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Multitone Curve-Fitting Algorithms for Communication Application ADC Testing

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SUMMARY

This paper describes multitone curve-fitting algorithms for accurate determination of intermodulation distortion products in the multitone testing of ADCs used in communication applications and the like. Accuracy of our curve-fitting algorithms for coherent sampling (input frequencies known) and incoherent sampling (input frequencies unknown) was validated by numerical simulations. We found that—especially for incoherent sampling—these algorithms provide better accuracy than conventional (single-tone) curve-fitting algorithms. © 2003 Wiley Periodicals, Inc. Electron Comm Jpn Pt 2, 86(8): 1–11, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ecjb.10148

Key words: ADC; sine curve fitting; intermodulation distortion; multitone signal; mixed-signal LSI tester.

1. Introduction

Multitone input signals

$$V_{in}(t) = \sum_{k=1}^{n} A_k \sin(\omega_k t + \theta_k) + C \qquad (1)$$

are used in evaluating the intermodulation distortion (IMD) and noise power ratio [1–4] of ADCs used in measuring equipment for communication applications, such as mobile phone receiver analog front ends. $n = 256$ is used for ADSL applications, for example. For simplicity we consider the two-tone case ($n = 2$):

$$V_{in}(t) = A_1 \cos(\omega_1 t) + B_1 \sin(\omega_1 t) + A_2 \cos(\omega_2 t) + B_2 \sin(\omega_2 t) + C$$

When the ADC has some nonlinearities, its output has frequency components of $p\omega_1 + q\omega_2$ where $p, q = 0, \pm1, \pm2, \pm3, \ldots$ The signal components are at ω_1 and ω_2 while the

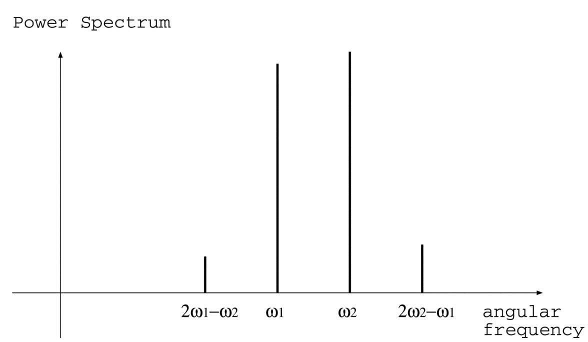

Fig. 1. Typical ADC output power spectrum for a two-tone input signal. Signal components are located at ω_1 and ω_2, while intermodulation components are at $m\omega_2 + n\omega_1$ ($m, n = 0, \pm1, \pm2, \pm3, \ldots$).

other components are mainly IMD due to ADC nonlinearity. The evaluation of third-order IMD components at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ is especially important because $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ can be close to the signal frequencies of ω_1 and ω_2 respectively when $\omega_1 \approx \omega_2$ (Fig. 1). However, testing methods for evaluating IMD of ADCs have not as yet been established; one reason is that there are no standard multitone signal generators, and another reason is that there are no good IMD evaluation algorithms. In this paper we will propose new algorithms which can evaluate IMD of ADCs very accurately.

2. FFT and Curve-Fitting Algorithms

The FFT method is successful for single-tone ADC testing [5–8], and is a good candidate for two-tone (or multitone) testing. However, it has the following drawbacks:

- Incoherent Sampling ADC Test Case [5, 6]:

Let us consider the case that the input signal and the sampling clock of the ADC are synchronized (Fig. 2). If ω_1 and ω_2 are integer multiples of ω_s/N, the FFT method can be used to evaluate IMD directly. (Here ω_s is the sampling angular frequency and N is the number of the captured data.) However, this condition (coherent sampling) is often difficult to satisfy; the incoherent sampling case is described below.

- Incoherent Sampling ADC Test Case [5, 6]:

Next let us consider the case that the input signal and the sampling clock of the ADC are *not* synchronized (Fig. 3). For example, when we test an ADC embedded in a system, its timebase may not be able to synchronize with the input signal. In such cases, a window function must be applied to the captured data before FFT [9], causing power spectrum skirts around the signal frequencies of ω_1, ω_2 which may hide the most important IMD components at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ (Fig. 4), because they are close to ω_1 and ω_2, respectively.

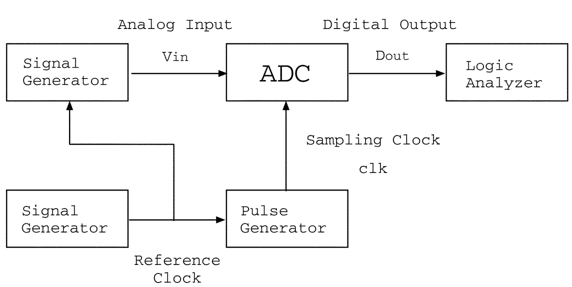

Fig. 2. An ADC test system using the coherent sampling method. A signal generator for the input signal and a pulse generator for the sampling clock are synchronized with the same reference clock.

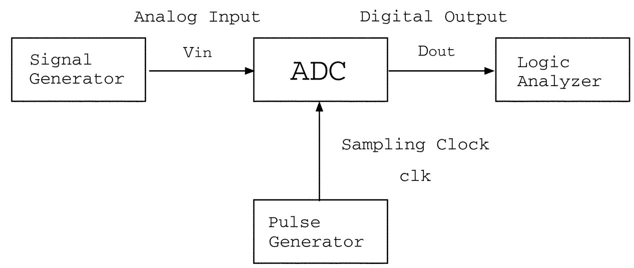

Fig. 3. An ADC test system using the incoherent sampling method. The input signal generator and the pulse generator for the sampling clock are not synchronized.

We note that in the incoherent sampling ADC test case, the signal generator for the analog inputs of ω_1, ω_2 and the pulse generator for the sampling clock of ω_s use different reference timing clocks (Fig. 3), whose timings can be slightly different. Hence, even if we set $\omega_1/(2\pi)$ of the signal generator to 1.0 MHz and $\omega_s/(2\pi)$ of the pulse generator to 1.0 MHz, the ratio of ω_1/ω_s is not exactly one.

To overcome these problems, we have developed two-tone (multitone) curve-fitting algorithms which are extensions of single-tone curve-fitting algorithms (Fig. 5) [5, 6, 8], and we have derived two algorithms for both coherent and incoherent sampling cases:

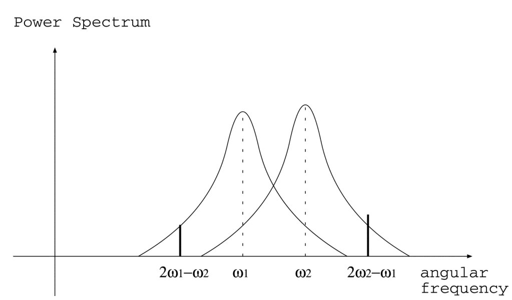

Fig. 4. ADC output power spectrum for two-tone signal after windowing. Intermodulation power spectrum components at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ are hidden in the power spectrum skirts of ω_1 and ω_2.

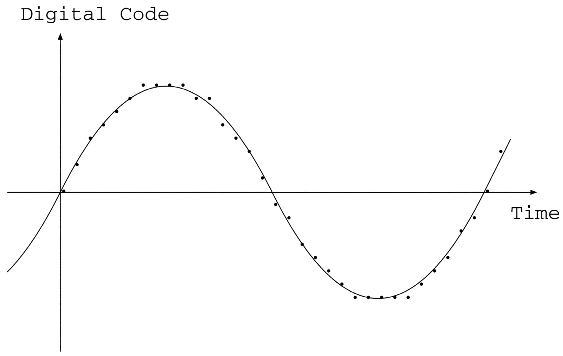

Fig. 5. Explanation of a single-tone curve-fitting algorithm. The dots show a typical ADC output for a sinusoidal input, and the solid line shows its fitted sine wave obtained by a sine curve-fitting algorithm. The fitted sine wave represents the signal component in the ADC output while the residual obtained by subtraction of the fitted sine wave from the ADC output represents noise and distortion components.

1. For coherent sampling ADC testing, we know exact ratios of the input angular frequencies to the sampling angular frequency $\omega_1/\omega_s$and ω_2/ω_s.

2. For incoherent sampling testing, exact ratios are not known (although we may have good estimates for their values).

In both cases window functions are unnecessary and we can evaluate IMD more precisely than with FFT methods. Section 3 describes the coherent sampling algorithm and Section 4 the incoherent sampling one. In both cases numerical simulations were performed to evaluate the effectiveness of the algorithms, and we found that—especially for incoherent sampling—our algorithms provide better accuracy than conventional (single-tone) curve-fitting algorithms.

3. Input Frequency Known Case

First let us consider the coherent sampling case (Fig. 2) where exact ratios of input frequencies to sampling frequency are known a priori.

3.1. Problem formulation

Let us assume the following multitone input to the ADC under test:

$$V_{in}(t) = \sum_{l=1}^{n}[A_l \cos(\omega_l t) + B_l \sin(\omega_l t)] + C \qquad (2)$$

Suppose that we have N samples of ADC output data $y(k)$ at time $2\pi k/\omega_s$ ($k = 0, 1, 2, \ldots, N-1$) for a multitone input of angular frequencies ω_l's, where the ratios of ω_l/ω_s's are known ($l = 0, 1, 2, \ldots, n-1$). We also assume that the ideal ADC output is given by

$$m(k) := \sum_{l=1}^{n}[a_l \cos(2\pi \frac{\omega_l}{\omega_s} k) + b_l \sin(2\pi \frac{\omega_l}{\omega_s} k)] + C \qquad (3)$$

Then we estimate a_l, b_l, and C from N samples of ADC output data record $y(k)$ according to the following least-squares-fit criteria:

$$P_e = \sum_{k=0}^{N-1}[y(k) - m(k)]^2 \rightarrow \text{ minimum} \qquad (4)$$

3.2. Solution

Since P_e in Eq. (4) is equal to

$$P_e = \sum_{k=0}^{N-1} \Big[y(k) - \sum_{l=1}^{n}[a_l \cos(2\pi \frac{\omega_l}{\omega_s} k) + b_l \sin(2\pi \frac{\omega_l}{\omega_s} k)] - C \Big]^2$$

and P_e should be minimized. Then

$$\frac{\partial P_e}{\partial a_l} = 0, \quad \frac{\partial P_e}{\partial b_l} = 0, \quad \frac{\partial e}{\partial C} = 0$$

where $l = 1, 2, \ldots, n$. Then we obtain the following algorithm:

$$\mathbf{x} = \mathbf{F}^{-1}\mathbf{y} \qquad (5)$$

Here

$$\mathbf{x} := (a_1, b_1, a_2, b_2, ..., a_n, b_n, C)^T$$

$$\mathbf{y} := \left(\sum_{k=0}^{N-1} y_k \alpha_{k1}, \sum_{k=0}^{N-1} y_k \beta_{k1}, \sum_{k=0}^{N-1} y_k \alpha_{k2}, \sum_{k=0}^{N-1} y_k \beta_{k2}, ..., \sum_{k=0}^{N-1} y_k \alpha_{kn}, \sum_{k=0}^{N-1} y_k \beta_{kn}, \sum_{k=0}^{N-1} y_k \right)^T$$

$$\mathbf{F} := (\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, ..., \mathbf{f}_{2n-1}, \mathbf{f}_{2n}, \mathbf{f}_{2n+1})$$

$$\mathbf{f}_1 := \left(\sum_{k=0}^{N-1} \alpha_{k1}^2, \sum_{k=0}^{N-1} \alpha_{k1}\beta_{k1}, \sum_{k=0}^{N-1} \alpha_{k1}\alpha_{k2}, \sum_{k=0}^{N-1} \alpha_{k1}\beta_{k2}, \right.$$

$$\ldots, \sum_{k=0}^{N-1} \alpha_{k1}\alpha_{kn}, \sum_{k=0}^{N-1} \alpha_{k1}\beta_{kn}, \sum_{k=0}^{N-1} \alpha_{k1} \Bigg)^T$$

$$\mathbf{f}_2 := \Bigg(\sum_{k=0}^{N-1} \beta_{k1}\alpha_{k1}, \sum_{k=0}^{N-1} \beta_{k1}^2, \sum_{k=0}^{N-1} \beta_{k1}\alpha_{k2}, \sum_{k=0}^{N-1} \beta_{k1}\beta_{k2}, \ldots, \sum_{k=0}^{N-1} \beta_{k1}\alpha_{kn}, \sum_{k=0}^{N-1} \beta_{k1}\beta_{kn}, \sum_{k=0}^{N-1} \beta_{k1} \Bigg)^T$$

$$\mathbf{f}_3 := \Bigg(\sum_{k=0}^{N-1} \alpha_{k2}\alpha_{k1}, \sum_{k=0}^{N-1} \alpha_{k2}\beta_{k1}, \sum_{k=0}^{N-1} \alpha_{k2}^2, \sum_{k=0}^{N-1} \alpha_{k2}\beta_{k2}, \ldots, \sum_{k=0}^{N-1} \alpha_{k2}\alpha_{kn}, \sum_{k=0}^{N-1} \alpha_{k2}\beta_{kn}, \sum_{k=0}^{N-1} \alpha_{k2} \Bigg)^T$$

$$\mathbf{f}_4 := \Bigg(\sum_{k=0}^{N-1} \beta_{k2}\alpha_{k1}, \sum_{k=0}^{N-1} \beta_{k2}\beta_{k1}, \sum_{k=0}^{N-1} \beta_{k2}\alpha_{k2}, \sum_{k=0}^{N-1} \beta_{k2}^2, \ldots, \sum_{k=0}^{N-1} \beta_{k2}\alpha_{kn}, \sum_{k=0}^{N-1} \beta_{k2}\beta_{kn}, \sum_{k=0}^{N-1} \beta_{k2} \Bigg)^T$$

$$\vdots$$

$$\mathbf{f}_{2n-1} := \Bigg(\sum_{k=0}^{N-1} \alpha_{kn}\alpha_{k1}, \sum_{k=0}^{N-1} \alpha_{kn}\beta_{k1}, \sum_{k=0}^{N-1} \alpha_{kn}\alpha_{k2}, \sum_{k=0}^{N-1} \alpha_{kn}\beta_{k} \ldots, \sum_{k=0}^{N-1} \alpha_{kn}^2, \sum_{k=0}^{N-1} \alpha_{kn}\beta_{k,n}, \sum_{k=0}^{N-1} \alpha_{kn} \Bigg)^T$$

$$\mathbf{f}_{2n} := \Bigg(\sum_{k=0}^{N-1} \beta_{kn}\alpha_{k1}, \sum_{k=0}^{N-1} \beta_{kn}\beta_{k1}, \sum_{k=0}^{N-1} \beta_{kn}\alpha_{k2}, \sum_{k=0}^{N-1} \beta_{kn}\beta_{k} \ldots, \sum_{k=0}^{N-1} \beta_{kn}\alpha_{kn}, \sum_{k=0}^{N-1} \beta_{kn}^2, \sum_{k=0}^{N-1} \beta_{kn} \Bigg)^T$$

$$\mathbf{f}_{2n+1} := \Bigg(\sum_{k=0}^{N-1} \alpha_{k1}, \sum_{k=0}^{N-1} \beta_{k1}, \sum_{k=0}^{N-1} \alpha_{k2}, \sum_{k=0}^{N-1} \beta_{k2}, \ldots, \sum_{k=0}^{N-1} \alpha_{kn}, \sum_{k=0}^{N-1} \beta_{kn}, N \Bigg)^T$$

$$\alpha_{kj} := \cos(2\pi \frac{\omega_j}{\omega_s} k), \quad \beta_{kj} := \sin(2\pi \frac{\omega_j}{\omega_s} k) \qquad j = 1, 2, \ldots, n$$

$\mathbf{F}^{-1}$ can be obtained from $\mathbf{F}$, for example, using the Cramer formula.

3.3. Algorithm evaluation

Next we will consider how to obtain the IMD from the algorithm.

Example 1: We have performed numerical simulations for the three-tone case ($n = 3$), where we use the model

$$m(k) = \sum_{l=1}^{3} A_l \sin(2\pi \frac{\omega_l}{\omega_s} k + \theta_l) + C$$

and estimate A_l, θ_l, and C. Then we consider the residual error

$$e(k) := y(k) - m(k)$$

and use the following model to estimate the third-order IMD:

$$\begin{aligned} m(k)' := & \\ & D_1 \sin(2\pi \frac{2\omega_1 - \omega_2}{\omega_s} k + \phi_1) \\ + \; & D_2 \sin(2\pi \frac{2\omega_1 - \omega_3}{\omega_s} k + \phi_2) \\ + \; & D_3 \sin(2\pi \frac{2\omega_2 - \omega_1}{\omega_s} k + \phi_3) \\ + \; & D_4 \sin(2\pi \frac{2\omega_2 - \omega_3}{\omega_s} k + \phi_4) \\ + \; & D_5 \sin(2\pi \frac{2\omega_3 - \omega_1}{\omega_s} k + \phi_5) \\ + \; & D_6 \sin(2\pi \frac{2\omega_3 - \omega_2}{\omega_s} k + \phi_6) \end{aligned}$$

Applying the least-squares-fit criteria,

$$\sum_{k=0}^{N-1} [e(k) - m(k)']^2 \rightarrow \text{ minimum}$$

we can estimate $D_1, \ldots, D_6, \phi_1, \ldots, \phi_6$ with the same algorithm as in Eq. (5). Table 1 shows numerical simulation results for $N = 8192$, $\omega_1/\omega_s = 0.09$, $\omega_2/\omega_s = 0.1006$, and $\omega_3/\omega_s = 0.1084$. We see that the algorithm in Eq. (5) can estimate the IMD components as well as the signal components with good accuracy.

Table 1. Simulation results of our proposed multitone curve-fitting algorithm for a three-tone input signal (input frequency known case)

parameter	actual value	estimated value
A_1	1.0	0.996654
A_2	1.0	0.995964
A_3	1.0	0.995191
θ_1	0.0 [deg]	-0.1
θ_2	45.0 [deg]	45.0021
θ_3	90.0 [deg]	90.1383
C	0.0	0.000286
D_1	0.3	0.299146
D_2	0.3	0.299598
D_3	0.3	0.299868
D_4	0.3	0.298836
D_5	0.3	0.299393
D_6	0.3	0.298914
ϕ_1	20 [deg]	20.0612
ϕ_2	40 [deg]	40.1761
ϕ_3	60 [deg]	59.9235
ϕ_4	80 [deg]	80.1635
ϕ_5	100 [deg]	99.9029
ϕ_6	120 [deg]	120.123

Example 2: Next we will consider the case that the ADC output has Gaussian noise of $n(k)$ (where the quantization noise of the ADC can be included):

$$y(k) = \sum_{l=1}^{3} A_l \sin(2\pi \frac{\omega_l}{\omega_s} k + \theta_l) + C + n(k)$$

Here $n(k)$ is Gaussian noise with zero mean and standard deviation of 0.125. Then we consider the following ADC output model:

$$m(k) = \sum_{l=1}^{3} A_l \sin(2\pi \frac{\omega_l}{\omega_s} k + \theta_l) + C$$

and we estimate A_1, A_2, A_3, θ_1, θ_2, θ_3, and C (Table 2).

Table 2. Simulation results of our proposed multitone curve-fitting algorithm for a three-tone input signal with Gaussian noise (input frequency known case)

parameter	actual value	estimated value
A_1	1.0	0.997322
A_2	1.0	0.995981
A_3	1.0	0.997945
θ_1	0.0 [deg]	-0.1800
θ_2	45.0 [deg]	45.0623
θ_3	90.0 [deg]	90.2995
C	0.0	0.002185

We see that, in both examples, our algorithm can estimate the parameter values very accurately.

Next let us consider how to obtain SNR using the estimated values of a_1, b_1, a_2, b_2, . . . , a_n, b_n, and C. The signal power P_s and noise power P_n in the ADC output are given by

$$P_s = \frac{1}{2}\sum_{l=1}^{n}(a_l^2 + b_l^2) + C^2, \qquad P_n = P_e/N$$

Then we have the following:

$$\begin{aligned} SNR &= 10\log_{10}\frac{P_s}{P_n} \quad [\text{dB}] \\ &= 10\log_{10}\frac{\sum_{l=1}^{n}(a_l^2 + b_l^2)/2 + C^2}{P_e/N} \quad [\text{dB}] \end{aligned}$$

4. Input Frequency Unknown Case

Next let us consider the incoherent sampling case (Fig. 3) where exact ratios of input frequencies to sampling frequency are not known a priori.

4.1. Problem formulation

Suppose that we have N samples of ADC output data $y(k)$ at time $2\pi k/\omega_s$ ($k = 0, 1, 2, \ldots, N-1$) for a two-tone input of ω_1 and ω_2 and exact ratios of ω_1/ω_s and ω_2/ω_s are unknown. We also assume that the ideal ADC output is given by

$$m(k) := \big[A_1 \sin(2\pi \frac{\omega_1}{\omega_s} k + \theta_1)$$

$$+A_2 \sin(2\pi \frac{\omega_2}{\omega_s} k + \theta_2)\big] + C \tag{6}$$

Then we estimate $A_1, A_2, \theta_1, \theta_2, \omega_1, \omega_2$ and C from N samples of ADC output data record $y(k)$ according to the criteria

$$P_e = \sum_{k=0}^{N-1} [y(k) - m(k)]^2 \rightarrow \text{ minimum} \tag{7}$$

4.2. Solution

Since P_e is equal to

$$P_e = \sum_{k=0}^{N-1} \Big[y(k) - A_1 \sin(2\pi \frac{\omega_1}{\omega_s} k + \theta_1) - A_2 \sin(2\pi \frac{\omega_2}{\omega_s} k + \theta_2) - C \Big]^2$$

and P_e should be minimized, then

$$\frac{\partial P_e}{\partial A_1} = 0, \quad \frac{\partial P_e}{\partial A_2} = 0, \quad \frac{\partial P_e}{\partial C} = 0$$

$$\frac{\partial P_e}{\partial \theta_1} = 0, \quad \frac{\partial P_e}{\partial \theta_2} = 0, \quad \frac{\partial P_e}{\partial \omega_1} = 0, \quad \frac{\partial P_e}{\partial \omega_2} = 0$$

Then we have the following:

$$\begin{aligned} 0 &= \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \alpha_{k1} \\ &- A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \alpha_{k1} \right\} \\ &- A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \alpha_{k1} \right\} \end{aligned}$$

$$\begin{aligned} 0 &= \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \alpha_{k2} \\ &- A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \alpha_{k2} \right\} \\ &- A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \alpha_{k2} \right\} \end{aligned}$$

$$\begin{aligned} 0 &= \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k1} \\ &- A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, k\beta_{k1} \right\} \\ &- A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, k\beta_{k1} \right\} \end{aligned}$$

$$\begin{aligned} 0 &= \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k2} \\ &- A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, k\beta_{k2} \right\} \\ &- A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, k\beta_{k2} \right\} \end{aligned}$$

$$\begin{aligned} 0 &= \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k1} \\ &- A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \beta_{k1} \right\} \\ &- A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \beta_{k1} \right\} \end{aligned}$$

$$\begin{aligned} 0 &= \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k2} \\ &- A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \beta_{k2} \right\} \\ &- A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \beta_{k2} \right\} \end{aligned}$$

Here

$$\bar{y} := \frac{1}{N} \sum_{k=0}^{N-1} y(k)$$

$$\bar{\alpha_j} := \frac{1}{N} \sum_{k=0}^{N-1} \alpha_{kj}, \quad \bar{\beta_j} := \frac{1}{N} \sum_{k=0}^{N-1} \beta_{kj}$$

$$\alpha_{kj} := \cos\left(2\pi \frac{\omega_j}{\omega_s} k + \theta_j\right)$$

$$\beta_{kj} := \sin\left(2\pi \frac{\omega_j}{\omega_s} k + \theta_j\right), \quad j = 1, 2$$

$$C = \bar{y} - \bar{\alpha_1} - \bar{\alpha_2}$$

These equations are nonlinear and we cannot solve them analytically; we have to solve them numerically. To do that, we will define R, S, T, U, V, and W (which indicate estimation errors) as follows:

$$R := \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \alpha_{k1} - A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \alpha_{k1} \right\} - A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \alpha_{k1} \right\}$$

$$S := \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \alpha_{k2} - A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \alpha_{k2} \right\} - A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \alpha_{k2} \right\}$$

$$T := \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k1} - A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, k\beta_{k1} \right\} - A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, k\beta_{k1} \right\}$$

$$U := \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k2} - A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, k\beta_{k2} \right\} - A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, k\beta_{k2} \right\}$$

$$V := \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k1} - A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \beta_{k1} \right\} - A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \beta_{k1} \right\}$$

$$W := \sum_{k=0}^{N-1} (y(k) - \bar{y})\, \beta_{k2} - A_1 \left\{ \sum_{k=0}^{N-1} (\alpha_{k1} - \bar{\alpha_1})\, \beta_{k2} \right\} - A_2 \left\{ \sum_{k=0}^{N-1} (\alpha_{k2} - \bar{\alpha_2})\, \beta_{k2} \right\}$$

When the estimated parameter values are equal to the actual values, all of R, S, T, U, V, and W are zeros. Now we will consider how to make an iteration algorithm to have the estimation errors of R, S, T, U, V, and W all be zero. Let us use a fitting function $z(k)$ defined as

$$z(k) := B_1 \sin(2\pi \frac{\psi_1}{\psi_s} k + \phi_1) + B_2 \sin(2\pi \frac{\psi_2}{\psi_s} k + \phi_2) + D$$

We will evaluate the fitting function $z(k)$ using the ADC output data. Letting $A_1, A_2, \omega_1, \omega_2, \theta_1, \theta_2$, and C be optimal estimates of parameter values (i.e., in this case, $R = S = T = U = V = W = 0$), the ADC output is approximated by

$$y(k) = A_1 \sin(2\pi \frac{\omega_1}{\omega_s} k + \theta_1) + A_2 \sin(2\pi \frac{\omega_2}{\omega_s} k + \theta_2) + C$$

Next we will derive an iteration algorithm using Taylor expansions:

$$R(B_1, B_2, \psi_1, \psi_2, \phi_1, \phi_2) := \left.\frac{\partial R}{\partial B_1}\right| (B_1 - A_1) + \left.\frac{\partial R}{\partial B_2}\right| (B_2 - A_2) + \left.\frac{\partial R}{\partial \psi_1}\right| (\psi_1 - \omega_1) + \left.\frac{\partial R}{\partial \psi_2}\right| (\psi_2 - \omega_2) + \left.\frac{\partial R}{\partial \phi_1}\right| (\phi_1 - \theta_1) + \left.\frac{\partial R}{\partial \phi_2}\right| (\phi_2 - \theta_2)$$

$$S(B_1, B_2, \psi_1, \psi_2, \phi_1, \phi_2) := \left.\frac{\partial S}{\partial B_1}\right| (B_1 - A_1) + \left.\frac{\partial S}{\partial B_2}\right| (B_2 - A_2) + \left.\frac{\partial S}{\partial \psi_1}\right| (\psi_1 - \omega_1) + \left.\frac{\partial S}{\partial \psi_2}\right| (\psi_2 - \omega_2) + \left.\frac{\partial S}{\partial \phi_1}\right| (\phi_1 - \theta_1) + \left.\frac{\partial S}{\partial \phi_2}\right| (\phi_2 - \theta_2)$$

$$T(B_1, B_2, \psi_1, \psi_2, \phi_1, \phi_2) := \left.\frac{\partial T}{\partial B_1}\right| (B_1 - A_1) + \left.\frac{\partial T}{\partial B_2}\right| (B_2 - A_2) + \left.\frac{\partial T}{\partial \psi_1}\right| (\psi_1 - \omega_1) + \left.\frac{\partial T}{\partial \psi_2}\right| (\psi_2 - \omega_2) + \left.\frac{\partial T}{\partial \phi_1}\right| (\phi_1 - \theta_1) + \left.\frac{\partial T}{\partial \phi_2}\right| (\phi_2 - \theta_2)$$

$$U(B_1, B_2, \psi_1, \psi_2, \phi_1, \phi_2) := \left.\frac{\partial U}{\partial B_1}\right| (B_1 - A_1) + \left.\frac{\partial U}{\partial B_2}\right| (B_2 - A_2) + \left.\frac{\partial U}{\partial \psi_1}\right| (\psi_1 - \omega_1) + \left.\frac{\partial U}{\partial \psi_2}\right| (\psi_2 - \omega_2) + \left.\frac{\partial U}{\partial \phi_1}\right| (\phi_1 - \theta_1) + \left.\frac{\partial U}{\partial \phi_2}\right| (\phi_2 - \theta_2)$$

$$V(B_1, B_2, \psi_1, \psi_2, \phi_1, \phi_2) := \left.\frac{\partial V}{\partial B_1}\right| (B_1 - A_1) + \left.\frac{\partial V}{\partial B_2}\right| (B_2 - A_2) + \left.\frac{\partial V}{\partial \psi_1}\right| (\psi_1 - \omega_1) + \left.\frac{\partial V}{\partial \psi_2}\right| (\psi_2 - \omega_2) + \left.\frac{\partial V}{\partial \phi_1}\right| (\phi_1 - \theta_1) + \left.\frac{\partial V}{\partial \phi_2}\right| (\phi_2 - \theta_2)$$

$$
\begin{aligned}
&W\left(B_1, B_2, \psi_1, \psi_2, \phi_1, \phi_2\right) \\
:=\ & \left.\frac{\partial W}{\partial B_1}\right|\left(B_1 - A_1\right) + \left.\frac{\partial W}{\partial B_2}\right|\left(B_2 - A_2\right) + \left.\frac{\partial W}{\partial \psi_1}\right|\left(\psi_1 - \omega_1\right) \\
+\ & \left.\frac{\partial W}{\partial \psi_2}\right|\left(\psi_2 - \omega_2\right) + \left.\frac{\partial W}{\partial \phi_1}\right|\left(\phi_1 - \theta_1\right) + \left.\frac{\partial W}{\partial \phi_2}\right|\left(\phi_2 - \theta_2\right)
\end{aligned}
$$

These equations are linear and we can estimate optimal values of B_1, B_2, ψ_1, ψ_2, ϕ_1, ϕ_2, and D. Letting the estimated values of B_1, B_2, ψ_1, ψ_2, ϕ_1, ϕ_2, and D be the new values of A_1, A_2, ω_1, ω_2, θ_1, θ_2, C in the above equations, we have the following iteration algorithm:

$$
\mathbf{x}_{(n+1)} = \mathbf{x}_{(n)} + \mathbf{F}_{(n)}^{-1} \cdot \mathbf{y}_{(n)} \tag{8}
$$

Here

$$
\mathbf{x}_{(n)} := \left(A_{1(n)}, A_{2(n)}, \omega_{1(n)}, \omega_{2(n)}, \theta_{1(n)}, \theta_{2(n)}\right)^T
$$

$A_{1(n)}$, $A_{2(n)}$, $\omega_{1(n)}$, $\omega_{2(n)}$, $\theta_{1(n)}$, and $\theta_{2(n)}$ are n-th iteration estimates for A_1, A_2, ω_1, ω_2, θ_1, and θ_2, respectively, and

$$
\mathbf{y}_{(n)} := \left(R_{(n)}, S_{(n)}, T_{(n)}, U_{(n)}, V_{(n)}, W_{(n)}\right)^T
$$

$$
\mathbf{F}_{(n)} :=
\begin{pmatrix}
\frac{\partial R_{(n)}}{\partial A_{1(n)}} & \frac{\partial R_{(n)}}{\partial A_{2(n)}} & \frac{\partial R_{(n)}}{\partial \omega_{1(n)}} & \frac{\partial R_{(n)}}{\partial \omega_{2(n)}} & \frac{\partial R_{(n)}}{\partial \theta_{1(n)}} & \frac{\partial R_{(n)}}{\partial \theta_{2(n)}} \\
\frac{\partial S_{(n)}}{\partial A_{1(n)}} & \frac{\partial S_{(n)}}{\partial A_{2(n)}} & \frac{\partial S_{(n)}}{\partial \omega_{1(n)}} & \frac{\partial S_{(n)}}{\partial \omega_{2(n)}} & \frac{\partial S_{(n)}}{\partial \theta_{1(n)}} & \frac{\partial S_{(n)}}{\partial \theta_{2(n)}} \\
\frac{\partial T_{(n)}}{\partial A_{1(n)}} & \frac{\partial T_{(n)}}{\partial A_{2(n)}} & \frac{\partial T_{(n)}}{\partial \omega_{1(n)}} & \frac{\partial T_{(n)}}{\partial \omega_{2(n)}} & \frac{\partial T_{(n)}}{\partial \theta_{1(n)}} & \frac{\partial T_{(n)}}{\partial \theta_{2(n)}} \\
\frac{\partial U_{(n)}}{\partial A_{1(n)}} & \frac{\partial U_{(n)}}{\partial A_{2(n)}} & \frac{\partial U_{(n)}}{\partial \omega_{1(n)}} & \frac{\partial U_{(n)}}{\partial \omega_{2(n)}} & \frac{\partial U_{(n)}}{\partial \theta_{1(n)}} & \frac{\partial U_{(n)}}{\partial \theta_{2(n)}} \\
\frac{\partial V_{(n)}}{\partial A_{1(n)}} & \frac{\partial V_{(n)}}{\partial A_{2(n)}} & \frac{\partial V_{(n)}}{\partial \omega_{1(n)}} & \frac{\partial V_{(n)}}{\partial \omega_{2(n)}} & \frac{\partial V_{(n)}}{\partial \theta_{1(n)}} & \frac{\partial V_{(n)}}{\partial \theta_{2(n)}} \\
\frac{\partial W_{(n)}}{\partial A_{1(n)}} & \frac{\partial W_{(n)}}{\partial A_{2(n)}} & \frac{\partial W_{(n)}}{\partial \omega_{1(n)}} & \frac{\partial W_{(n)}}{\partial \omega_{2(n)}} & \frac{\partial W_{(n)}}{\partial \theta_{1(n)}} & \frac{\partial W_{(n)}}{\partial \theta_{2(n)}}
\end{pmatrix}
$$

$$
\begin{aligned}
R_{(n)} &:= \sum_{k=0}^{N-1} \left(y(k) - \bar{y}\right) \alpha_{k1(n)} \\
&- A_{1(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k1(n)} - \bar{\alpha}_{1(n)}\right) \alpha_{k1(n)} \right\} \\
&- A_{2(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k2(n)} - \bar{\alpha}_{2(n)}\right) \alpha_{k1(n)} \right\}
\end{aligned}
$$

$$
\begin{aligned}
S_{(n)} &:= \sum_{k=0}^{N-1} \left(y(k) - \bar{y}\right) \alpha_{k2(n)} \\
&- A_{1(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k1(n)} - \bar{\alpha}_{1(n)}\right) \alpha_{k2(n)} \right\} \\
&- A_{2(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k2(n)} - \bar{\alpha}_{2(n)}\right) \alpha_{k2(n)} \right\}
\end{aligned}
$$

$$
\begin{aligned}
T_{(n)} &:= \sum_{k=0}^{N-1} \left(y(k) - \bar{y}\right) \beta_{k1(n)} \\
&- A_{1(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k1} - \bar{\alpha}_{1(n)}\right) k\beta_{k1(n)} \right\} \\
&- A_{2(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k2(n)} - \bar{\alpha}_{2(n)}\right) k\beta_{k1(n)} \right\}
\end{aligned}
$$

$$
\begin{aligned}
U_{(n)} &:= \sum_{k=0}^{N-1} \left(y(k) - \bar{y}\right) \beta_{k2(n)} \\
&- A_{1(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k1(n)} - \bar{\alpha}_{1(n)}\right) k\beta_{k2(n)} \right\} \\
&- A_{2(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k2(n)} - \bar{\alpha}_{2(n)}\right) k\beta_{k2(n)} \right\}
\end{aligned}
$$

$$
\begin{aligned}
V_{(n)} &:= \sum_{k=0}^{N-1} \left(y(k) - \bar{y}\right) \beta_{k1(n)} \\
&- A_{1(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k1(n)} - \bar{\alpha}_{1(n)}\right) \beta_{k1(n)} \right\} \\
&- A_{2(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k2(n)} - \bar{\alpha}_{2(n)}\right) \beta_{k1(n)} \right\}
\end{aligned}
$$

$$
\begin{aligned}
W_{(n)} &:= \sum_{k=0}^{N-1} \left(y(k) - \bar{y}\right) \beta_{k2(n)} \\
&- A_{1(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k1(n)} - \bar{\alpha}_{1(n)}\right) \beta_{k2(n)} \right\} \\
&- A_{2(n)} \left\{ \sum_{k=0}^{N-1} \left(\alpha_{k2(n)} - \bar{\alpha}_{2(n)}\right) \beta_{k2(n)} \right\}
\end{aligned}
$$

$$
\bar{\alpha}_{j(n)} := \frac{1}{N} \sum_{k=0}^{N-1} \alpha_{kj(n)}, \quad \bar{\beta}_{j(n)} := \frac{1}{N} \sum_{k=0}^{N-1} \beta_{kj(n)}
$$

$$
\alpha_{kj(n)} := \cos\left(2\pi \frac{\omega_{j(n)}}{\omega_s} k + \theta_{j(n)}\right)
$$

$$
\beta_{kj(n)} := \sin\left(2\pi \frac{\omega_{j(n)}}{\omega_s} k + \theta_{j(n)}\right)
$$

$$
j = 1, 2
$$

$$
\bar{y} = \frac{1}{N} \sum_{k=0}^{N-1} y(k), \qquad C = \bar{y} - \bar{\alpha}_1 - \bar{\alpha}_2
$$

Note that as $A_{1(n)}$, $A_{2(n)}$, $\omega_{1(n)}$, $\omega_{2(n)}$, $\theta_{1(n)}$, and $\theta_{2(n)}$ converge to their corresponding actual values of A_1, A_2, ω_1, ω_2, θ_1, and θ_2, then all of $R_{(n)}$, $S_{(n)}$, $T_{(n)}$, $U_{(n)}$, $V_{(n)}$, and $W_{(n)}$ converge to zero.

4.3. Algorithm evaluation

Tables 3 and 4 show numerical simulation results for two-tone ADC testing in the input frequency unknown case, where "actual value" means the actual parameter value used in the simulation, "estimation" means the final estimated value obtained by the algorithm, and "initial guess" means the initial value used for the iteration to solve Eq. (8). (Table 4 is the case where Gaussian noise with zero mean, and standard deviation of 0.125, is added to the ADC output.) Tables 3(a) and 4(a) show the case where our two-tone curve-fitting algorithm is applied, and Tables 3(b) and 4(b) show the case where a conventional single-tone curve-fitting algorithm (Fig. 5) is applied iteratively. We see that our multitone curve-fitting algorithm can estimate the parameter values more accurately; we confirm this by several examples.

Table 3. Simulation results for two-tone input (input frequency unknown case, $N = 8192$)

	actual value	estimation	initial guess
ω_1/ω_s	2.2 x 10^{-4}	2.200 x 10^{-4}	2.0 x 10^{-4}
ω_2/ω_s	5.8 x 10^{-4}	5.800 x 10^{-4}	6.0 x 10^{-4}
A_1	1.0	1.0000	1.0
A_2	1.0	1.0000	1.0
θ_1	45.0 [deg]	45.0000	0.0
θ_2	90.0 [deg]	90.0000	0.0

(a) Estimation of $\omega_1/\omega_s, \omega_2/\omega_s$ components using our proposed multi-tone curve fitting algorithm.

	actual value	estimation	initial gues
ω_1/ω_s	2.2 x 10^{-4}	2.121x 10^{-4}	2.0 x 10^{-4}
ω_2/ω_s	5.8 x 10^{-4}	5.922x 10^{-4}	6.0 x 10^{-4}
A_1	1.0	0.9650	1.0
A_2	1.0	0.9670	1.0
θ_1	45.0 [deg]	59.1427	0.0
θ_2	90.0 [deg]	74.0683	0.0

(b) Estimation of $\omega_1/\omega_s, \omega_2/\omega_s$ components using iterative usage of a single-tone curve fitting algorithm. First, ω_1/ω_s component is estimated, and then the ω_2/ω_s component is estimated from the residue.

Table 4. Simulation results of our proposed two-tone curve-fitting algorithm when Gaussian noise is added (input frequency unknown case)

	actual value	estimated value	initial value
ω_1/ω_s	2.2 x 10^{-4}	2.200 x 10^{-4}	2.0 x 10^{-4}
ω_2/ω_s	5.8 x 10^{-4}	5.798 x 10^{-4}	6.0 x 10^{-4}
A_1	1.0	1.0011	1.0
A_2	1.0	1.0001	1.0
θ_1	45.0 [deg]	44.8025	0.0
θ_2	90.0 [deg]	90.2496	0.0

(a) In the case that our proposed multi-tone curve fitting algorithm is used.

	actual value	estimated value	initial value
ω_1/ω_s	2.2 x 10^{-4}	2.120x 10^{-4}	2.0 x 10^{-4}
ω_2/ω_s	5.8 x 10^{-4}	5.920x 10^{-4}	6.0 x 10^{-4}
A_1	1.0	0.9650	1.0
A_2	1.0	0.9663	1.0
θ_1	45.0 [deg]	59.1427	0.0
θ_2	90.0 [deg]	74.3681	0.0

(b) In the case that a conventional single-tone curve fitting algorithm is used iteratively.

We can also estimate the IMD components by subtracting the estimated two-tone curves from the ADC output data and applying the multitone curve-fitting algorithm to the residual (which is similar to the input known case in Section 3).

5. Conclusions

We have developed multitone curve-fitting algorithms for accurate determination of intermodulation distortion products in the multitone testing of ADCs used in communication applications. Accuracy of our curve-fitting algorithms for coherent sampling (input frequencies known) and incoherent sampling (input frequencies unknown) was validated by numerical simulations. We will implement these algorithms in mixed-signal LSI testers [10] after completing the following work:

- Improvement of our algorithms (especially in input frequency unknown case) to reduce calculation load.

- Validation of our algorithms by applying them to measured ADC data.

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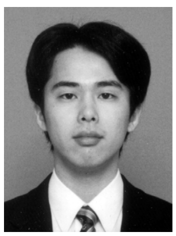 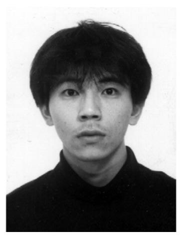  

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