Analysis of Coupled Inductors for Low-Ripple Fast-Response Buck Converter

Graduate School of Engineering, Gunma University
Santhos A. Wibowo, Zhang Ting, Masashi Kono, Tetsuya Taura, Yasunori Kobori, and Haruo Kobayashi

Research Purpose

Microprocessor Power Supply
⇒ Low-ripple Fast-response Power Converter.

Coupled inductors in multiphase DC-DC converter.

Clarification of its characteristics in theory, simulation and experiments.

Tradeoff between Efficiency and Transient-Response

- Small Inductor

×Large current ripple	◎Fast response

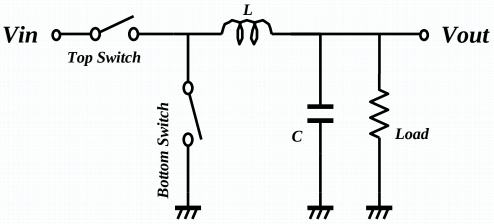
- Large Inductor

◎Small current ripple	×Slow response

- **Coupled Inductor**

◎Small current ripple	**◎Fast response**

DC-DC Buck Converter



◆ Top Switch ON, Bottom Switch OFF :

$$\Delta I_{L\_Top} = \frac{V_{in} - V_{out}}{L} T_{on}$$

◆ Bottom Switch ON, Top Switch OFF :

$$\Delta I_{L\_Bottom} = \frac{-V_{out}}{L} T_{off}$$

◆ $\Delta I_{L\_Top} = \Delta I_{L\_Bottom}$

$$V_{out} = \frac{T_{on}}{T_{on} + T_{off}} V_{in} = \frac{T_{on}}{T} V_{in}$$

Coupled Inductor in Two-phase DC-DC Buck Converter

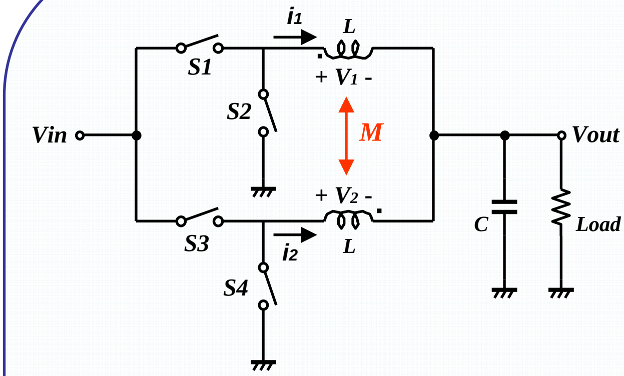


Fig. 1. Two-phase coupled-inductor buck converter

◆

$$V_1 = L\frac{di_1}{dt} + M\frac{di_2}{dt} .$$

$$V_2 = L\frac{di_2}{dt} + M\frac{di_1}{dt} .$$

◆ Mutual inductance:

$$M = k \times L . \quad (-1 \leq k \leq 0)$$

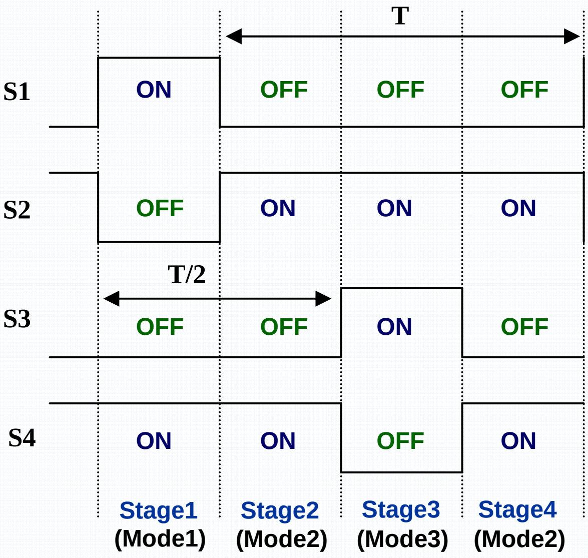


Fig. 2. Operating waveforms of interleaved two-phase coupled inductor buck converter (Duty-cycle $D \leq 0.5$)

Steady-state Analysis

Derivation of Steady-state Average Equivalent Inductance

Mode1 : $V_1 = \frac{L^2 - M^2}{L + M \cdot \frac{D}{1-D}} \frac{di_1}{dt}.$ ➡ $L_{eq1} = L \frac{1-k^2}{1 + k \cdot \frac{D}{1-D}}.$

Mode2: $V_1 = (L+M)\frac{di_1}{dt}.$ ➡ $L_{eq2} = L(1+k).$

Mode3: $V_1 = \frac{L^2 - M^2}{L + M \cdot \frac{1-D}{D}} \frac{di_1}{dt}.$ ➡ $L_{eq3} = L \frac{1-k^2}{1 + k \cdot \frac{1-D}{D}}.$

- **Average inductor current in one switching period:**

$$\Delta i_L = \frac{\overline{V_1} \cdot D}{L_{eq1}} + \frac{\overline{V_1} \cdot (1-2D)}{L_{eq2}} + \frac{\overline{V_1} \cdot D}{L_{eq3}}$$

$$\frac{\overline{V_1}}{\overline{L_{eq}}} = \frac{\overline{V_1} \cdot D}{L_{eq1}} + \frac{\overline{V_1} \cdot (1-2D)}{L_{eq2}} + \frac{\overline{V_1} \cdot D}{L_{eq3}}.$$

- **Duty-cycle *D ≤ 0.5***

$L_{eq1} \rangle L$ ➡ **Low ripple**

- **Duty-cycle *D ≥ 0.5***

$L_{eq3} \rangle L$ ➡ **Low ripple**

Current Ripple

- **Peak to peak current ripple (Duty-cycle *D ≤ 0.5*):**

$$I_{pp-coupled} = \frac{di_L}{dt} \times T_{off}.$$
$$= (1-D)T \times \frac{Vout}{L_{eq1}}.$$

Minimum current ripple Vs Coupling coefficient

- **Derivative functions with *A=D/(1-D)* :**

$F = \frac{\overline{L_{eq}}}{L} = \frac{1-k^2}{1+Ak}.$ ➡ **Minimum**

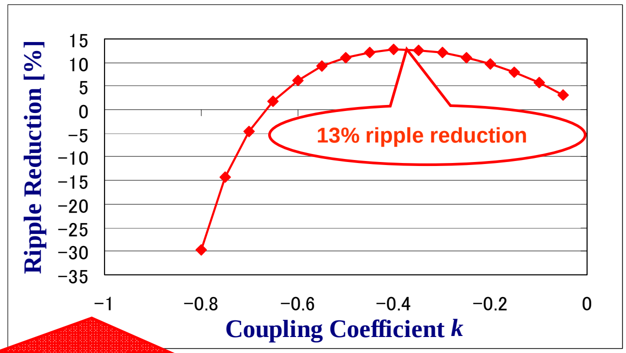
Duty cycle	0.1	0.2	0.3	0.4	0.5
k	-0.056	−0.128	−0.225	−0.382	−1.00

Current Ripple Reduction

- **Per-phase current ripple reduction:**

$$\frac{\left|I_{pp-uncoupled} - I_{pp-coupled}\right|}{I_{pp-uncoupled}} = -\frac{k\left[k + \frac{D}{1-D}\right]}{1-k^2}.$$

Coupling Coefficient Vs Ripple Reduction for D=40%



Transient Analysis

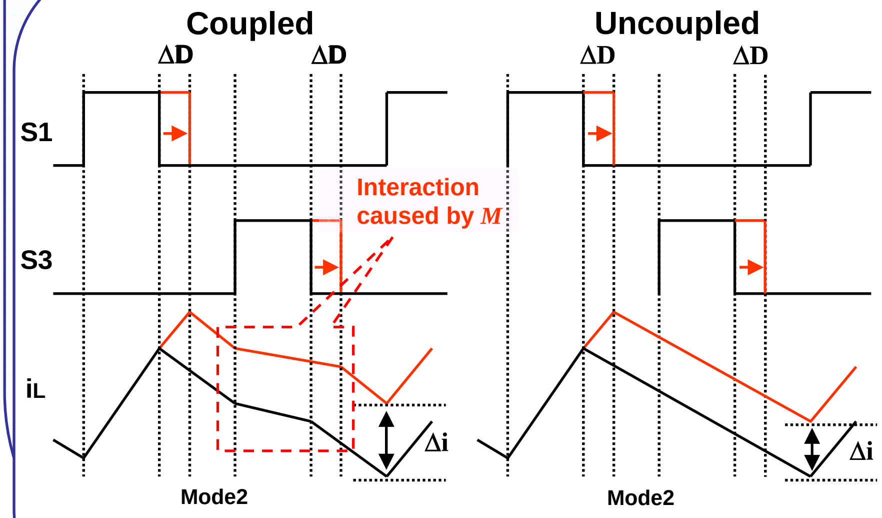


Fig. 3. Comparison of load current transient response of buck converters with and without inductor coupling.

- **Output voltage in real circuits:**

$$Vout = (Vin - Io \cdot Rs)D.$$

- **Current change Δi in coupled inductor circuits:**

$$\Delta i = \frac{Vin \cdot T}{L_{eq2}} \Delta D.$$

- $L_{eq2} = L(1+k) \langle L$

Fast transient

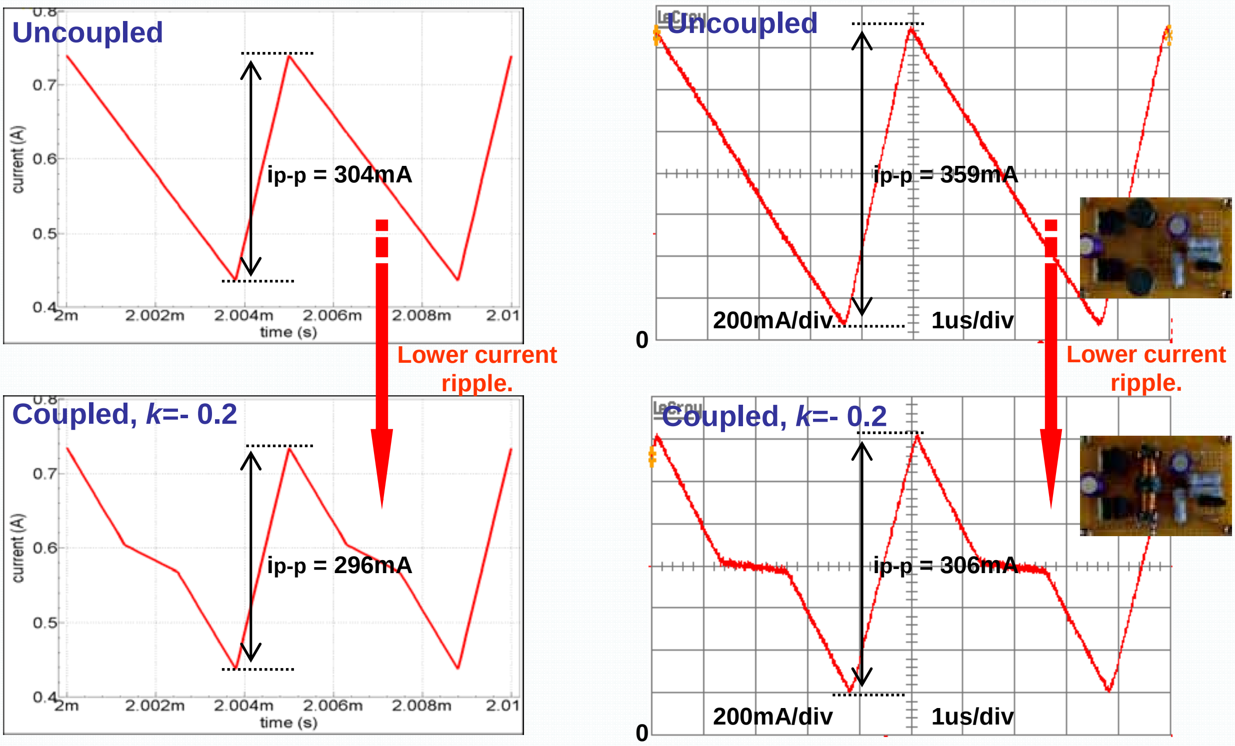
Simulation and Experimental Verifications

Vin = 5V *Duty* = 24% *Fs* = 200KHz *Cout* = 220uF *L* = 15uH *k* = -0.2 *Load transients* : 1.2A ⇔ 3.6A

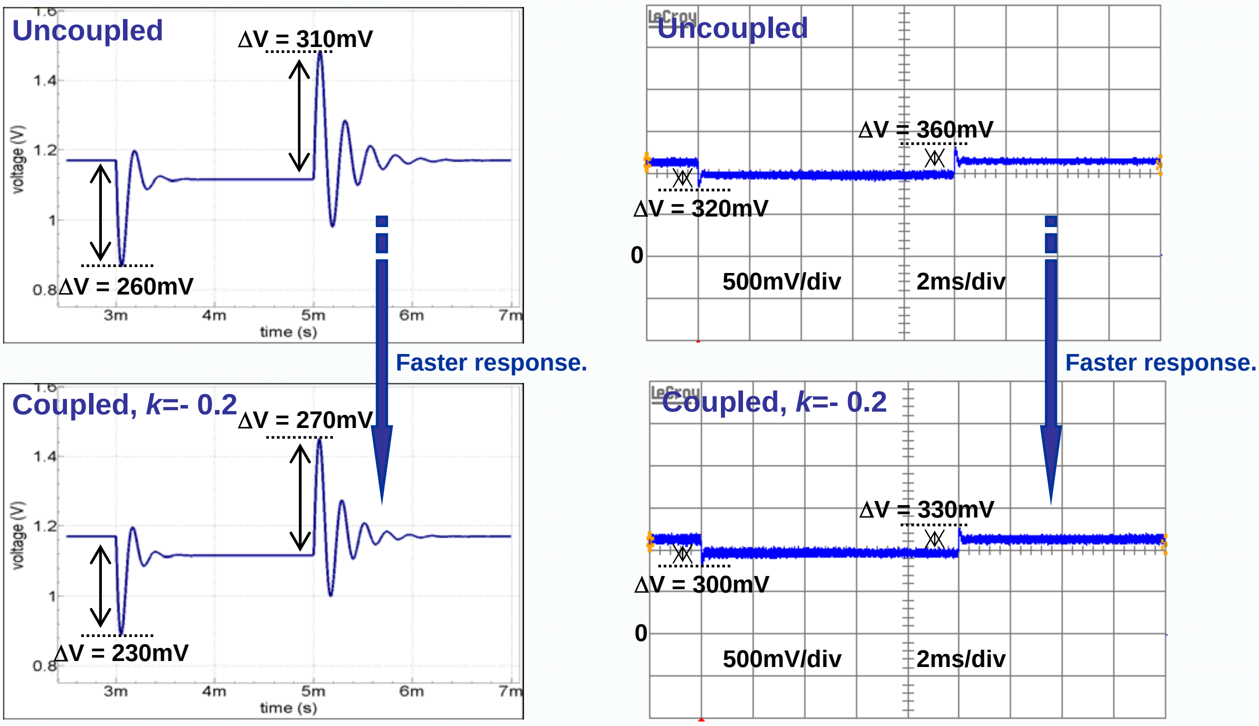
Simulation

Experiments

◆ Inductor current ripple in steady state.



◆ Output voltage for load transients.



Summary

- ◆ Analysis of coupled inductors in multiphase DC-DC buck converter.
  - ✓Lower per-phase ripple current ➡ reduced switching losses
  - ✓Faster transient response
- ◆ Clarification by theory analysis, simulation and experiments.