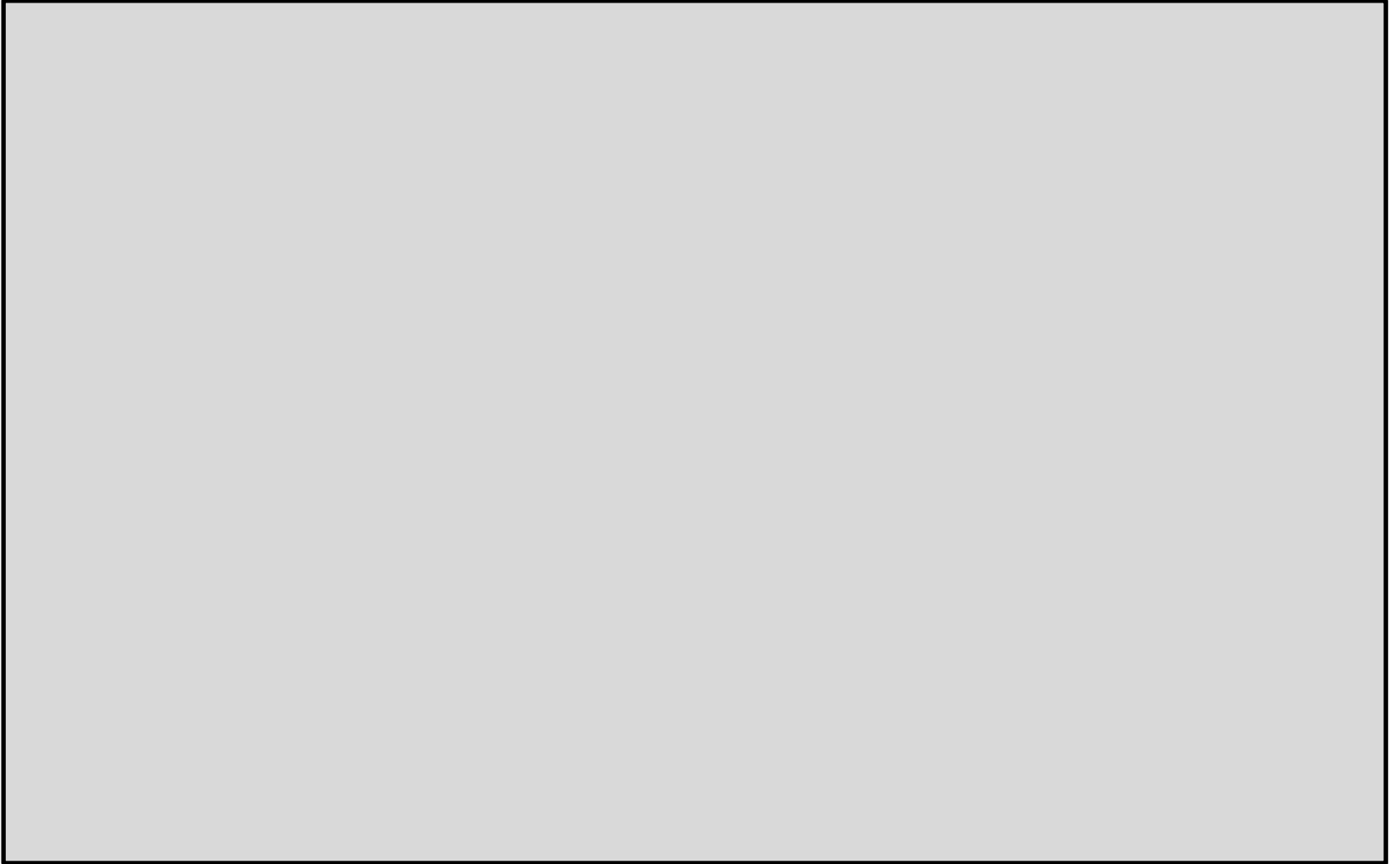
- Sampling accuracy : **High** to **Low**
- Sampling efficiency : **High** to **Low**
- Stability : **Low**



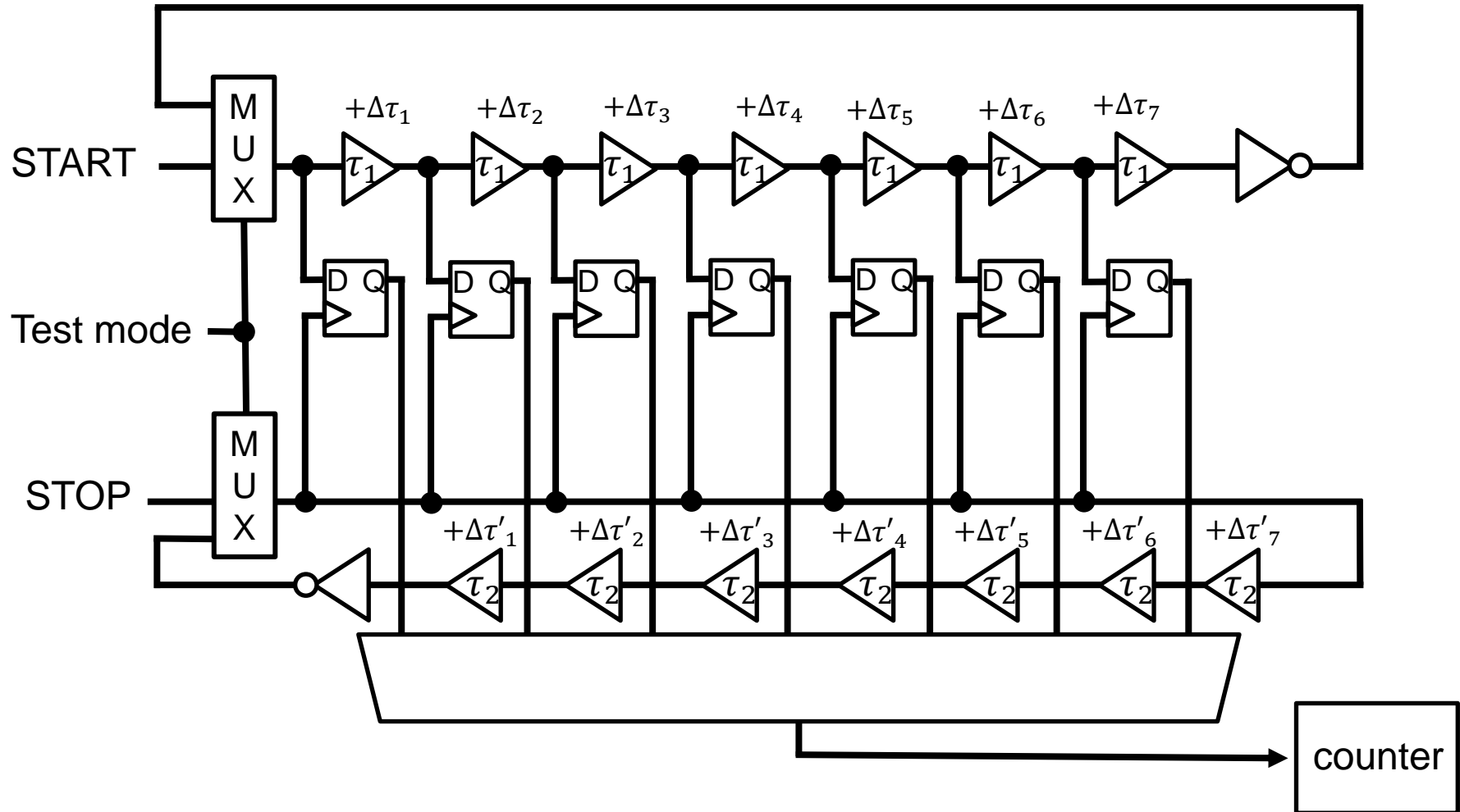
Metallic ratio sampling is **useful** and provides a **guideline**



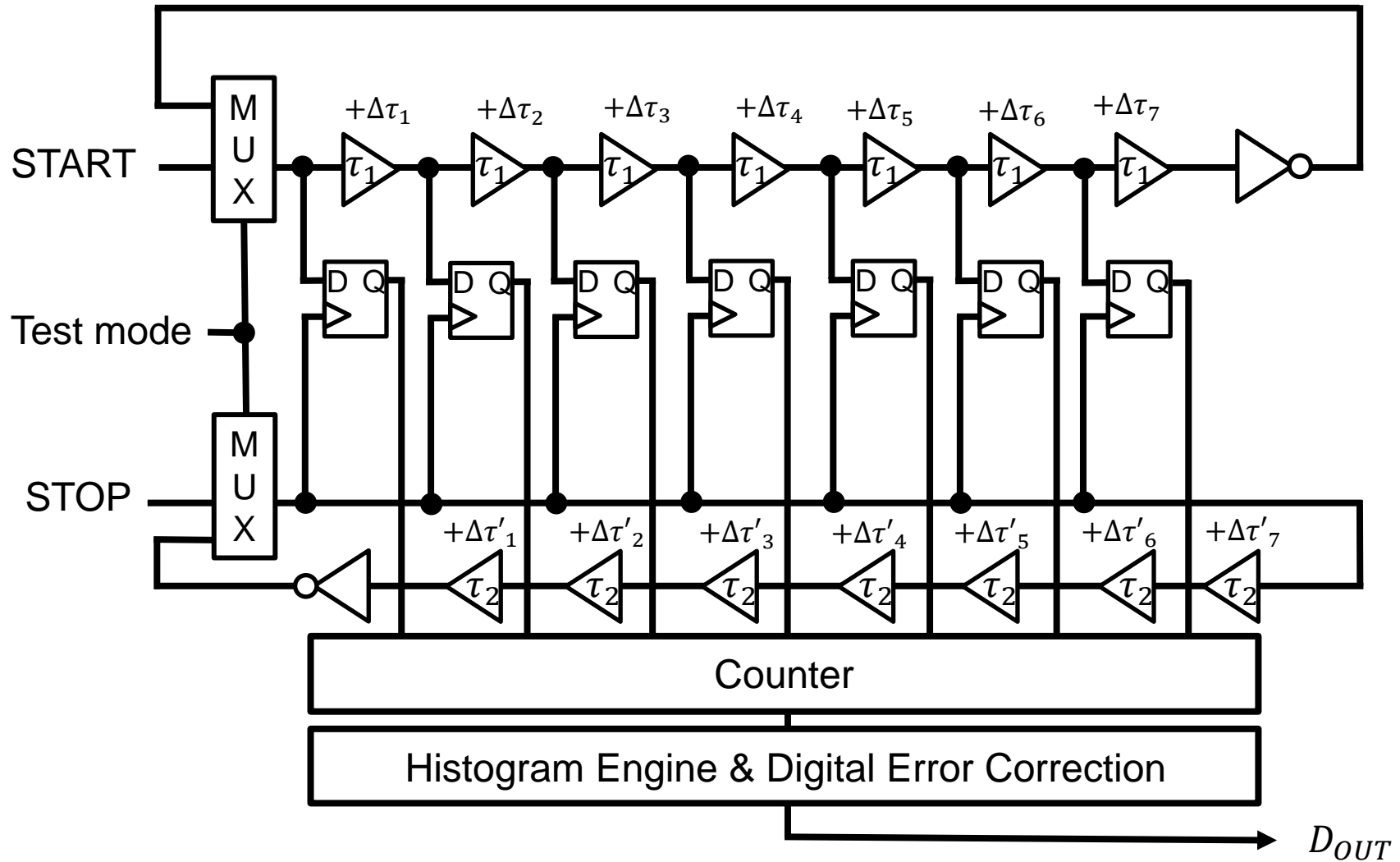
Appendix



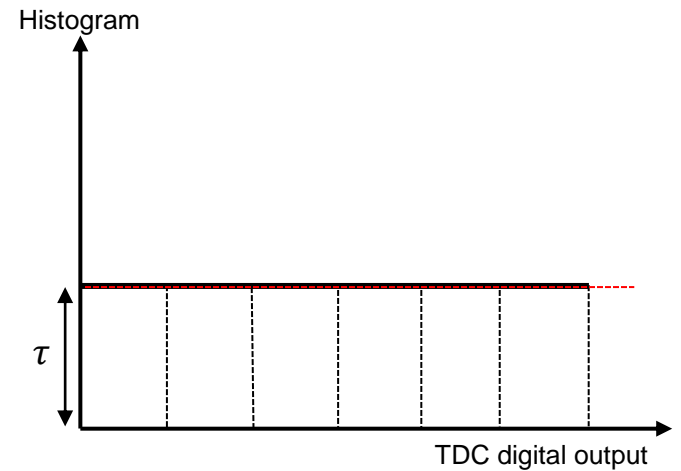
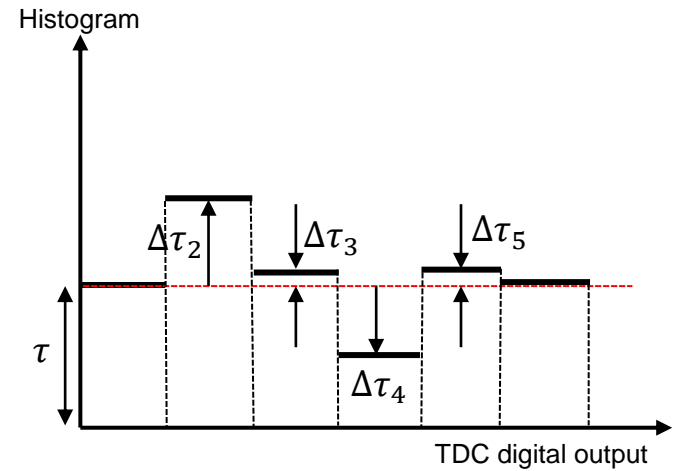
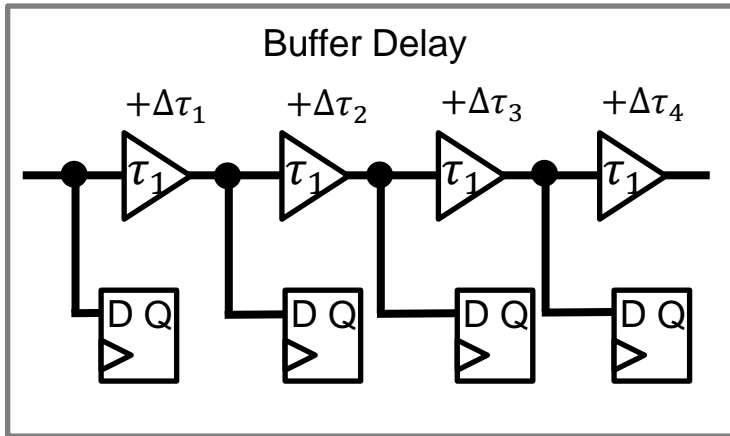
Appendix



Appendix

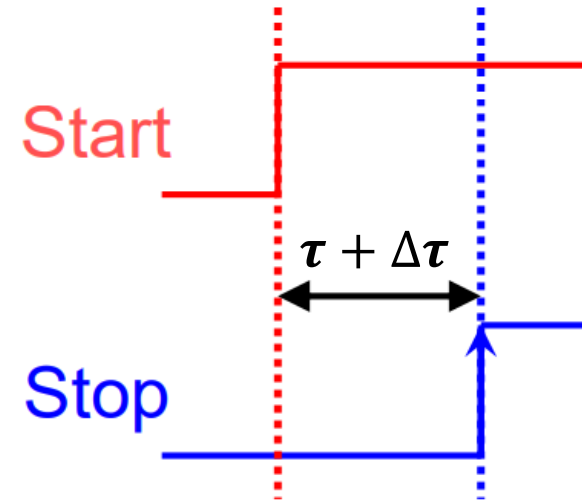
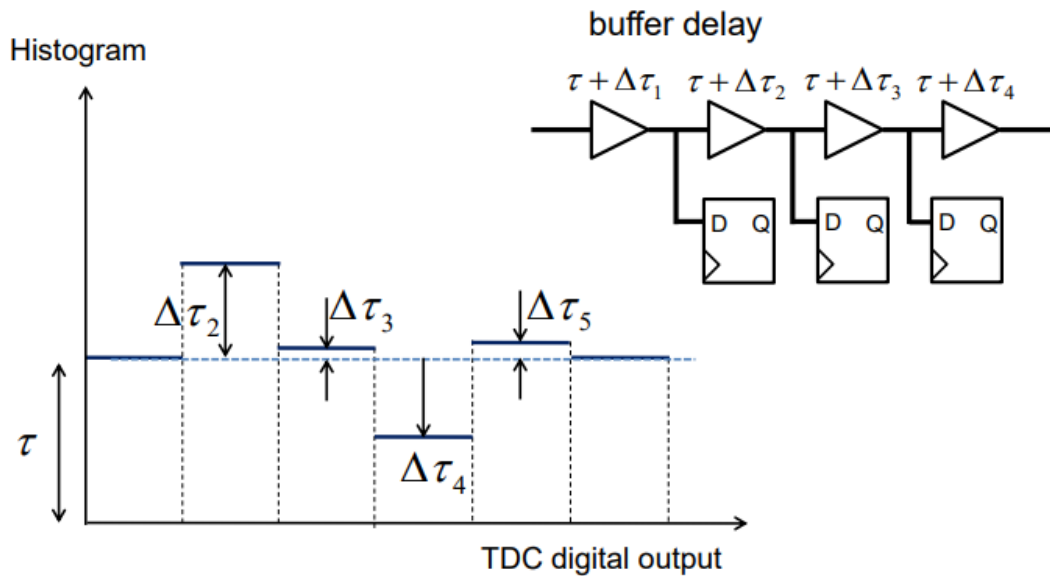


Appendix



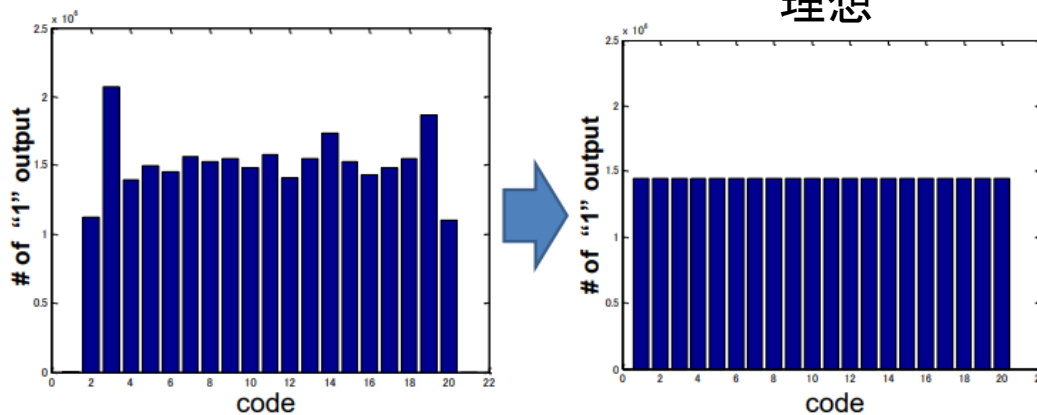
Appendix

TDC is non-linear



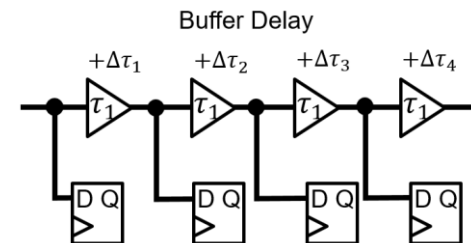
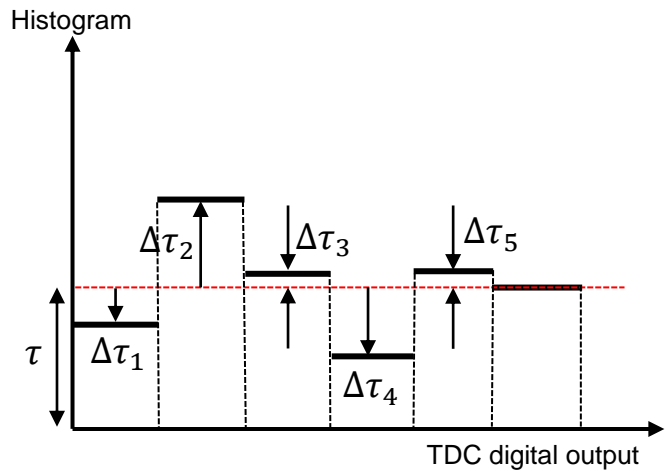
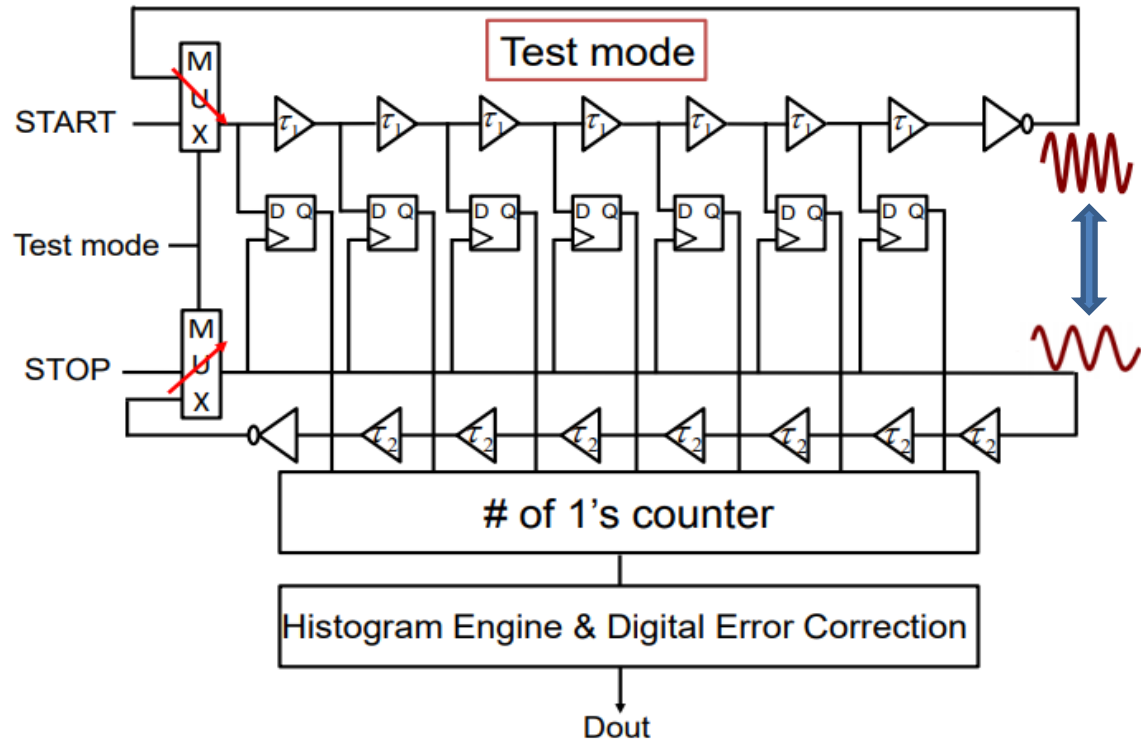
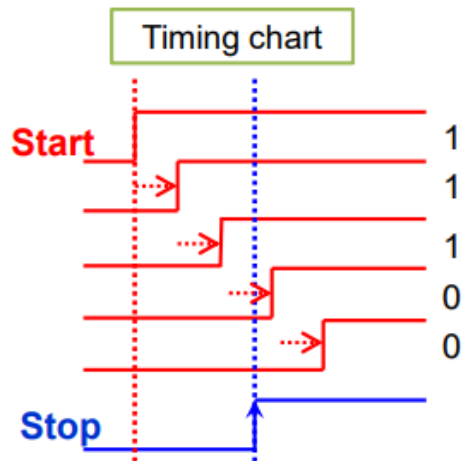
取得時間がずれる。

理想



測定を高効率で高い精度での
時間取得を期待。

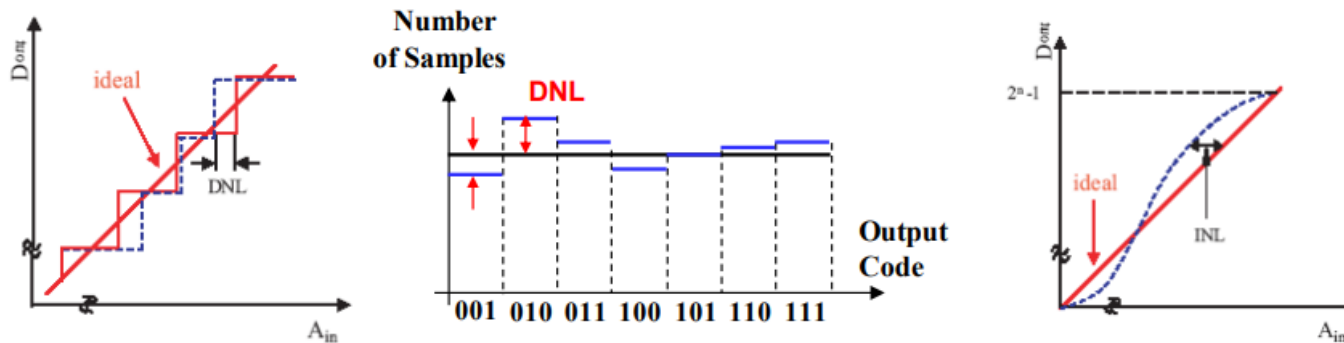
Appendix



Appendix

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DNL & INL



- Important ADC testing items

DNL : Difference between
actual step width and ideal value

INL : Deviation from ideal conversion line

$$INL(k) = \sum_{i=1}^k DNL(i)$$

