Distributed Arithmetic for Taylor-Series Expansion

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This paper describes the digital arithmetic that reduces the calculation for Taylor series expansion [1] in Eq. (1) by applying the distributed (bit-serial) arithmetic [2] with the proposed term division method.

$$f(x) = f(\alpha) + \frac{f'(\alpha)}{1!}(x-\alpha) + \frac{f''(\alpha)}{2!}(x-\alpha)^2 + \frac{f'''(\alpha)}{3!}(x-\alpha)^3 + \cdots \quad (1)$$

Suppose that α=0, $f(x)$ has 10 terms and each coefficient is given by a_n, Eq. (1) yields to the following:

$$f(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 \quad (2)$$

Let us consider calculating the polynomial given by Eq. (2) efficiently.

(i) Direct calculation given by Fig. 1 (a) requires 9 times of multiplications and 9 times of additions.

(ii) Our proposed distributed arithmetic with term division in Fig. 1 (b) requires the following: 2 times of distributed arithmetic for $a_0+a_2x^2+a_4x^4+a_6x^6+a_8x^8$ and $a_1+a_3x^2+a_5x^4+a_7x^6+a_9x^8$, 5 times of multiplications for x^2, x^4, x^6, x^8 as well as $x \times A$ (where $A = a_1+a_3x^2+a_5x^4+a_7x^6+a_9x^8$) and 1 addition.

Thus, the above investigated method (ii) can reduce the number of multiplications approximately by half compared to the direct method (i), though 2 distributed arithmetic calculations are required.

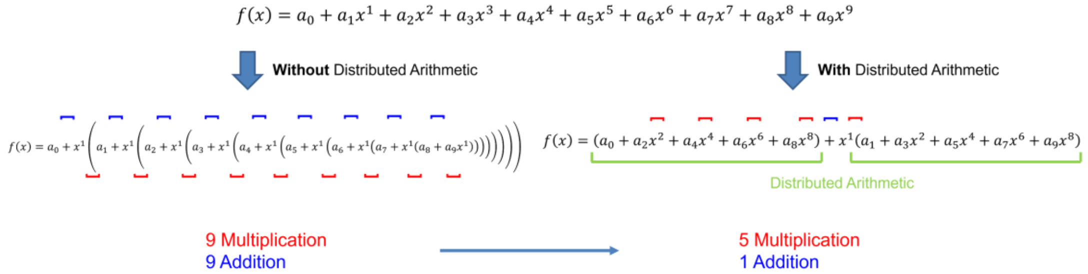

Fig. 1: How to calculate $f(x)$. (a) Direct calculation. (b) Distributed arithmetic with term division.

Fig. 2 (a) shows the distributed arithmetic circuit composed of a Look Up Table (LUT), a bit shifter, an adder and registers. Fig. 2 (b) shows the LUT (1024 words) without term division, while Fig. 2 (c) shows the ones (32 words x 2 = 64 words) with term division; our term division method reduces the LUT sizes as well as the number of multiplications. Notice that in Fig. 2 (b), its LUT address is given by (x^0, x^1, x^2, ..., x^9), and e. g., in case its address is (0100000000), its output data is a_1.

As the number of terms for Taylor series expansion increases, the increase of the number of term divisions for distributed arithmetic would be more effective for calculation and hardware efficiency.

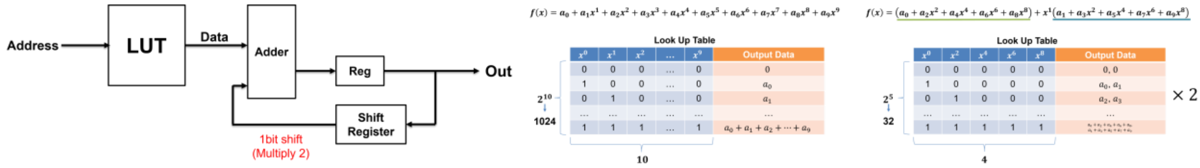

Fig. 2: Distributed arithmetic. (a) Circuit. (b) LUT for no term division. (c) LUTs for term division by 2.

References: [1] J. Wei, A. Kuwana, H. Kobayashi, K. Kubo, “Divide and Conquer: Floating-Point Exponential Calculation Based on Taylor-Series Expansion”, IEEE 14th International Conference on ASIC (Oct. 2021).

[2] R. Bala, S. Aktar, “Fast Fourier Transformation Realization with Distributed Arithmetic”, International Journal of Computer Applications, vol. 102, no. 15, pp. 22-25 (Sept. 2014).