

Track 1: Analog/Mixed-Signal Circuit Design and Related Technologies

Invited

# Revisit to RC Linear Circuit Theory

Haruo Kobayashi

K. Otomo, L. Nengvang, M. Chiba, S. Katayama,  
K. Yoshihiro, A. Kuwana, T. Ooide, **H. Tanimoto**

*Gunma University*  
*Kitami Institute of Technology*



# Outline

- Objective of This Paper
- Active Resistor Network
  - Spatial and Temporal Dynamics
  - Three New Property Findings
- ReDAC with RC Filter
  - Conventional ReDAC with LPF
  - Proposed ReDAC with HPF
- Conclusion

**ReDAC:** Relaxation Digital-to-Analog Converter

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# Objective of This Paper

To show **research for RC linear circuit has not ended yet.**

[RC Network Theory]

Spatial and temporal dynamics of **active** resistor network.

**Active** resistor network: **positive** and **negative** resistors

[Application of RC Circuit]

Relaxation DAC with RC high-pass filter  
for positive and negative polarity output

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# Research Outline of Active Network Dynamics

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Our previous theorem:

Spatial and temporal stability conditions are equivalent for **uniform** active resistor network



This paper:

Investigation of spatial and temporal dynamics for **non-uniform** active resistor network

⇒ Three new findings

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# Retina Chip with Positive Resistor Network

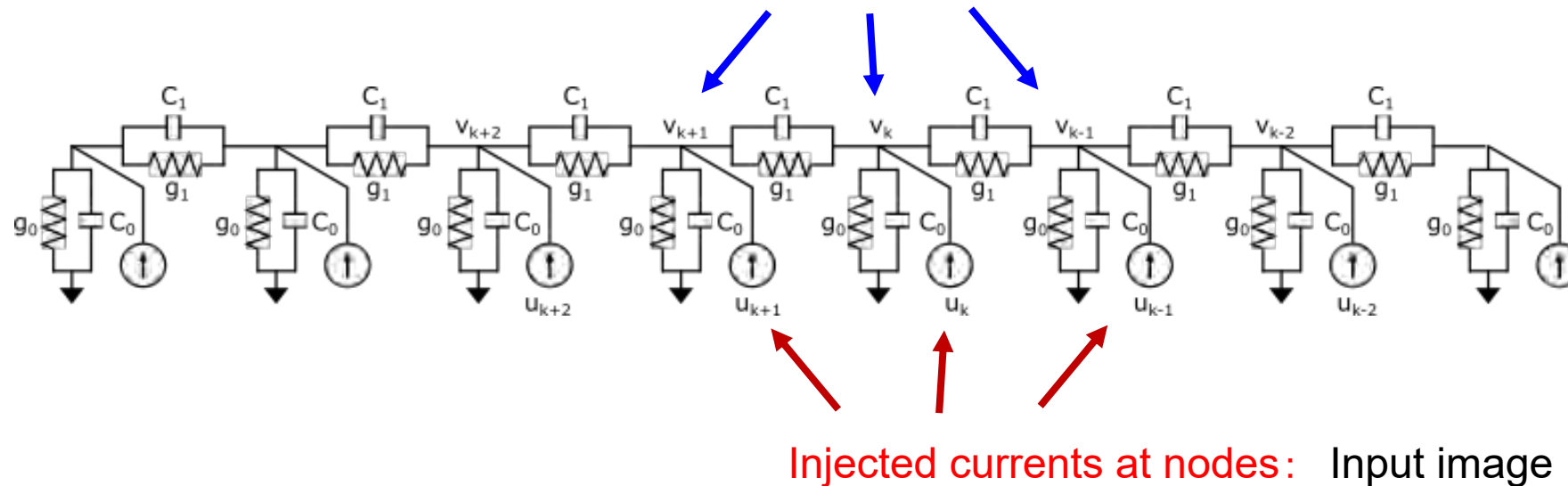
## High-speed analog image-smoothing processor

1D image case

$$g_0 = 1/R_0$$

$$g_1 = 1/R_1$$

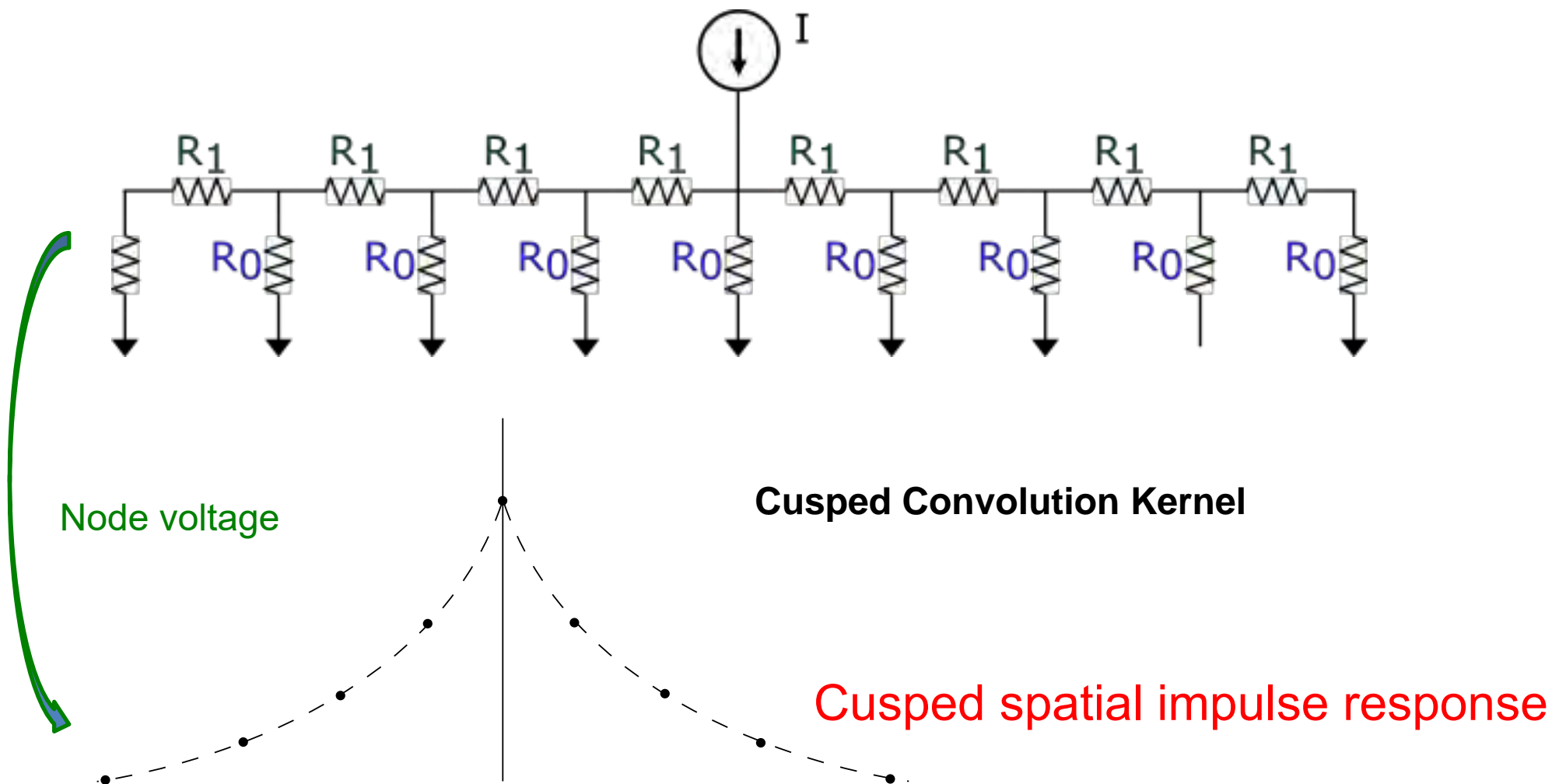
Node voltages: Output image



[1] C. A. Mead, Analog VLSI and Neural Systems, Addison Wesley, 1989



# Spatial Impulse Response of Retina Chip



# Gaussian Chip with Active Resistor Network

## High-speed analog image processor

$$g_0 = 1/R_0$$

$$g_1 = 1/R_1$$

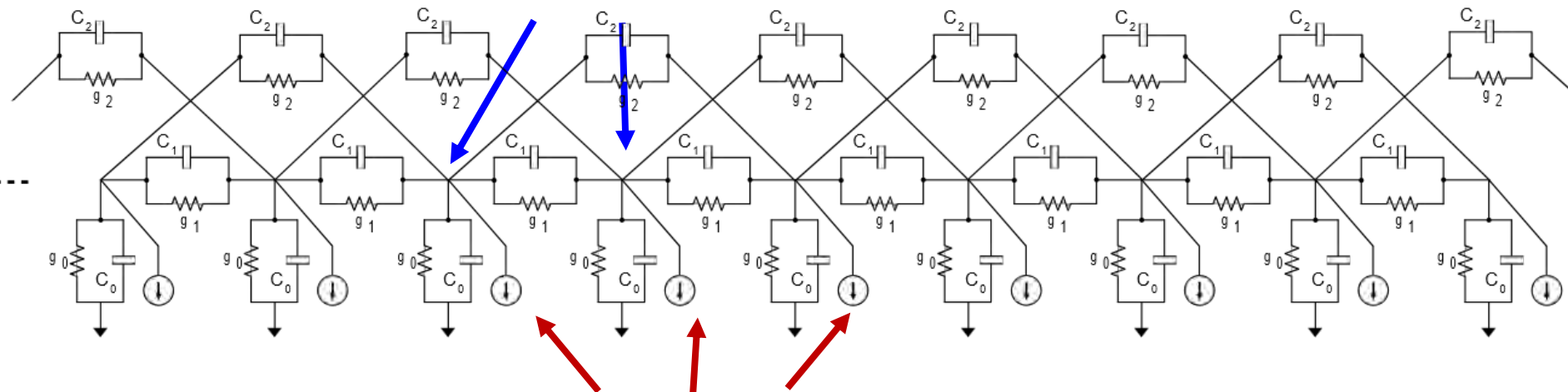
$$g_2 = 1/R_2$$

1D image case

Negative resistor

$$R_2 = -4R_1 < 0$$

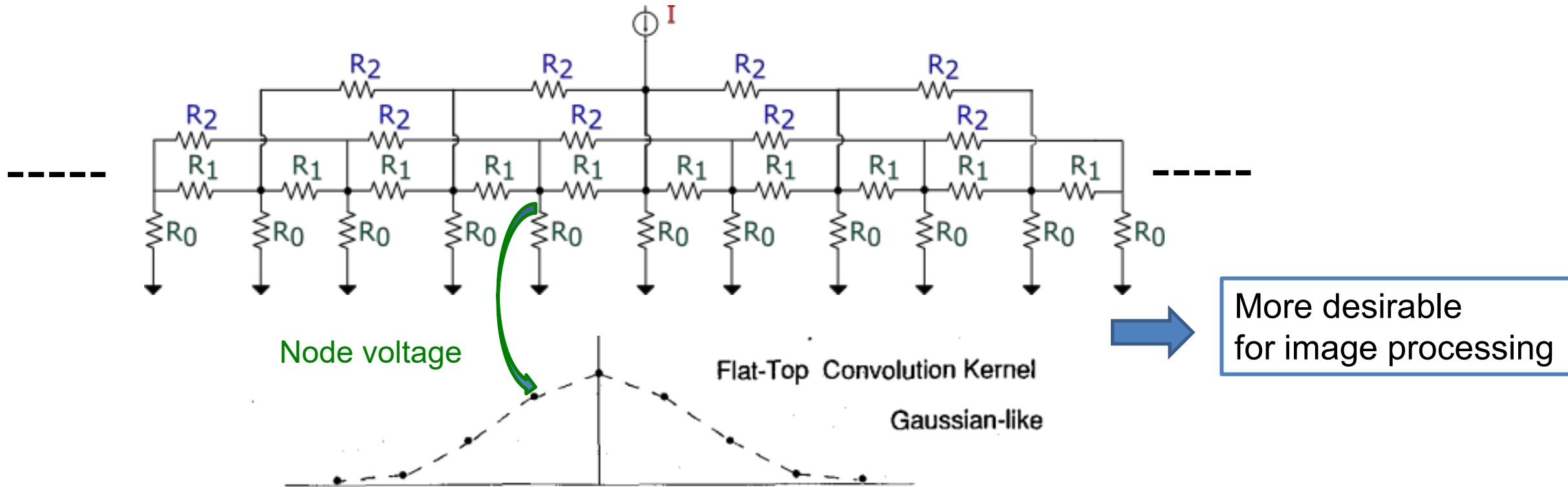
Node voltages: Output image



Injected currents at nodes: Input image

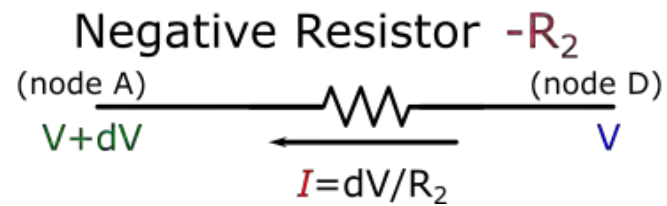
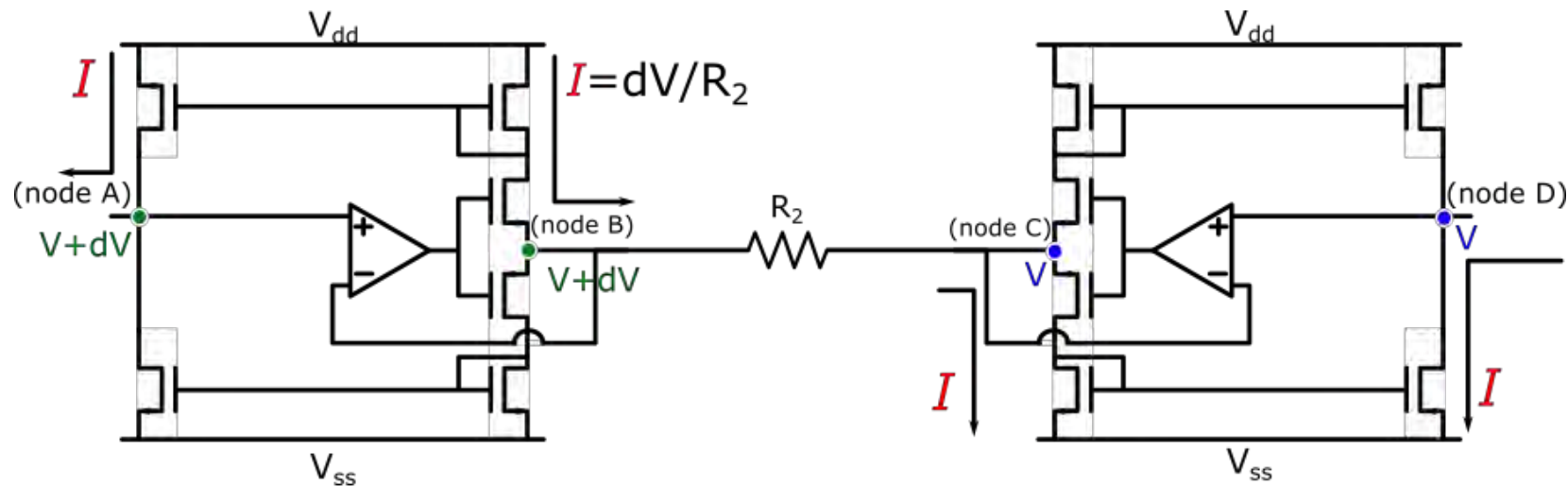
[2] H. Kobayashi, J. L. White, A. A. Abidi, "An Active Resistor Network for Gaussian Filtering of Images", IEEE Journal of Solid-State Circuits (May 1991)

# Spatial Impulse Response of Gaussian Chip



Flat-top spatial impulse response

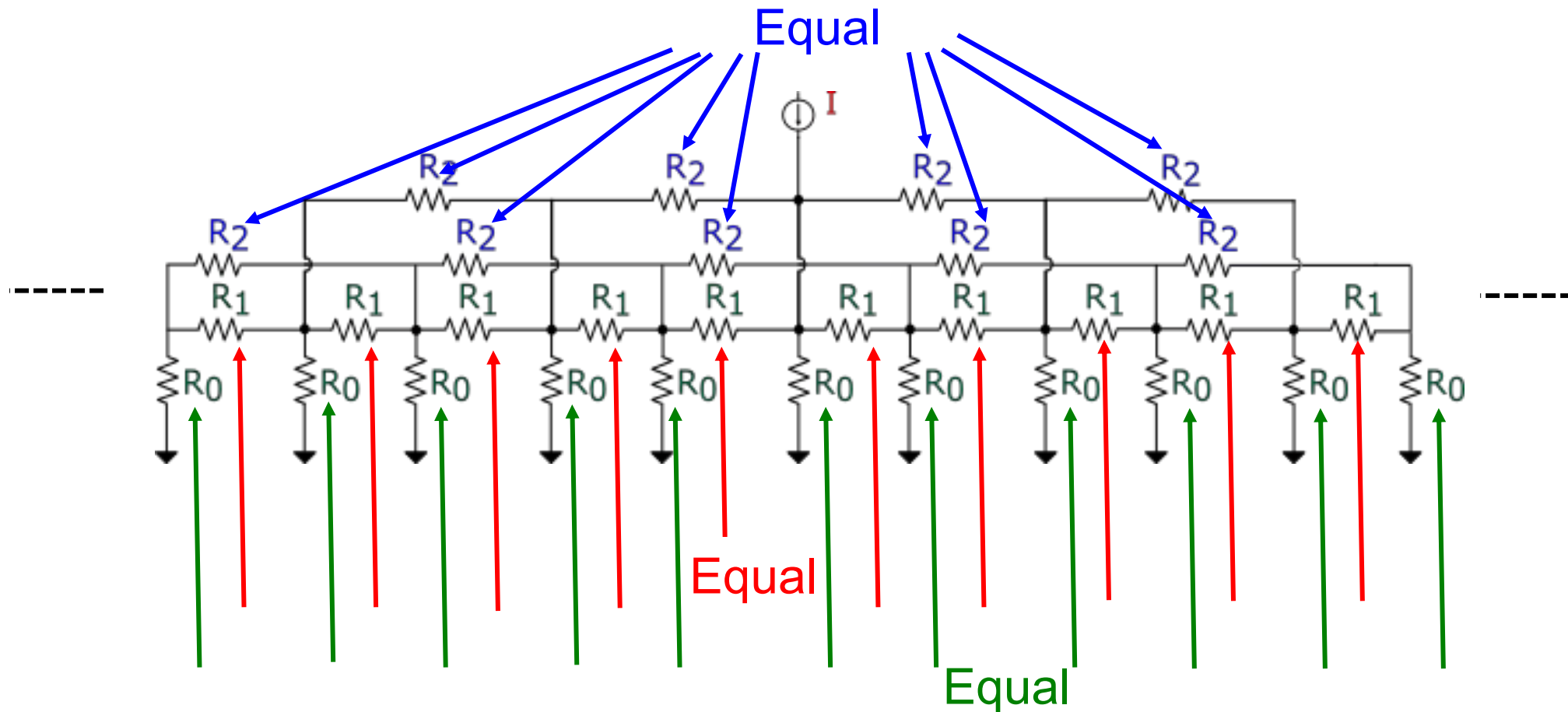
# Negative Resistor with Standard CMOS



$V+dV$  @ Node A  $>$   $V$  @ Node B

Current  $I = \frac{dV}{R_2}$

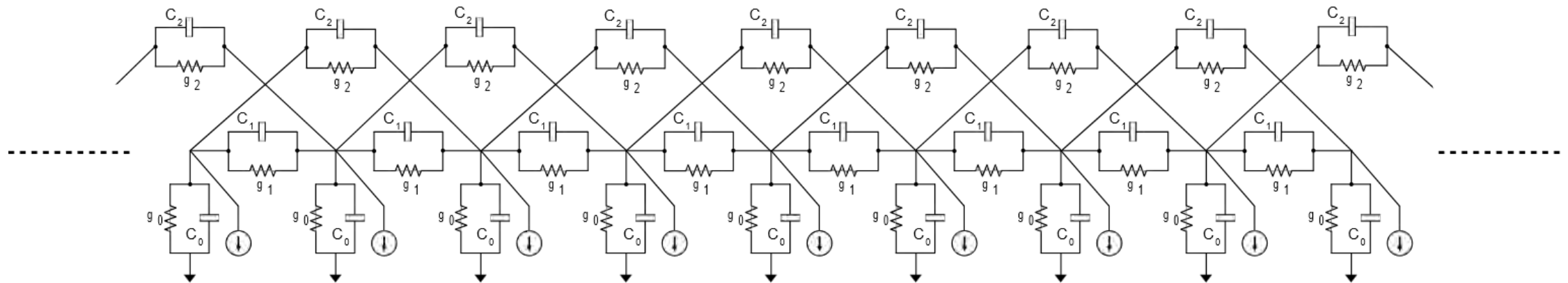
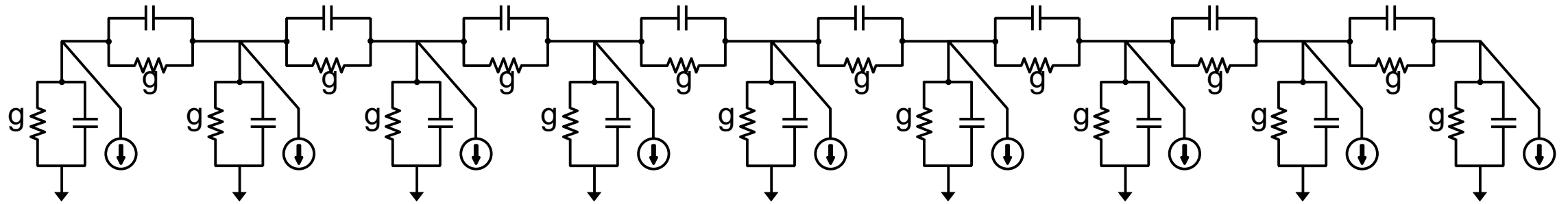
# Uniform Resistor Network for Spatial Dynamics



- Shift invariant
- Spatial transfer function can be defined

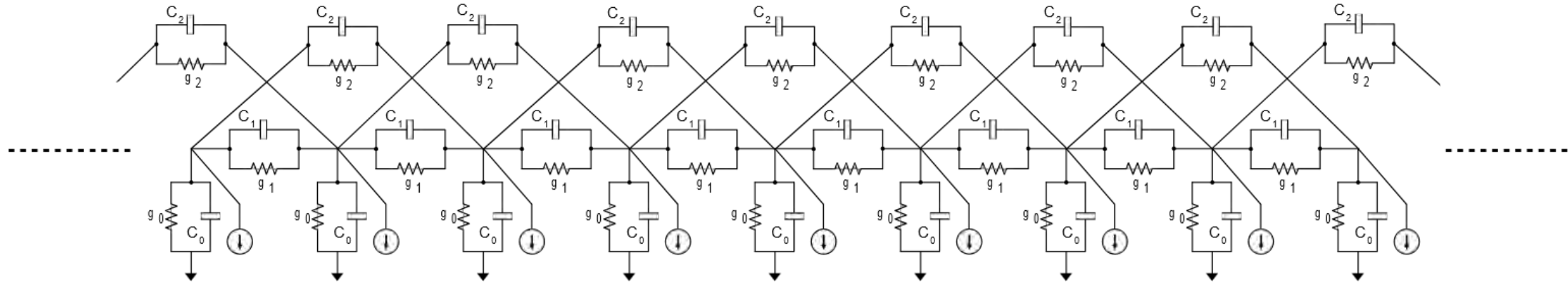
# Temporal Dynamics with R, C

Capacitances are considered for temporal dynamics



# Simulation Results: Spatial Temporal Stabilities

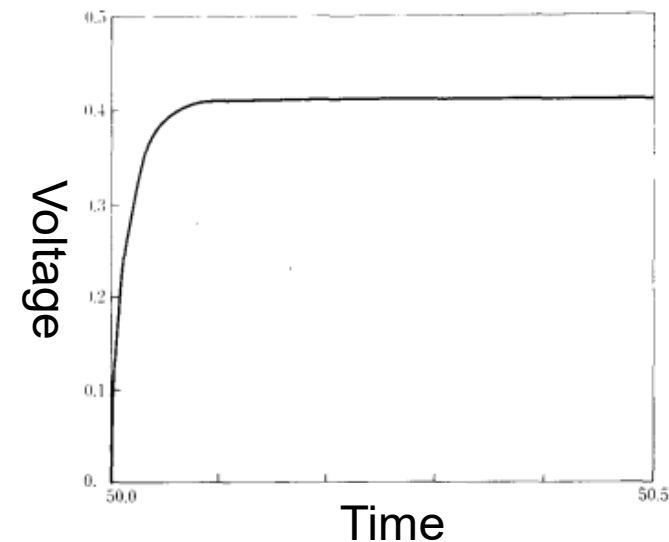
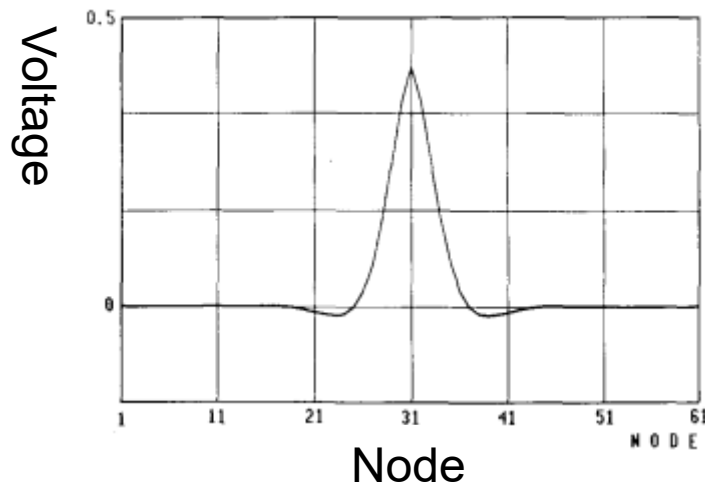
$$R_0 = 1/g_0 = 200\text{k}\Omega, \quad R_1 = 1/g_1 = 5\text{k}\Omega, \quad R_2 = 1/g_2 = -20\text{k}\Omega$$



Spatially stable

Temporally stable

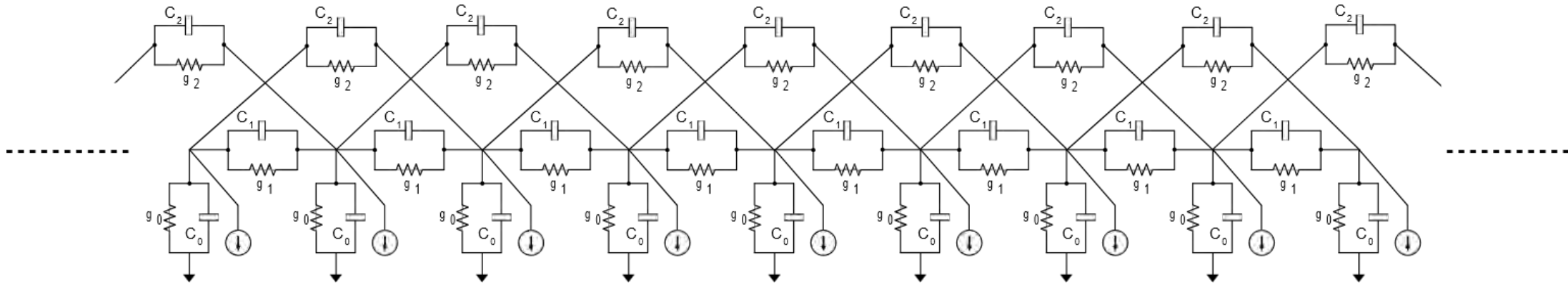
Spatial  
impulse  
response



Temporal  
step  
response

# Simulation Results: Spatial Temporal **Instabilities**

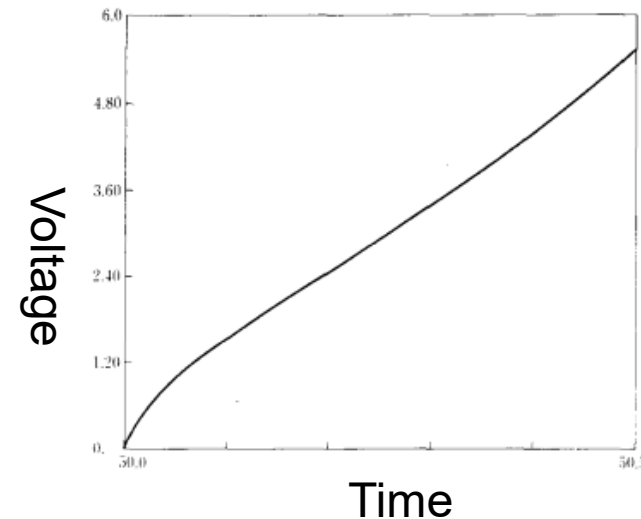
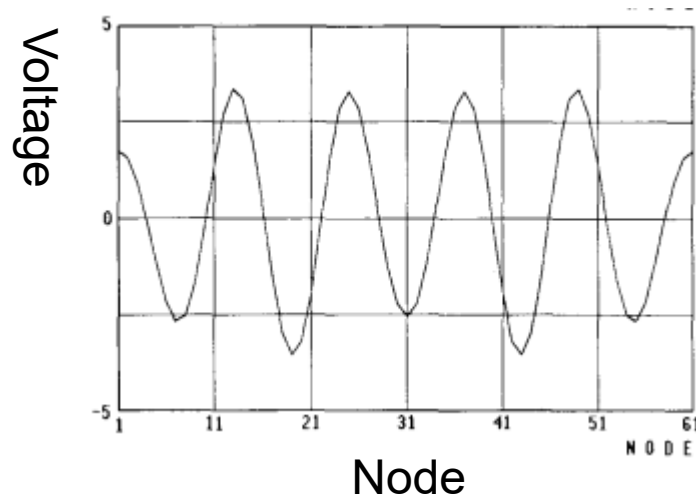
$$R_0 = 1/g_0 = 200\text{k}\Omega, \quad R_1 = 1/g_1 = 5\text{k}\Omega, \quad R_2 = 1/g_2 = -17\text{k}\Omega$$



**Spatially unstable**

**Temporally unstable**

**Spatial  
impulse  
response**



**Temporal  
step  
response**



# Circuit Network Theorem

For **uniform** network with **positive** and **negative** resistors, spatial and temporal stability conditions are equivalent.

[3] T. Matsumoto, H. Kobayashi, Y. Togawa,

“Spatial Versus Temporal Stability Issues in Image Processing Neuro chips ”,  
IEEE Trans. Neural Networks, (July 1992).

[4] H. Kobayashi, T. Matsumoto, J. Sanekata,

“Two-Dimensional Spatio-Temporal Dynamics of Analog Image Processing Neural Networks”,  
IEEE Trans. Neural Networks (Oct. 1995).

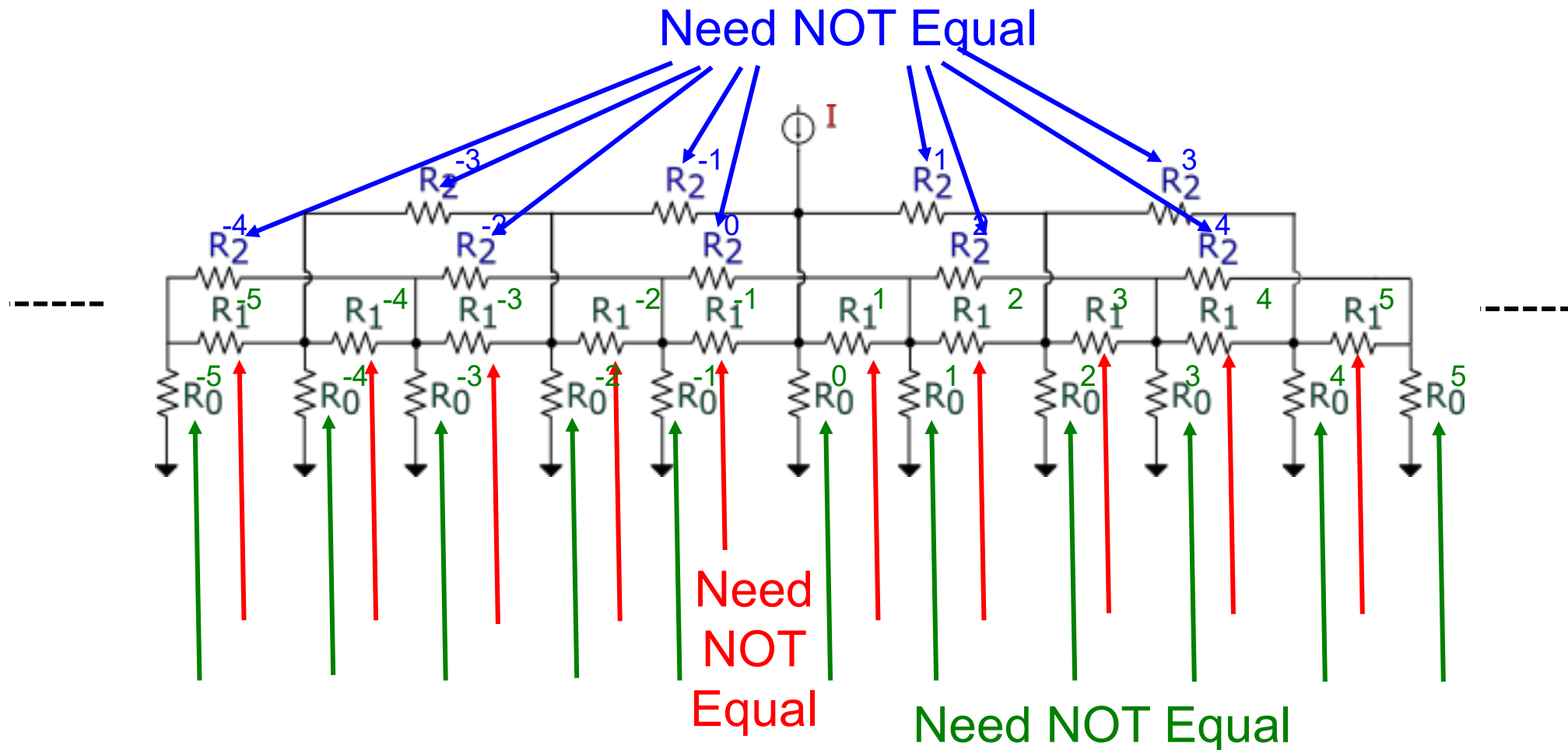
How about **non-uniform** network ?

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# Non-Uniform Resistor Network

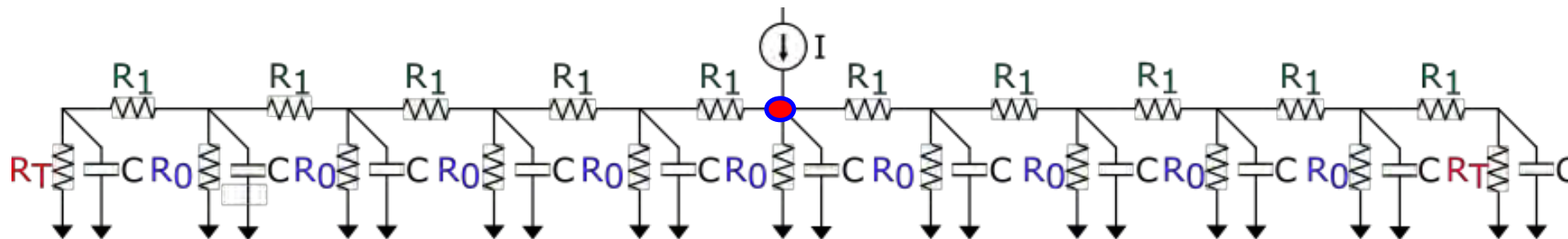


- Shift **variant**
- Spatial transfer function **CANNOT** be defined

# Finding 1

If there is a node where the input current is injected and its node voltage as the spatial impulse response is *negative*,

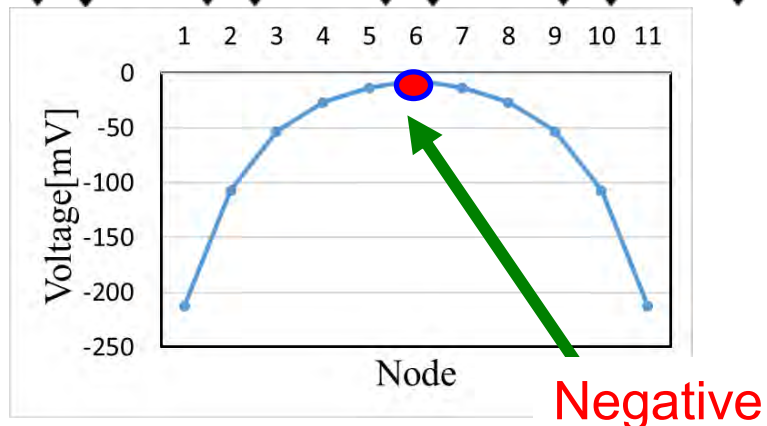
➔ the network is *temporally unstable*



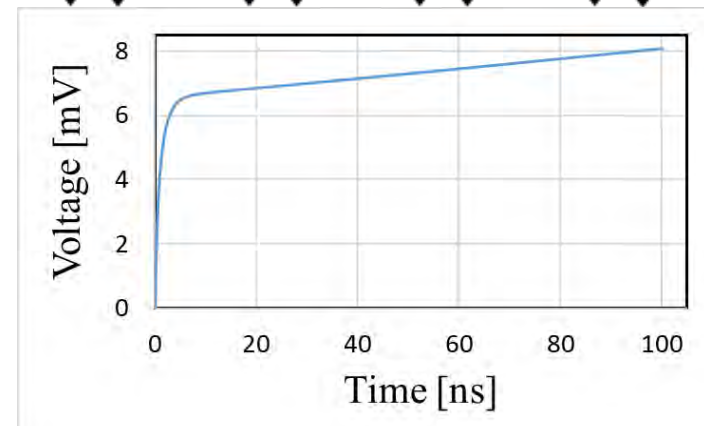
$$R_0 = 2\text{k}\Omega$$

$$R_1 = 1\text{k}\Omega$$

$$R_T = -2\text{k}\Omega$$

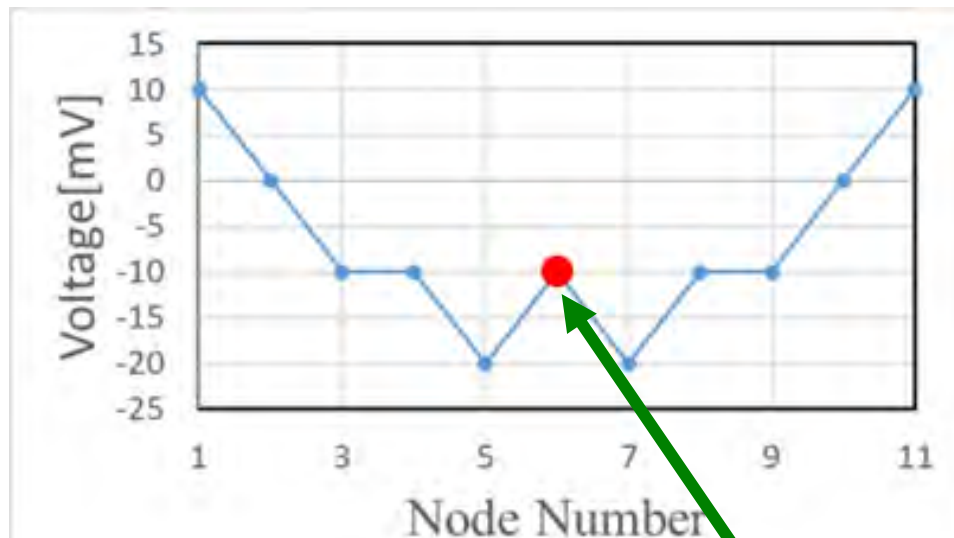
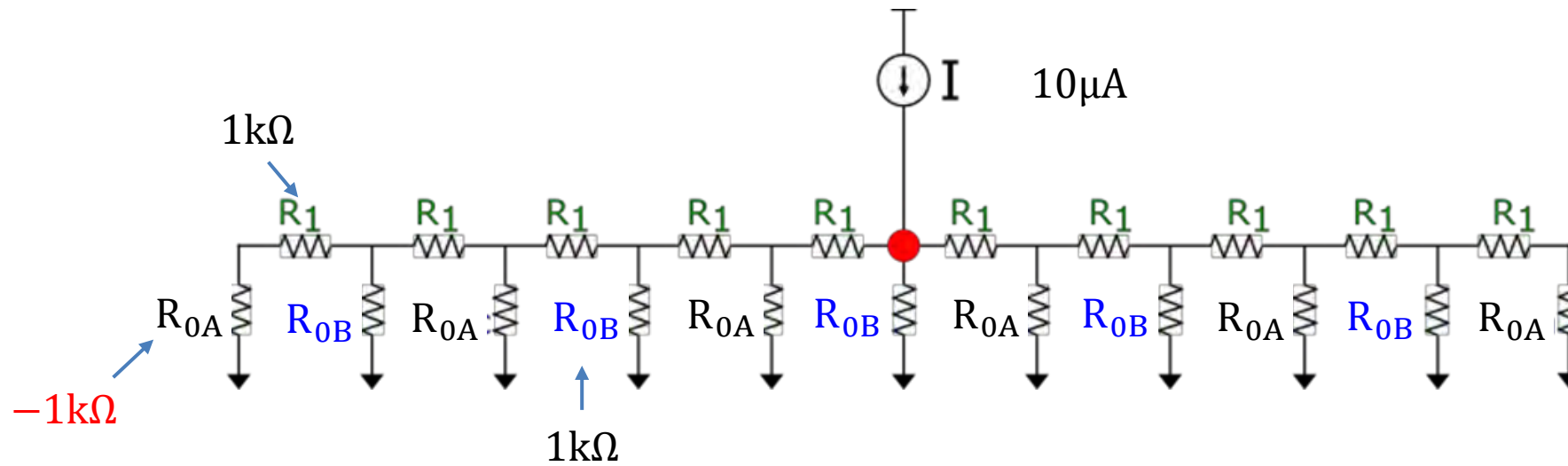


Spatial Impulse Response

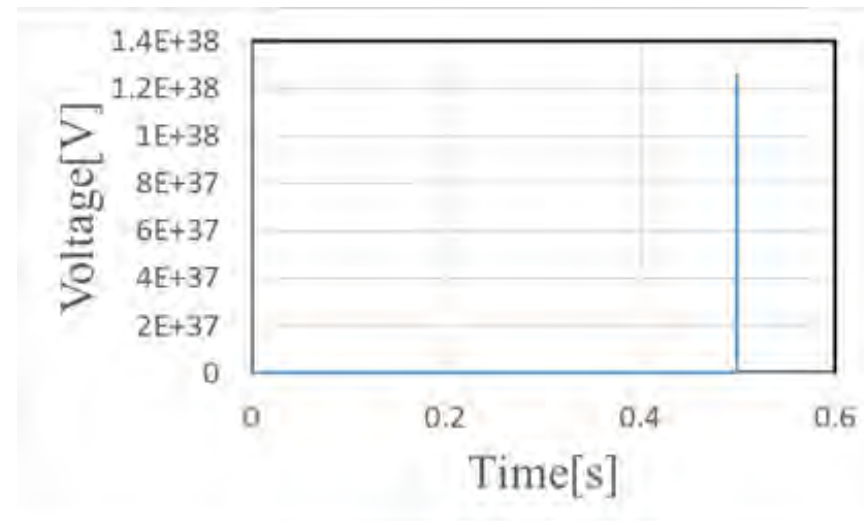


Temporally unstable

# 1<sup>st</sup> Nearest Connection (2)

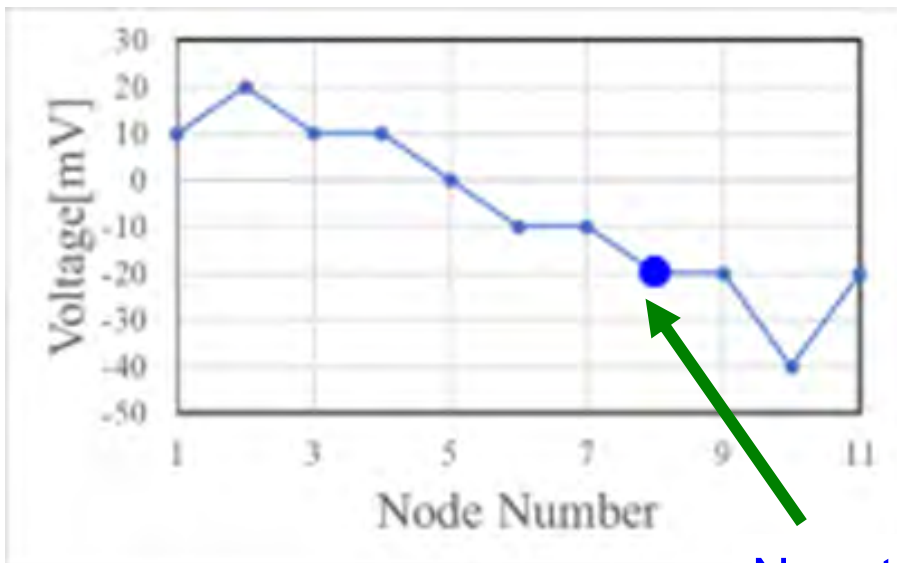
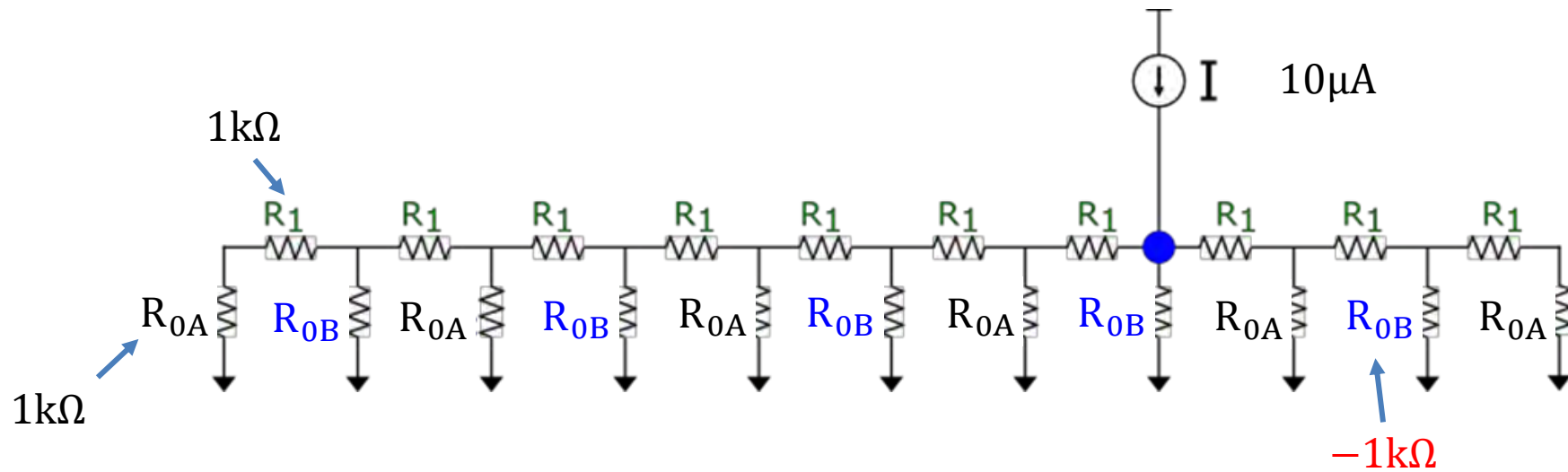


Negative

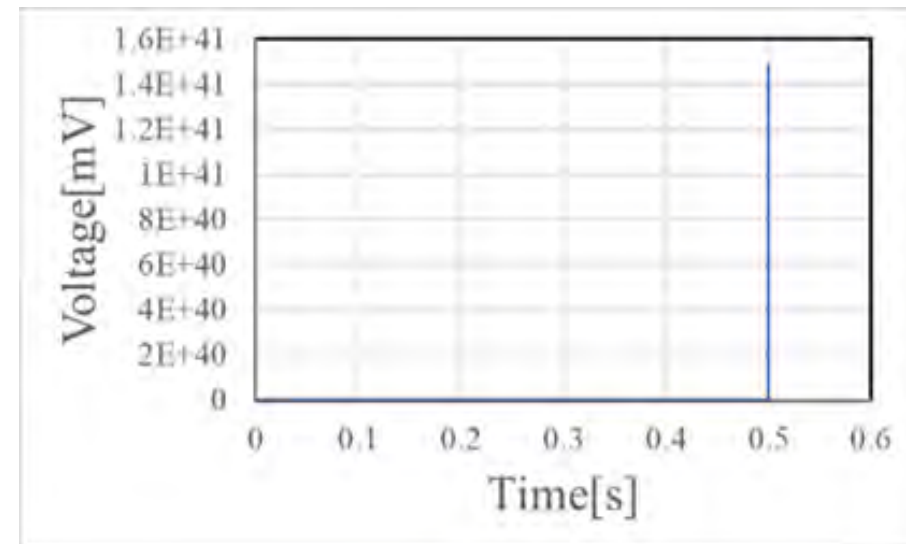


Temporally Unstable

# 1<sup>st</sup> Nearest Connection (3)

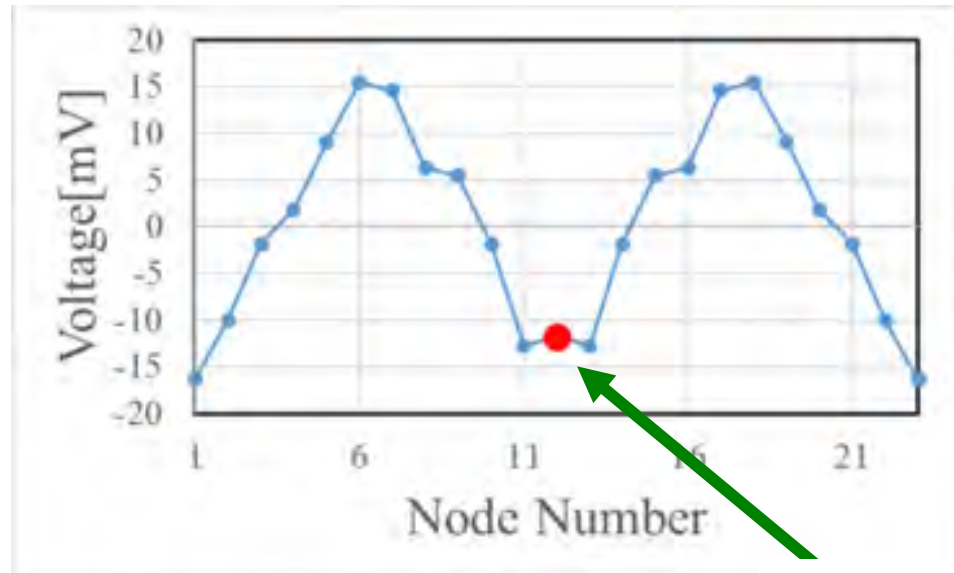
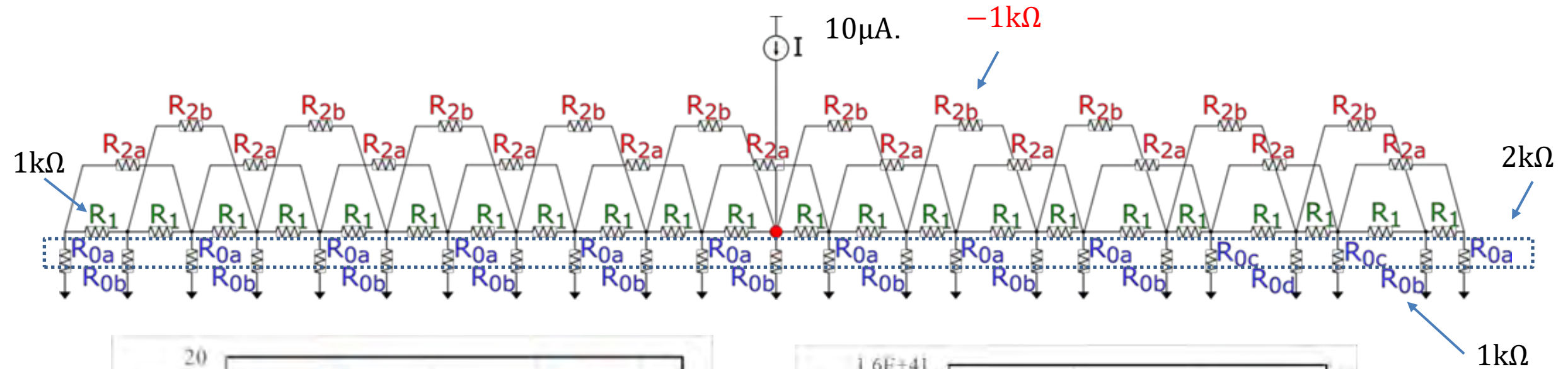


Negative

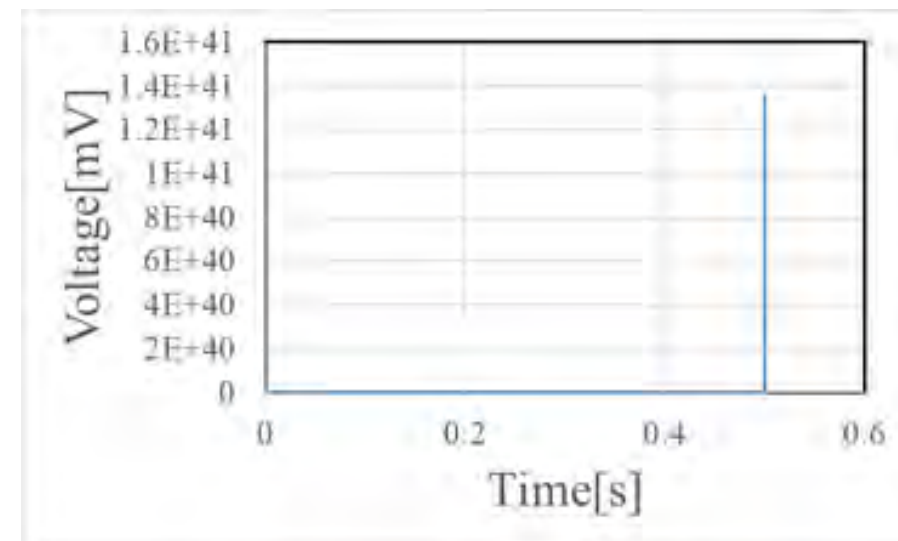


Temporally Unstable

# 2<sup>nd</sup> Nearest Connection (1)

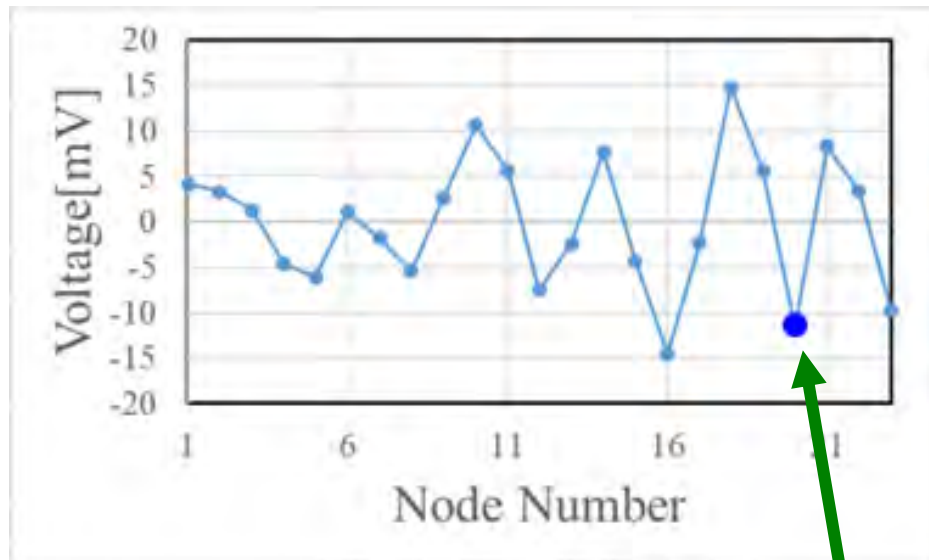
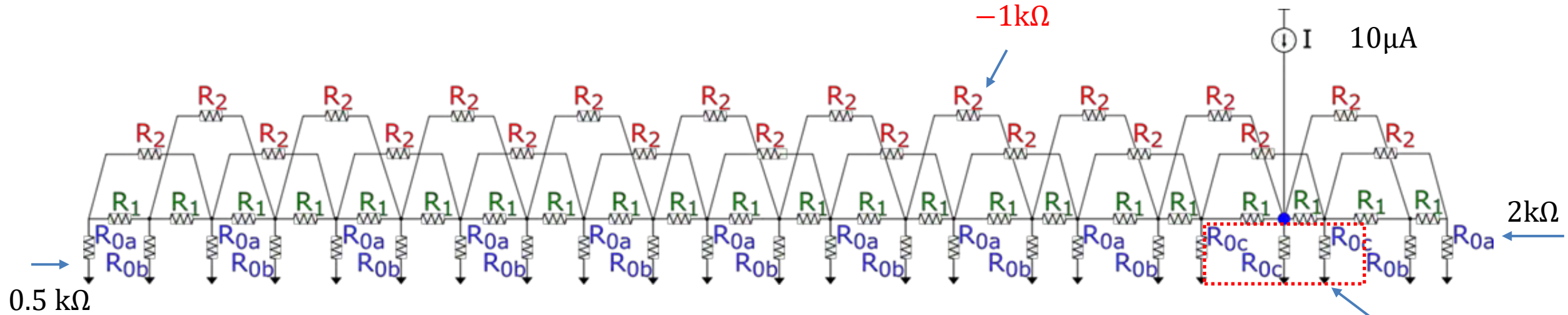


Negative

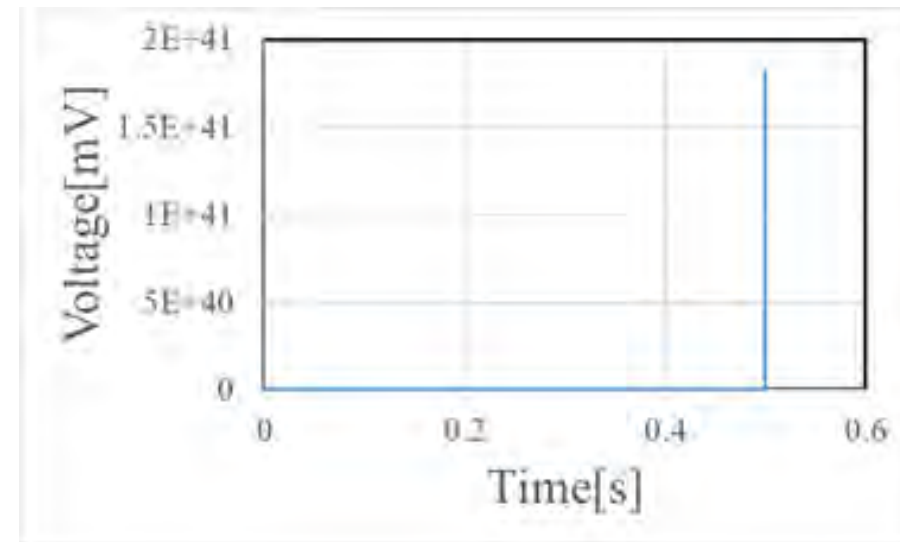


Temporally Unstable

# 2<sup>nd</sup> Nearest Connection (2)



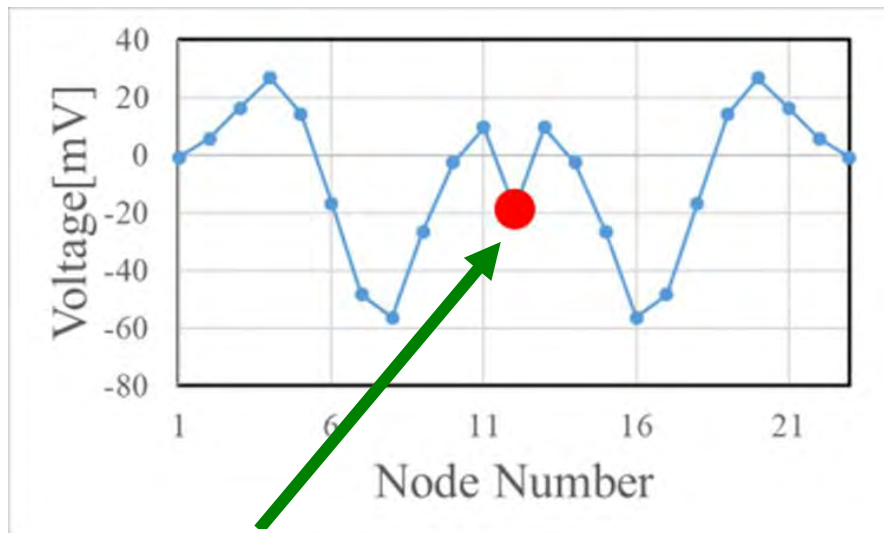
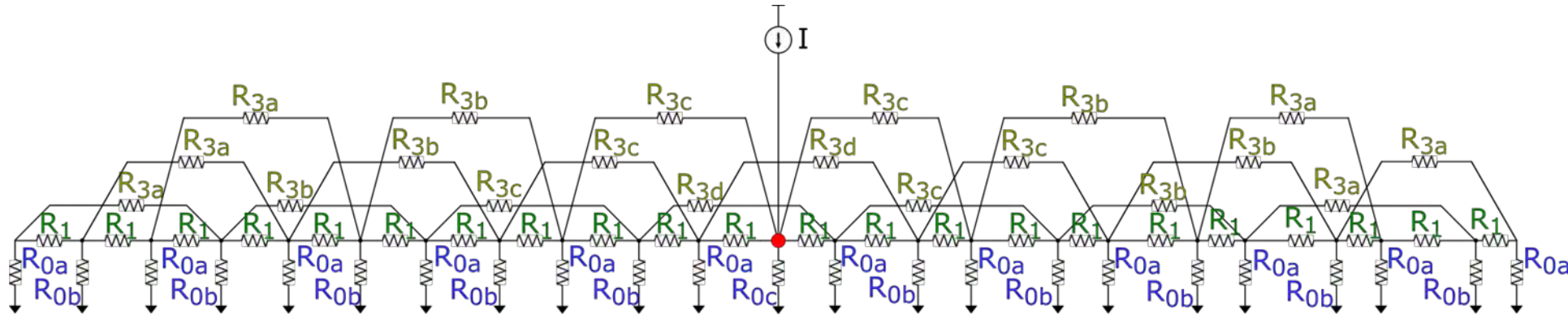
Negative



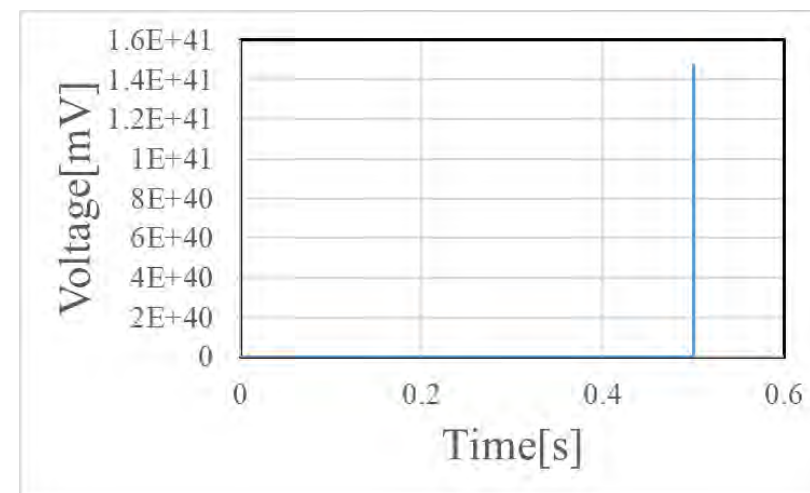
Temporally Unstable



# 3<sup>rd</sup> Nearest Connection



Negative



Temporally Unstable

$$R_{0a} = 2\text{k}\Omega, R_{0b} = 3\text{k}\Omega, R_{0c} = -0.25\text{k}\Omega, R_1 = 1\text{k}\Omega, R_{3a} = -4\text{k}\Omega$$

$$R_{3b} = -3\text{k}\Omega, R_{3c} = -2\text{k}\Omega, R_{3d} = -1\text{k}\Omega, I = 10\mu\text{A}$$

## Finding 2

*For all nodes,*

*“The input current is injected at each one node and its node voltage as the spatial impulse response is **positive**”,*

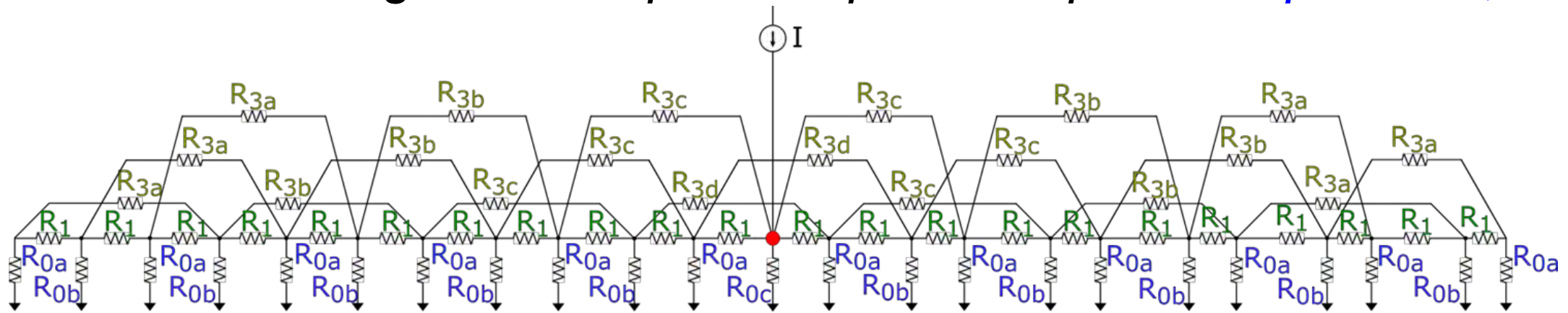
 *Some networks can be **temporally unstable**.*

We have found such an example by simulation

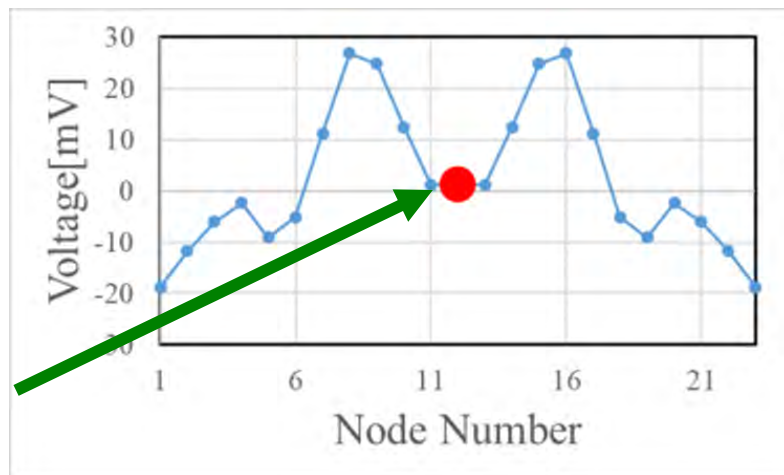
Theoretical analysis is left.

# 3<sup>rd</sup> Nearest Connection

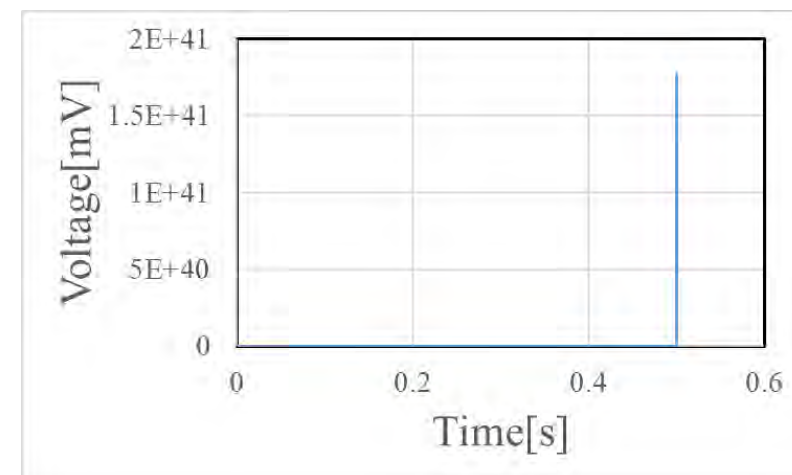
For all nodes, “the input current is injected at a node and its node voltage as the spatial impulse response is *positive*”,



$$R_{0a} = 2\text{k}\Omega, R_{0b} = 3\text{k}\Omega, R_{0c} = -0.25\text{k}, R_1 = 1\text{k}\Omega, R_{3a} = -1\text{k}, R_{3b} = -2\text{k}, R_{3c} = -3\text{k}, R_{3d} = -4\text{k}\Omega, I = 10\mu\text{A}$$



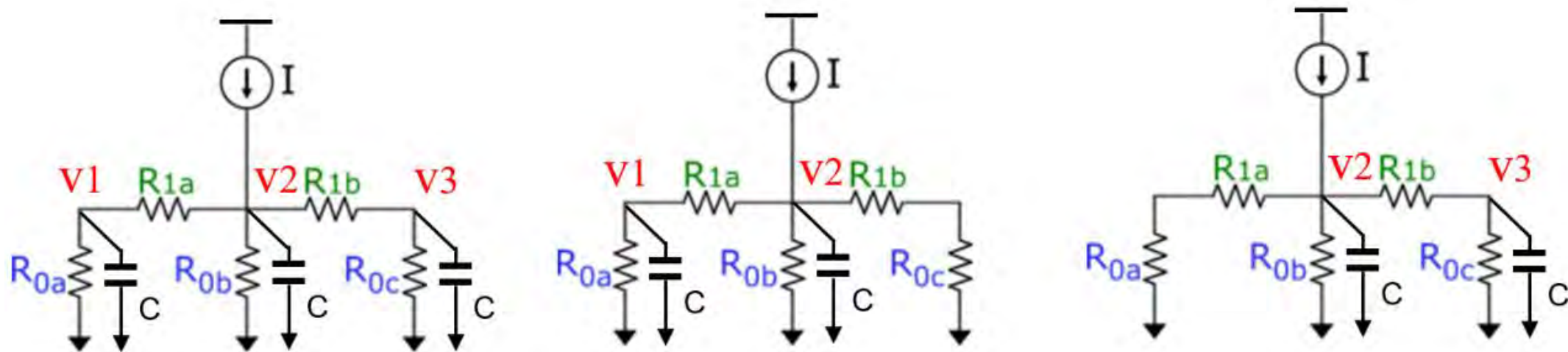
Positive



Temporally  
Unstable

# Finding 3

*Temporal stability and instability can depend on capacitors (C) from nodes to ground.*



$$R_{0a} = -1 \text{ k}\Omega, \quad R_{0b} = 1 \text{ k}\Omega, \quad R_{0c} = 0.5 \text{ k}\Omega, \quad R_{1a} = -1 \text{ k}\Omega, \quad R_{1b} = 0.5 \text{ k}\Omega, \quad I = 10 \text{ }\mu\text{A},$$

Temporally Unstable

Unstable

Stable



Proved with theory

# Our Three New Findings

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For general non-uniform active resistor network dynamics, three new properties have been found with simulation and theoretical analysis.



Their rigorous proof has NOT been completed yet.

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- Conclusion

**ReDAC: Relaxation Digital-to-Analog Converter**

# Outline

## From Politecnico di Torino, Italy

[5] P. S. Croveti, et. al.,

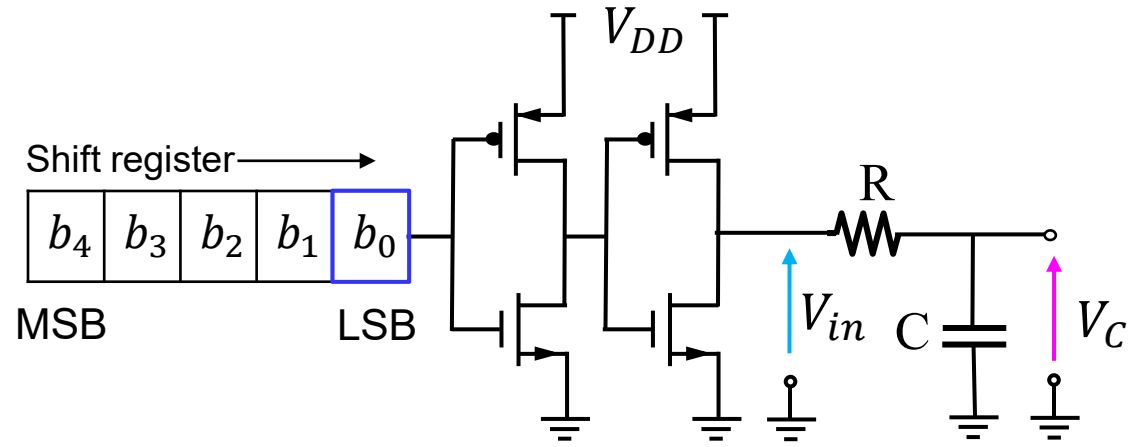
“Relaxation Digital-to-Analogue Converter,” Electronics Letters, 2009.

[6] R. Rubino, et al.

“Design of Relaxation Digital-to-Analogue Converters for IoT Applications in 4 nm CMOS,” APCCAS 2019.

- ReDAC with RC Filter
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# Configuration and Operation of Conventional (1)



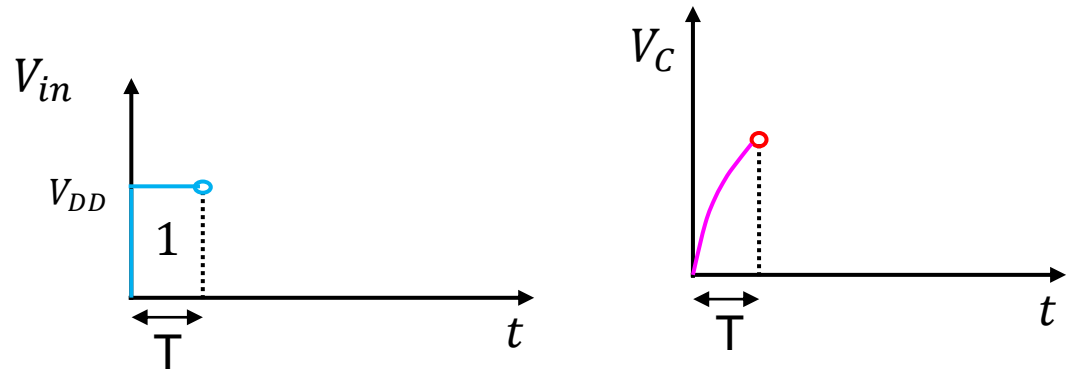
■ Digital input bit stream  $b_{n-1} \dots b_0$   
 ➔ RC LPF network  
 LSB-first

Digital Input code

$$N = \sum_{i=0}^{n-1} b_i 2^i$$

Ex:  $N=5$  ( $b_4 b_3 b_2 b_1 b_0 = 00101$ )

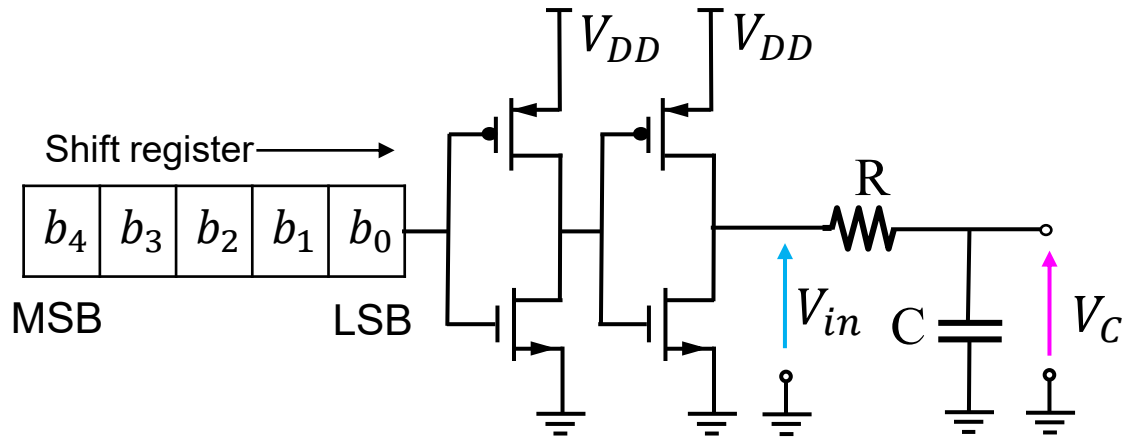
$$N = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 + 0 \cdot 2^4 = 5$$



Condition  $T = RC \cdot \log 2$

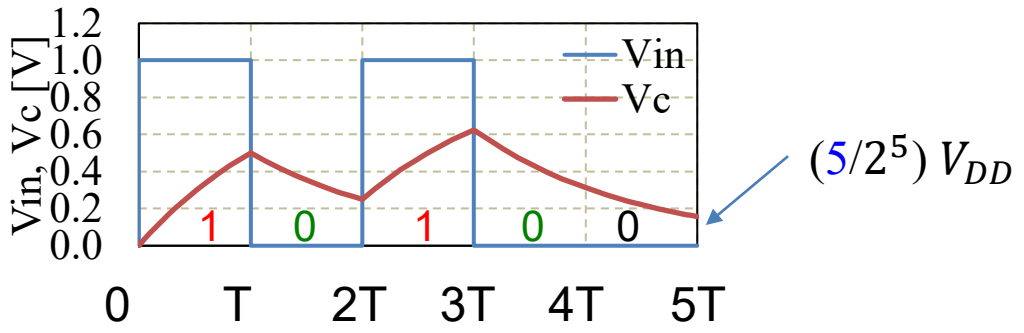


# Configuration and Operation of Conventional (2)



RC LPF  
 $\tau = RC$

Ex: N=5 ( $b_4b_3b_2b_1b_0=00101$ )



Final capacitor voltage  
 → Conversion result

$$V_C(nT) = V_{DD} (1 - e^{-\frac{T}{\tau}}) \cdot \sum_{i=0}^{n-1} b_i e^{-\frac{(n-i-1)T}{\tau}}$$

Condition

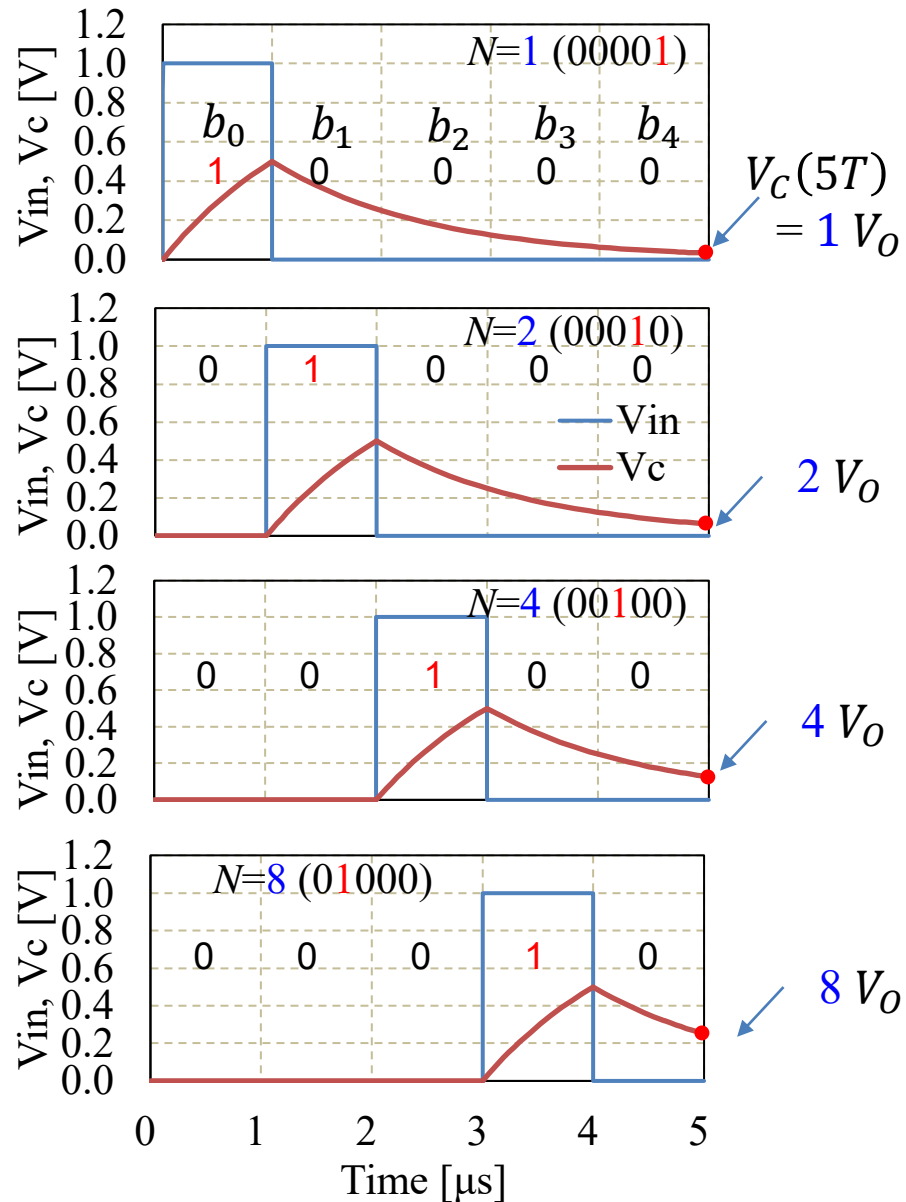
$$e^{-\frac{T}{\tau}} = \frac{1}{2} \Rightarrow T = \tau \cdot \log 2$$



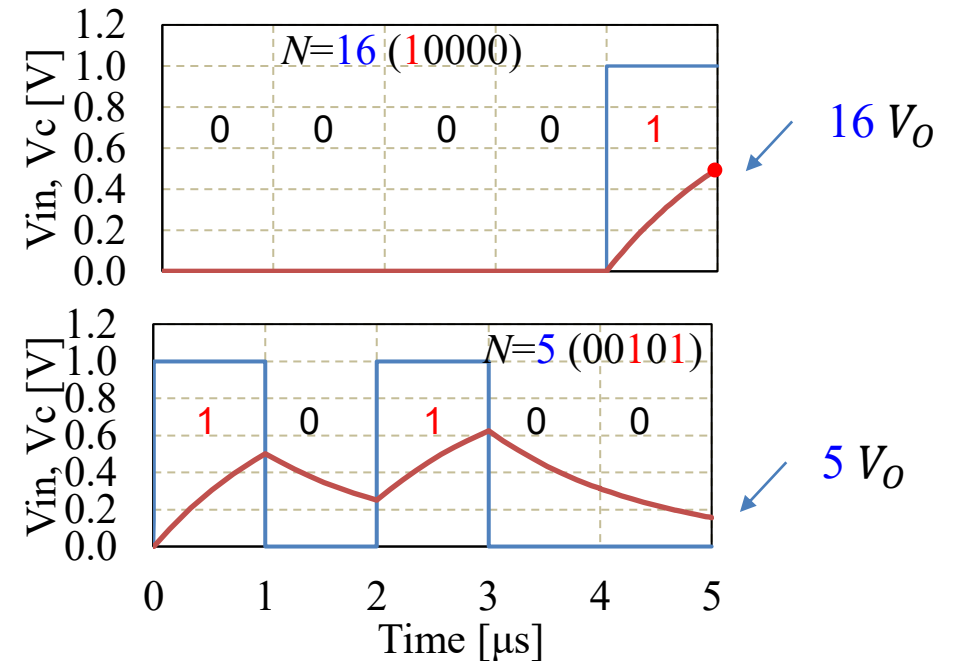
$$V_C(nT) = \frac{V_{DD}}{2^n} \sum_{i=0}^{n-1} b_i 2^i = \frac{N}{2^n} \cdot V_{DD}$$

Output voltage proportional to N

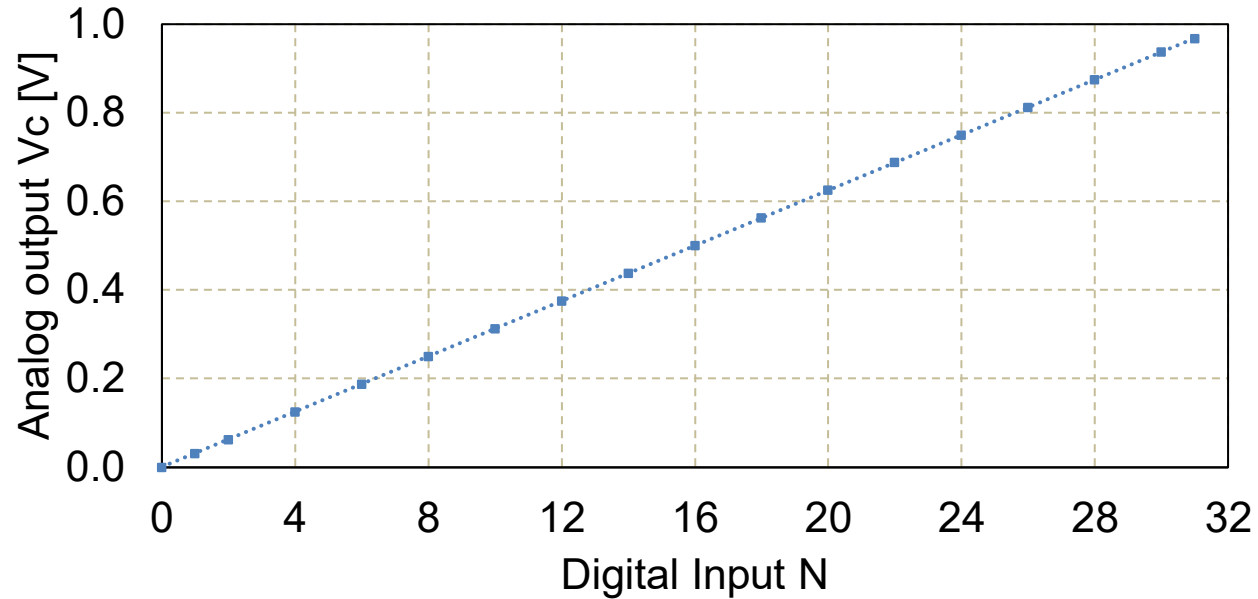
# Simulation Verification of Conventional (1)



parameter	value
$R$	1 k $\Omega$
$C$	1.443 nF
$T$	1 $\mu s$
$V_{DD}$	1 V



# Simulation Verification of Conventional (2)



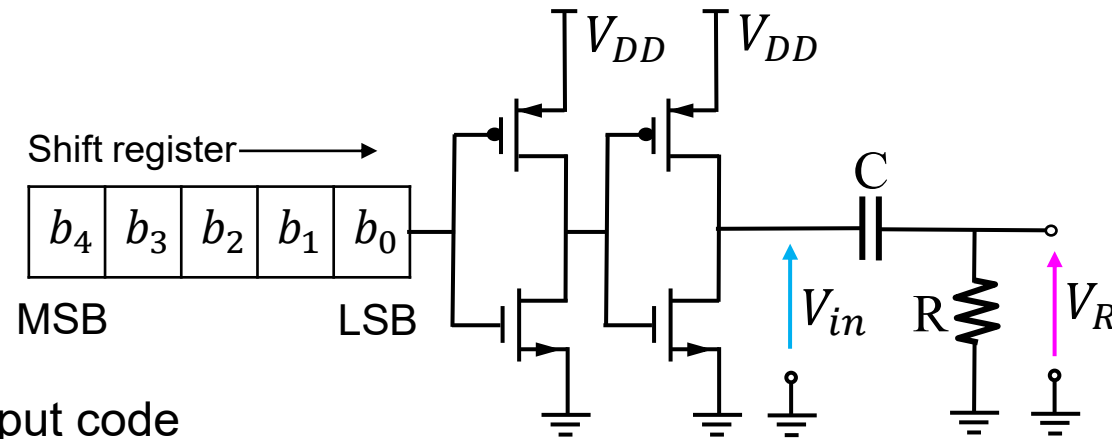
- Analog output is proportional to digital input data  $N$ .
  - Analog output generates with **only positive polarity**.
- ➔ **Negative digital input data is not available.**

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# Configuration and Operation of Proposal (1)



■ Digital input bit stream  $b_{n-1} \dots b_0$

➔ RC HPF network  
LSB-first

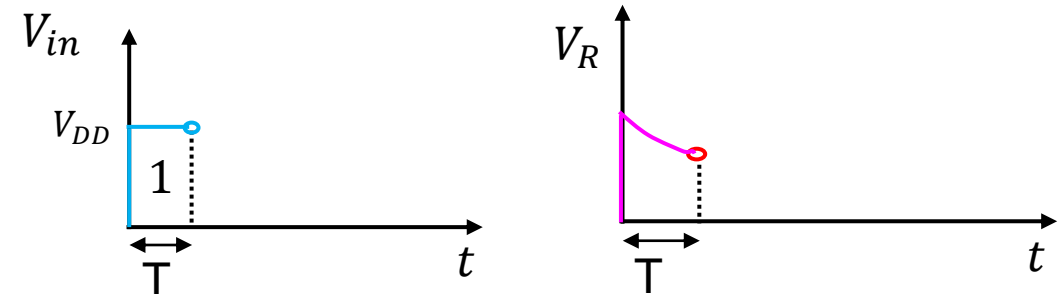
Digital input code

$$N = -b_{n-1}2^{n-1} + \sum_{i=0}^{n-2} b_i 2^i$$

Two's complement format

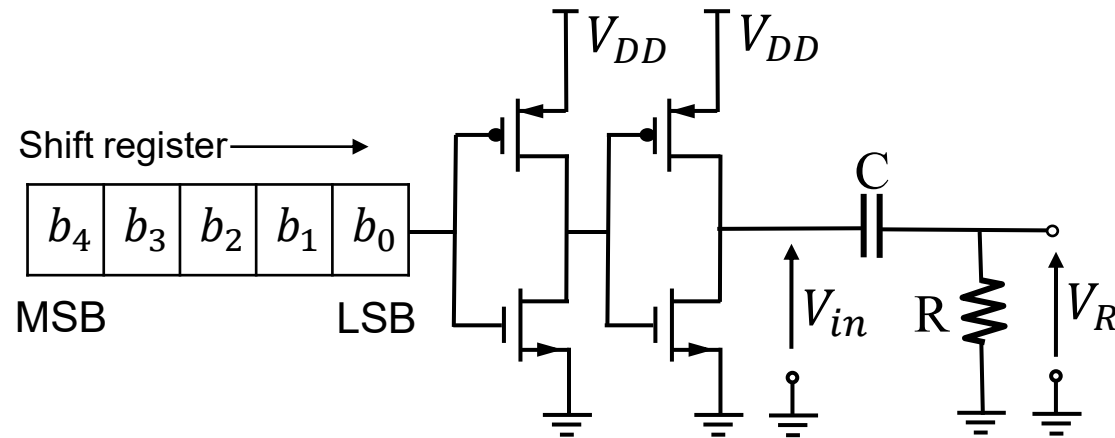
Ex:  $N = -11$  ( $b_4 b_3 b_2 b_1 b_0 = 10101$ )

$$N = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 - 1 \cdot 2^4 = -$$



Condition  $T = RC \cdot \log 2$

# Configuration and Operation of Proposal (2)



RC HPF  
 $\tau = RC$

Final resistor voltage  
→ Conversion result

$$V_R(nT) = -V_{DD} \left( -b_{n-1} e^{\frac{(n-1)T}{\tau}} + \sum_{i=0}^{n-2} b_i e^{-\frac{(n-i-1)T}{\tau}} \right)$$

Condition

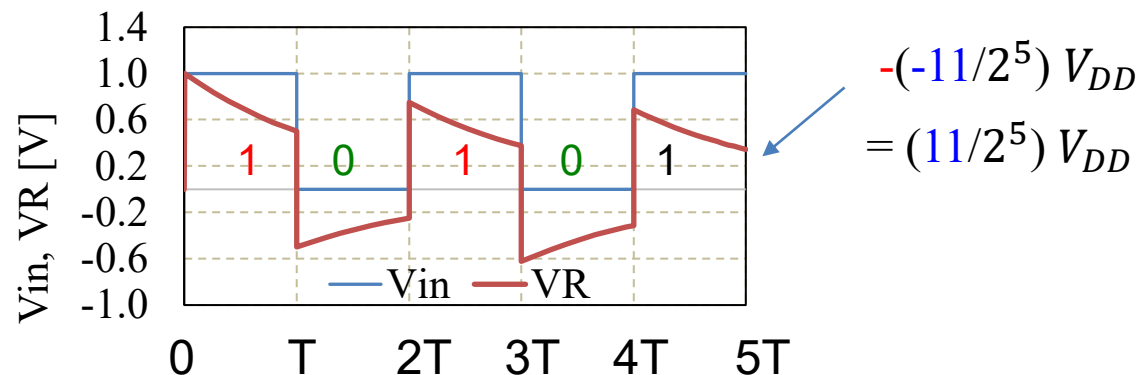
$$e^{-\frac{T}{\tau}} = \frac{1}{2} \Rightarrow T = \tau \cdot \log 2$$



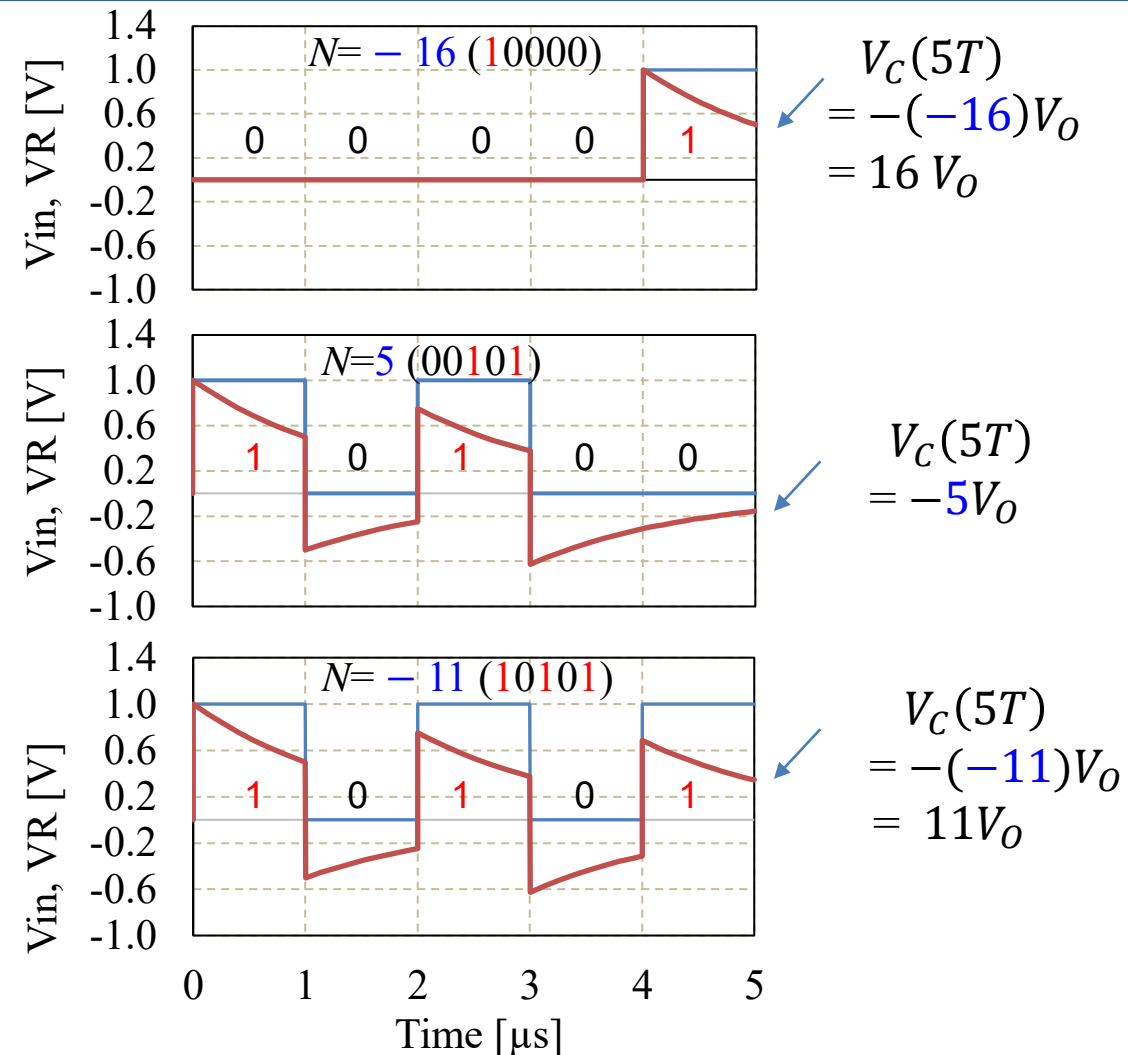
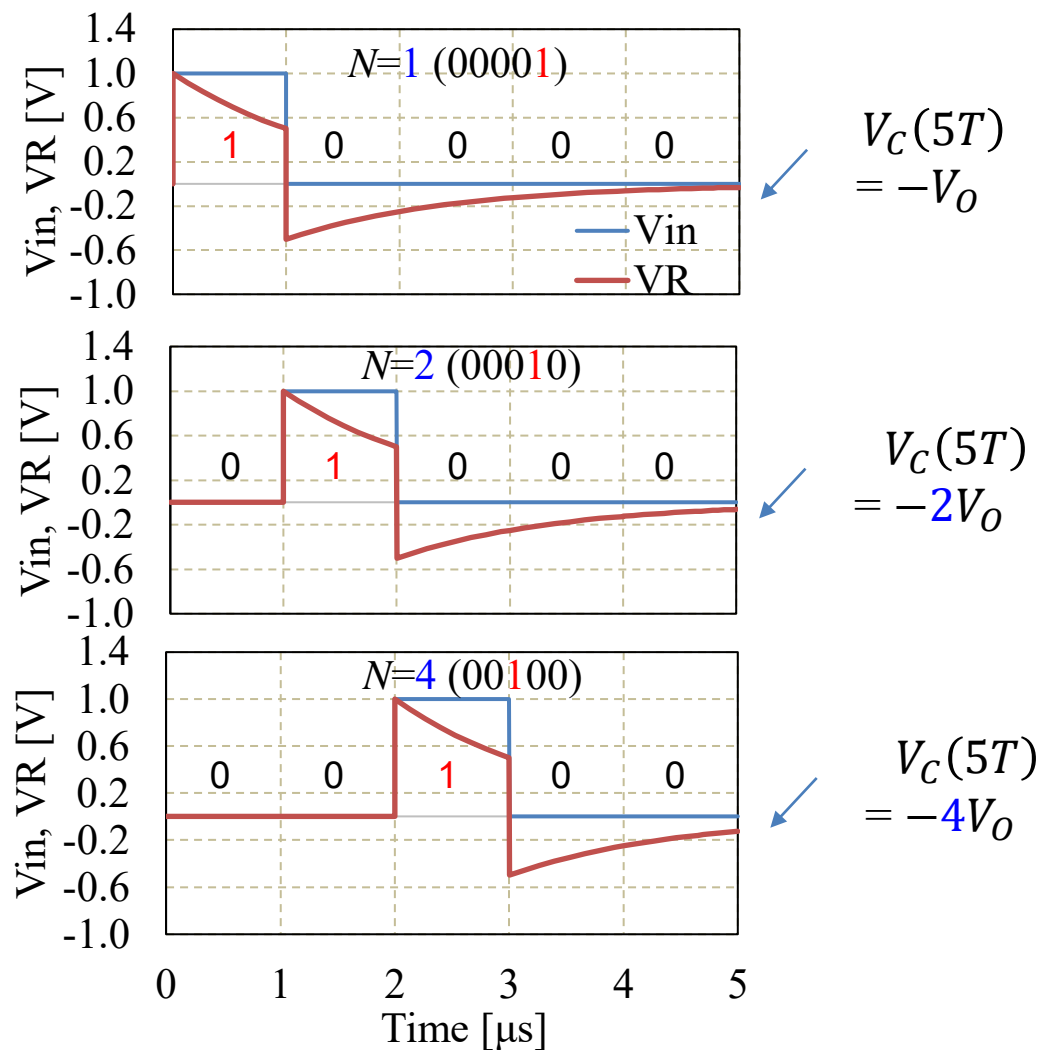
$$\begin{aligned} V_R(nT) &= -\frac{V_{DD}}{2^n} \left( -b_{n-1} 2^{n-1} + \sum_{i=0}^{n-2} b_i 2^i \right) \\ &= -\frac{N}{2^n} \cdot V_{DD} \end{aligned}$$

Output Voltage proportional to  $N$

Ex:  $N = -11$  ( $b_4 b_3 b_2 b_1 b_0 = 10101$ )



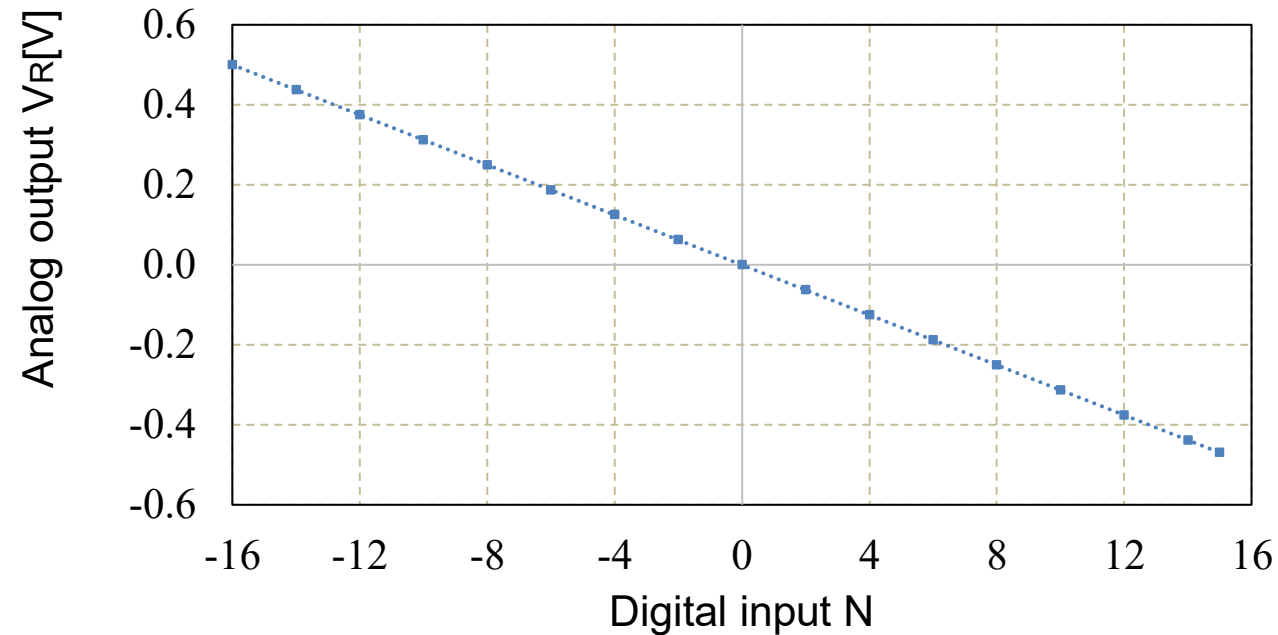
# Simulation Verification of Proposal (1)



■  $V_R(5T)|_{N=5} = V_R(5T)|_{N=1} + V_R(5T)|_{N=4}$

■  $V_R(5T)|_{N=-11} = V_R(5T)|_{N=1} + V_R(5T)|_{N=4} + V_R(5T)|_{N=-16}$

# Simulation Verification of Proposal (2)



- Analog output is proportional to the digital input data  $N$ .
- **Positive and negative polarity** output can be generated.
- Digital input data is in **two's complement format**.





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# Conclusion

We have shown the following:

- Spatial and temporal dynamics of active resistor network have close relationships  Theoretical analysis is left.
  - Relaxation DAC with RC HPF produces positive and negative polarity output for digital input data in two's complement format.
-  RC linear networks still have challenges in circuit theory and application

# References

## Active Resistor Network Dynamics

- [1] M. Chiba, H. Kobayashi, et. al., "Spatial and Temporal Dynamics of Non-Uniform Active Resistor Networks", IEEE 16th ICSICT (Oct. 2022).
- [2] K. Otomo, H. Kobayashi, et. al., "Conjecture on Spatial-Temporal Response Relationship for Spatially Shift-Variant Networks with Positive and Negative Resistors", 6<sup>th</sup> ICTSS (Dec. 2022).

## ReDAC with HPF

- [3] L. Nengvang, H. Kobayashi, et. al., "Relaxation DAC with Positive and Negative Polarity Output using High-Pass Filter", IEICE Electronics Express (Feb. 2023).