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高速応答・低リップル電源用カップルド・インダクタの解析

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Analysis of Coupled Inductor for Fast-Response Low-Ripple Buck Converter

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Abstract： This paper presents analysis on characteristics of coupled inductor in multi-phase buck converter application. Calculation of equivalent inductance in steady state that provides a low ripple current per-phase, fast transient response in transient-state, and transfer function using state space averaging method are discussed. Moreover, characteristics of coupled inductor, low ripple current per-phase in steady-state and fast response in transient-state, are examined and verified by circuit simulation and experiment.

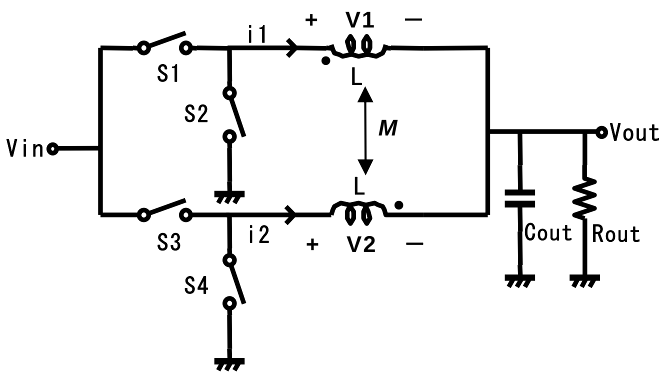
Keyword： Coupled Inductor, Multi-phase, Buck Converter, DC-DC Converter

1. Introduction

In recent years, demand for high-current low-voltage high efficiency DC-DC converter with fast-response for powering digital system such a microprocessor has become very high. One of the proven methods to obtain a high efficiency DC-DC converter is by using coupled inductors as the magnetic components for multiphase buck converters. Using coupled inductors on multiphase buck converters not only can reduce response times in transient state, it also can reduce current ripple in each phase. In inverse coupling with 1-to-1 winding ratio, the greater value of coupling coefficient is, the faster a response times can be obtain. In spite of that, each switching duty cycle has their best value of coupling coefficient to obtain the lowest current ripple in each phase. There has been no any detail analysis about these characteristics of multiphase coupled inductor buck converters. Therefore, the purpose of this research is to clarify the basic characteristic of coupled inductors, using two-phase (with 180 degrees between phases) buck converter as an analysis and experiment tools. However, the same method of analysis can be used for buck converters with greater number of phases.

2. Steady-state and Transient-state Analysis

The basic schematic diagram of two-phase coupled inductor buck converter is shown in Fiq.1.



Fiq.1 Two-phase Coupled Inductor Buck Converter

Inductors are coupled together and the mutual inductance M represents the coupling effect between the two inductors. Assuming that both inductors have the same values, the voltages applied across the two inductors are related to the currents as follows:

$$\begin{aligned} V_1 &= L\frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2 &= L\frac{di_2}{dt} + M\frac{di_1}{dt} \end{aligned} \tag{1}$$

With the mutual inductance,

$$M = k \cdot L \tag{2}$$

Note that k coupling coefficient is between -1 to 0 because M is negative (inverse coupling).

Fig.2 shows a graph illustrating typical waveforms of two-phase interleaved buck converter with duty cycle less than 50%.

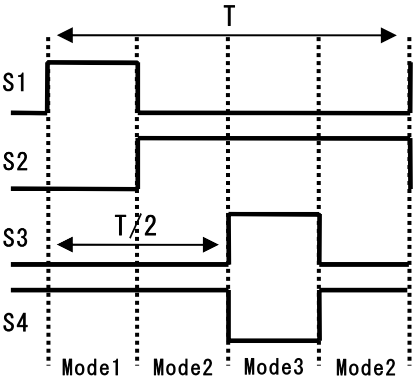


Fig.2 Operating Waveforms of Interleaved Two-phase Coupled Inductor Buck Converter (Duty cycle $D \leq 0.5$)

One switching period includes four stages of operation which is divided into 3 kinds of mode, which first stage begins when S1 and S4 are turned ON to connect the first phase to the power supply and the second phase connects to the ground. In this stage, the voltage applied to the inductor in the first phase is given in equation:

$$V1 = \frac{L^2 - M^2}{L + M \cdot \dfrac{D}{1-D}} \frac{di_1}{dt} \tag{3}$$

The circuit enters second stage of operation when both, first phase and second phase is connected to the ground, or S1 and S3 are turned OFF, S2 and S4 are turned ON (Note that duty cycle $\leq$ 0.5). In this stage, the voltage across the inductor in the first phase is given in equation:

$$V1 = (L + M)\frac{di_1}{dt} \tag{4}$$

In the third stage of operation, S2 and S3 are turned ON, S1 and S4 are turned OFF to connect first phase to the ground and second phase to the power supply. In this stage the voltage applied to the inductor in the first phase is given in equation:

$$V1 = \frac{L^2 - M^2}{L + M \cdot \dfrac{1-D}{D}} \frac{di_1}{dt} \tag{5}$$

Finally, to complete one cycle of switching period, the circuit enters fourth stage which has the same operation with the second stage.

Using equations (3), (4), and (5), the equivalent inductances in each mode can be written as:

$$\begin{aligned} L_{eq1} &= L\frac{1-k^2}{1 + k \cdot \dfrac{D}{1-D}} \\ L_{eq2} &= L(1+k) \\ L_{eq3} &= L\frac{1-k^2}{1 + k \cdot \dfrac{1-D}{D}} \end{aligned} \tag{6}$$

Where *Leq1*, *Leq2*, *Leq3* are equivalent inductances in mode1, mode2, and mode3, respectively.

A steady-state average equivalent inductance can be obtained based on the average inductor current in one switching period. The average inductor current is given in equation:

$$\begin{aligned} \Delta i_L &= \frac{\overline{V_1} \cdot D}{L_{eq1}} + \frac{\overline{V_1} \cdot (1-2D)}{L_{eq2}} + \frac{\overline{V_1} \cdot D}{L_{eq3}} \\ \frac{\overline{V_1}}{\overline{L_{eq1}}} &= \frac{\overline{V_1} \cdot D}{L_{eq1}} + \frac{\overline{V_1} \cdot (1-2D)}{L_{eq2}} + \frac{\overline{V_1} \cdot D}{L_{eq3}} \end{aligned} \tag{7}$$

By calculating equation above, an average equivalent inductance when the duty cycle is less than 50% can be written as:

$$\overline{L_{eq}} = L\frac{1-k^2}{1 + k \cdot \dfrac{D}{1-D}} \tag{8}$$

It is equal to the equivalent inductance in the first mode, or it can be said that current ripple per-phases are determined by the value of equivalent inductance in the first mode or condition where the first phase is connected to the power supply and second phase is connected to the ground.

The steady-state average equivalent inductance for circuits with duty-cycles more than 50% can be calculated with the same method above. This calculation result is given in equation:

$$\overline{L_{eq}} = L\frac{1-k^2}{1 + k \cdot \dfrac{1-D}{D}} \tag{9}$$

As can be seen, the steady-state average equivalent inductance in this case is equal to Leq3. Therefore, the current-ripple per-phases for more than 50% duty-cycles are based on the value of equivalent inductance in the third mode or condition where the first phase is connected to the ground and second phase is connected to the power supply.

As can be seen in equations (8) and (9), the magnitude of average equivalent inductances is dependent on the coupling coefficient k and the duty cycle D. Therefore, the best coupling coefficient to obtain lowest current ripple are different for varied duty cycles. The maximum values of average equivalent inductance, to obtain the lowest current-ripple, can be calculated using derivative function:

$$F = \frac{\overline{L_{eq}}}{L} = \frac{1-k^2}{1+Ak}$$
$$\frac{dF}{dk} = -A(k^2 + \frac{2}{A}k + 1) = 0 \tag{10}$$

Where A is $D/(1\text{-}D)$.

The calculation result is shown in Table 1. Where we can see that for less than 10% duty cycles, the lowest current ripple can be achieved limited in a very small coupling coefficient.

Duty-cycle	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
K	**-0.056**	**-0.128**	**-0.225**	**-0.382**	**-1**

Table1. Duty cycles and values of coupling coefficient to obtain the lowest current ripple per-phase

Furthermore, the peak to peak current ripple on each-phase for less than 50% duty-cycle is given by equation (10), where Ts is the switching period.

$$I_{pp-coupled} = \frac{di_L}{dt} \times T_{off} = (1-D)Ts \cdot \frac{Vout}{\overline{L_{eg}}} \tag{11}$$

Using equations (8) and (9), the current ripple per-phase reduction between coupled and uncoupled case can be written in equation:

$$\frac{\left|I_{pp-uncoupled} - I_{pp-coupled}\right|}{I_{pp-uncoupled}} = \frac{k\left(k + \frac{D}{1-D}\right)}{1-k^2} \tag{12}$$

From equation above, we can see that the current ripple reduction is dependent on the steady-state average equivalent inductance.

During the transient-state, circuit enters mode2, where S1 and S3 are turned ON (D=100%), both phases are connected to the power supply, or S2 and S4 are turned ON (D=0%), both phases are connected to the ground. Therefore, the equivalent inductance in the transient-state is equal to *Leq2*, and as a result the transient response time is determine only by the value of *L(1+k)*. The transient response times are directly proportional with the value of coupling coefficients.

Next, by applying the space state averaging method, transfer function between input voltage *Vin*, output current *iout*, and output voltage *Vout* can be calculated. Dividing circuit operation into 4 stages, assuming that inductor and switch have no resistance and there is no ESR and ESL on the capacitor, and by multiply each stage equation by it's time ratio and sum it, the relationship between *iout*, *Vin* and *Vout* can be written as:

$$\frac{d}{dt}\begin{bmatrix} i_{out} \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0 & -2\dfrac{\left\{1+k\left(\dfrac{D}{1-D}\right)\right\}}{L(1-k^2)} \\ \dfrac{1}{C} & -\dfrac{1}{CR} \end{bmatrix}\begin{bmatrix} i_{out} \\ V_{out} \end{bmatrix} + 2\begin{bmatrix} \dfrac{D + k\dfrac{D^2}{1-D}}{L(1-k^2)} \\ 0 \end{bmatrix} Vin \tag{13}$$

Noted that *iout* is a sum between inductor currents in the first and second phase (*iout=i1+i2*), and duty-cycle is less than 50%.

In the steady-state, condition when the energy stored in each component is the same at the beginning and at the end of a switching period, the equations above can be calculated and written as:

$$\begin{bmatrix} i_{out} \\ V_{out} \end{bmatrix} = \begin{bmatrix} \dfrac{D}{R} \\ D \end{bmatrix} Vin \tag{14}$$

From equation above, we can see that coupling coefficient does not have any affect on the steady-state input to output transfer function.

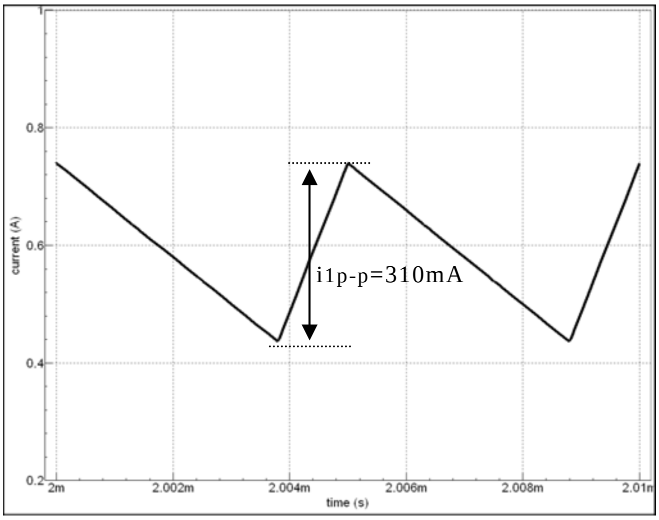
3. Simulation Verification

Circuit simulation is performed on SmartSPICE software based on next specification and circuit parameters.

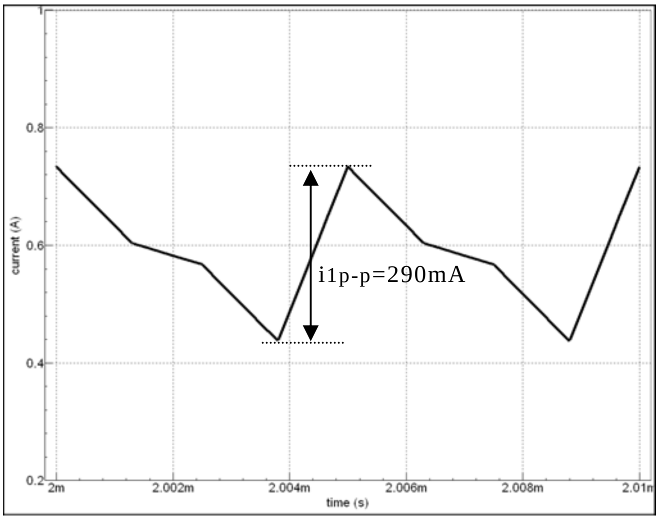
Vin	Input Voltage	5V
Vout	Output Voltage	1.2V
Fs	Switching Frequency	200KHz
Cout	Output capacitor	220uF
L	Inductor	15uH
k	Coupling Coefficient	-0.2

The circuit is operated at open loop and constant duty-cycle. Coupling coefficient value is set to -0.2 so buck converter with coupled inductor can achieve both low-ripple and fast-response characteristics, considering that for duty-cycles between 0.2 and 0.3, values of coupling coefficient to obtain the lowest current ripple is -0.128 and -0.225, respectively (discussed in the steady-state analysis). Simulation results at steady-state is shown in Fig.3, where i1p-p is the peak to peak inductor current ripple in the first phase and Fig.3(a) shows inductor ripple current in buck converter with uncoupled inductor and Fig.3(b) shows inductor ripple current in buck converter with coupled inductor. Compared Fig.3(a) and Fig.3(b), buck converter with coupled inductor achieves a lower peak to peak current ripple per-phase than those with the uncoupled inductors.

Transient-states are simulated at load changes from 1.2[A] to 3.6[A]. Output voltages at load transient-states are shown in Fig.4, where Fig.4(a) shows simulation result in uncoupled case and Fig.4(b) shows simulation result in coupled case. From these waveforms, we can see that under-shoot of output voltage when the loads step-up and over-shoot of output voltage when the loads step down is better in buck converter with coupled inductor. Therefore, it can be concluded that coupled inductor not only can improve the efficiency of circuit but also can reduce the transient response time.

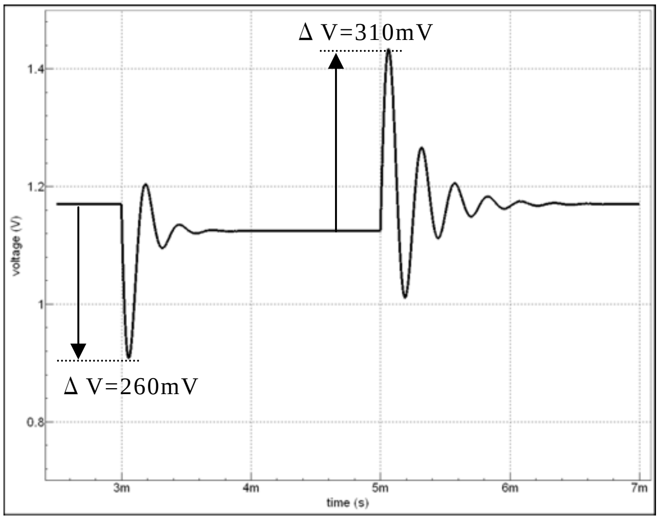


(a) Uncoupled inductor case.

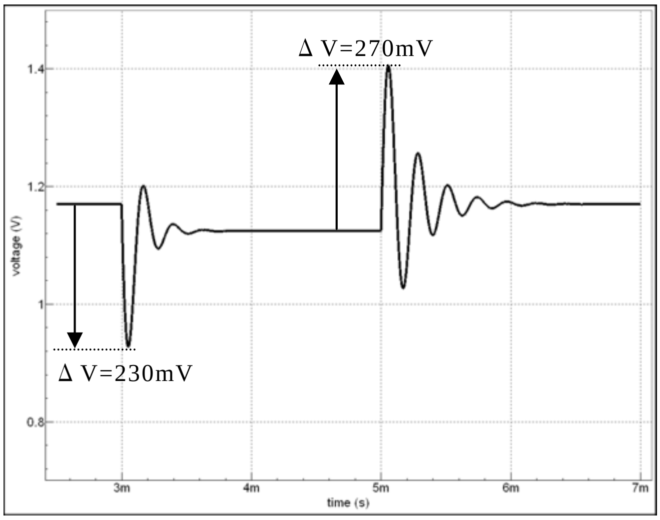


(b) Coupled inductor case with *k*=-0.2.

Fig.3 Simulation results of inductor current in the first phase at steady-state period.



(a) Uncoupled inductor case.



(b) Coupled inductor case with *k*=-0.2.

Fig.4 Simulation results of output voltage at load transient-state period.

4. Experiment Verification

The circuit for experiment is built and measured with specifications and parameters near to the circuit simulation specifications and parameters. Coupled inductor is made by cleaving two separate inductor bobbin with 15uH inductance on each inductor. Coupling coefficient value is -0.2 tested by an LCR tester.

The steady-state experiment results is shown in Fig.5, with Fig.5(a) and Fig.5(b) are experiment results of the first phase inductor current in two-phase buck converter with uncoupled and coupled inductor, respectively. These experiment results show that coupled inductor has a low ripple current per-phase characteristic at the steady-state. However, this characteristic is limited at duty-cycles with limited coupling coefficients (discussed in the steady-state analysis). Waveforms are with the same scales of 200mA/div and 5us/div.

The load transient response is measured at load changes from 1.2[A] to 3.6[A]. Fig.6(a) and Fig.6(b) show the output voltage (500mV/div) at load transient-state of two-phase buck converter with uncoupled and coupled inductor, respectively. These experiment results show that coupled inductor can improve the transient response during step-up and step-down load transients compare to uncoupled case. Transient response in a multi-phase buck converter with coupled inductor is independent from switching duty-cycle and directly proportional with the value of mutual inductance (discussed in the transient-state analysis). The time scale is 200us/div. Differences between the simulation and experiment results are due to conduction losses.

5. Conclusions

Analysis of coupled inductor, a proven method to achieve high efficiency buck converters has been presented. Analytical calculations show that coupled inductors allow power converters to achieve a low ripple-current in each phase, which leads to reducing switching losses, and a fast transient response. It also shows that in steady-state, average equivalent inductance is varied by duty-cycles and coupling coefficients. As a result, this method is limited in that it offers a current ripple reduction at some duty cycles for limited amounts coupling coefficient. The characteristics of coupled inductors are verified using circuit simulation and experiment.

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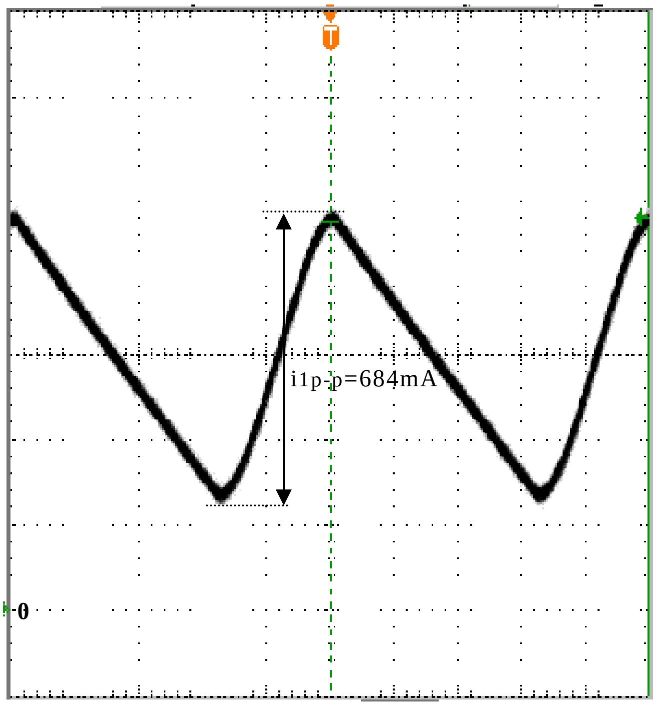
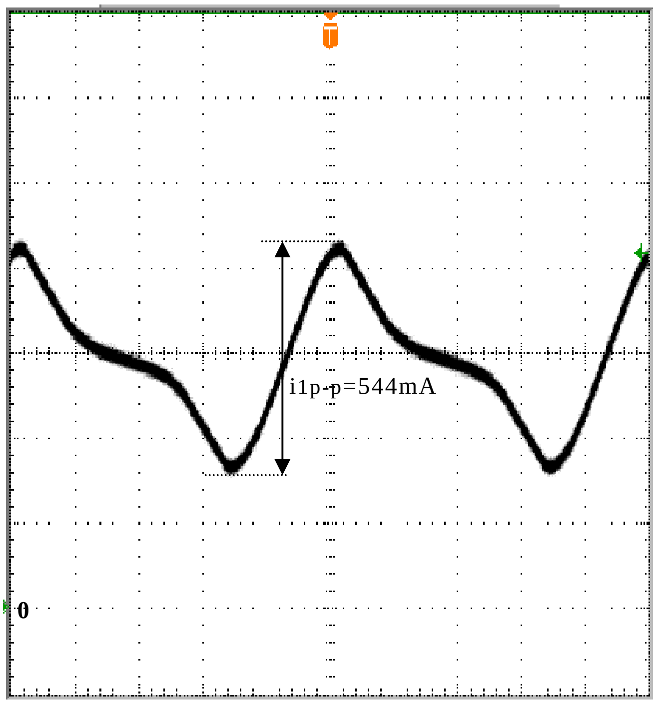
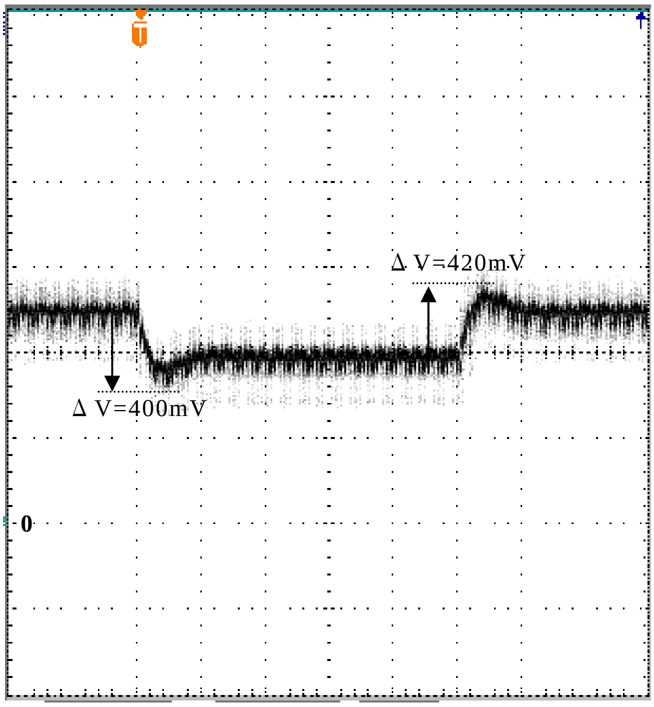


(a) Uncoupled inductor case.

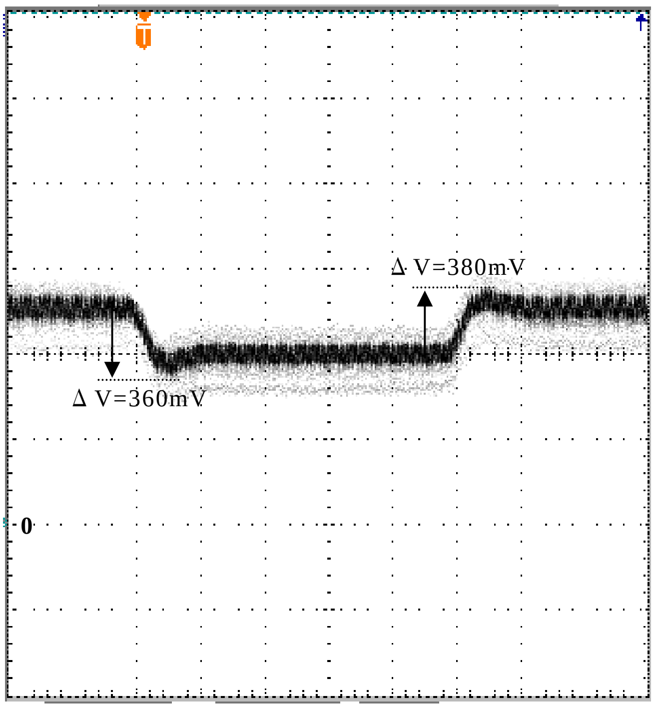


(b) Coupled inductor case with k=-0.2.

Fig.5 Experiment results of inductor current in the first phase at steady-state period.



(a) Uncoupled inductor case.



(b) Coupled inductor case with *k*=-0.2

Fig.6 Experiment results of output voltage at load transient-state period.