ＲＣポリフェーズ・フィルタの解析と設計

- 入力インピーダンス、出力終端、素子ばらつきの影響の解析、通過域平坦利得フィルタ設計 -

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あらまし　携帯電話等の無線送受信機のアナログフロントエンド部で、直交信号 (I, Q 信号) 発生、イメージ除去のため用いられる RC ポリフェーズ・フィルタの設計論の確立のための解析を行ない、またＲＣパラメータ値の一設計法を提案する。具体的には、入力インピーダンスの導出と出力を終端したときの伝達関数の導出を行い、また抵抗値 R, 容量値 C の相対ばらつきの影響を解析し、これらの相対ばらつきによりイメージ信号成分が発生することを示した。さらに２次 RC ポリフェーズ・フィルタで通過域のゲインが平坦になるような R, C のパラメータ値の設計法を提案し、そのときのフィルタのイメージ除去比の公式を導出した。

キーワード　ポリフェーズ・フィルタ、イメージ信号除去、RF 回路、無線送受信器、複素信号

Analysis and Design of RC Polyphase Filters

- Input Impedance, Output Termination, Component Mismatch Effects, Flat-Passband Filter Design -

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Abstract　This paper analyzes first-, second- and third-order RC polyphase filters; these are important components used for image rejection and in quadrature signal generation circuits in the analog front-ends of wireless transceivers. We clarify their input impedance and the effects of output termination, parasitic capacitance, and component mismatches. We also propose a design algorithm for a flat-passband second-order filter.

Key words　Polyphase Filter, Image Rejection, RF Circuit, Wireless Transceiver, Complex Signal

1. Introduction

RC polyphase filters are important components in analog front-ends of wireless transceivers; they are used for In-Phase and Quadrature (I and Q) signal generation and for image rejection [1]〜[4]. In our previous papers, we have analyzed their transfer functions including parasitic capacitance effects [5]〜[7]. In this paper we analyze their characteristics in more detail; we describe their input impedance, and clarify the effects of output termination, parasitic capacitance, and component mismatches. Also, based on this explicit analysis, we propose a design algorithm for a flat-passband second-order filter.

In this paper, we use the following notation:

$$\omega_k := 1/(R_k C_k), \quad \omega_{ij} := 1/(R_i C_j) \quad (k, i, j = 1, 2, 3).$$

2. Input Impedance

We will consider the input impedance of RC polyphase filters. Let J_{I+}, J_{I-}, J_{Q+} and J_{Q-} respectively be the currents which flow from the input voltage sources I_{in+}, I_{in-}, Q_{in+}

and Q_{in-} into the RC polyphase filter. Then we have the following (complex) input impedance $Z_k(j\omega)$, $(k = 1, 2, 3)$:

$$Z_k(j\omega) := \frac{I_{in}(j\omega) + jQ_{in}(j\omega)}{J_I(j\omega) + jJ_Q(j\omega)}.$$

Here $J_{Iin}(j\omega) := J_{I+}(j\omega) - J_{I-}(j\omega)$,

$J_{Qin}(j\omega) := J_{Q+}(j\omega) - J_{Q-}(j\omega)$.

Fact 1 Input impedance $Z_1(j\omega)$ of the first-order filter (Fig.1) is given by

$$Z_1(j\omega) = \frac{1}{2j\omega C_1}\frac{1 + j\omega R_1 C_1}{1 + j(1 + \omega R_1 C_1)}.$$

Fact 2 Input impedance $Z_2(j\omega)$ of the second-order filter is given by

$$Z_2(j\omega) = \frac{1}{2}\frac{N_{Z2R}(\omega) + jN_{Z2I}(\omega)}{D_{Z2R}(\omega) + jD_{Z2I}(\omega)}.$$

Here $N_{Z2R}(\omega) :=$

$$(R_1 + R_2) - \omega^2(R_1^2R_2C_1^2 + R_1R_2^2C_2^2 + 4R_1^2R_2C_1C_2 + 2R_1R_2^2C_1C_2 + 2R_1^2R_2C_2^2),$$

$N_{Z2I}(\omega) :=$

$$\omega(R_1^2C_1 + 2R_1R_2C_1 + 4R_1R_2C_2 + R_2^2C_2 + 2R_1^2C_2) - \omega^3(2R_1^2R_2^2C_1^2C_2 + R_1^2R_2^2C_1C_2^2),$$

$D_{Z2R}(\omega) :=$

$$\omega(R_1C_1 + R_2C_2) + \omega^2(R_1^2C_1^2 + R_2^2C_2^2 + 6R_1C_1R_2C_2 + R_1^2C_1C_2 + R_1R_2C_1^2 + R_1R_2C_2^2 + R_2^2C_1C_2,) + \omega^3(R_1^2R_2C_1^2C_2 + R_1R_2^2C_1C_2^2),$$

$D_{Z2I}(\omega) :=$

$$-\ \omega(R_1C_1 + R_1C_2 + R_1C_2 + R_2C_1) + \omega^3(R_1C_1R_2^2C_2^2 + R_1^2R_2C_1^2C_2 + R_1R_2^2C_1^2C_2 + R_1^2R_2C_1C_2^2).$$

Fact 3 Input impedance $Z_3(j\omega)$ of the third-order filter is given by

$$Z_3(j\omega) = R_1^2R_3\frac{N_{Z3R}(\omega) + jN_{Z3I}(\omega)}{D_{Z3R}(\omega) + jD_{Z3I}(\omega)}.$$

Here $N_{Z3R}(\omega) :=$

$$\begin{aligned}
&1 - \omega^2(R_1R_2C_1C_2 + R_1R_3C_1C_2 + R_2R_3C_2C_3\\
&+ 2R_1R_2C_2C_3 + 2R_1R_3C_2C_3 + 2R_1R_3C_1C_3 + R_2^2C_2^2)\\
&+ \omega^4(R_1R_2^3C_1C_2^3 + R_2^3R_3C_2^3C_3 + R_1R_2^2R_3C_1C_2^3\\
&+ 2R_1R_2^3C_2^3C_3 + 2R_1R_2^2R_3C_2^3C_3 + +R_1R_2^2R_3C_1C_2^2C_3),
\end{aligned}$$

$N_{Z3I}(\omega) :=$

$$\begin{aligned}
&\omega(R_1C_1 + R_2C_2 + R_3C_3 + 2R_1C_2 + 2R_1C_3 + 2R_2C_3)\\
&- \omega^3(R_1R_2R_3C_1C_2C_3 + R_1R_2^2C_1C_2^2 + R_2^3C_2^3 + R_2^2R_3C_2^2C_3\\
&+ 2R_1R_2^2C_2^3 + 2R_1R_2^2C_2^2C_3 + 2R_2^3C_2^2C_3)\\
&+ \omega^5R_1R_2^3R_3C_1C_2^3C_3,
\end{aligned}$$

$$\begin{aligned}
D_{Z3R}(\omega) :=\ &-R_1R_2 - R_1R_3 - R_2R_3 - R_2^2\\
&- 2\omega(R_1R_2R_3C_3 + R_1R_2^2C_2 + R_1^2R_2C_1)\\
&+ \omega^2\big[R_1R_2^3(C_1C_2 - C_2^2 + 2C_1C_3 + 2C_2C_3)\\
&+ R_1R_2^2R_3(2C_1C_2 + 3C_2^2 + 4C_1C_3 + 4C_2C_3 + 2C_3^3)\\
&+ R_1R_2R_3^2(C_1C_3 + 3C_2C_3 - C_3^2) + R_2^3R_3(4C_2C_3 + C_2^2 + 2C_3^2)\\
&+ R_2^2R_3^2(2C_2 + C_3)C_3 + R_1^3R_3C_1(C_1 + 2C_2 + 2C_3)\\
&- R_1^3R_2C_1^2 + R_1^2R_2R_3(C_1^2 + 2C_1C_2 + 2C_2C_3)\\
&+ 2R_1^2R_3^2(C_1 + C_2)C_3 + R_1^2R_2^2C_1(C_1 - 4C_2)\big]\\
&- \omega^3\big[2R_1R_2^3(R_1C_1 + R_3C_3)C_2^2 + 2R_1^3R_2(R_2C_2 + R_3C_3)C_1^2\\
&+ 2R_1^2R_2R_3C_1C_3(4R_2C_2 + R_3C_3)\big]\\
&- \omega^4\big[R_1R_2^3R_3^2C_2C_3(C_1C_2 + C_1C_3 + C_2C_3 + C_2^2)\\
&+ R_1^3R_2^2R_3(2C_1C_3^3 + 2C_1C_2^2C_3 + 2C_1C_2^3\\
&+ 4C_1^3C_2 + 3C_1^2C_2^2 + 12C_1^2C_2C_3 + 2C_1^2C_3^2)\\
&+ R_1^3R_2^3C_1(C_1^2C_2 + 2C_1^2C_3 + C_1C_2^2 + 2C_1C_2C_3)\\
&+ R_1^3R_2R_3^2C_1^2(C_1C_3 + 3C_2C_3 + C_3^2)\\
&+ R_1^2R_2^3R_3(C_1^2C_2^2 + 2C_1C_2^3 + 2C_1C_3^3\\
&+ 4C_1^2C_2C_3 + 8C_1C_2^2C_3 + 2C_2^3C_3)\big]\\
&- R_1^2R_2^2R_3^2(C_1^2C_3^2 + 2C_1^2C_2C_3 + 4C_1C_2C_3^2\\
&+ 2C_1C_2^2C_3 + 2C_2^3C_3)\big]\\
&- \omega^5R_1^2R_2^2R_3C_1C_2C_3(R_1R_2C_1C_2 + R_1R_3C_1C_3 + R_2R_3C_2C_3)\\
&+ \omega^6R_1^3R_2^3R_3^2C_1^2C_2C_3(C_1 + C_2)(C_2 + C_3),
\end{aligned}$$

$D_{Z3I}(\omega) :=$

$$\begin{aligned}
&- \omega\big[R_1R_2^2(C_1 + 2C_2 + 2C_3) + R_1R_2R_3(C_1 + 3C_2 + 4C_3)\\
&+ R_2^3(C_2 + 2C_3) + 2R_2^2R_3(C_2 + 2C_3) + 2R_1^2R_3(C_1 + C_2 + C_3)\\
&+ (R_1 + R_2)R_3^2C_3\big] - \omega^2R_1^2R_2R_3C_1C_2\\
&+ \omega^3\big[R_1^3R_2^2C_1^2(C_1 + 2C_2 + 2C_3) + R_1^2R_2^3C_1^2(C_2 + 2C_3)\\
&+ R_1^3R_3^2C_1(C_1 + 2C_2)C_3 + R_2^3R_3^2C_2(C_2 + C_3)C_3\\
&+ 2R_1^2R_2^2R_3(C_1^2C_2 + C_1C_2^2 + 2C_1^2C_3 + C_2^2C_3 + 2C_2^3)\\
&+ R_1^2R_2R_3^2C_1(C_1 + 2C_2)C_3\\
&+ R_1^3R_2R_3C_1(C_1^2 + 2C_2C_3 + 3C_1C_2 + 4C_1C_3)\\
&+ R_1R_2^2R_3^2(C_1C_3^2 + 2C_1C_2C_3 + 3C_2^2C_3 + 4C_2C_3^2)\\
&+ R_1R_2^3R_3(C_1C_2^2 + 2C_1C_3^2 + 4C_1C_2C_3 + C_2^3 + 4C_2^2C_3 + 2C_2C_3^2)\big]\\
&- \omega^5\big[R_1^3R_2^3R_3C_1(2C_2^3C_3 + 4C_1^2C_2C_3 + C_1^2C_2^2\\
&+ 2C_1^2C_3^2 + C_1C_2^3 + 4C_1C_2^2C_3 + 2C_1C_2C_3^2)\\
&+ R_1^3R_2^2R_3^2C_1C_2C_3(2C_1^2C_2 + C_1^2C_3 + 3C_1C_2^2 + 2C_1C_2C_3 + 2C_2^3)\\
&+ R_1^2R_2^3R_3^2C_1C_2C_3(C_1C_2 + C_1C_3 + 2C_2^2)\big].
\end{aligned}$$

3. Output Termination

Sometimes the outputs of the RC polyphase filters are connected as shown in Fig.4 [2]; Q_{out+} and I_{out+} are connected, and Q_{out-} and I_{out+} are connected. Letting

$$\begin{aligned}
V_{outk+} &:= Q_{out+} = I_{out+}\\
V_{outk-} &:= Q_{out-} = I_{out-}\\
V_{outk} &:= V_{outk+} - V_{outk-} \quad (k = 1, 2, 3)
\end{aligned}$$

then we obtain the following input/output relationships:

Fact 4 First-order RC polyphase filter case:

$$V_{out1}(j\omega) = \frac{1}{2}I_{in}(j\omega) - \frac{1}{2}\frac{1 - j\omega R_1C_1}{1 + j\omega R_1C_1}Q_{in}(j\omega).$$

Note that $\omega_1 := 1/(R_1C_1)$,

$$V_{out1}(j\omega_1) = \tfrac{1}{2}[I_{in}(j\omega_1) + jQ_{in}(j\omega_1)],$$
$$V_{out1}(-j\omega_1) = \tfrac{1}{2}[I_{in}(-j\omega_1) - jQ_{in}(-j\omega_1)].$$

In other words,

$$V_{out1}(j\omega_1) = a_1 I_{in}(j\omega_1) + b_1 Q_{in}(j\omega_1),$$
$$\angle a_1 - \angle b_1 = -\frac{\pi}{2}, \qquad |a_1| = |b_1| = \frac{1}{2},$$

$$V_{out1}(-j\omega_1) = c_1 I_{in}(-j\omega_1) + d_1 Q_{in}(-j\omega_1),$$
$$\angle c_1 - \angle d_1 = \frac{\pi}{2}, \qquad |c_1| = |d_1| = \frac{1}{2}.$$

Fact 5 Second-order RC polyphase filter case:

$$\begin{aligned} V_{out2}(j\omega) = \\ & \big[(1+\omega^2 R_1R_2C_1C_2) + j\omega(R_1C_1 + R_2C_2)]I_{in}(j\omega) \\ - & [(1+\omega^2 R_1R_2C_1C_2) - j\omega(R_1C_1 + R_2C_2)]Q_{in}(j\omega)\big] \\ / & \big[2\big(1-\omega^2 R_1R_2C_1C_2) + j\omega(R_1C_1 + R_2C_2 + 2R_1C_2)\big)\big]. \end{aligned}$$

Note that when $R_1C_1 = R_2C_2$ and $\omega_1 := 1/(R_1C_1)$,

$$V_{out2}(j\omega_1) = \tfrac{1}{4}[(1-j)I_{in}(j\omega_1) + (1+j)Q_{in}(j\omega_1)],$$
$$V_{out2}(-j\omega_1) = \tfrac{1}{4}[(1+j)I_{in}(-j\omega_1) + (1-j)Q_{in}(-j\omega_1)].$$

In other words,

$$V_{out2}(j\omega_1) = a_2 I_{in}(j\omega_1) + b_2 Q_{in}(j\omega_1)],$$
$$\angle a_2 - \angle b_2 = -\frac{\pi}{2}, \qquad |a_2| = |b_2| = \frac{1}{2\sqrt{2}},$$

$$V_{out2}(-j\omega_1) = c_2 I_{in}(-j\omega_1) + d_2 Q_{in}(-j\omega_1)],$$
$$\angle c_2 - \angle d_2 = \frac{\pi}{2}, \qquad |c_2| = |d_2| = \frac{1}{2\sqrt{2}}.$$

Fact 6 Third-order RC polyphase filter case:

$$V_{out3}(j\omega) = \frac{1}{2}\frac{[A(\omega) + jB(\omega)]I_{in} - [C(\omega) + jD(\omega)]Q_{in}}{D_{V3R}(\omega) + jD_{V3I}(\omega)}$$

Here $A(\omega) := R_2 + R_3$
$$+\ \omega^2(R_1R_2^2C_1C_2 + R_1R_3^2C_1C_3 - R_2^2R_3C_2^2 - R_2R_3^2C_3^2)$$
$$-\ \omega^4 R_1R_2^2R_3^2C_1C_2C_3(C2 + C_3),$$

$B(\omega) :=$
$$\omega(R_1R_2C_1 + R_2^2C_2 + 2R_2R_3C_3 + R_1R_3C_1 + 2R_2R_3C_2$$
$$+R_3^2C_3) + \omega^3[2(R_2+R_3)R_1R_2R_3C_1C_2C_3 +$$
$$R_2^2R_3^2C_2C_3(C_2+C_3) + R_1R_2R_3C_1(R_2C_2 + R_3C_3)]$$

$C(\omega) := R_2 + R_3$
$$+\ \omega^2[R_1R_2^2C_1C_2 + R_1R_3^2C_1C_3 + R_2^2R_3C_2^2 + R_2R_3^2C_3^2$$
$$+\ 2R_2R_3(R_1C_1C_2 + R_1C_1C_3 + R_2C_2C_3 + R_3C_2C_3)]$$
$$+\ \omega^4 R_1R_2^2R_3^2C_1C_2C_3(C_2+C_3),$$

$D(\omega) :=$
$$-\ \omega(R_1R_2C_1 + R_1R_3C_1 + R_2^2C_2 + R_3^2C_3)$$
$$+\ \omega^3(R_1R_2^2R_3C_1C_2^2 + R_2^2R_3^2C_2^2C_3 + R_2^2R_3^2C_2C_3^2$$
$$+\ R_1R_2R_3^2C_1C_3^2),$$

$D_{V3R}(\omega) := R_2 + R_3$
$$-\ \omega^2[R_1R_2^2C_1C_2 + R_1R_3^2C_1C_3 + R_1R_2^2C_2C_3$$
$$+\ R_1R_2R_3C_3^2 + R_2R_3^2C_3^2 + R_2^2R_3C_2^2$$
$$+\ 2(R_1R_2R_3C_1C_2 + R_2R_3^2C_2C_3 + R_1R_2^2C_1C_3$$
$$+\ R_1R_3^2C_2C_3 + R_1R_2^2C_2C_3 + R_1R_2R_3C_2^2 + R_2^2R_3C_3^2$$
$$+\ R_1R_3^2C_3^2) + 4(R_1R_2R_3C_1C_3 + R_2^2R_3C_2C_3)$$
$$+\ 7R_1R_2R_3C_2C_3 - R_1R_3^2C_3^2]$$
$$+\ \omega^4 R_1R_2^2R_3^2C_1C_2C_3(C_2+C_3),$$

$D_{V3I}(\omega) :=$
$$\omega[R_1R_2C_1 + R_1R_3C_1 + R_1R_3C_3 + R_2^2C_2 + R_3^2C_3$$
$$+\ 2(R_2R_3C_2 + R_1R_3C_2 + R_1R_2C_2 + R^2C_3)$$
$$+\ 3R_1R_2C_3 + 4R_2R_3C_3]$$
$$-\ \omega^3[R_1R_2^2R_3C_1C_2C_3 + R_2^2R_3^2C_2^2C_3 + R_2^2R_3^2C_2C_3^2$$
$$+\ R_1R_2^2R_3C_2C_3^2 + R_1R_2^2R_3C_1C_2^2 + R_1R_2R_3^2C_2C_3^2$$
$$+\ R_1R_2R_3^2C_1C_3^2 + 2(R_1R_2R_3^2C_1C_2C_3 + R_1R_2R_3^2C_2^2C_3$$
$$+\ R_1R_2^2R_3C_2^2C_3 + R_1R_2^2R_3C_2C_3^2 + R_1R_2^2R_3C_1C_3^2)$$
$$+\ 3R_1R_2^2R_3C_1C_2C_3].$$

Note that when $R_1 = R_2 = R_3$, $C_1 = C_2 = C_3$ and $\omega_1 := 1/(R_1C_1)$,

$$V_{out3}(j\omega_1) = -\frac{j}{4}[I_{in}(j\omega_1) + jQ_{in}(j\omega_1)],$$
$$V_{out3}(-j\omega_1) = \frac{j}{4}[I_{in}(-j\omega_1) - jQ_{in}(-j\omega_1)].$$

In other words,

$$V_{out3}(j\omega_1) = a_3 I_{in}(j\omega_1) + b_3 Q_{in}(j\omega_1),$$
$$\angle a_3 - \angle b_3 = -\frac{\pi}{2}, \qquad |a_3| = |b_3| = \frac{1}{4},$$

$$V_{out3}(-j\omega_1) = c_3 I_{in}(-j\omega_1) + d_3 Q_{in}(-j\omega_1),$$
$$\angle c_3 - \angle d_3 = \frac{\pi}{2}, \qquad |c_3| = |d_3| = \frac{1}{4}.$$

4. Component Mismatch Effects

Next we will study component mismatch effects. Suppose that the resistors (four R_1's) and capacitors (four C_1's) of the first-order RC polyphase filter in Fig.1) have (relative) component mismatches described as follows:

$$R_{1Q+} := R_1 + \Delta R_{1Q+}, \quad R_{1Q-} := R_1 + \Delta R_{1Q-},$$
$$R_{1I+} := R_1 + \Delta R_{1I+}, \quad R_{1I-} := R_1 + \Delta R_{1I-},$$
$$C_{1Q+} := C_1 + \Delta C_{1Q+}, \quad C_{1Q-} := C_1 + \Delta C_{1Q-},$$
$$C_{1I+} := C_1 + \Delta C_{1I+}, \quad C_{1I-} := C_1 + \Delta C_{1I-},$$

where

$$R_1 := (R_{1Q+} + R_{1I+} + R_{1Q-} + R_{1I-})/4,$$
$$C_1 := C_{1Q+} + C_{1I+} + C_{1Q-} + C_{1I-})/4,$$
$$\Delta R_{1Q+} + \Delta R_{1I+} + \Delta R_{1Q-} + \Delta R_{1I-} = 0,$$
$$\Delta C_{1Q+} + \Delta C_{1I+} + \Delta C_{1Q-} + \Delta C_{1I-} = 0.$$

Then we have the following:

Fact 6

$$V_{out}(j\omega) = G_1(j\omega)V_{in}(j\omega) + E_1(j\omega)\cdot\Delta X\cdot\overline{V_{in}(j\omega)}.$$

$$\overline{V_{out}(j\omega)} = \overline{G_1(j\omega)}\cdot\overline{V_{in}(j\omega)} + \overline{E_1(j\omega)}\cdot\Delta X\cdot V_{in}(j\omega).$$

Here

$$\overline{V_{in}} := I_{in} - jQ_{in}, \quad \overline{V_{out}} := I_{out} - jQ_{out},$$

$$\Delta X := (\frac{\Delta R_{I+}}{R_1} + \frac{\Delta R_{I-}}{R_1} + \frac{\Delta C_{I+}}{C_1} + \frac{\Delta C_{I-}}{C_1}),$$

$$G_1(j\omega) = \frac{1+\omega R_1C_1}{1+j\omega R_1C_1},$$

$$E_1(j\omega) := -\frac{(1+j)\omega R_1C_1}{2(1+j\omega R_1C_1)^2},\ \overline{E_1(j\omega)} := -\frac{(1-j)\omega R_1C_1}{2(1-j\omega R_1C_1)^2}.$$

The above fact tells us that when there are component mismatches among resistors and capacitors, the input image signal of $\overline{V_{in}(j\omega)}$ affects the output of $V_{out}(j\omega)$, and also the input signal of $V_{in}(j\omega)$ causes the output image signal of $\overline{V_{out}(j\omega)}$; these effects are the most significant at $\omega = \pm 1/(R_1C_1)$ because $|E_1(j\omega)|$ has a maximum value at $\omega = \pm 1/(R_1C_1)$. We will call $E_1(j\omega)$ an *image transfer function*; Table 1 and Fig.5 describe its characteristics.

Table 1: Characteristics of $E_1(j\omega)$.

ω	$E_1(j\omega)$	$\lvert E_1(j\omega)\rvert$
$-\infty$	0	0
$(-1-\sqrt{2})\omega_1$	- 1/4	1/4
$-\omega_1$	- 1/4 + j/4	$\sqrt{2}/4$
$(1-\sqrt{2})\omega_1$	j/4	1/4
0	0	0
$(-1+\sqrt{2})\omega_1$	- 1/4	1/4
ω_1	- 1/4 + j/4	$\sqrt{2}/4$
$(1+\sqrt{2})\omega_1$	j/4	1/4
∞	0	0

5. Design Algorithm for Flat-Passband Second-Order Filters

In this section, we propose a design algorithm for second-order filters (Fig.2) to make their passband gain flat. Let us consider the design of a second-order RC polyphase filter for image rejection; we will determine the four parameter values R_1, R_2, C_1 and C_2.

Flat Gain Second-order RC Polyphase Filter Design Algorithm :

- Filter Specification :

Stop band: $-\omega_a < \omega < -\omega_b$

Pass band: $\omega_b < \omega < \omega_a$.

Here

$$\frac{\omega_a}{\omega_b} < 12.63556. \tag{1}$$

Gain $|G_2(\jmath\omega)|$ is very flat in pass band.
$\big(|G_2(\jmath\omega)| \approx$ constant for $\min(\omega_1,\omega_2) < \omega < \max(\omega_1,\omega_2)\big)$.

- Design Algorithm:

Choose the values of R_1, R_2, C_1 and C_2 to satisfy

$$\frac{1}{R_1C_1} = \omega_a,\ \ \frac{1}{R_2C_2} = \omega_b,\ \ \frac{1}{R_2C_1} = \frac{-\beta+\sqrt{\beta^2-4\alpha\gamma}}{2\alpha} \tag{2}$$

or

$$\frac{1}{R_1C_1} = \omega_b,\ \ \frac{1}{R_2C_2} = \omega_a,\ \ \frac{1}{R_2C_1} = \frac{-\beta+\sqrt{\beta^2-4\alpha\gamma}}{2\alpha}. \tag{3}$$

Here $\quad \alpha := 6\omega_1^2 + 6\omega_2^2 + 4\omega_1\omega_2 - 8\sqrt{\omega_1\omega_2}(\omega_1+\omega_2),\ (4)$

$$\beta := 6\omega_1^3+6\omega_2^3+10\omega_1\omega_2(\omega_1+\omega_2)-8\sqrt{\omega_1\omega_2}(\omega_1+\omega_2)^2, \tag{5}$$

$$\gamma := \omega_1^4+\omega_2^4+2\omega_1\omega_2(\omega_1^2+\omega_2^2+5\omega_1\omega_2) - 4\sqrt{\omega_1\omega_2}(\omega_1^3+\omega_1^2\omega_2+\omega_1\omega_2^2+\omega_2^3). \tag{6}$$

- Image Rejection Ratio (IRR)

When eq.(2) or eq.eq.(3) is satisfied, the image rejection ratio is given by

$$IRR := 20\log_{10}\frac{|G_2(j\omega)|_{\omega_b<\omega<\omega_a}}{\max_{-\omega_a<\omega<-\omega_b}|G_2(j\omega)|}$$
$$= 20\log_{10}\frac{(\sqrt{\omega_a}+\sqrt{\omega_b})^2}{(\sqrt{\omega_a}-\sqrt{\omega_b})^2}\ [dB].$$

Remark The number of parameters (R_1,R_2,C_1,C_2) is 4 while the number of their constraint equations (eq.(2) or eq.(3)) is 3. So the designer can add one more constraint arbitrarily, for example, by considering their physical implementation.

Now we will explain why the proposed algorithm can make the passband gain flat. Note that the transfer function of the second-order filter is given by

$$G_2(j\omega) = \frac{(1+\omega R_1C_1)(1+\omega R_2C_2)}{1-\omega^2R_1C_1R_2C_2+j\omega(C_1R_1+C_2R_2+2R_1C_2)} \tag{7}$$

It follows from eq.(7) that $|G_2(\jmath\omega_1)| = |G_2(\jmath\omega_2)|$ is always satisfied, and

$$|G_2(j\omega_1)| = |G_2(j\omega_2)| = \frac{\sqrt{2}(\omega_1+\omega_2)}{\sqrt{\omega_1^2+\omega_2^2+2\omega_{21}(\omega_1+\omega_2+\omega_{21})}} \tag{8}$$

$$|G_2(j\sqrt{\omega_1\omega_2})| = \frac{(\sqrt{\omega_1}+\sqrt{\omega_2})^2}{\omega_1+\omega_2+2\omega_{21}}. \tag{9}$$

Recall Fact 8 in [7] and look at the Nyquist chart of $G_2(j\omega)$ in Fig.6 (b). Then we will propose the following algorithms: *For given ω_1 and ω_2, choose $\omega_{21} := 1/(R_2C_1)$ such that*

$$|G_2(j\omega_1)|(=|G_2(j\omega_2)|) = |G_2(j\sqrt{\omega_1\omega_2})|. \tag{10}$$

Using eqs.(8) and (9), we have the following equation to satisfy eq.(10):

$$\alpha\omega_{21}^2 + \beta\omega_{21} + \gamma = 0. \tag{11}$$

Here α, β and γ are defined by eqs.(4), (5), (6) respectively. Note that

$$\alpha \geqq 2(\omega_1-\omega_2)^2 \geqq 0, \qquad \beta \geqq 2(\omega_1-\omega_2)^2(\omega_1+\omega_2) \geqq 0,$$

we see that the condition that "eq.(11) has a positive real solution of ω_{21}" is "$\gamma < 0$". We have obtained the following from numerical calculation:

$$\gamma < 0 \quad \text{for } 0.079142 < \omega_1/\omega_2 < 12.63556.$$

In this case, we have

$$\omega_{21} = \frac{-\beta+\sqrt{\beta^2-4\alpha\gamma}}{2\alpha} > 0$$

and we obtain the above-mentioned proposed algorithm.

Our simulation showed that the passband gain becomes very flat (its ripple is less than 0.1%) when these conditions are satisfied (Figs.7, 8).

Next we will consider the image rejection ratio (IRR) when eq.(10) is satisfied. It follows from eq.(7) and Fig.6 (b) that

$$\max_{-\omega_a<\omega<-\omega_b} |G_2(j\omega)| = |G_2(-j\sqrt{\omega_1\omega_2})| = \frac{(\sqrt{\omega_1}-\sqrt{\omega_2})^2}{\omega_1+\omega_2+2\omega_{21}}.$$

Hence we have obtained the following IRR:

$$IRR := 20\log_{10}\frac{|G_2(j\omega)|_{\omega_b<\omega<\omega_a}}{\max_{-\omega_a<\omega<-\omega_b}|G_2(j\omega)|}$$

$$= 20\log_{10}\frac{|G_2(j\sqrt{\omega_1\omega_2})|}{|G_2(-j\sqrt{\omega_1\omega_2})|}$$

$$= 20\log_{10}\frac{(\sqrt{\omega_1}+\sqrt{\omega_2})^2}{(\sqrt{\omega_1}-\sqrt{\omega_2})^2}\ [dB].$$

We see that in order to have a large IRR, ω_1/ω_2 has to be close to one (in other words, it should not be that $\omega_1/\omega_2 << 1$ or $\omega_1/\omega_2 >> 1$) in any case; hence in our algorithm, eq.(1) is not a serious restriction.

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文 献

[1] S. Sheng and R. Broderson, *Low Power Wireless Communication Applications - A Wideband CDMA System Design - ,* Kluwer Academic Publishers (1998).

[2] F. Behbahani, Y. Kishigami, J. Leete and A. A. Abidi, "CMOS Mixers and Polyphase Filters for Large Image Rejection," *IEEE J. Solid-State Circuits,* vol.36, no.6, pp.873-887 (June 2001).

[3] A. Rofougaran, G. Chang, J. J. Rael, J. Y.-C. Chang, M. Rofougaran, P. J. Chang, M. Djafari, M.-K.Ku, E. R. Roth, A. A. Abidi and H. Samueli, "A Single-Chip 9–MHz Spread-Spectrum Wireless Transceiver in 1-μm CMOS - Part I; Architecture and Transmitter Design," *IEEE J. Solid-State Circuits,* vol.33, no.4, pp.515-534 (April 1998).

[4] M. Steyeart, M. Borremans, J. Janssens, B. D. Muer, N. Itoh, J. Carnicky, J. Crols, E. Morifuji, H. S. Momose and W. Sansen, "A Single-Chip CMOS Transceiver for DCS-1800 Wireless Communications," *ISSCC Digest of Technical Papers,* vol.41, pp.48-49 (Feb. 1998).

[5] J. Kang, H. Kobayashi, T. Kitahara, S. Takigami and H. Sadamura, "Explicit Analysis of RC Polyphase Filter for I, Q Signal Generation and Image Rejection," *The 15th Workshop on Circuits and Systems in Karuizawa,* pp.53-58 (April 2002).

[6] H. Kobayashi, J. Kang, T. Kitahara, S. Takigami and H. Sadamura," Explicit Transfer Function of RC Polyphase Filter for Wireless Transceiver Analog Front-End, *"2002 IEEE Asia-Pacific Conference on ASICs*, pp.137-140, Taipei, Taiwan (Aug. 2002).

[7] N. Yamaguchi, H. Kobayashi, J. Kang, Y. Niki and T. Kitahara, "Analysis of RC Polyphase Filters - Higher-Order Filter Transfer Function, Nyquist Chart, Parasitic Capacitance Effects - ", *IEICE Technical Meeting of Circuits and Systems,* Wakayama (Jan. 2003).

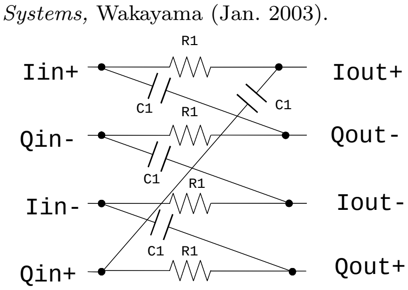

Fig.1: The first-order RC polyphase filter.

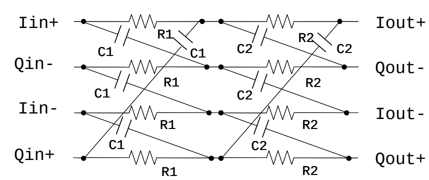

Fig.2: The second-order RC polyphase filter.

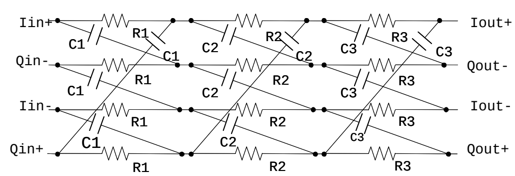

Fig.3: The third-order RC polyphase filter.

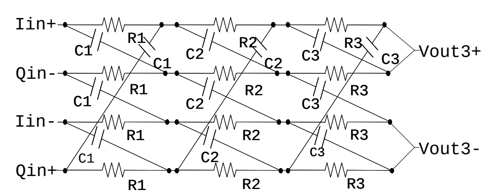

Fig.4: Output termination of the third-order RC polyphase filter.

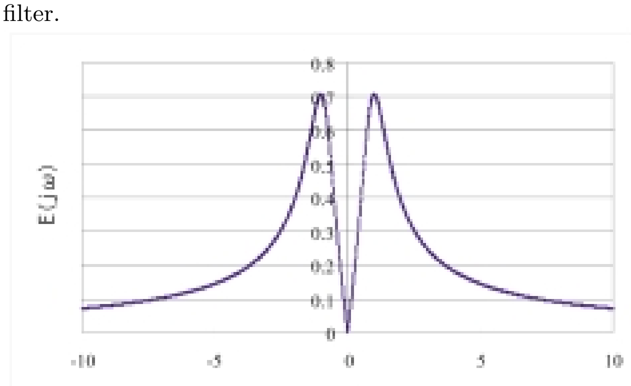

Fig.5 (a): Gain characteristics of an image transfer function $E_1(j\omega)$ with $R_1C_1 = 1$.

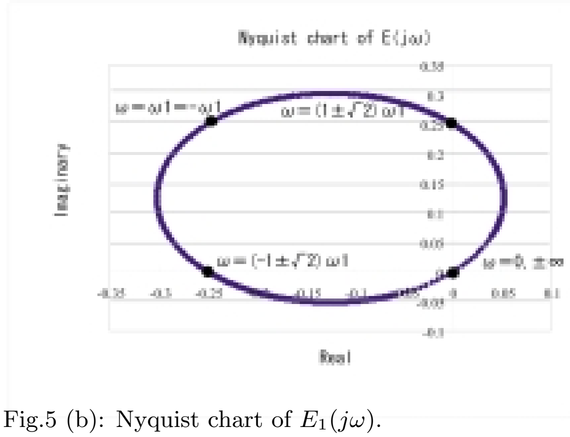

Fig.5 (b): Nyquist chart of $E_1(j\omega)$.

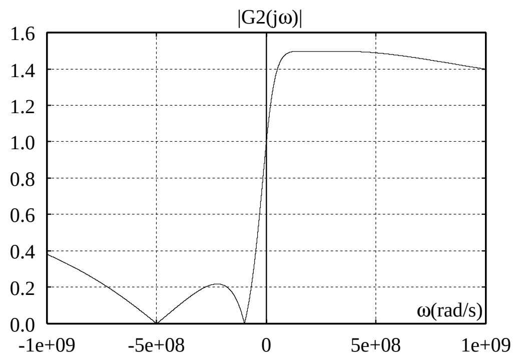

Fig.6 (a): Gain characteristics of $G_2(j\omega)$.

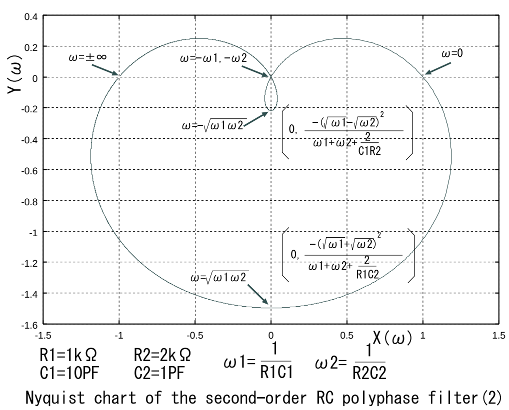

Fig.6 (b): Nyquist chart of $G_2(j\omega) := X_2(\omega) + jY_2(\omega)$.

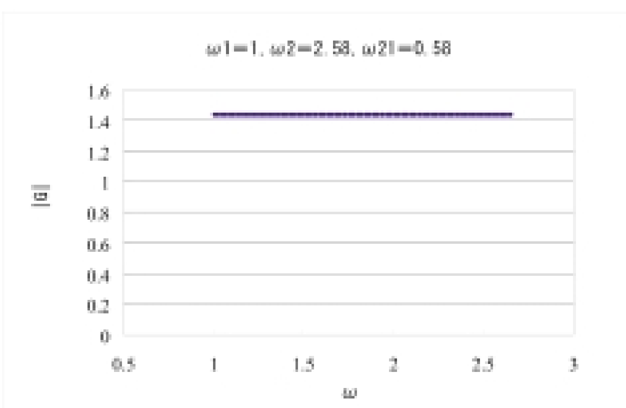

Fig.7 (a) : Example 1 ($\omega_1 = 1, \omega_2 = 2.58, \omega_{21} = 0.58$).

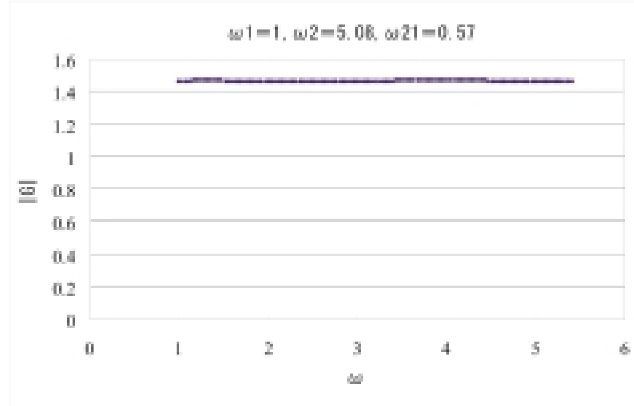

Fig.7 (b) : Example 2 ($\omega_1 = 1, \omega_2 = 5.08, \omega_{21} = 0.57$).

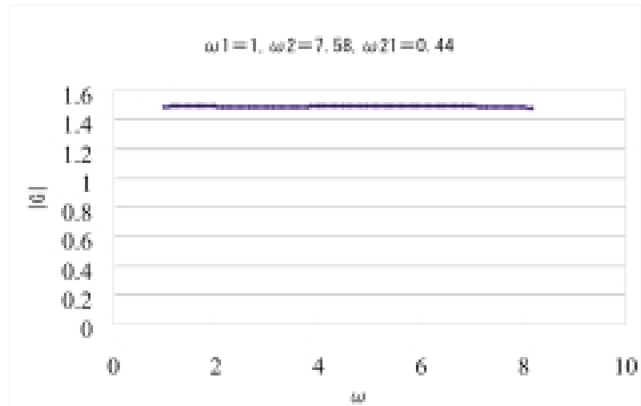

Fig.7 (c) : Example 3 ($\omega_1 = 1, \omega_2 = 7.58, \omega_{21} = 0.44$).

Fig.7 : Gain characteristics of second-order RC polyphase filters for passband ($\omega_1 < \omega < \omega_2$) using the proposed flat-passband algorithm.

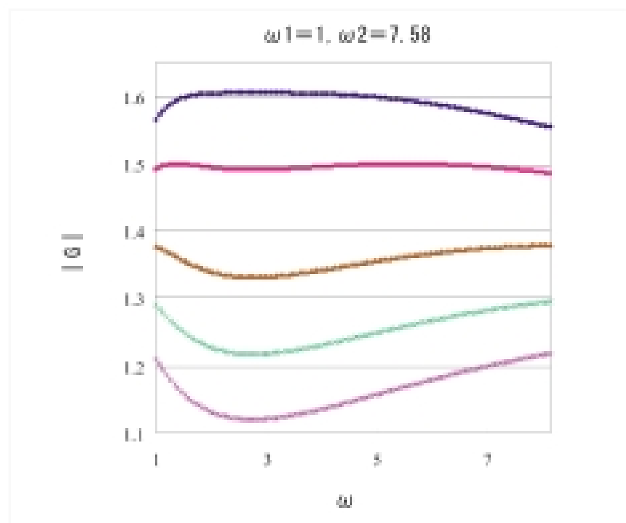

Fig.8 : Passband gain characteristics of several second-order RC polyphase filters with $\omega_1 = 1.0$, $\omega_2 = 7.58$. From the top to bottom lines, $\omega_{21} = 0.10$, 0.44, 1.0, 1.5, 2.0 respectively. We see that the passband gain is very flat for $\omega_{21} = 0.44$ which is obtained by our proposed algorithm.