

Spatial and Temporal Dynamics of Vision Chips Including Parasitic Inductances and Capacitances

Haruo KOBAYASHI[†] and Takashi MATSUMOTO^{††}, *Members*

SUMMARY There are two dynamics issues in vision chips: (i) The *temporal* dynamics issue due to the parasitic capacitors in a CMOS chip, and (ii) the *spatial* dynamics issue due to the regular array of processing elements in a chip. These issues are discussed in [1]–[3] for the resistor network with only associated parasitic capacitances. However, in this paper we consider also parasitic *inductances* as well as parasitic capacitances for a more precise network dynamics model. We show that in some cases the temporal stability condition for the network with parasitic inductances and capacitances is equivalent to that for the network with only parasitic capacitances, but in general they are not equivalent. We also show that the spatial stability conditions are equivalent in both cases.

key words: *network stability, vision chip, neuro chip, neural network, negative resistor*

1. Introduction

This study has been motivated by spatial versus temporal stability issues of analog image-processing neuro chips (vision chips). The image-smoothing vision chip in [4], for instance, consists of a regular array of photo-sensors with conductances $g_0 > 0$, $g_1 > 0$, $g_2 < 0$ (Fig.1). We refer the reader to [4] for the chip details. Since the chip involves negative conductances g_2 , both spatial and temporal stability issues naturally arise. There are two intriguing elements. First, our earlier numerical experiments suggested that generally a vision chip is temporally stable if and only if it is spatially stable, where spatial stability means that the node voltage distribution behaves “properly.” Second, spatial dynamics naturally induces a discrete linear dynamical system so that its stability should be checked by its eigenvalues. “A discrete linear dynamical system is stable if and only if all the eigenvalues lie inside the unit circle of the complex plane.” This statement turned out to be false. Namely, due to the noncausal nature of the dynamics, if λ is an eigenvalue, so is $1/\lambda$, and hence the stability condition for causal linear systems is never satisfied.

Most of the fundamental issues involving these two elements have been settled in [1], [2] for 1D and 2D ar-

ray cases. For instance, a network is temporally stable if and only if it is spatially stable, except for a set of Lebesgue measure zero in the parameter space. Another fundamental result was that a network is spatially stable if and only if the eigenvalues of the dynamics are off the unit circle, even though they can be outside the unit circle. These results are far from trivial. One of reasons that makes these results crucial is the boundary conditions associated with the finiteness of a network. Even if the eigenvalue conditions are satisfied, solutions can oscillate or explode if the boundary conditions are inappropriate.

Although the previous results in [1], [2] are completely rigorous, the results are for the resistor network with only associated parasitic capacitances; associated parasitic inductances are neglected as a first-order approximation. However, in this paper we consider parasitic inductances as well as parasitic capacitances for a more precise network dynamics model. We show in some cases that the temporal stability condition for the network with parasitic inductances and capacitances is equivalent to that for the network with only parasitic capacitances, but in general they are not equivalent. We also show that the spatial stability conditions are equivalent in both cases.

Our approach in this paper is a systematic exploitation of the circulant network structure for 1D cases; speaking roughly, a circulant network has a “ring” structure as shown in Fig. 2. The validity of such

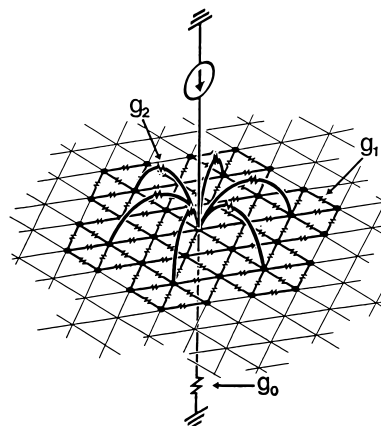


Fig. 1 The image-smoothing neuro chip. Only one unit is shown. © IEEE 1992.

Manuscript received June 20, 1998.

Manuscript revised September 30, 1998.

[†]The author is with the Department of Electronic Engineering, Faculty of Engineering, Gunma University, Kiryushi, 376-8515 Japan.

^{††}The author is with the Department of Electrical, Electronics and Computer Engineering, Waseda University, Tokyo, 169-8555 Japan.

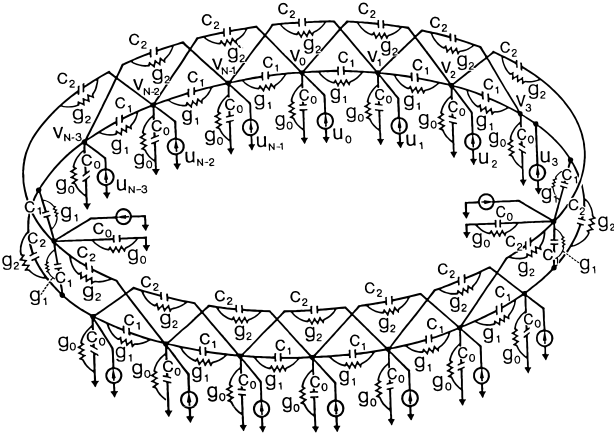


Fig. 2 A 1D circulant network with $m = 2$. © IEEE 1995.

an approach has already been discussed in [2], and we also remark that our results here can be extended to 2D cases in a similar manner to [2].

We also remark that the analysis method of resistor networks described here as well as [1]–[3], [5] may be useful not only for vision chips but also for the substrate noise analysis of the integrated circuits, where their substrates are modeled with 2D or 3D resistive networks [6].

2. Problem Formulation

Now let us consider a 1D network with N nodes numbered 0 through $N - 1$, where each node k is excited by a current source u_k and has an admittance y_0 to ground, and an admittance y_p to nodes $(k + p)$ for $p = \pm 1, \pm 2, \dots, \pm m$. Note that $y_p = y_{-p}$ because node k connects to node $k + p$ with y_p whereas node $(k + p)$ connects to node $((k + p) - p)$, i.e., node k with y_{-p} and hence $y_p = y_{-p}$. The network is said to be circulant if the rightmost and leftmost nodes are connected together, and thus the network is of a ring structure. Figure 2 shows a circulant network where $m = 2$ and the admittance y_p is composed of a conductance g_p and a capacitance c_p in parallel ($p = 0, 1, 2$, Fig. 3). Then we obtain the following equation from Kirchhoff's current law at node k :

$$-\left(y_0 + 2 \sum_{p=1}^m y_p\right) v_k + \sum_{p=1}^m y_p (v_{k-p} + v_{k+p}) + u_k = 0.$$

Then letting

$$\begin{aligned} \mathbf{v} &:= (v_0, v_1, \dots, v_{N-1})^T, \\ \mathbf{u} &:= (u_0, u_1, \dots, u_{N-1})^T, \\ \alpha_0 &:= -\left(y_0 + 2 \sum_{p=1}^m y_p\right), \end{aligned}$$

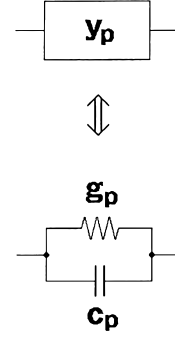


Fig. 3 The admittance y_p in Fig. 2 consists of a conductance g_p and a capacitance c_p in parallel ($p = 0, 1, 2$).

$$\alpha_p = y_p, \quad p = 1, 2, \dots, m,$$

the state equation is given by

$$Y\mathbf{v} + \mathbf{u} = \mathbf{0} \quad (1)$$

where

$$Y := \text{circl}(\alpha_0, \alpha_1, \dots, \alpha_m, 0, \dots, 0, \alpha_m, \dots, \alpha_1)$$

and $\text{circl}()$ denotes a circulant matrix [7]. Let F be a Fourier matrix with size $N \times N$ and note that $F^*F = I$, then Eq. (1) leads to

$$F^*YFF^*\mathbf{v} + F^*\mathbf{u} = \mathbf{0}. \quad (2)$$

It follows from [7] that F^*YF is diagonalized as follows:

$$F^*YF := \Lambda = \text{diag}(\lambda_0, \lambda_1, \dots, \lambda_{N-1}),$$

where

$$\begin{aligned} \lambda_k &:= \alpha_0 + 2 \sum_{p=1}^m \alpha_p \cos(2\pi pk/N) \\ k &= 0, 1, 2, \dots, N - 1. \end{aligned}$$

Letting

$$-F^*\mathbf{v} := \mathbf{o} = (o_0, o_1, \dots, o_{N-1})^T,$$

$$F^*\mathbf{u} := \mathbf{i} = (i_0, i_1, \dots, i_{N-1})^T,$$

then Eq. (2) reads

$$\Lambda \mathbf{o} = \mathbf{i}.$$

Thus if Λ is nonsingular, the followings are obtained:

$$\frac{o_0}{i_0} = \frac{1}{\lambda_0}, \quad \frac{o_1}{i_1} = \frac{1}{\lambda_1}, \dots, \quad \frac{o_{N-1}}{i_{N-1}} = \frac{1}{\lambda_{N-1}}.$$

Note that transfer functions can be defined as follows:

$$G_0(s) := \frac{o_0}{i_0}, \quad G_1(s) := \frac{o_1}{i_1}, \dots, \quad G_{N-1}(s) := \frac{o_{N-1}}{i_{N-1}}. \quad (3)$$

We see that the network is temporally stable if and only if all the transfer functions of $G_0(s), G_1(s), \dots, G_{i_{N-1}}(s)$ are stable, i.e., all of their poles are located in the left-half of the s -plane.

This statement is very general for the temporal stability of the network and is consistent to the previous results [1]–[3].

3. RCL Network

Now consider the case that the admittance y_p consists of a conductance g_p , a capacitance c_p and an inductance l_p as shown in Fig. 4, where g_p, c_p and l_p can be *negative*. Then the admittance y_p is given by

$$y_p = \frac{1}{1/g_p + sl_p} + sc_p = \frac{g_p + sc_p + s^2 l_p g_p c_p}{1 + sl_p g_p} \quad (4)$$

The reader may wonder why g_p, c_p and l_p can be negative. For the image processing purpose, some of g_p have to be negative [4] and this negative conductance $g_p < 0$ can be implemented with a positive conductance $g'_p (= -g_p) > 0$ and two admittance inverters as shown in Fig. 5. Let c'_p and l'_p be parasitic capacitance and inductance associated with g'_p . Even if c'_p and l'_p are positive, these can be effectively negative between the nodes A and B due to the two admittance inverters; the effective admittance y_p between the nodes A and B is equal to $-y'_p$. Note also that in the previous cases [1]–[3], the parasitic inductances were neglected as shown in Fig. 3 where $l_p = 0$.

Proposition 1 Consider the *RCL* network where y_p is given in Fig. 4 and also the following restriction is satisfied:

$$l_p g_p = d > 0 \quad \text{for } p = 0, 1, 2, \dots, m, \quad (5)$$

where d is a positive constant. In this case the temporal stability condition of the *RCL* network is equivalent to that of the *RC* network (where L is neglected as shown in Fig. 3.)

Proof : It follows from Eqs. (4) and (5) that α_p 's are given by

$$\alpha_0 := -\frac{(g_0 + 2 \sum_{p=1}^m g_p) + (s + s^2 d)(c_0 + 2 \sum_{p=1}^m c_p)}{1 + sd},$$

$$\alpha_p := y_p = \frac{g_p + sc_p + s^2 dc_p}{1 + sd} \quad \text{for } p = 1, 2, \dots, m.$$

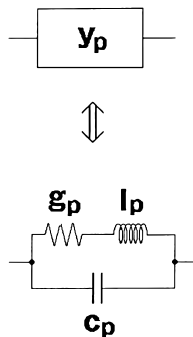


Fig. 4 The admittance y_p consists of a conductance g_p , a capacitance c_p and an inductance l_p .

Then the transfer functions of $G_k(s)$ described in Eq. (3) are given by

$$G_k(s) = \frac{-(1 + sd)}{-\mu_k + s\nu_k + s^2 d\nu_k} \quad (6)$$

for $k = 0, 1, 2, \dots, N - 1$, where

$$\mu_k := -\left(g_0 + 2 \sum_{p=1}^m g_p\right) + 2 \sum_{p=1}^m g_p \cos(2\pi pk/N)$$

$$\nu_k := \left(c_0 + 2 \sum_{p=1}^m c_p\right) - 2 \sum_{p=1}^m c_p \cos(2\pi pk/N).$$

From the Routh-Hurwitz stability criteria, we obtain the following temporal stability condition:

$$-\mu_k > 0, \quad \nu_k > 0, \quad d\nu_k > 0, \quad d\nu_k^2 > 0$$

for $k = 0, 1, 2, \dots, N-1$. Then the above conditions yield to the following:

$$\mu_k < 0, \quad \nu_k > 0, \quad d > 0 \quad (7)$$

for $k = 0, 1, 2, \dots, N - 1$. Let us compare this result (Eq. (7)) to the *RC* network case in [1], [2]. “ $\mu_k < 0$ for all $k = 0, 1, 2, \dots, N - 1$ ” means that the system matrix A [1], [2] is negative definite, and “ $\nu_k > 0$ for all $k = 0, 1, 2, \dots, N - 1$ ” is equivalent to that the capacitance matrix B [1], [2] is positive definite. We see that the temporal stability condition of the *RCL* network which satisfies Eq. (5) is equivalent to that of the *RC* network. (Q.E.D.)

Remark (i) In general, when y_p is given by Eq. (4), the transfer functions of (3) become so complicated that they are difficult to solve analytically. However, if we assume Eq. (5), we can solve them as shown above.

(ii) The assumption given by Eq. (5) is, in some sense,

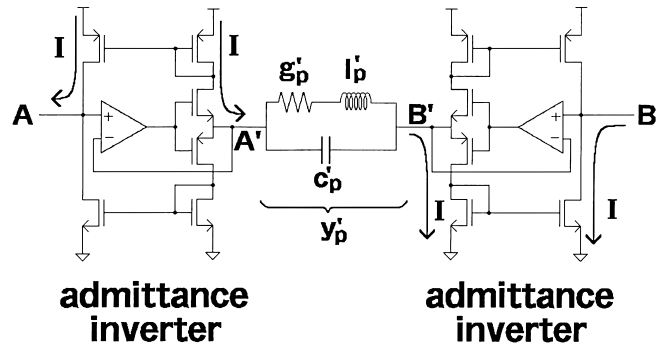


Fig. 5 Admittance inverters can realize *negative* conductances, capacitances and inductances. Suppose that V_A, V_B, V'_A and V'_B are node voltages of A, B, A' and B' , and also $V_A > V_B$. Then, due to the voltage followers, $V_A \approx V'_A$ and $V_B \approx V'_B$, and the current I flows from node A' to B' with $I = y'_p(V'_A - V'_B)$. We see that the current I effectively flows from node B to node A with $I = y'_p(V_A - V_B)$ and the admittance y_p between node A and B is effectively equal to $-y'_p$.

reasonable; in many conductors, their conductance values g_p are inversely proportional to their length while their inductance values l_p are proportional to their length. Hence if all conductances for $p = 0, 1, \dots, m$ are made with the same material, $l_p g_p$ yields to a positive constant d .

Lemma 1 When Eq. (5) is satisfied and the network is stable, the transfer function given by Eq. (6) is minimum-phase-shift.

Proof : Since $d > 0$, the zero of the transfer function given by Eq. (6) is $-1/d < 0$. Then its all poles and zero are located in the left-half of the s -plane. (Q.E.D.)

When Eq. (5) is satisfied and the network is stable, the transfer function of Eq. (6) can be rewritten as follows:

$$\begin{aligned} G_k(s) &= \frac{-(1+sd)}{-\mu_k + s\nu_k + s^2 d\nu_k} \\ &:= \frac{-K_k(1+sd)}{\omega_k^2 + 2\xi_k \omega_k s + s^2} \end{aligned} \quad (8)$$

where

$$\begin{aligned} K_k &:= \frac{1}{d\nu_k}, & \omega_k &:= \sqrt{\frac{-\mu_k}{d\nu_k}}, \\ \xi_k &:= \frac{1}{2} \sqrt{\frac{\nu_k}{-d\mu_k}} \end{aligned} \quad (9)$$

for $k = 0, 1, 2, \dots, N-1$. Then, according to the theory of the second-order system, we obtain the following:

Lemma 2 (i) When $0 < \xi_k < 1$ (i.e., $0 < \nu_k < -4d\mu_k$), the step response of the transfer function (8) shows *no* overshoot.

(ii) When $\xi_k = 1$ (i.e., $0 < \nu_k = -4d\mu_k$), its step response behaves as critical damping.

(iii) When $1 < \xi_k$ (i.e., $0 < -4d\mu_k < \nu_k$), its step response shows overshoot.

The circuit which consists of inductors, capacitors, and positive & negative resistors is extensively used as a sinusoidal signal oscillator in communication circuits. However the following lemma shows that the network which satisfies Eqs. (2) and (5) can not work as an oscillator.

Lemma 3 When Eq. (5) is satisfied, the transfer function of Eq. (8) can not be of the form

$$G_k(s) = \frac{-K_k(1+sd)}{\omega_k^2 + s^2}, \quad (10)$$

for $k = 0, 1, 2, \dots, N-1$.

Proof : If Eq. (10) is satisfied, ξ_k has to be zero. However, if $\xi_k = 0$, then $\nu_k = 0$ (see Eq. (9)) and the transfer function of Eq. (6) yields to

$$G_k(s) = \frac{1+sd}{\mu_k}$$

which is not of the form in Eq. (10). (Q.E.D.)

Proposition 2 If $l_p g_p \neq l_q g_q$ for some $0 \leq p, q \leq$

m , then the temporal stability condition of the *RCL* network is *not* necessarily equivalent to that of the *RC* network.

Proof : Consider the case $m = 1$ and $l_0 g_0 \neq l_1 g_1$. Then it follows from Eq. (4) that

$$\begin{aligned} y_0 &= \frac{g_0 + sc_0 + s^2 d_0 c_0}{1 + sd_0}, \\ y_1 &= \frac{g_1 + sc_1 + s^2 d_1 c_1}{1 + sd_1}, \end{aligned}$$

where $d_0 := l_0 g_0$, $d_1 := l_1 g_1$ and $d_0 \neq d_1$. Then the transfer functions described in Eq. (3) are given by

$$\begin{aligned} G_k(s) &= \frac{1}{\alpha_0 + 2\alpha_1 \cos(2\pi k/N)} \\ &= \frac{1}{-(y_0 + 2y_1) + 2y_1 \cos(2\pi k/N)} \\ &= -\frac{(1+sd_0)(1+sd_1)}{n_k(s)}, \end{aligned}$$

where

$$n_k(s) := e_0 + se_1 + s^2 e_2 + s^3 e_3,$$

$$e_0 := g_0 + \beta_k g_1,$$

$$e_1 := d_1 g_0 + \beta_k d_0 g_1 + c_0 + \beta_k c_1,$$

$$e_2 := (d_0 + d_1)(c_0 + \beta_k c_1),$$

$$e_3 := d_0 d_1 (c_0 + \beta_k c_1),$$

$$\beta_k := 2(1 - \cos(2\pi k/N)).$$

It follows from the Routh-Hurwitz stability criteria that the network temporal stability condition is given by

$$e_0 > 0, \quad e_1 > 0, \quad e_2 > 0, \quad e_3 > 0,$$

$$e_1 e_2 - e_0 e_3 > 0.$$

This result means that in addition to the negative definiteness of the system matrix A , the positive definiteness of the capacitance matrix B and $d_0 > 0, d_1 > 0$, we need to satisfy the following conditions:

$$d_1^2 g_0 + \beta_k d_0^2 g_1 + (d_0 + d_1)(c_0 + \beta_k c_1) > 0, \quad (11)$$

$$d_1 g_0 + \beta_k d_0 g_1 + c_0 + \beta_k c_1 > 0. \quad (12)$$

If $d_0 = d_1 > 0$ (i.e. Eq. (5) is satisfied), the negative definiteness of A and the positive definiteness B automatically lead to the above conditions (11), (12), however, if $d_0 \neq d_1$, they do not. Hence in this case the temporal stability condition is more strict than that in the *RC* network. (Q.E.D.)

Proposition 3 The spatial stability condition of the *RCL* network is equivalent to that of the *RC* network.

Proof : We need to consider the equilibrium point for the spatial stability, and at the equilibrium, the admittance y_p is given by

$$y_p|_{s=0} = \frac{1}{1/g_p + sl_p} + sc_p|_{s=0} = g_p.$$

This is the same as y_p at the equilibrium in the *RC* network, and hence the spatial stability conditions are equivalent in both cases. (Q.E.D)

Lemma 4 In the *RCL* network where y_p consists of g_p , c_p and l_p as shown in Fig. 4 and Eq. (5) is satisfied, the spatial and temporal stability conditions are virtually equivalent.

Proof : Note that the spatial and temporal stability conditions of the *RC* network are virtually equivalent [1], [2]. Then we see that according to this fact and Propositions 1, 3, the above statement is valid. (Q.E.D.)

4. Conclusion

We have analyzed the spatial and temporal dynamics for the positive and negative resistor network with parasitic inductances as well as parasitic capacitances. We have derived that in some cases the temporal stability condition for the network with the parasitic inductances and capacitances is equivalent to that for the network with only the parasitic capacitances, but in general they are not equivalent. We have also shown that the spatial stability conditions are equivalent in both cases.

Acknowledgement

We would like to thank Y. Togawa and K. Wilkinson for valuable discussions. This work was partially supported by the Sumitomo Foundation and Gunma University Foundation for Science and Technology.

References

- [1] T. Matsumoto, H. Kobayashi, and Y. Togawa, "Spatial versus temporal stability issues in image processing neuro chips," *IEEE Trans. Neural Networks*, vol.3, no.4, pp.540–569, July 1992.
- [2] H. Kobayashi, T. Matsumoto, and J. Sanekata, "Two-dimensional spatio-temporal dynamics of analog image processing neural networks," *IEEE Trans. Neural Networks*, vol.6, no.5, pp.1148–1164, Sept. 1995.
- [3] J.L. White and A.N. Willson, Jr., "On the equivalence of spatial and temporal stability for translation invariant linear resistive networks," *IEEE Trans. Circuits and Systems-I*, vol.39, no.9, pp.734–743, Sept. 1992.
- [4] H. Kobayashi, J.L. White, and A.A. Abidi, "An active resistor network for Gaussian filtering of images," *IEEE J. Solid-State Circuits*, vol.26, no.5, pp.738–748, May 1991.
- [5] H. Kobayashi and T. Matsumoto, "Spatial and temporal stability of vision chips including parasitic inductances and capacitances," *Proc. IEEE International Conference on*

Acoustics, Speech and Signal Processing, vol.2, pp.1089–1092, Seattle, May 1998.

- [6] A. Iwata and M. Nagata, "Substrate coupling noise simulation for AD mixed LSIs," *Proc. 11th Workshop on Circuits and Systems in Karuizawa*, pp.1–6, April 1998.
- [7] P.J. Davis, "Circulant Matrices," John Wiley & Sons, 1979.

Figure Credits

Fig. 1: Reprinted with permission from [1].

Fig. 2: Reprinted with permission from [2].



Haruo Kobayashi received the B.S. and M.S. degrees in information physics and mathematical engineering from University of Tokyo in 1980 and 1982 respectively, the M.S. degree in electrical engineering from UCLA in 1989, and the Dr.Eng. degree in electrical engineering from Waseda University in 1995. He joined Yokogawa Electric Corp. Tokyo, Japan in 1982, where he was engaged in the research and development related to

measuring instruments and mini-supercomputers. In 1997 he joined Gunma University and presently is an Associate Professor there. He was also an adjunct lecturer at Waseda University from 1994 to 1997. His research interests include analog & digital integrated circuit design and signal processing algorithms. He is a recipient of the 1994 Best Paper Award from the Japanese Neural Network Society.



Takashi Matsumoto received his B.Eng. degree in Electrical Engineering from Waseda University, Tokyo, Japan, M.S. degree in Applied Mathematics from Harvard University, Cambridge, MA, and Dr.Eng. degree in Electrical Engineering from Waseda University, Tokyo, Japan. He is presently Professor of Electrical, Electronics and Computer Engineering, Waseda University. His research interests include hierarchical Bayes approach to

nonlinear time series prediction, bifurcations/chaos, on-line hand writing recognition and CMOS circuit design. He is currently the Chairperson of the Special Committee on "Chaos/Mathematics and New Technology" of the Institute of Electrical Engineers of Japan, where he organizes various workshops on bifurcations, chaos, fractals, and their applications. He serves on the editorial board of *Circuits, Systems and Signal Processing*. He is a recipient of the 1994 Best Paper Award from the Japanese Neural Network Society. He is a coauthor of the book "Bifurcations," Springer-Verlag, 1993. He is a member of IEEE (Fellow), INNS, APS and IEEJ.