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# Cross-Noise-Coupled Architecture of Complex Bandpass $\Delta\Sigma AD$ Modulator

SUMMARY Complex bandpass  $\Delta\Sigma AD$  modulators can provide superior performance to a pair of real bandpass  $\Delta\Sigma AD$  modulators of the same order. They process just input I and Q signals, not image signals, and AD conversion can be realized with low power dissipation, so that they are desirable for such low-IF receiver applications. This paper proposes a new architecture for complex bandpass  $\Delta\Sigma AD$  modulators with cross-noisecoupled topology, which effectively raises the order of the complex modulator and achieves higher SQNDR (Signal to Quantization Noise and Distortion Ratio) with low power dissipation. By providing the cross-coupled quantization noise injection to internal I and Q paths, noise coupling between two quantizers can be realized in complex form, which enhances the order of noise shaping in complex domain, and provides a higher-order NTF using a lower-order loop filter in the complex  $\Delta\Sigma$ AD modulator. Proposed higher-order modulator can be realized just by adding some passive capacitors and switches, the additional integrator circuit composed of an operational amplifier is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation. We have performed simulation with MATLAB to verify the effectiveness of the proposed architecture. The simulation results show that the proposed architecture can achieve the realization of higher-order enhancement, and improve SQNDR of the complex bandpass  $\Delta\Sigma AD$  modulator.

key words: complex bandpass  $\Delta\Sigma AD$  modulator, noise coupling, feedforward, multibit

# 1. Introduction

Recently, the research for complex bandpass  $\Delta\Sigma$ ADCs has become popular for their applications to RF receivers in wireless communication systems. Shifting the ADC towards the antenna side in receiver architecture relaxes the requirements placed on analog circuits at the expense of more complicated digital circuit, and allows more digital integration of analog function on a single chip, and as such results in a cheaper system with a higher level of integration. However, ADCs with high linearity, large dynamic range, bandwidth and strong image rejection capabilities are required, and a complex bandpass  $\Delta\Sigma$ ADC is one of their candidates. In the RF receiver of communication systems such as cellular phones and wireless LANs, low-IF receiver architecture is frequently used so that more receiver functions, such as multi-standard and automatic gain control, can be moved to the digital part to provide more programmability. In conventional low-IF receiver architectures, two real (one input and one output)  $\Delta\Sigma AD$  modulators are used for in-phase (I) and

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quadrature (Q) paths. Their disadvantage is that not only input signals but also image signals are converted by ADCs. On the other hand, a complex bandpass  $\Delta\Sigma$ AD modulator can provide superior performance to a pair of real bandpass  $\Delta\Sigma$ AD modulators of the same order. It processes just input I and Q signals, not image signals, and AD conversion can be realized with low power dissipation. Thus, they are desirable for such low-IF receiver applications [1]–[6].

In a  $\Delta\Sigma$ AD modulator, oversampling and noise-shaping techniques are used to achieve high accuracy. In order to realize higher SQNDR, higher oversampling ratio (OSR) is needed which demands higher sampling rate, and/or a highorder filter inside a modulator (as well as a high-order digital filter following the  $\Delta\Sigma$ AD modulator) is required, which need more hardware. However, either of above techniques for higher SQNDR will cause more power dissipation for the modulator. The best solution to the problems is at system level. By applying a complex noise-coupled structure to the front-end of internal ADCs, the order of the complex modulator will be effectively raised. The complex noisecoupled structure can be realized just by adding some passive capacitors and switches, the additional active circuits are not necessary. Therefore, it can achieve higher SQNDR with low power dissipation.

#### 2. Complex Bandpass ΔΣAD Modulator

A complex bandpass  $\Delta\Sigma$ AD modulator gains its advantage by implementing the poles and zeros of its loop filter without conjugates, which effectively cancels the leakeage in the image band for a complex single-side band signal. Figure 1 shows the signal-flow-graph (SFG) of complex bandpass  $\Delta\Sigma$ AD modulator [2], [3], and Fig. 2 shows its simplified structure, which is composed of a complex bandpass filter, two internal quantizers(ADCs) and two DACs. When input signal X(z), output signal Y(z) and quantizer noise  $E_q(z)$  are given by complex form,



Fig. 1 SFG of complex bandpass  $\Delta\Sigma AD$  modulator.

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Fig. 2 Complex bandpass  $\Delta\Sigma AD$  modulator structure.

$$X(z) = I_{in} + jQ_{in}$$
  

$$Y(z) = I_{out} + jQ_{out}$$
  

$$E(z) = E_I + jE_Q$$

then, the transfer function of this complex modulator can be expressed as:

$$I_{out} + jQ_{out} = (I_{in} + jQ_{in}) + \frac{1}{1 + H(z)}(E_I + jE_Q) \quad (1)$$

Here, H(z) is a complex filter transfer function, and then we have signal transfer function STF(z) and noise transfer function NTF(z) as follows:

$$STF(z) = 1 \tag{2}$$

$$NTF(z) = \frac{1}{1 + H(z)}.$$
 (3)

Equation (1) shows that in a complex bandpass  $\Delta\Sigma$ AD modulator with two inputs and outputs of I and Q signal paths, two analog input signals are modulated by complex form, and converted to two digital output signals. Quantization noise of two ADCs  $E(z) = E_I + jE_O$  is noise shaped in complex form according to the NTF(z) (expressed by Eq. (3)) of the modulator. Complex bandpass filter in the modulator has asymmetrical frequency characteristics to the axis of  $\omega = 0$ , which is different from a real bandpass filter. It has opposite frequency characteristics for  $\omega > 0$  and  $\omega < 0$ , where one side is signal-band (passband), the other side is image-band (attenuation band). Therefore, a complex bandpass  $\Delta\Sigma AD$  modulator performs AD conversion effectively only for the positive frequency of I, Q input signals in a low-IF receiver, and hence it can be realized with lower power dissipation than a pair of real bandpass  $\Delta\Sigma AD$  modulators which perform AD conversion for the negative frequency (image signal) as well as the positive frequency.

#### 3. Self-Noise-Coupled $\Delta\Sigma$ AD Modulator

Figure 3 shows a lowpass  $\Delta\Sigma AD$  modulator with selfcoupled noise injection [7], which is a full feedforward  $\Delta\Sigma AD$  modulator with an additional error-feedback structure of quantization noise. Notice the error-feedback structure surrounded by dotted line, we see that:



Fig. 3 Self-noise-coupled lowpass  $\Delta\Sigma$ AD modulator.

$$A(z) = Y(z) - E(z)$$
$$B(z) = E(z)$$

which means that the quantization noise E(z) is obtained by subtracting the internal ADC's input from the DAC output; after though a filter  $z^{-1}$ , a delayed replica of E(z) is fed back to the input node of the ADC again [8]. For simplification, the two  $z^{-1}$  filters at ADC and DAC paths respectively are merged into one filter after subtraction. While the noise transfer function of  $\Delta\Sigma$ AD modulator without additional error-feedback structure is given by NTF(z), the transfer function of input and output of self-noise-coupled  $\Delta\Sigma$ AD modulator shown in Fig. 3 can be written as follows:

$$Y(z) = X(z) + NTF'(z)E(z)$$
  

$$NTF'(z) = NTF(z)(1 - z^{-1}).$$
(4)

As shown in Eq. (6), by providing an additional self-noisecoupled structure with the error-feedback topology, the NTF'(z) of  $\Delta\Sigma$ AD modulator increments the NTF(z) by an extra  $(1 - z^{-1})$  factor, the order of the modulator is increased by one, which is equivalent to obtaining more noise shaping in low frequency signal band, and achieving higher SQNDR of modulator.

In a noise-coupled  $\Delta\Sigma AD$  modulator, the injection method of the quantization noise to the modulator is similar to a second-stage cascade (or MASH) modulator, which provides a higher-order noise shaping using a lower-order loop filter. However, in a second-stage MASH structure, a higher SONDR is achieved by accurate cancellation of the firststage quantization noise. Any mismatch errors for analog implementation will change the transfer function, and cause the noise leakage in the MASH modulator. As contrast, in a self-noise-coupled structure, higher-order noise shaping is realized by injection the quantization noise to the modulator again, there is no mismatch error of the noise leakage at all. Furthermore, while multi-bit quantizer is used for the modulator, the quantization noise can be assumed under busy signal conditions. Then the injected noise also acts as merely as a dither signal, which reduces tones and harmonic spurs. Thus, the noise coupling method can raise the order of a noise transfer function, at the same time, and the stability condition of the original modulator is preserved [9].

#### 4. Proposal of Cross-Noise-Coupled Complex Bandpass ΔΣΑD Modulator

In this paper, we propose a complex bandpass  $\Delta\Sigma AD \mod$ 



Fig. 4 SFG of cross-noise-coupled complex bandpass  $\Delta\Sigma$ AD modulator.

ulator with complex noise coupling, which extends the lowpass self-noise-coupled  $\Delta\Sigma AD$  modulator to complex domain [10].

Figure 4 shows the SFG of the proposed complex bandpass noise-coupled  $\Delta\Sigma$ AD modulator. Compare with the SFG of the lowpass  $\Delta\Sigma$ AD modulator shown in Fig. 3, and we see that the proposed complex modulator with two input and output signals in complex domain is almost the same as the lowpass one, except that the filter at error-feedback structure is  $jz^{-1}$ . According to Eq. (6), if we can realize the  $jz^{-1}$  in the modulator, then  $z^{-1}$  factor in this equation can be replaced by  $jz^{-1}$ , so that we will get the noise transfer function by extra  $(1 - jz^{-1})$  factor.

The noise transfer function of the conventional 2ndorder  $\Delta\Sigma AD$  modulator without additional error-feedback structure in Fig. 4 can be written as:

$$NTF(z) = (1 - jz^{-1})^2.$$
 (5)

Then, the transfer function of the proposed complex bandpass  $\Delta\Sigma AD$  with noise-coupled image rejection shown in Fig. 4 can be written as follows:

$$Y(z) = X(z) + NTF'(z)E(z)$$
  

$$NTF'(z) = NTF(z)(1 - jz^{-1}).$$
(6)

According to Eq. (5), we obtain that

$$NTF'(z) = (1 - jz^{-1})^3$$
(7)

We see from Eqs. (6) and (7) that, by providing this additional noise-coupled structure with the error-feedback topology, the NTF'(z) of the proposed  $\Delta\Sigma$ AD modulator increased by an extra  $(1 - jz^{-1})$  factor, which has a complex zero at z = -j. Therefore, the NTF of the proposed modulator becomes third-order.

Figure 5 shows the realization structure of the proposed modulator with complex noise coupling. Similar to a lowpass self-noise-coupled modulator, the proposed modulator is a conventional feedforward complex  $\Delta\Sigma$ AD modulator with an additional error-feedback structure in complex domain. Notice the complex error-feedback structure of I and Q paths surrounded by dotted line, we see that:

$$I_a = I_{out} - E_I, \qquad Q_a = Q_{out} - E_Q$$

and

$$I_b = E_I, \qquad \qquad Q_b = E_Q$$



Fig. 5 Cross-noise-coupled complex bandpass  $\Delta\Sigma AD$  modulator structure.

Above equations mean that the quantization noises  $E_I$  and  $E_Q$  of two ADCs are obtained by subtracting the internal ADCs' input from the DACs' output, respectively; after though the filter  $z^{-1}$ , delayed replica of the quantization noise  $E_I$  and  $E_Q$  are different from self-noise-coupled lowpass modulator, they are cross-noise-coupled to the input node of ADCQ and ADCI, but not ADCI and ADCQ, respectively. The proposed cross-noise-coupled error-feedback structure is equivalent to the realization of j factor to the complex signals (with 90° phase-shifted), then we get the following:

$$I_{b} + jQ_{b} = (-Ia + I_{out}) + j(-Qa + Q_{out})$$
  
=  $E_{I} + jE_{Q}$ . (8)

While the noise transfer function of the original complex bandpass  $\Delta\Sigma AD$  modulator without additional errorfeedback structure is NTF(z), the transfer function of crossnoise-coupled complex  $\Delta\Sigma AD$  modulator shown in Fig. 5 can be written as follows:

$$Y(z) = STF(z) \cdot X(z) + NTF'(z) \cdot E(z)$$
  

$$NTF'(z) = NTF(z) \cdot (1 - jz^{-1}).$$
(9)

Compare Eq. (9) to Eq. (6), and we know the following:

- For a self-noise-coupled lowpass  $\Delta\Sigma AD$  modulator shown in Fig. 3, delayed quantization noise is selfcoupled to the input node of internal ADC, so that the NTF'(z) of  $\Delta\Sigma AD$  modulator increments the NTF(z) by an extra  $(1 - z^{-1})$  factor, which means that the order of noise shaping is increased by one for low frequency signal band.
- For the proposed complex bandpass  $\Delta\Sigma$ AD modulator with cross-noise-coupling shown in Fig. 4, two delayed quantization noises of ADCI and ADCQ are cross-coupled (NOT self-coupled) to the different input nodes of ADCQ and ADCI with different polarities. Therefore, the *NTF*(*z*) of proposed  $\Delta\Sigma$ AD modulator increments the NTF(*z*) by an extra  $(1 - jz^{-1})$ factor, which means that the order of noise shaping is increased by one for intermediate frequency (IF) frequency signal band in complex domain.

According to the above discussion, we know that the complex noise-coupled structure can be realized simply by cross coupling the two quantization noises. In the circuit implementation, the proposed structure can be realized just by adding some passive capacitors and switches, the additional complex integrator circuit (including amplifiers) is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation.

Same as the self-noise-coupled lowpass  $\Delta\Sigma AD$  modulator, multibit ADCs/DACs are required for the proposed cross-noise-coupled complex bandpass  $\Delta\Sigma AD$  modulator, so that the additional noise coupling have not any damage to the stability of the modulator. On the other hand, multibit DACs cannot be made perfectly linear and their nonlinearity in the feedback paths are equivalent to errors added directly to the input signals; hence, they may degrade the SQNDR of the  $\Delta\Sigma AD$  modulator. However, a complex data-weighted averaging (DWA) algorithm can be provided for the modulator to suppress nonlinearity effects of multibit DACs in a complex form [11].

### 5. Simulation Results

We have conducted MATLAB simulations to evaluate the effectiveness of the proposed complex bandpass  $\Delta\Sigma AD$  modulator with cross-noise-coupling architecture. We made comparison of behavioral models which are shown in Fig. 2 and Fig. 5. In the behavioral model of Fig. 2, a second-order full-feedforward complex bandpass  $\Delta\Sigma AD$  modulator with 3-bit internal ADCs/DACs is used, and in the behavioral model of the proposed modulator shown in Fig. 5, we just add the complex noise coupling structure to Fig. 2.

Figure 6 shows simulation result comparison of output power spectrum between behavioral models of Fig. 2 and Fig. 5. Around IF input signal band of Fin = Fs/4 ( $F_s$  is sampling frequency of  $\Delta\Sigma$ AD modulator), the signal power of the proposed modulator is the same as conventional one, but for the proposed modulator, the noise floor is lower than conventional architecture, which means that the noise power can be suppressed well in the proposed modulator.

Figure 7 shows simulation result comparison of SQNDR vs. OSR which are calculated from above of their output power spectrum between behavioral models of Fig. 2 and Fig. 5. For the conventional complex bandpass  $\Delta\Sigma$ AD modulator shown in Fig. 2, the SQNDR increases by 15 dB/Oct while OSR is increased, which shows 2ndorder characteristics of  $\Delta\Sigma AD$  modulator. On the other hand, for the proposed complex bandpass  $\Delta\Sigma AD$  modulator with cross-noise-coupled architecture shown in Fig. 5, the SQNDR increases by 21 dB/Oct while OSR is increased, which shows 3rd-order characteristics of  $\Delta\Sigma$ AD modulator. It suggests that the proposed modulator realizes high order of noise shaping by complex noise-coupled architecture, it can effectively raise the order of the modulator, and suppress the noise power of interest band. Cross-coupled noise injection provides an efficient way to realize higher-order complex bandpass  $\Delta\Sigma AD$  modulators. The SQNDR of the



Fig. 7 Simulation results comparison of SQNDR-OSR.

proposed complex bandpass  $\Delta\Sigma$ AD modulator can be higher than the conventional one.

#### 6. Discussion of Parameter Variations in Cross-Noise-Coupled Architecture

Cross-noise-coupling technique realized an error-feedback structure for a complex bandpass  $\Delta\Sigma AD$  modulator. Normally, the error-feedback structure is often applied for the digital loops in  $\Delta\Sigma DA$  modulators, and it is not practical for  $\Delta\Sigma AD$  modulators, since it is very sensitive to variations of its parameters in analog implementation. However, in the proposed cross-noise-coupled modulaltor architecture, the error-feedback structure is in the backend of feedback loop. Any influence from the variations of analog parameters will be suppressed by the feedback loop, and be noise-shaped as the same as quantization noise.

We also performed the MATLAB simulation with behavioral models which are shown in Fig. 8 to verify the suppression effect on variations of analog parameters. We assume that the elements (e.g. capacitors in I and Q paths) realizing the coefficient of 1.0 have  $\pm 5\%$  mismatch error,



Fig. 8 Cross-noise-coupled complex bandpass  $\Delta\Sigma AD$  modulator with coefficient mismatch.



**Fig. 9** Comparison of power spectrum ( $F_{in} = F_s/4$ ).



Fig. 10 Simulation results comparison of SQNDR-OSR.

a = 1.05 and b = 0.95 for the proposed modulator shown in Fig. 8.

Figure 9 shows simulation result comparison of output power spectrum, and Fig. 10 shows simulation result comparison of SQNDR vs. OSR for behavioral models of Fig. 2, Fig. 5 and Fig. 8. We see that, the noise floor of the proposed cross-noise-coupled complex bandpass  $\Delta\Sigma$ AD modulator with coefficient mismatch is almost the same as its ideal case, lower than conventional architecture, which means that the noise power can be suppressed well even there are  $\pm 5\%$  analog parameter mismatch in the proposed modulator. And the SQNDR of the proposed modulator with coefficient mismatch increases by 21 dB/Oct while OSR is increased, which shows 3rd-order characteristics of  $\Delta\Sigma$ AD modulator, the same as its ideal case. The SQNDR drops only by 3 dB while OSR = 128 even there are  $\pm 5\%$  analog parameter mismatch in the error-feedback structure. It suggests that the proposed modulator realizes high order of noise shaping and is less sensitive to the parameter variations for their analog implementations.

#### 7. Conclusion

We have proposed a new complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled architecture. By providing the crosscoupled quantization noise injection between internal I and Q paths, complex noise coupling of two quantization noises can be realized, which effectively enhances the order of the complex modulator and achieves higher-order noise shaping. Proposed complex noise coupling structure can be realized just by adding some passive capacitors and switches. As a result, the proposed complex modulator provides a higher-order NTF using a lower-order loop filter, the additional integrator circuit which consists of an operational amplifier is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation. The MATLAB simulation results with behavioral model show that the proposed architecture can effectively raise the order of the modulator, and improve the SQNDR of a complex bandpass  $\Delta \Sigma AD$  modulator.

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