A High-Precision AC Wheatstone Bridge Strain Gauge

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ABSTRACT

This paper describes high-precision dynamic strain measurement bridge circuits with on-line calibration of parasitic capacitance effects. The proposed calibration system is very reliable and robust against temperature change and aging because most of the calibration is done in digital domain.

Keywords: Strain Measurement, Strain Gauge, Bridge Circuit, Calibration.

1. Introduction

Recently much attention is being paid to sensor technology for automotive applications. In this paper we focus on high-precision strain measurement technology (Figs.2 and 3, [1]-[8]) for such applications. Strain measurement methods can be classified into DC (Fig.4) and AC (Fig.5) methods. DC methods are simple, but suffer from low-frequency noise (such as 50Hz or 60Hz hum noise from power supply), drift, and thermal electromotive force (emf), and hence cannot achieve high precision. On the other hand, AC methods suffer from parasitic capacitance effects [1,2], even though they are not affected by low-frequency noise, drift, and thermal emf. In this paper, we describe how we have attempted to solve this problem of parasitic capacitance effects, which have been a problem for a long time in AC methods of strain measurement, using ADC and modern digital technology.



Fig. 1. Explanation of "Strain".

2. Principle of Strain Measurement: with Strain Gauge and Bridge Circuit

When a material is stretched (or compressed), the force used generates a corresponding stress σ inside the material. This stress in turn generates a proportional tensile strain (or compressive strain) which deforms the material by $L + \Delta L(orL - \Delta L)$, where L is the original length of the material. When this occurs, the strain is the radio of ΔL to L (Fig.1).



Fig. 2. Strain gauge and Wheatstone bridge circuit (quarter bridge 2-wire system).

Fig.2 shows an example of a strain gauge and a Wheatstone bridge circuit (There are several types of their combinations according to applications). Suppose that $V_{in} = 2[V]$ is applied, and further suppose



Fig. 3. Strain gauges and Wheatstone bridge circuit (half bridge 2-active method).



Fig. 4. DC-type dynamic strain measurement system.



Fig. 5. AC-type dynamic strain measurement system.

that the applied strain changes the gauge resistance from R to $R + \Delta R$.

Then the Wheatstone bridge circuit output voltage ΔV is given by

$$\Delta V = \frac{\Delta R}{2R + \Delta R} = \frac{\frac{\Delta R}{R}}{2R + \frac{\Delta R}{R}} [V],$$

and we can obtain the strain value ε as

$$\varepsilon = \frac{2}{k} * \frac{\Delta V}{1 - \Delta V} \varepsilon \simeq \Delta V$$

where $\Delta V = \frac{k\varepsilon}{2 + k\varepsilon} [V].$



Fig. 6. A simplified model of a half bridge circuit with parasitic capacitances.

3. Analysis of Parasitic Capacitance Influences

In applications where the strain gauge must be at some distance from the strain measuring instrument, and so long wires must be used to connect them, the parasitic capacitance associated with the wires limits the accuracy of the measurement. In this section we analyze the parasitic capacitance effects. Fig.6 shows a bridge circuit with parasitic capacitances, and its transfer function $H(j\omega)$ is given by :

$$H(j\omega) = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

where
$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2}.$$

The transfer function $H(j\omega)$ can be rewritten as

$$\begin{split} H(j\omega) &= \frac{H_N(j\omega)}{H_D(j\omega)} , \text{ where} \\ H_N &= R_2^2 - R_1^2 + \omega^2 R_1^2 R_2^2 (C_1^2 - C_2^2)^2 \\ &+ 2j\omega R_1 R_2 (R_1 C_1 - R_2 C_2), \\ H_D &= (R_2 + R_1)^2 + \omega^2 R_1^2 R_2^2 (C_1 + C_2)^2. \end{split}$$

We have checked the above equations by SPICE simulation (Figs.6 and 7). Simulation condition is R1 = 350 , R2 = 353 , C1 = 4,000 pF, C2 = 3,500 pF, f = 20 kHz, $V_{in} = 2V$.

Letting ε be the strain value (to be measured), k be the gauge ratio (a given value), σ be the Poisson



Fig. 7. SPICE simulation results of input and output waveforms in the bridge circuit with parasitic capacitances in Fig.6.



Fig. 8. Effect of parasitic capacitance effects on the real part of H(jw).

ratio (a given value), then resistor values R_1, R_2 in the above equation for are replaced as follows

 $R_1 \rightarrow R_1(1 + k\varepsilon), R_2 \rightarrow R_2(1 - L\varepsilon), L = k\sigma.$ Then $H(j\omega)$ can be rewritten as $H(j\omega) = H_R(j\omega) + jH_I(j\omega)$, where

$$\begin{aligned} H_R(j\omega) &= R_2^2(1+k\varepsilon\sigma)^2 - R_1^2(1+k\varepsilon)^2 \\ &+ \omega^2 R_1^2 R_2^2(1+k\sigma)^2(1+k\varepsilon\sigma)^2(C_1^2-C_2^2) \\ &+ 2j\omega^2 R_1 R_2(1+k\varepsilon)(1-k\sigma\varepsilon) \\ &\quad \{R_1 C_1(1+k\varepsilon) - R_2 C_2(1-k\varepsilon\sigma)\}, \\ H_I(j\omega) &= \{\{R_2(1-k\varepsilon\sigma) + R_1(1+k\varepsilon)\}^2 \\ &+ \omega^2 R_1^2 R_2^2(1+k\varepsilon)^2(1-k\varepsilon\sigma)^2(C_1+C_2)^2 \end{aligned}$$

We see that both the real and imaginary parts of $H(j\omega)$ are affected by parasitic capacitances C_1, C_2 (Fig.8) and they are also functions of ε , ε^2 and ε^3 .



Fig. 9. An analog method for parasitic capacitances C_1 compensation.

4. Conventional Strain Measurement Method

Conventional AC dynamic strain measurement systems measure the real part $H_R(j\omega)$ of the bridge output voltage by phase detection and calculate the strain, assuming that C_1, C_2 are small enough to be neglected, and also that squared and cubed can be neglected. However, in recent applications where parasitic capacitances are not negligible and very high precision measurement is demanded, these assumptions are not valid any more. Sometimes an analog calibration method is used to compensate for parasitic capacitances as shown in Fig.9, but due to temperature change and aging effects this method cannot satisfy more demanding requirements.

5. Proposed Parasitic Capacitance Cancellation System

5.1 Configuration

Fig.10 shows our proposed strain measurement system, which has the following features:

(a) Oscillators of two different frequencies (ω₁, ω₂) are used. (ω₁/2π, ω₂/2π) are on the order of 10kHz.
(b) The bridge output is amplified by an AC amplifier which does not suffer from low frequency noise.

(c)The input and output signals of the bridge circuit are converted to digital data (with accuracy of 16 bits or greater) by delta-sigma AD modulators, and are stored in memory.

(d)A digital signal processor compensates for parasitic capacitance and calculates ε taking into account ε^2 , ε^3 effects.



Fig. 10. Proposed dynamic strain measurement system.

5.2 Algorithm

The strain calculation algorithm is as follows:

(1) The input $cos(\omega_1 t) + cos(\omega_2 t)$ is applied to the bridge circuit, and its input and output are converted to digital signals. Digital filter operation is performed to them so that the component of $cos(\omega_1 t)$ is extracted from the input part and also ω_1 component is obtained from the output. By multiplying the input $cos(\omega_1 t)$ and output ω_1 component signal, and taking the time average of the multiplication result in the digital domain, we have the value of $H_R(j\omega_1) = V_{cr1}$.

In other words, if the output ω_1 component signal $V_{out}1(t) = a_1 cos(\omega_1 t) + b_1 sin(\omega_1 t)$,

$$cos(\omega_1 t) \times V_{out}(t) = \frac{a_1}{2} + \frac{a_1}{2}cos(2\omega_1 t) + \frac{b_1}{2}sin(2\omega_1 t),$$

then $H_R(j\omega_1) = \frac{a_1}{2} = V_{cr1}.$

(2) In the same way, the input $cos(\omega_1 t) + cos(\omega_2 t)$ is applied to the bridge circuit, and its input and output are converted to digital signals. Digital filter operation is performed to them so that the component of $cos(\omega_1 t)$ is extracted from the input part and also ω_1 component is obtained from the output. The input $cos(\omega_1 t)$ is phase-shifted by 90 degrees in the digital domain, to obtain $sin(\omega_1 t)$ precisely. (Accurate 90-degree phase-shift in digital domain is one of advantages of our proposed system.) Then by multiplying $sin(\omega_1 t)$ and the output ω_1 component, and taking the time average of the multiplication result in the digital domain, we have the value of $H_I(j\omega_1) = V_{ci}$.

$$sin(\omega_1 t) \times V_{out}(t) = \frac{b_1}{2} - \frac{b_1}{2}cos(2\omega_1 t)$$

+
$$\frac{a_1}{2}sin(2\omega_1 t),$$

then $H_I(j\omega_1) = \frac{b_1}{2} = V_{ci}.$

(3) Next we consider ω_2 component. The input $\cos(\omega_1 t) + \cos(\omega_2 t)$ is applied to the bridge circuit, and its input and output are converted to digital signals. Digital filter operation is performed to them so that the component of $\cos(\omega_2 t)$ is extracted from the input part and also ω_2 component is obtained from the output. By multiplying the input and the output, and taking the time average of the multiplication result in the digital domain, we have the value of $H_R(j\omega_2) = V_{cr2}$.

(4) Then we have the following three equations (1),(2),(3). Since the number of unknown parameters (ε, C_1, C_2) is three and there are three equations, this problem can be solved.

$$V_{cr1} = \frac{R_2^2(1-k\varepsilon\sigma)^2 - R_1^2(1+k\varepsilon)^2 + \omega_1^2 R_1^2 R_2^2(1+k\sigma)^2(1-k\varepsilon\sigma)^2(C_1^2-C_2^2)}{\{R_2(1-k\varepsilon\sigma) + R_1(1+k\varepsilon)\}^2 + \omega_1^2 R_1^2 R_2^2(1+k\varepsilon)^2(1-k\varepsilon\sigma)^2(C_1+C_2)^2}$$
(1)

$$V_{cr2} = \frac{R_2^2(1-k\varepsilon\sigma)^2 - R_1^2(1+k\varepsilon)^2 + \omega_2^2 R_1^2 R_2^2(1+k\sigma)^2(1-k\varepsilon\sigma)^2(C_1^2-C_2^2)}{\{R_2(1-k\varepsilon\sigma) + R_1(1+k\varepsilon)\}^2 + \omega_2^2 R_1^2 R_2^2(1+k\varepsilon)^2(1-k\varepsilon\sigma)^2(C_1+C_2)^2}$$
(2)

$$V_{ci} = \frac{2\omega_1^2 R_1 R_2(1+k\varepsilon)(1-k\sigma\varepsilon)\{R_1 C_1(1+k\varepsilon) - R_2 C_2(1-k\varepsilon\sigma)\}}{\{R_2(1-k\varepsilon\sigma) + R_1(1+k\varepsilon)\}^2 + \omega_1^2 R_1^2 R_2^2(1+k\varepsilon)^2(1-k\varepsilon\sigma)^2(C_1+C_2)^2}$$
(3)

Digital domain calculation enables taking care of ε^2 , ε^3 terms. Our numerical calculations with typical parameter values show that ignoring ε^3 terms causes about 0.001% error , and ignoring both ε^2 , and ε^3 terms causes about 0.14% error. Hence, for greater than 16-bit accuracy, both terms must be taken into account. If we consider ε , ε^2 and ε^3 terms, the above equations are a third-order polynomial in ε , and hence we have three possible solutions for ε . However, we have developed an algorithm which chooses the correct value from the three values.

6. Advantages

Parasitic capacitance values are unknown and can change according to temperature and aging. Analog signal processing circuits may generate additional noises inside them. However the above-mentioned system and algorithm are digital, and they can reliably compensate for parasitic capacitance on-line. Delta-sigma ADCs with greater than 16bit accuracy and a few tens kilo Hertz bandwidth are now commercially available, and which, together with recent rapid progress of DSP, enable realization of our proposed system at low cost.

7. Conclusions

We have proposed a high-precision strain measurement system based on ADC and digital technology. The effectiveness of this system was investigated by simulation and by numerical calculation. Next we plan to implement the proposed system.

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Appendix

This appendix describes a conventional analog method to compensate parasitic capacitances C_1 (Fig.9). C_1 effects are compensated when the ratio of r_1 to r_2 is adjusted so that the current I_c which flows through C_1 is equal to I_3 , and then I_2 and the current which flows through R_1 are equal. (In other words, parasitic capacitance C1 is compensated by C_c, r_1, r_2 and the operational amplifier). Currents I_1, I_2, I_3 and voltages V_D, V_M are given by

$$I_{1} = I_{2} + I_{3},$$

$$I_{1} = \frac{V_{in} - V_{B}}{Z_{1}}, \quad I_{2} = \frac{V_{B}}{R_{2}}, \quad I_{3} = \frac{V_{B} - V_{M}}{Z_{c}},$$

$$V_{M} = \frac{r_{2}}{r_{1} + r_{2}} V_{in}.$$

$$V_{B} = \frac{R_{2} + j\omega R_{1}R_{2}(C_{1} + \alpha C_{c})}{R_{1} + R_{2} + j\omega R_{1}R_{2}(C_{1} + C_{c})} V_{in}.$$

$$\left(\therefore z_{1} = \frac{R_{1}}{1 + j\omega R_{1}C_{1}}, \quad Z_{c} = \frac{1}{j\omega C_{c}} \right)$$

Hence, the current I_c which flows C_1 is given by

$$I_{c} = \frac{R_{1} + j\omega R_{1}R_{2}C_{c}(1-\alpha)}{R_{1} + R_{2} + j\omega R_{1}R_{2}(C_{1}+C_{c})}j\omega C_{1}V_{in}.$$

Also I_3 is can be rewritten as

$$I_3 = \frac{-\alpha R_1 - R_2(1-\alpha) + j\omega R_1 R_2 C_c(1-\alpha)}{R_1 + R_2 + j\omega R_1 R_2 (C_1 + C_c)} j\omega C_2 V_{in}$$

Here, $\alpha = r_2/(r_1 + r_2)$. Then we can derive that the condition for $I_c = I_3$ is

$$\frac{r_2}{r_1} = \frac{C_c R_c - C_1 R_1}{R_1 (C_c + C_1)}$$

With this condition, the parasitic capacitance C_1 effects are compensated. However, the value of C_1 may be unknown and also it may change according to temperature and aging, and furthermore automatic adjustment of the ratio r_2/r_1 is difficult; thus its complete cancellation with this analog method is very difficult.