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**Paper**

# Analysis and Design of Operational Amplifier Stability Based on Routh-Hurwitz Stability Criterion

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This paper proposes to use Routh-Hurwitz stability criterion for analysis and design of the opamp stability, when its small equivalent circuit is derived; this can lead to explicit stability condition derivation for opamp circuit parameters. In the theoretical part, we describe the equivalence between Nyquist and Routh-Hurwitz stability criteria under some conditions, and we deduce the relationship that between parameters of Routh-Hurwitz stability criterion and phase margin of the operational amplifier. In the verification part, our statement is confirmed with LTspice simulations at transistor level opamp circuits.

**Keywords** : operational amplifier, stability, nyquist stability criterion, Routh-Hurwitz criterion, phase margin

## 1. Introduction

We proposed to use the Routh-Hurwitz stability criterion for operational amplifier stability analysis and design, to obtain explicit stability conditions for operational amplifier circuit parameters(1)-(4); this has not been described in any operational design book/paper, to the best of our knowledge(5)-(13). In this paper, we demonstrate that the respective mathematical foundations of Nyquist and Routh-Hurwitz stability criteria are equivalent, and we deduce the relationship between Routh-Hurwitz stability criterion parameters with phase margin of the operational amplifier as theoretical support and perfection for the proposed method, and then, we verify our proposed method with some amplifier models. Our SPICE simulation results show good agreements with our theoretical analysis based on the proposed method. We emphasize that our proposed method using the Routh-Hurwitz stability criterion can derive explicit stability/phase-margin-improvement equations using small signal circuit parameters, and the operational amplifier designer can understand which parameter values should be increased or decreased to obtain stability and phase margin. Even though small parasitic components in the small circuit model are neglected, these equations would be helpful for the designer.

In the control theory field, there are many criteria for judging the stability of the feedback system(13). For example, Nyquist stability criterion and Routh-Hurwitz (R-H) stability criterion are widely utilized. The Nyquist stability criterion is a graphical technique for determining the stability of a dynamical system, and the Bode plot and Nyquist plot which be well known and used are all application examples based on the principle of Nyquist stability criterion. In the electronic circuit design field, Bode plot for the open-loop frequency characteristic is the most frequently used by circuit designers(5)-(12), while Nyquist plot is occasionally used(14). However, strangely enough, according to our survey of the related texts about analog electronic circuits(1)-(9), the Routh-Hurwitz method(12)-(14) is rarely mentioned in analysis and design of the operational amplifier stability. It seems that even some mature analog designers are not familiar with the R-H stability criterion. On this account, we have made attempts to introduce the R-H stability criterion into electronic circuit design field and begin with used for judging stability of operational amplifier.

This paper is an extended version of our conference papers(1)-(4), and organized as follows: Section 2 briefly introduces Nyquist stability criterion and Routh-Hurwitz stability criterion. Section 3 presents detailed derivation process of the proposed criterion with application to the small signal models of the three selected amplifiers. Section 4 deduces respective mathematical foundations of these criteria, and their equivalency is demonstrated. In Section 5, we deduce the relationship that between Routh-Hurwitz stability criterion parameters with phase margin (PM). In Section 6, we select some amplifiers to verify our proposed method with theoretical analysis and SPICE simulations. Section 7 presents some discussions, and Section 8 provides conclusion.

## 2. Nyquist and Routh-Hurwitz Stability Criteria

Let us consider the stability of the linear feedback system in Fig.1, and its stability criteria may be classified into the following:

(1) Nyquist stability criterion using open-loop frequency characteristics of $fA(j\omega)$.
 ・Bode plots (gain and phase plots) of $fA(j\omega)$.
 ・Nyquist plot of $fA(j\omega)$.
 ・Nicholas plot of $fA(j\omega)$.

(2) Routh-Hurwitz stability criterion based on the closed-loop transfer function $A(s)/(1+fA(s))$, when it is represented as $N(s)/D(s)$, where,

$$N(s) = b_n s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0$$
$$D(s) = \alpha_n s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0 \quad \cdots\cdots (1)$$

and m<n.

**2.1 Nyquist Plot** The Nyquist plot is a frequency response plot in Gaussian plane, widely used in automatic control and signal processing (Fig.2 (a))(14). The most common use of

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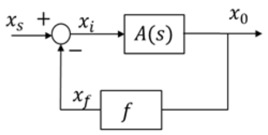
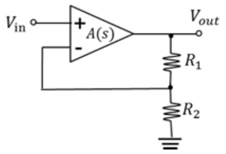
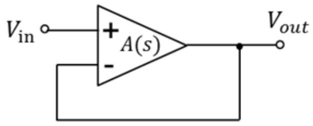



(a) Block diagram of the feedback system

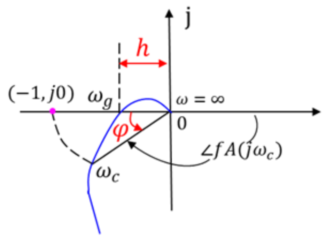



(b) Circuit example 1 with $f = R_2/(R_1 + R_2)$

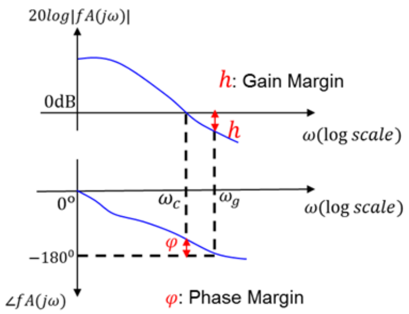



(c) Circuit example 2 with $f = 1$ (voltage follower)

Fig. 1. Feedback system.




(a) Nyquist plot of an open-loop system




(b) Bode plots of the loop gain for stable system

Fig. 2. Nyquist plot and Bode plots.

Nyquist plot is for assessing the stability of a system with feedback.

Necessary and sufficient condition for the closed-loop system stability is given as follows:

$$\text{when, } \omega = 0 \to \infty, \quad \text{N} = \frac{P}{2}$$

Here, N is the number of Nyquist plot anti-clockwise encircle point $(-1, j0)$, and $P$ is the number of positive roots of the open-loop characteristic equation.

**2.2 Bode Plot** In electrical engineering and control theory, the Bode plots are graphs of the frequency responses (gain and phase) of the open-loop characteristics of the feedback system, and they can show gain margin and phase margin (Fig.2 (b)) required to maintain feedback system stability under variations in circuit characteristics[(5)-(13)]. Circuit designers can routinely use the Bode plots to determine the bandwidth and frequency stability of the operational amplifier circuits.

Table 1. Routh table.

| | | | | | |
|---|---|---|---|---|---|
| $s^n$ | $\alpha_n$ | $\alpha_{n-2}$ | $\alpha_{n-4}$ | $\alpha_{n-6}$ | … |
| $s^{n-1}$ | $\alpha_{n-1}$ | $\alpha_{n-3}$ | $\alpha_{n-5}$ | $\alpha_{n-7}$ | … |
| $s^{n-2}$ | $\beta_1 = \frac{\alpha_{n-1}\alpha_{n-2} - \alpha_n\alpha_{n-3}}{\alpha_{n-1}}$ | $\beta_2 = \frac{\alpha_{n-1}\alpha_{n-4} - \alpha_n\alpha_{n-5}}{\alpha_{n-1}}$ | $\beta_3$ | $\beta_4$ | … |
| $s^{n-3}$ | $\gamma_1 = \frac{\beta_1\alpha_{n-3} - \alpha_{n-1}\beta_2}{\beta_1}$ | $\gamma_2 = \frac{\beta_1\alpha_{n-5} - \alpha_{n-1}\beta_3}{\beta_1}$ | $\gamma_3$ | $\gamma_4$ | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $s^0$ | $\alpha_0$ | | | | |

**2.3 Routh-Hurwitz Stability Criterion** In the time domain analysis of the control system theory, the Routh–Hurwitz stability criterion is a mathematical test that is the necessary and sufficient condition for the stability of a linear time invariant control system[(13)]. It uses the ideas above to determine whether a given polynomial has roots in the right half-plane.

Suppose that characteristic equation of the closed-loop transfer function is as follows:

$$D(s) = \alpha_n s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0 = 0 \quad \cdots\cdots (2)$$

Necessary and sufficient condition of the stability is that all real parts of the solutions of (2) are negative, which is equivalent to the following:

$\alpha_i > 0$ for $i = 0, 1, \ldots, n$, and all values of the first column parameters in Routh table (Table 1) are positive.

In the first column of the Routh table, the number of times for the coefficient sign changes is equal to the number of the system characteristic equation solutions with the positive real part.

## 3. Small Signal Model of Amplifier and Proposed Criterion

This section shows several examples of operational amplifiers and applications of the proposed stability criterion to them.

$R_1, R_2$ are equivalent resistors, $C_1, C_2$ are equivalent capacitances, $G_{m1}, G_{m2}$ are transconductances, $C_{r1}$ is compensation capacitance.

Example 1: Two-pole operational amplifier with C compensation.

Consider the two-pole amplifier in Fig.3 whose open-loop transfer function is given by:

$$G(s) = K\frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}\ . \quad \cdots\cdots (3)$$

Here, $b_1 = -\frac{C_{r1}}{G_{m2}}$, $K = G_{m1}G_{m2}R_1R_2$

$$a_1 = R_1C_1 + R_2C_2 + \left(R_1 + R_2 + R_1G_{m2}R_2\right)C_{r1},$$
$$a_2 = R_1R_2C_2\left[C_1 + \left(1 + \frac{C_1}{C_2}\right)C_{r1}\right] \quad \cdots\cdots (4)$$

Fig.1 (b), (c) show feedback amplifiers using the operational amplifier in Fig.3, and their closed-loop transfer function is obtained as follows:

$$\frac{G(s)}{1 + fG(s)} = \frac{K(1 + b_1 s)}{1 + fK + (a_1 + fKb_1)s + a_2 s^2}. \quad \cdots\cdots (5)$$

Here $f = \frac{R_2}{R_1 + R_2}$ for Fig.1 (b) and $f = 1$ for Fig.1 (c).

Application of proposed criterion

Then we set a parameter θ as follows:

$$\theta = a_1 + fKb_1. \quad \cdots\cdots (6)$$

Using equations of (4), the parameter θ is obtained as follows

$$\theta = R_1C_1 + R_2C_2 + \left(R_1 + R_2\right)C_{r1} + \left(G_{m2} - fG_{m1}\right)R_1R_2C_{r1}. \quad \cdots\cdots (7)$$

Based on the R-H stability criterion, we can obtain the following as the necessary and sufficient condition for the operational amplifier feedback circuit stability:

$$\theta > 0. \quad \cdots\cdots (8)$$

Note that the explicit stability condition in Eqs. (7), (8) cannot be found out in any analog circuit design book(6)-(13), to the best of our knowledge. We can see from Eqs. (7), (8) which parameter values should be increased or decreased to obtain the feedback stability.

Example 2: Two-pole operational amplifier with R, C compensation.

$R_1, R_2$ are equivalent resistors, $C_1, C_2$ are equivalent capacitances, $G_{m1}, G_{m2}$ are transconductances, $C_{r1}$ is compensation capacitance, $R_r$ is compensation resistor.

The closed-loop transfer function of the feedback amplifier using the operational amplifier in Fig.4 is given by

$$\frac{G(s)}{1+fG(s)} = \frac{K\left(1+b_1 s\right)}{1+fK+\left(a_1+fKb_1\right)s+a_2s^2+a_3s^3}. \quad \cdots\cdots (9)$$

Here, $b_1 = -\left(\dfrac{C_{r2}}{G_{m2}} - R_rC_{r2}\right)$,

$$\begin{aligned} K &= G_{m1}G_{m2}R_1R_2, \quad a_3 = R_1R_2R_rC_1C_2C_{r2}, \\ a_1 &= R_1C_1 + R_2C_2 + \left(R_1 + R_2 + R_r + R_1R_2G_{m2}\right)C_{r2}, \quad \cdots\cdots (10) \\ a_2 &= R_1R_2\left(C_2C_{r2} + C_1C_2 + C_1C_{r2}\right) + R_rC_{r2}. \end{aligned}$$

Then we can obtain the parameter $\alpha_1$ as follows:

$$\begin{aligned} \alpha_1 &= \left(a_1 + fKb_1\right) = R_1C_1 + R_2C_2 + \left(R_1 + R_2 + R_r\right)C_{r2} \\ &\quad + \left(G_{m2} - fG_{m1} + fG_{m1}G_{m2}R_r\right)R_1R_2C_{r2}. \quad \cdots\cdots (11) \end{aligned}$$

and the Routh table's parameter $\beta_1$ is given by

$$\begin{aligned} \beta_1 &= \frac{\left(a_1 + fKb_1\right)a_2 - a_3\left(1+fK\right)}{a_2} \\ &= R_1C_1 + R_2C_2 + \left(R_1 + R_2 + R_r\right)C_{r2} \\ &\quad + \left(G_{m2} - fG_{m1} + fG_{m1}G_{m2}R_r\right)R_1R_2C_{r2} \\ &\quad - \frac{R_1R_2C_1C_2R_rC_{r2}\left(1 + fG_{m1}G_{m2}R_1R_2\right)}{R_1R_2\left(C_2C_{r2} + C_1C_2 + C_1C_{r2}\right) + R_rC_{r2}\left(R_1C_1 + R_2C_2\right)}. \quad \cdots\cdots (12) \end{aligned}$$

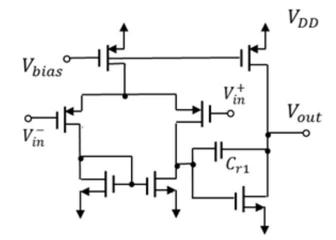
The stability condition is as follows:

$$\alpha_1 > 0, \quad \beta_1 > 0. \quad \cdots\cdots (13)$$

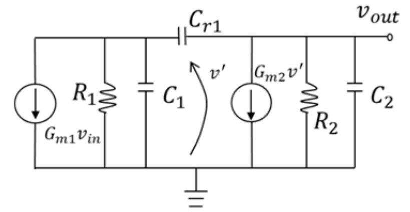
Again, the explicit stability condition in Eqs. (11), (12), (13) cannot be found out in any analog circuit design book(6)-(13), to the best of our knowledge, and we understand from Eqs. (11), (12), (13) which parameter values should be increased or decreased to obtain the feedback stability.

Example 3: Three-pole operational amplifier.

$R_1, R_2, R_3$ are equivalent resistors, $C_1, C_2, C_3$ are equivalent capacitances, $G_{m1}, G_{m2}, G_{m3}$ are transconductances, $C_{r3}, C_{r4}$ are compensation capacitances.

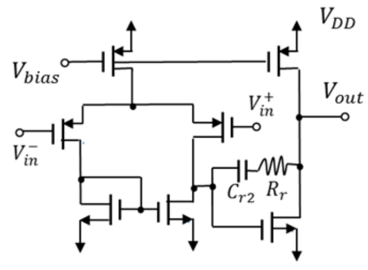



(a) Transistor level circuit

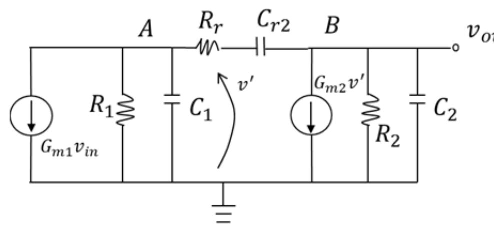



(b) Small-signal model

Fig. 3. Two-pole amplifier with inter-stage capacitance.




(a) Transistor level circuit




(b) Small-signal model

Fig. 4. Two-pole amplifier with compensation of Miller right-half-plane zero.

The closed-loop transfer function of the feedback amplifier using the operational amplifier in Fig.5 is given by

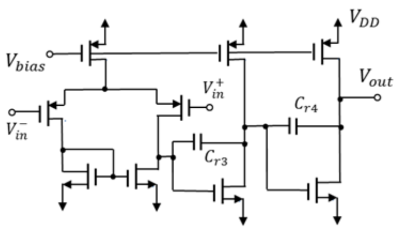
$$\frac{G(s)}{1+fG(s)} = \frac{K\left(1+b_1s+b_2s^2\right)}{1+fK+\left(a_1+fKb_1\right)s+\left(a_2+fKb_2\right)s^2+a_3s^3}. \quad \cdots\cdots (14)$$

Where, $K = G_{m1}G_{m2}G_{m3}R_1R_2R_3$,

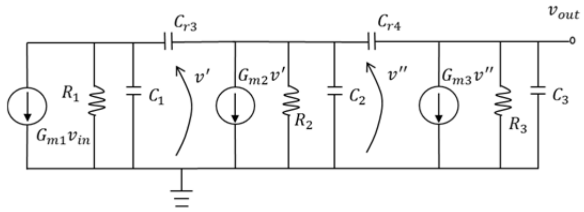
$$b_1 = -\left(\frac{C_{r3}}{G_{m2}} + \frac{C_{r4}}{G_{m3}}\right), \quad b_2 = \frac{C_{r3}C_{r4}}{G_{m2}G_{m3}},$$

$$\begin{aligned} a_1 &= C_{r3}\left(R_1 + R_2 + G_{m2}R_1R_2\right) \\ &\quad + C_{r4}\left(R_2 + R_3 + G_{m3}R_2R_3\right) + R_1C_1 + R_2C_2 + R_3C_3. \end{aligned}$$

$$\begin{aligned} a_2 &= C_{r3}(G_{m2}R_1R_2R_3C_3 + (R_1 + R_2)R_3C_3 + R_1R_2(C_1 + C_2)) \\ &\quad + C_{r4}(G_{m3}R_1R_2R_3C_1 + (R_2 + R_3)R_1C_1 + R_2R_3(C_2 + C_3)) \\ &\quad + C_{r3}C_{r4}\left(\left(G_{m2} + G_{m3}\right)R_1R_2R_3 + R_1R_2 + R_2R_3 + R_1R_3\right) \\ &\quad + R_1R_2C_1C_2 + R_2R_3C_2C_3 + R_1R_3C_1C_3. \end{aligned}$$




(a) Transistor level circuit




(b) Small-signal model

Fig. 5. Three-pole amplifier with inter-stage capacitance.

$$a_3 = R_1R_2R_3\left[C_{r3}(C_2C_3 + C_1C_3) + C_{r2}(C_1C_2 + C_1C_3) + C_{r1}C_{r4}(C_1 + C_2 + C_3) + C_1C_2C_3\right]. \quad (15)$$

Then we can obtain the parameter $\partial_2$:

$$\begin{aligned}\partial_2 &= a_1 + fKb_1 \\ &= C_{r3}(R_1 + R_2 + G_{m2}R_1R_2) \\ &\quad + C_{r4}(R_2 + R_3 + G_{m3}R_2R_3) + R_1C_1 + R_2C_2 + R_3C_3 \\ &\quad - fG_{m1}G_{m2}G_{m3}R_1R_2R_3\left(\frac{C_{r3}}{G_{m2}} + \frac{C_{r4}}{G_{m3}}\right).\end{aligned} \quad (16)$$

and the Routh table's parameter $\beta_2$:

$$\beta_2 = \frac{(a_1 + fKb_1)(a_2 + fKb_2) - a_3(1 + fK)}{a_2 + fKb_2}. \quad (17)$$

The stability condition is as follows:

$$\alpha_2 > 0, \quad \beta_2 > 0. \quad (18)$$

Again, the explicit stability condition in Eqs. (16), (17), (18) cannot be found out in any analog circuit design book[(6)-(13)], to the best of our knowledge.

In this section, we select three circuit configurations as examples for deducing the explicit stability condition based on the proposed method. For other circuit configuration, the R-H method would can be applied in the condition that if we can derive its characteristic equation of the closed-loop transfer function as (2) and Routh table as Table 1.

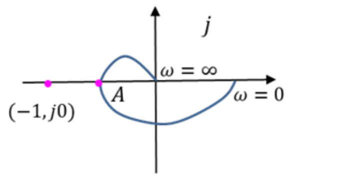
## 4. Equivalence at Mathematical Foundations

This section shows the equivalency between the Nyquist stability criterion and the R-H stability criterion in some conditions.

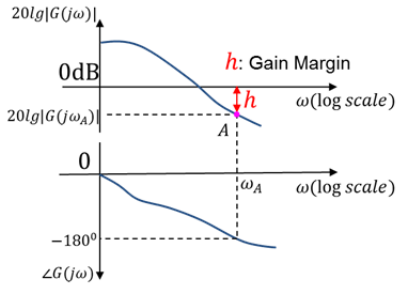
Example 1: Select one amplifier whose open-loop transfer function is given by

$$G(s) = \frac{K}{1 + a_1s + a_2s^2 + a_3s^3} \quad (19)$$

Fig.1 (c) shows a feedback amplifier (voltage follower) using this operational amplifier, and the closed-loop transfer function is obtained as follows:




(a) Sketch of Nyquist plot




(b) Sketch of Bode plot

Fig. 6. Sketch diagram of the open loop transfer function.

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{1 + K + a_1s + a_2s^2 + a_3s^3} \quad (20)$$

Based on the R-H stability criterion, we can obtain the following as the necessary and sufficient condition for the operational amplifier feedback circuit stability:

$$\frac{a_2a_1 - a_3(1 + K)}{a_2} > 0 \quad (21)$$

Since $a_2 > 0$ for stability, we can deduce the stability condition as following:

$$a_2a_1 > a_3(1 + K) \quad (22)$$

In frequency domain, Eq. (19) is represented as:

$$\begin{aligned}G(j\omega) &= \frac{K}{1 + a_1(j\omega) + a_2(j\omega)^2 + a_3(j\omega)^3} \\ &= \frac{K\left[(1 - a_2\omega^2) - j(a_1\omega - a_3\omega^3)\right]}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}\end{aligned} \quad (23)$$

According to the explanation of Nyquist plot introduced in Section 2.1, and based on the Nyquist plot sketch as shown in Fig.6 (a), we can find out that if the open-loop system is stable ($P = 0$), the Nyquist plot must not encircle the plot $(-1, j0)$, so the stability condition is given as following:

$$\angle G(j\omega_1) = -\pi \quad (24)$$

$$|G(j\omega_1)| < 1 \quad (25)$$

Here, $\omega_1$ is the frequency at point $A$ in Fig.6.

Also according to the explanation of Bode plot that has been introduced in Section 2.2, and based on the Bode plot sketch as shown in Fig.6 (b), we can find out that if the open-loop system is stable, the Bode plot should satisfy the following conditions:

$$\angle G(j\omega_1) = -\pi \quad (26)$$

$$\mathrm{GM} = 0 - 20lg|G(j\omega_1)| > 0 \quad (27)$$

By simple derivation, we can found out the stability condition that the respective based on Nyquist plot and Bode plot as shown in

Eqs. (24), (25) and Eqs. (26), (27) is actually identical.

Considering Eqs. (23), (24) and (26), we can obtain:

$$\omega_1^2 = \frac{a_1}{a_3} \quad (28)$$

Hence, the gain value at point $A$ is:

$$\left|G(j\omega_1)\right| = \left|\frac{K\sqrt{(1-a_2\omega_1^2)^2 + (a_1\omega_1 - a_3\omega_1^3)^2}}{(1-a_2\omega_1^2)^2 + (a_1\omega_1 - a_3\omega_1^3)^2}\right| = \frac{K}{\left|1 - a_2\frac{a_1}{a_3}\right|} \quad (29)$$

Based on calculation of Eq. (29) and conditions of Eqs. (25) and (27), we can obtain the following inequality expression ultimately:

$$a_1a_2 - a_3 < Ka_3 < a_3 - a_1a_2, \text{ in case } a_3 - a_1a_2 > 0$$

$$a_3 - a_1a_2 < Ka_3 < a_1a_2 - a_3, \text{ in case } a_3 - a_1a_2 < 0 \quad (30)$$

Obviously, inequality expressions of Eqs. (22) and (30) are equivalent under some conditions. So, we can say that mathematical foundations of Nyquist and R-H stability criteria are equivalent for G(s) in Eq. (19).

Example 2: Select one amplifier whose open-loop transfer function is given by

$$G(s) = \frac{K(1+b_1s)}{1+a_1s+a_2s^2} \quad (31)$$

Fig.1 (c) shows a feedback amplifier (voltage follower) using this operational amplifier, and its closed-loop transfer function can be obtained as follows:

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{K+Kb_1s}{1+K+(a_1+Kb_1)s+a_2s^2} \quad (32)$$

Based on the R-H stability criterion, we can also deduce the stability condition as following:

$$1+K>0, \quad a_1+Kb_1>0, \quad a_2>0 \quad (33)$$

We can obtain the stability condition:

$$K < -\frac{a_1}{b_1}, \text{ in case } b_1<0$$

$$K > -\frac{a_1}{b_1}, \text{ in case } b_1>0 \quad (34)$$

In frequency domain, Eq. (31) is represented as:

$$G(j\omega) = \frac{K(1+b_1(j\omega))}{1+a_1(j\omega)+a_2(j\omega)^2} = \frac{K(1-a_2\omega^2+b_1a_1\omega^2) + jK(b_1\omega - a_1\omega - a_2b_1\omega^3)}{(1-a_2\omega^2)^2 + a_1^2\omega^2} \quad (35)$$

According to the explanation of Nyquist plot that has been introduced in Section 2.1, and based on the Nyquist plot sketch as shown in Fig.6 (a), we can find out that if the open-loop system is stable ($P=0$), the Nyquist plot must not encircle the plot $(-1, j0)$. So the stability condition is given as follows:

$$\angle G(j\omega_2) = -\pi \quad (36)$$

$$\left|G(j\omega_2)\right| < 1 \quad (37)$$

Here, $\omega_2$ is the frequency at point $A$.

Also according to the explanation of Bode plot that has been introduced in Section 2.2, and based on the Bode plot sketch as shown in Fig.6 (b), we can find out that if the open-loop system is stable, the Bode plot should satisfy the following conditions:

$$\angle G(j\omega_1) = -\pi \quad (38)$$

$$\text{GM} = 0 - 20lg\left|G(j\omega_1)\right| > 0 \quad (39)$$

By simple derivation, we can found out the stability condition that the respective based on Nyquist plot and Bode plot as shown in Eqs. (36), (37) and (38), (39) is actually identical.

Considering Eqs. (35), (36) and (38), we can obtain:

$$\omega_2^2 = \frac{1}{a_2}\left(1 - \frac{a_1}{b_1}\right) \quad (40)$$

Hence, the gain value at point $A$ is:

$$\left|G(j\omega_2)\right| = \left|\frac{K(1-a_2\omega_2^2+b_1a_1\omega_2^2)}{(1-a_2\omega_2^2)^2 + a_1^2\omega_2^2}\right| = \frac{K\left|\frac{a_1}{b_1} + \frac{a_1}{a_2}(b_1-a_1)\right|}{\left|\left(\frac{a_1}{b_1}\right)^2 + \frac{a_1}{a_2}\frac{a_1}{b_1}(b_1-a_1)\right|} = K\left|\frac{b_1}{a_1}\right| \quad (41)$$

Based on the calculation of Eq. (41) and conditions of Eqs. (37) and (39), we can obtain the following inequality expression ultimately:

$$-\frac{a_1}{b_1} < K < \frac{a_1}{b_1}, \text{ in case } a_1b_1>0$$

$$\frac{a_1}{b_1} < K < -\frac{a_1}{b_1}, \text{ in case } a_1b_1<0 \quad (42)$$

Clearly, inequality expressions of Eqs. (34) and (42) are equivalent under some conditions. So, we can say that mathematical foundations of Nyquist and R-H stability criteria are equivalent.

Example 3: Select one amplifier whose open-loop transfer function is given by

$$G(s) = \frac{K(1+b_1s)}{1+a_1s+a_2s^2+a_3s^3} \quad (43)$$

Fig.1 (c) show a feedback amplifier (voltage follower) using this operational amplifier, and the closed-loop transfer function is obtained as follows:

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{K+Kb_1s}{1+K+(a_1+Kb_1)s+a_2s^2+a_3s^3} \quad (44)$$

Based on the R-H stability criterion, we also can deduce the stability condition as following:

$$1+K>0, \quad a_1+Kb_1>0, \quad a_2>0, \quad a_3>0,$$

$$\frac{a_2(a_1+Kb)-a_3(1+K)}{a_2} > 0. \quad (45)$$

We can obtain stability condition:

$$K > \frac{a_3-a_1a_2}{a_2b-a_3}, \text{ in case } a_2b-a_3>0$$

$$K < \frac{a_3 - a_1 a_2}{a_2 b - a_3}, \text{ in case } a_2 b - a_3 < 0 \quad \cdots\cdots (46)$$

In frequency domain, equation (43) is represented as:

$$G(j\omega) = \frac{K\left(1 + b_1(j\omega)\right)}{1 + a_1(j\omega) + a_2(j\omega)^2 + a_3(j\omega)^3} = \frac{K\left[\left(1 - a_2\omega^2 + a_1 b_1\omega^2 - a_3 b\omega^4\right) + j\left(b_1\omega - a_2 b_1\omega^3 - a_1\omega + a_3\omega^3\right)\right]}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2} \quad \cdots\cdots (47)$$

According to the explanation of Nyquist plot that has been introduced in Section 2.1, and based on the Nyquist plot sketch as shown in Fig.6 (a), we can find out that if the open-loop system is stable ($P = 0$), the Nyquist plot should not encircle the plot $(-1, j0)$, so the stability condition is given as follows:

$$\angle G(j\omega_3) = -\pi \quad \cdots\cdots (48)$$

$$\left|G(j\omega_3)\right| < 1 \quad \cdots\cdots (49)$$

Here, $\omega_3$ is the frequency at point $A$.

Also according to the explanation of Bode plot that has been introduced in Section 2.2, and based on the Bode plot sketch as shown in Fig.6 (b), we can find out that if the open-loop system is stable, the Bode plot should satisfy the following conditions:

$$\angle G(j\omega_1) = -\pi \quad \cdots\cdots (50)$$

$$\mathrm{GM} = 0 - 20lg\left|G(j\omega_1)\right| > 0 \quad \cdots\cdots (51)$$

By simple derivation, we can found out the stability condition that the respective based on Nyquist plot and Bode plot as shown in (48), (49) and (50), (51) is actually identical.

Considering that (47), (48) and (50), we can obtain:

$$\omega_3^2 = \frac{a_1 - b_1}{a_3 - a_2 b_1} \quad \cdots\cdots (52)$$

Hence, the gain value at point $A$ is:

$$\left|G(j\omega_3)\right| = \left|\frac{K\left(1 - a_2\omega_3^2 + a_1 b_1\omega_3^2 - a_3 b_1\omega_3^4\right)}{(1 - a_2\omega_3^2)^2 + (a_1\omega - a_3\omega_3^3)^2}\right| = K\left|\frac{a_3 - a_2 b_1}{a_3 - a_1 a_2}\right| \quad \cdots\cdots (53)$$

Based on calculation of (53) and conditions (49) and (51), we can obtain the following inequality expressions ultimately:

$$\frac{a_3 - a_1 a_2}{a_2 b - a_3} < K < \frac{a_3 - a_1 a_2}{a_3 - a_2 b}, \text{ in case } (a_3 - a_1 a_2)(a_3 - a_2 b) > 0$$

$$\frac{a_3 - a_1 a_2}{a_3 - a_2 b} < K < \frac{a_3 - a_1 a_2}{a_2 b - a_3}, \text{ in case } (a_3 - a_1 a_2)(a_3 - a_2 b) < 0 \quad \cdots\cdots (54)$$

Clearly, inequality expressions (54) and (46) are equivalent under some conditions. So, we can say that mathematical foundations of Nyquist and R-H stability criteria are equivalent.

Example 4: Select one amplifier whose open-loop transfer function is given by

$$G(s) = \frac{K\left(1 + b_1 s + b_2 s^2\right)}{1 + a_1 s + a_2 s^2 + a_3 s^3} \quad \cdots\cdots (55)$$

Fig.1 (c) show a feedback amplifier (voltage follower) using this operational amplifier, and the closed-loop transfer function is obtained as follows:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K + Kb_1 s + Kb_2 s^2}{1 + K + (a_1 + Kb_1)s + \left(a_2 + Kb_2\right)s^2 + a_3 s^3} \quad \cdots\cdots (56)$$

Based on the R-H stability criterion, we also can deduce the stability condition as follows:

$$\left(a_2 + Kb_2\right)\left(a_1 + Kb_1\right) - a_3\left(1 + K\right) > 0 \quad \cdots\cdots (57)$$

Let set one function:

$$f(K) = \left(a_2 + Kb_2\right)\left(a_1 + Kb_1\right) - a_3\left(1 + K\right) = K^2 b_1 b_2 + Ka_1 b_2 + Ka_2 b_1 - Ka_3 + a_1 a_2 - a_3 \quad \cdots\cdots (58)$$

- Domain of definition $K \in (0, +\infty)$
- Initial value: $f(0) = a_1 a_2 - a_3$ $\cdots\cdots$ (59)
- Derived function:

$$f'(\mathrm{K}) = 2Kb_1 b_2 + a_1 b_2 + a_2 b_1 - a_3 \quad \cdots\cdots (60)$$

For get to the stability condition (57), the following condition should be satisfied:

$$f(0) \geq 0, \text{ and } f'(\mathrm{K}) > 0 \quad \cdots\cdots (61)$$

Thus, the stability condition is becoming:

$$2Kb_1 b_2 + a_1 b_2 + a_2 b_1 - a_3 > 0 \quad \cdots\cdots (62)$$

at condition: $a_1 a_2 - a_3 > 0$.

In frequency domain, equation (55) is represented as:

$$G(j\omega) = \frac{K\left(1 + b_1(j\omega) + b_2(j\omega)^3\right)}{1 + a_1(j\omega) + a_2(j\omega)^2 + a_3(j\omega)^3} = \frac{K\left(1 - a_2\omega^2 - b_2\omega^2 + a_2 b_2\omega^4 + a_1 b_1\omega^2 - a_3 b_1\omega^4\right) + jK\left(a_3\omega^3 - a_1\omega + a_1 b_2\omega^3 - a_3 b_2\omega^5 + b_1\omega - a_2 b_1\omega^3\right)}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2} \quad \cdots\cdots (63)$$

According to the explanation of Nyquist plot that has been introduced in Section 2.1, and based on the Nyquist plot sketch as shown in Fig.6 (a), we can find out that if the open-loop system is stable ($P = 0$), the Nyquist plot must not encircle the plot $(-1, j0)$, so the stability condition as following:

$$\angle G(j\omega_4) = -\pi \quad \cdots\cdots (64)$$

$$\left|G(j\omega_4)\right| < 1 \quad \cdots\cdots (65)$$

Here, $\omega_4$ is the frequency at point $A$.

Also according to the explanation of Bode plot that has been introduced in Section 2.2, and based on the Bode plot sketch as shown in Fig.6 (b), we can find out that if the open-loop system is stable, the Bode plot should satisfy the following conditions:

$$\angle G(j\omega_1) = -\pi \quad \cdots\cdots (66)$$

$$\mathrm{GM} = 0 - 20lg\left|G(j\omega_1)\right| > 0 \quad \cdots\cdots (67)$$

By simple derivation, we can found out the stability condition that the respective based on Nyquist plot and Bode plot as shown in (64), (65) and (66), (67) is actually identical.

Considering that (63), (64) and (66), we can obtain:

$$a_3\omega_4^3 - a_1\omega_4 + a_1 b_2\omega_4^3 - a_3 b_2\omega_4^5 + b_1\omega_4 - a_2 b_1\omega_4^3 = 0. \quad \cdots\cdots (68)$$

After transformation, we can obtain:

$$1 - a_2\omega_4^2 = \frac{\left(1 - b_2\omega_4^2\right)\left(a_1 - a_3\omega_4^2\right)}{b_1} \quad \cdots\cdots (69)$$

Hence, the gain value at point $A$ is:

$$\left|G\left(j\omega_4\right)\right| = \left|\frac{K1 - a_2\omega_4^2 - b_2\omega_4^2 + a_2 b_2 \omega_4^4 + a_1 b_1 \omega_4^2 - a_3 b_1 \omega_4^4}{(1 - a_2\omega_4^2)^2 + (a_1\omega_4 - a_3\omega_4^3)^2}\right|$$

$$= \cdots = \frac{Kb_1}{\left|a_1 - a_3\omega_4^2\right|} \quad \cdots\cdots(70)$$

It follows from (68) that we have

$$a_3 b_2 \omega_4^4 + (a_2 b_1 - a_1 b_2 - a_3)\omega_4^2 + a_1 - b_1 = 0 \quad \cdots\cdots (71)$$

Solution of Eq. (71) is given by

$$\omega^2 = \frac{a_3 + a_1 b_2 - a_2 b_1 \pm \sqrt{(a_2 b_1 - a_1 b_2 - a_3)^2 - 4 a_3 b_2 \left(a_1 - b_1\right)}}{2 a_3 b_2}$$

$$\approx \frac{a_3 + a_1 b_2 - a_2 b_1}{2 a_3 b_2} \quad \cdots\cdots(72)$$

It follows from Eqs. (72), (70) and condition (57) that we have

$$\left|G\left(j\omega_4\right)\right| = \frac{K\left|b_1\right|}{\left|a_1 - a_3\omega_4^2\right|} = \frac{K\left|b_1\right|}{\left|a_1 - a_3 \dfrac{a_3 + a_1 b_2 - a_2 b_1}{2 a_3 b_2}\right|}$$

$$= \frac{K\left|2 b_1 b_2\right|}{\left|a_1 b_2 + a_2 b_1 - a_3\right|} < 1 \quad \cdots\cdots (73)$$

By calculation, we can obtain the following inequality expression ultimately:

$$a_3 - a_1 b_2 - a_2 b_1 < 2K b_1 b_2 < a_1 b_2 + a_2 b_1 - a_3$$

in case $a_1 b_2 + a_2 b_1 - a_3 > 0$

$$a_1 b_2 + a_2 b_1 - a_3 < 2K b_1 b_2 < a_3 - a_1 b_2 - a_2 b_1$$

in case $a_1 b_2 + a_2 b_1 - a_3 < 0$ $\cdots\cdots$ (74)

Clearly, inequality expressions of Eqs. (62) and (74) are equivalent under some conditions. So, we can say that mathematical foundations of Nyquist and R-H stability criteria are equivalent.

## 5. Relationship Between R-H Parameters and Phase Margin

Example 1: Consider the two-pole amplifier as shown in Fig.3. Accordingly, Fig.1 (b) shows a feedback amplifier using this operational amplifier, and its closed-loop transfer function is shown as Eq. (5). Based on the R-H stability criterion, we can obtain the explicit stability condition is shown as Eq. (8).

We define the R-H parameter θ as one time dimension parameter. Using the parameter values of short-channel CMOS devices, and calculating the values of parameter θ and the corresponding operational amplifier system phase margin (PM), gain margin (GM), $F_{gm}$ and $F_{pm}$ at various feedback factor $f$ conditions, using MATLAB. $F_{gm}$ is the frequency where the gain margin is measured, which is a −180° phase crossing frequency in Bode plot, and $F_{pm}$ is the frequency where the phase margin is measured, which is the 0 dB gain crossing frequency in Bode plot. For example, when feedback factor $f = 0.01$, we can obtain the values as Table 2.

Using the polyfit function of MATLAB, we can obtain the fitted curve which can indicate the relationship between parameter θ with phase margin as shown in Fig.7 under various feedback factor conditions. In feedback factor $f = 0.01$ condition, we can obtain the fitted curve as shown in Fig.8, and the corresponding relation function is given as follows:

Table 2. Data collection.

| $f$=0.01 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_{r1}$ [fF] | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90… |
| θ [uS] | 0.11 | 0.18 | 0.25 | 0.32 | 0.39 | 0.46 | 0.53 | 0.60 | 0.67… |
| PM [degree] | 16 | 19 | 22 | 24 | 27 | 29 | 31 | 33 | 34… |
| GM [dB] | 9.1 | 7.6 | 7.0 | 6.6 | 6.4 | 6.3 | 6.2 | 6.0 | 6.0… |
| $F_{gm}$ [GHz] | 4.5 | 3.4 | 2.9 | 2.6 | 2.3 | 2.1 | 2.0 | 1.9 | 1.8… |
| $F_{pm}$ [GHz] | 2.6 | 2.1 | 1.8 | 1.5 | 1.4 | 1.2 | 1.1 | 1.0 | 9.4… |

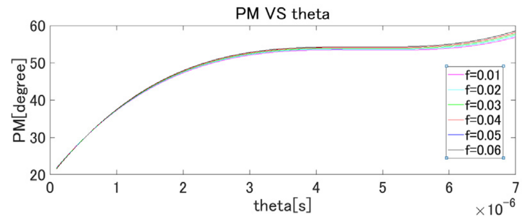



Fig. 7. Relationship between PM and parameter θ in various feedback factor conditions.

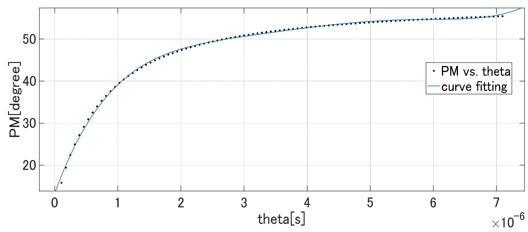



Fig. 8. Relationship between PM and parameter θ in feedback factor $f = 0.01$ condition.

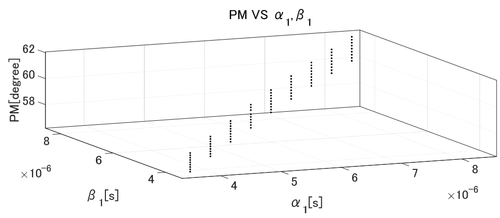



Fig. 9. Relationship between PM with parameter $\alpha_1, \beta_1$ at feedback factor $f = 0.01$ condition.

$$PM = 2.601e^{28}\theta^5 - 5.616e^{23}\theta^4 + 4.683e^{18}\theta^3$$
$$-1.915e^{13}\theta^2 + 4.076e^{28}\theta + 13.38 \quad \cdots\cdots (75)$$

As shown in Fig.7 and Fig.8, the PM and the R-H parameter θ have the linear relationship, following with the increase of parameter value θ, the phase margin will be increased; in other words, the feedback system will be more stable.

We can calculate a required value of the compensation capacitor, for a given operational amplifier PM, based on the calculated value of the parameter θ.

Example 2: Consider the two-pole amplifier as shown in Fig.4, whose open-loop transfer function is given by

$$G\left(s\right) = \frac{K\left(1 + b_1 s\right)}{1 + a_1 s + a_2 s^2 + a_3 s^3} \quad \cdots\cdots (76)$$

Accordingly, Fig.1 (b) shows a feedback amplifier using this operational amplifier, and its closed-loop transfer function is shown as (9). Based on the R-H stability criterion, we can obtain the stability condition is shown as (13).

We also define the R-H parameters $\alpha_1, \beta_1$ as time dimension parameters. Using the parameter values of short-channel CMOS devices, and calculating the values of parameters $\alpha_1, \beta_1$ and the corresponding feedback system's PM, at variation feedback factor $f$ conditions by MATLAB. At feedback factor $f = 0.01$ condition, we can obtain the relation function in Fig.9, when parameters $\alpha_1, \beta_1$ as independent variables and PM as dependent variable by using interpolation function in curve fitting tool of MATLAB.

As shown in Fig.9, the relationship between R-H parameter $\alpha_1, \beta_1$ with PM is linear one, and following with the increase of parameter's value, the phase margin will be increased, in other words, the feedback system will be more stable.

## 6. Verification with SIPCE Simulation

We calculate the values of the parameters θ, $\alpha_1, \beta_1$ as shown in Eqs. (7), (11), (12) and depict Bode plots using SPICE for judging stability of the amplifier with the voltage follower configuration (Fig.1 (c)) for amplifiers 1, 2. See Table 3, Figs. 10, 11, 12 as amplifier 1, Table 4, Figs. 13, 14, 15 as amplifier 2.

Then we show analysis between their simulation results and the parameter values of θ, $\alpha_1$ and $\beta_1$. We found out the following: when θ, $\alpha_1$ and $\beta_1$ are greater than 0, less than 0 and approximated to 0, then the corresponding amplifier with the voltage follower configuration in Fig.1 (b) is stable, unstable and critical stable, respectively.

We can distinctly find that the amplifier stability depends on the parameters θ, $\alpha_1$, $\beta_1$, and the feedback system is stable if and only if the parameters θ, $\alpha_1$ and $\beta_1$ are positive.

Using the parameter values of short-channel CMOS devices, and calculating inequality Eqs. (8) and (13), we can obtain the value range of the compensation capacitor $C_{r1}$:

$$C_{r1} > 79.57\text{fF} \quad \cdots\cdots (77)$$

Table 3. Parameter values of the amplifier 1.

| | Parameter values | | | | | | | R-H criterion | Bode plot |
|---|---|---|---|---|---|---|---|---|---|
| case | $R_1$ | $C_1$ | $R_2$ | $C_2$ | $G_{m1}$ | $G_{m2}$ | $C_{r1}$ | $\theta$ | SPICE simulation |
| (1) | $50k$ | $10f$ | 10k | $0.1p$ | 0.01 | $8m$ | $1p$ | $< 0$ | unstable |
| (2) | $50k$ | $1f$ | 10k | $10f$ | 0.01 | $8m$ | $0.1p$ | $< 0$ | unstable |
| (3) | $100k$ | $100f$ | 10k | $1f$ | $9m$ | $4m$ | $0.1p$ | $< 0$ | unstable |
| (4) | $100k$ | $5f$ | $90k$ | $3f$ | $8m$ | $7.5m$ | $0.9p$ | $\approx 0$ | critical stable |
| (5) | $100k$ | $3f$ | $50k$ | $1f$ | $8.5m$ | $8m$ | $0.5p$ | $\approx 0$ | critical stable |
| (6) | $1meg$ | $6f$ | $500k$ | $0.5f$ | $80u$ | $70u$ | $1f$ | $\approx 0$ | critical stable |
| (7) | $50k$ | $10f$ | 100 | $0.1p$ | 0.01 | $8m$ | $1p$ | $> 0$ | stable |
| (8) | $100k$ | $5f$ | $90k$ | $3f$ | $80u$ | $70u$ | $0.9p$ | $> 0$ | stable |
| (9) | $150k$ | $6f$ | $100k$ | $1.5f$ | $80u$ | $70u$ | $0.5p$ | $> 0$ | stable |

Table 4. Parameter values of the amplifier 2.

| | Parameter values | | | | | | | | R-H criterion | | Bode plot |
|---|---|---|---|---|---|---|---|---|---|---|---|
| case | $R_1$ | $C_1$ | $R_2$ | $C_2$ | $G_{m1}$ | $G_{m2}$ | $R_r$ | $C_{r2}$ | $\partial_1$ | $\beta_1$ | SPICE simulation |
| (1) | $115k$ | $5f$ | $100k$ | $80f$ | $9m$ | $8m$ | 5 | $0.5p$ | $< 0$ | $< 0$ | unstable |
| (2) | $50k$ | $5f$ | $10k$ | $10f$ | $9m$ | $8m$ | 2 | $0.2p$ | $< 0$ | $< 0$ | unstable |
| (3) | $150k$ | $5f$ | $100k$ | $10f$ | $9m$ | $8m$ | 1 | $0.8p$ | $< 0$ | $< 0$ | unstable |
| (4) | $110k$ | $10f$ | $10k$ | $3f$ | 0.01 | $8m$ | 5 | $0.5f$ | $\approx 0$ | $\approx 0$ | critical |
| (5) | $115k$ | $10f$ | $100k$ | $3f$ | 0.01 | $8m$ | 5 | $0.5f$ | $\approx 0$ | $\approx 0$ | critical |
| (6) | $150k$ | $8f$ | $100k$ | $50f$ | $7m$ | $8m$ | 10 | $0.6p$ | $> 0$ | $> 0$ | stable |
| (7) | $100k$ | $8f$ | $80k$ | $50f$ | $6m$ | $8m$ | 5 | $0.6p$ | $> 0$ | $> 0$ | stable |
| (8) | $200k$ | $5f$ | $150k$ | $10f$ | $5m$ | $7m$ | 2.5 | $0.6p$ | $> 0$ | $> 0$ | stable |

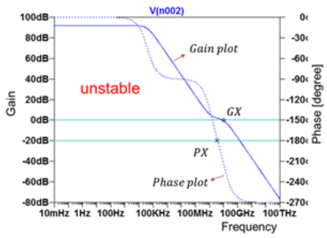



Fig. 10. Bode plots for case (1) of unstable amplifier 1.

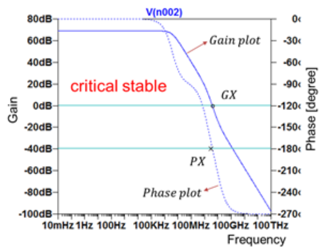



Fig. 11. Bode plot for case (6) of the critical stable amplifier 1.

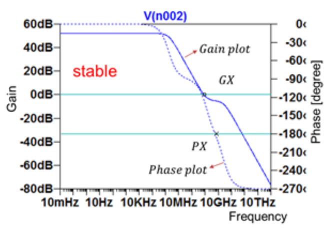



Fig. 12. Bode plot for case (8) of the stable amplifier 1.

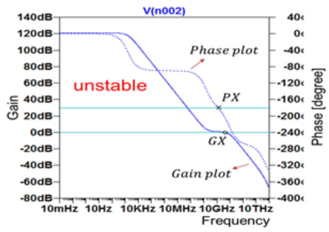



Fig. 13. Bode plot for case (3) of the unstable amplifier 2.

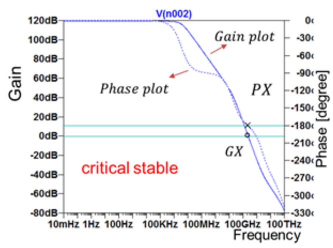



Fig. 14. Bode plot for case (5) of the critical stable amplifier 2.

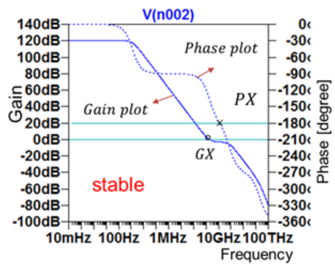



Fig. 15. Bode plots for case (8) of the stable amplifier 2.

and obtain inequality expression:

$$3.5\times10^{-8}+3.7\times10^{10}C_{r2}+R_rC_{r2}+831.7R_r > \frac{4.3\times10^{-8}R_rC_{r2}}{5.1\times10^{-17}+4.3\times10^{-3}C_{r2}+3.5\times10^{-8}R_rC_{r2}} \quad \cdots\cdots (78)$$

Let,

$$X=3.5\times10^{-8}+3.7\times10^{10}C_{r2}+R_rC_{r2}+831.7R_r$$

$$Y=\frac{4.3\times10^{-8}R_rC_{r2}}{5.1\times10^{-17}+4.3\times10^{-3}C_{r2}+3.5\times10^{-8}R_rC_{r2}} \quad \cdots\cdots\cdots (79)$$

We select several values of the parameters in Eqs. (77), (78) and depict their Bode plots using SPICE (LTspice) simulator for judging stability of the amplifier with voltage follower configuration. See Table 5, Fig.16~Fig.21 as amplifier 3, Table 6, Fig.22~Fig.27 as amplifier 4. The frequency in these transient analysis simulations is $1\times10^5$ Hz.

Consider the two-pole amplifier in Fig.28. Based on the

Table 5. Parameter values of the amplifier 3.

| case | $C_{r1}$ | SPICE |
|---|---|---|
| (1) | 150fF | stable |
| (2) | 79.57fF | critical stable |
| (3) | 10fF | unstable |

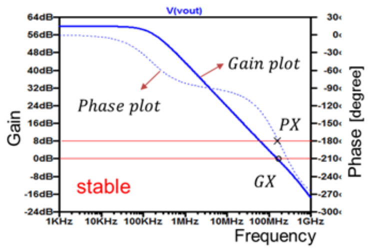



Fig. 16. Bode plot for case (1) of the stable amplifier 3.

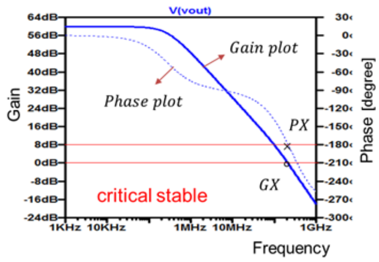



Fig. 17. Bode plot for case (2) of the critical stable amplifier 3.

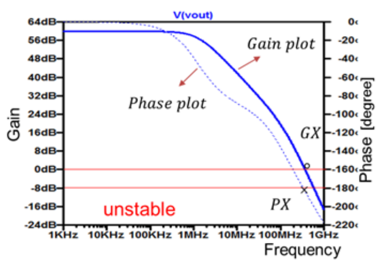



Fig. 18. Bode plot for case (3) of the unstable amplifier 3.

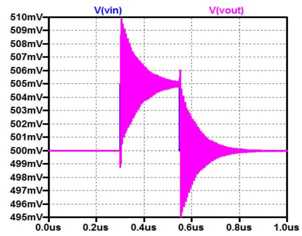



Fig. 19. Pulse response for case (1) of the stable amplifier 3.

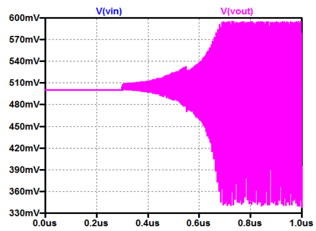



Fig. 20. Pulse response for case (2) of the critical stable amplifier 3.

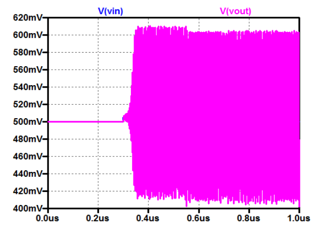



Fig. 21. Pulse response for case (1) of the unstable amplifier 3.

Table 6. Parameter values of the amplifier 4.

| | parameter values | | | | R-H criterion | Bode plot |
|---|---|---|---|---|---|---|
| case | $R_r$ | $C_{r2}$ | $X$ | $Y$ | | SPICE simulation |
| (1) | 6.5k | 2.4p | $1.41\times10^{-5}$ | $6.13\times10^{-8}$ | $X>Y$ | stable |
| (2) | 1 | 2.4p | $1.10\times10^{-6}$ | $9.94\times10^{-12}$ | $X>Y$ | stable |
| (3) | 7k | 10f | $9.8\times10^{-8}$ | $3.10\times10^{-8}$ | $X\approx Y$ | critical |

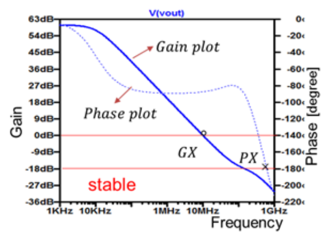



Fig. 22. Bode plot for case (1) of the stable amplifier 4.

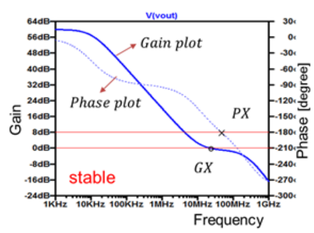



Fig. 23. Bode plot for case (2) of the stable amplifier 4.

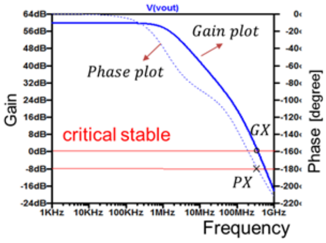



Fig. 24. Bode plot for case (3) of the critical stable amplifier 4.

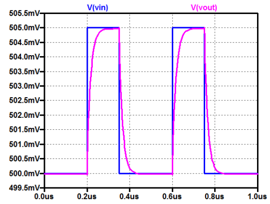



Fig. 25. Pulse response for case (1) of the stable amplifier 4.

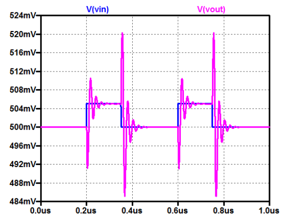



Fig. 26. Pulse response for case (2) of the stable amplifier 4.

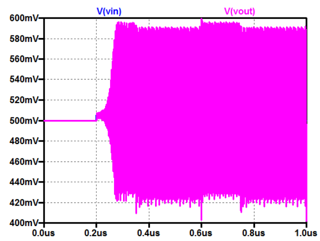
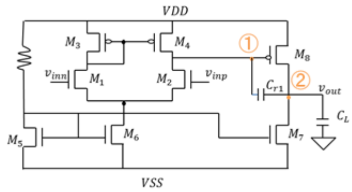
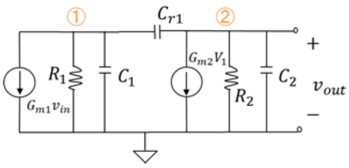



Fig. 27. Pulse response for case (3) of the critical stable amplifier 4.




(a) Transistor level circuit



(b) Small-signal model

Fig. 28. Two-pole amplifier with an inter-stage capacitance.

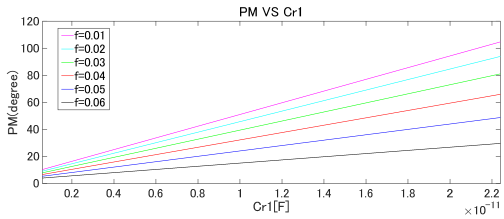
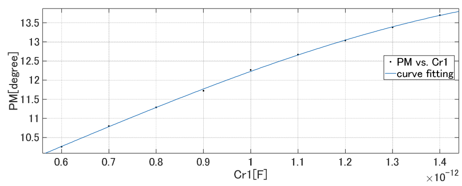
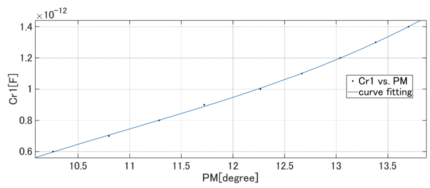



Fig. 29. Relationship between PM with compensation capacitor $C_{r1}$ in various feedback factor $f$ conditions.




(a) Compensation capacitor $C_{r1}$ as an independent variable and PM as a dependent variable




(b) PM as an independent variable and compensation capacitor $C_{r1}$ as a dependent variable

Fig. 30. Relationship between PM with compensation capacitor $C_{r1}$ at feedback factor $f = 0.01$ condition.

principle and processing described in Section 5, we obtain the parameter θ as shown in Eq. (7). We can calculate a required value of the compensation capacitor, for a given operational amplifier PM, based on the calculated value of the parameter θ. Using the polyfit function, we can obtain the curves which can indicate the relationship between capacitor $C_{r1}$ and phase margin as shown in Fig.29.

$R_1, R_2$ are equivalent resistors, $C_1, C_2$ are equivalent capacitances, $G_{m1}, G_{m2}$ are transconductances, $C_{r1}$ is compensation capacitance.

At feedback factor $f = 0.01$ condition, we can obtain the fitted curve as Fig.30 and the relation function between PM with capacitor as following:

$$PM = -1.026e^{36}C_{r1}^{\ 3} + 1.52e^{24}C_{r1}^{\ 2} + 4.488e^{12}C_{r1} + 7.24 \quad \cdots\cdots (80)$$

$$Cr1 = 6.343e^{-15}PM^3 - 2.091e^{-13}PM^2 + 2.493e^{-12}PM - 9.822e^{-12} \quad \cdots\cdots (81)$$

If we want to obtain 45° phase margin, the required corresponding capacitor value is 0.25694nF by calculated from Eq. (81).

In order to verify this result, we have conducted simulation for the amplifier circuit shown in Fig.28, the feedback system circuit shown in Fig.1 (b) when the feedback factor $f = 0.01$, and compensation capacitance is 0.25694nF. The simulation result is shown in Fig.31. The phase margin result is $180° - 133° = 47°$ obtained from LTspice simulation, which matches the result of 45° obtained from Eq. (80).

Although the relationship between $C_{r1}$ and the phase margin (corresponding to Fig.29) can be obtained by using the small

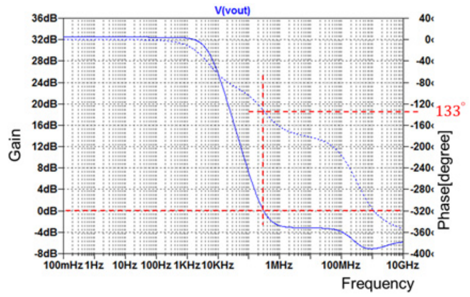



Fig. 31. LTspice simulation result with conditions of feedback factor $f = 0.01$ and compensation capacitor of 0.25694nF.

equivalent circuit which can indicate the variation tendency of stability following circuit parameter variation. But as we see, this relationship only can reflect the impact from single circuit parameter on stability. The advantages of the proposed method are the explicit stability conditions (8), (13), (18) and the relationship between parameter and phase margin (corresponding function (75) and Fig.8); we can overall consider multiple circuit parameters one time as well as the trade-off analysis between the influences on system stability from every single circuit parameter.

## 7. Discussions

According to the above consideration, we propose the following for operational amplifier stability analysis and design:

(1) Depict a small signal equivalent circuit for the operational amplifier circuit in open-loop structure.
(2) Derive its open-loop transfer function.
(3) Derive its closed-loop transfer function and obtain its characteristic equation.
(4) Apply the R-H stability criterion and obtain the relation function between the R-H parameter with phase margin. (which is not easy to obtain with Bode plot)
(5) Then use this relation function for circuit parameters.

The R-H method would be effective especially for multi-stage operational amplifiers (high-order systems).

It may be true that derivation of precise explicit transfer function with polynomials of s is difficult due to many parasitic components in the operational amplifier circuit. However, even if the derived equivalent circuit or transfer function uses only major components and neglects parasitic components, the R-H method provides the information about which major parameter values should be increased or decreased for stability; its usage together with the Bode plot would be effective.

## 8. Conclusion

This paper proposes to use a new stability analysis and design method for the operational amplifier feedback circuit based on principle of the Routh-Hurwitz stability criterion. We have shown the equivalence between Nyquist and Routh-Hurwitz stability criteria for analysis and design of the operational amplifier stability under some conditions, and have deduced the relationship between Routh-Hurwitz stability criterion parameters with phase margin of the operational amplifier. We have shown that they are monotonic relationship. In the verification and simulation parts, we have confirmed with SPICE simulation that this method is equivalent to the Bode plot method, and satisfactory results have been obtained with LTspice simulations at transistor level circuit. The acquisition and application of the relationship between R-H stability criterion parameters with phase margin demonstrate the feasibility of proposed method on both side of theory and practice.

Compared to the conventional Bode plot method which only can qualitatively judge the stability, the proposed method not only can judge the stability but also can do further quantitative analysis; this clarifies which circuit parameters influence the operational amplifier stability, and we know whether these circuit parameters should be increased or decreased.

The R-H method has an advantage of being able to obtain explicit stability condition for circuit parameters; hence we expect that the R-H method can be practically used together with the Bode plot method.

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