IEICE TRANS. FUNDAMENTALS, VOL.E92–A, NO.2 FEBRUARY 2009

LETTER *Special Section on Analog Circuit Techniques and Related Topics*

Analysis of Coupled Inductors for Low-Ripple Fast-Response Buck Converter

Santhos A. WIBOWO†, **Zhang TING**†, ***Nonmembers***, **Masashi KONO**†, ***Member***, **Tetsuya TAURA**†, ***Nonmember***, **Yasunori KOBORI**†, ***Member***, **Ken-ichi ONDA**††, ***Nonmember***, ***and*** **Haruo KOBAYASHI**†a), ***Member***

SUMMARY This letter presents an analysis of characteristics of multiphase buck converters with coupled inductors. We derive equivalent inductances that provide both low per-phase steady-state ripple current and fast transient response. The characteristics of coupled-inductor circuits — low per-phase ripple current and fast response — were examined and verified by circuit simulation and experiments.

key words: *coupled inductor, multiphase, buck converter, DC-DC converter*

1. Introduction

The optimizing of inductor values for the best tradeoff between efficiency and transient response is a key item in switching DC-DC converter design. In DC-DC converters, using a smaller inductor improves the response to load changes but increases steady-state current ripple and decreases circuit efficiency. Conversely, larger inductors reduce current ripple but adversely affect transient response. One proven method to solve this tradeoff issue and achieve both high efficiency and fast transient response is by using coupled inductors as the magnetic component in multiphase buck DC-DC converters [1]–[4]. This method can reduce current ripple in each phase — thus reducing switching losses — as well as improving transient response, so we expect that it will be used in next-generation microprocessor power supply circuits. For inductors with inverse coupling and 1-to-1 winding ratio, the higher the coupling coefficient, the faster the response time that can be achieved. However, the optimum value of coupling coefficient to achieve minimum current ripple in each phase depends on switching duty cycle. We have not seen any detailed analysis of this for multiphase coupled-inductor buck converters. The purpose of this letter is to clarify the basic characteristics of such coupled-inductor circuits by analyzing a two-phase buck converter with 180 degrees between phases [1]. The same methods of analysis can be used for buck converters with a greater number of phases.

Manuscript received June 25, 2008.
Manuscript revised September 8, 2008.

†The authors are with the Department of Electronic Engineering, Graduate School of Engineering, Gunma University, Kiryu-shi, 376-8515 Japan.
††The author is with Hitachi, Ltd., Hitachi Research Laboratory, Hitachi-shi 319-1292 Japan.
a) E-mail: k_haruo@el.gunma-u.ac.jp

DOI: 10.1587/transfun.E92.A.451

2. Steady State and Transient Analysis of Coupled Inductors

The basic schematic diagram of a two-phase coupled-inductor buck converter is shown in Fig. 1. The mutual inductance M represents the coupling between the two inductors. Assuming that both inductors have the same inductance, the voltages across the two inductors are related to the currents through them as follows:

$$V_1 = L\frac{di_1}{dt} + M\frac{di_2}{dt}. \quad (1)$$

$$V_2 = L\frac{di_2}{dt} + M\frac{di_1}{dt}. \quad (2)$$

With the mutual inductance,

$$M = k \cdot L. \quad (3)$$

Note that k coupling coefficient is between -1 and 0 because M is negative (inverse coupling).

Figure 2 shows typical waveforms in a two-phase interleaved buck converter with duty cycle less than 50%. Each switching cycle comprises a sequence of four stages of operation, two of which are identical, i.e. it has three modes. The first stage (mode 1) begins when S1 and S4 are turned ON, connecting the first phase to the power supply and connecting the second phase to ground. In this mode, the voltage across the inductor in the first phase is given by:

$$V_1 = \frac{L^2 - M^2}{L + M \cdot \frac{D}{1-D}}\frac{di_1}{dt}. \quad (4)$$

In the second stage of operation (mode 2), both first and second phases are connected to ground, i.e. S1 and S3 are turned OFF but S2 and S4 are turned ON (note that duty

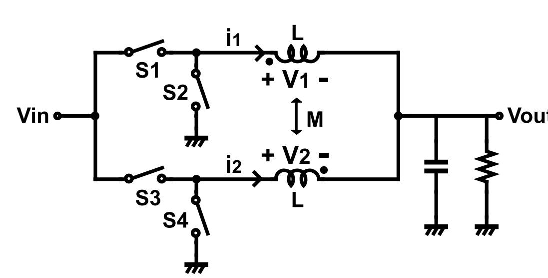


Fig. 1 Two-phase coupled-inductor buck converter.

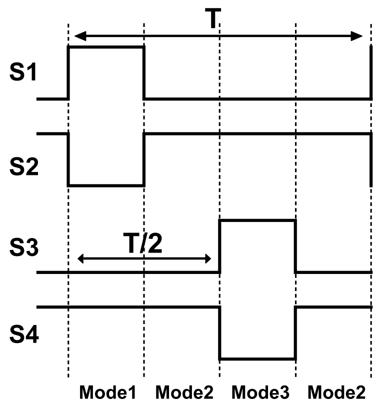


Fig. 2 Operating waveforms of interleaved two-phase coupled-inductor buck converter (Duty-cycle $D \le 0.5$).

cycle $D \le 0.5$). In this mode, the voltage across the inductor in the first phase is given by:

$$V_1 = (L + M)\frac{di_1}{dt}. \quad (5)$$

In the third stage of operation (mode 3), S2 and S3 are turned ON but S1 and S4 are turned OFF, connecting the first phase to ground and the second phase to the power supply. In this stage, the voltage across the inductor in the first phase is given by:

$$V_1 = \frac{L^2 - M^2}{L + M \cdot \frac{1-D}{D}} \frac{di_1}{dt}. \quad (6)$$

Finally, to complete one complete switching cycle, the circuit enters the fourth stage, which is the same as the second (mode 2). Using Eqs. (4), (5), and (6), the equivalent inductances in each of the three modes can be written as:

$$L_{eq1} = L\frac{1-k^2}{1 + k \cdot \frac{D}{1-D}} \quad (7)$$

$$L_{eq2} = L(1 + k) \quad (8)$$

$$L_{eq3} = L\frac{1-k^2}{1 + k \cdot \frac{1-D}{D}}. \quad (9)$$

Where L_{eq1}, L_{eq2}, L_{eq3} are equivalent inductances in mode1, mode2, and mode3, respectively.

We can derive a steady-state average equivalent inductance based on the average inductor current in one switching period. The average inductor current in the first phase is given by:

$$\Delta i_L = \frac{\overline{V_1} \cdot D}{L_{eq1}} + \frac{\overline{V_1} \cdot (1-2D)}{L_{eq2}} + \frac{\overline{V_1} \cdot D}{L_{eq3}} \quad (10)$$

$$\frac{\overline{V_1}}{\overline{L_{eq}}} = \frac{\overline{V_1} \cdot D}{L_{eq1}} + \frac{\overline{V_1} \cdot (1-2D)}{L_{eq2}} + \frac{\overline{V_1} \cdot D}{L_{eq3}}. \quad (11)$$

Where, $\overline{V_1}$ is the average voltage across the inductor in the first phase. By solving the above equations, the average equivalent inductance when the duty cycle is less than 50% is derived as:

$$\overline{L_{eq}} = L\frac{1-k^2}{1 + k \cdot \frac{D}{1-D}}. \quad (12)$$

The steady-state average equivalent inductance in this case is equal to L_{eq1}. Therefore, the per-phase current-ripple for duty cycle less than 50% depends on the equivalent inductance in the first mode where the inductor in the first phase is connected to the power supply and the inductor in the second phase is connected to ground. For duty cycles of greater than 50%, the same method of analysis can be applied, resulting in steady-state average equivalent inductance being equal to L_{eq3}.

As can be seen from Eq. (12), the average equivalent inductance depends on coupling coefficient k and duty cycle D. Thus the optimum coupling coefficient for minimum current ripple varies with duty cycle. Using derivative functions we obtain (maximum) values of average equivalent inductance that minimize current ripple:

$$F = \frac{\overline{L_{eq}}}{L} = \frac{1-k^2}{1 + Ak}. \quad (13)$$

$$\frac{dF}{dk} = \frac{-A(k^2 + \frac{2}{A}k + 1)}{(1 + AK)^2} = 0. \quad (14)$$

Where A is $D/(1 - D)$. Calculation results are shown in Table 1. We can see that for duty cycles of 10% or less, minimum current ripple can be achieved with very small coupling coefficients.

Table 1 Duty cycles and values of coupling coefficient to minimize per-phase current ripple.

Dutycycle	0.1	0.2	0.3	0.4	0.5
k	−0.056	−0.128	−0.225	−0.382	−1

Furthermore, for duty cycles less than 50%, peak-to-peak current ripple in each phase is given by Eq. (15), where Ts is the switching period.

$$I_{pp-coupled} = \frac{di_L}{dt} \times T_{off}. = (1-D)Ts \cdot \frac{Vout}{\bar{L}_{eq}}. \quad (15)$$

Using equation (12), comparing cases with and without coupling between inductors, the per-phase current ripple reduction can be written as:

$$\frac{|I_{pp-uncoupled} - I_{pp-coupled}|}{I_{pp-uncoupled}} = -\frac{k[k + \frac{D}{1-D}]}{1-k^2}. \quad (16)$$

From the equation above, we can see that the current ripple reduction depends on the steady-state average equivalent inductance.

In an ideal lossless circuit, the output current would be independent of the duty cycle — however in real circuits, with voltage drop caused by MOSFET ON resistances Rs or other parasitic resistances, the output current Io is dependent on the duty cycle D as follows:

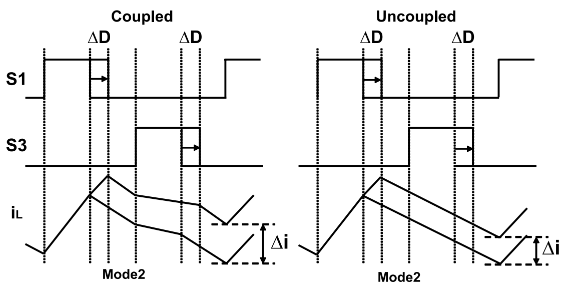


Fig. 3 Comparison of load-current transient responses of buck converters with and without inductor coupling.

$$Vo = (Vin - Io \cdot Rs)D. \quad (17)$$

Therefore, it is necessary for duty cycle D to change when the output current changes. Comparison of load-current transient response of inductor currents in buck converters with and without coupling between inductors is shown in Fig. 3. Here, we see that the current change Δi for buck converters with coupled inductors is larger than that with no inductor coupling. In other words, coupled inductors enable faster current-change response. When a load current change occurs, the period of operation of mode 2 changes. Let this period change be ΔD. In the coupled inductor case, the relation of current change Δi to load current change can be written as:

$$\Delta i = \frac{Vin \cdot Ts}{L_{eq2}} \Delta D. \quad (18)$$

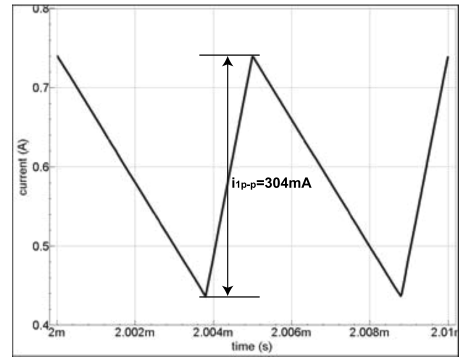
From the above equation, it can be concluded that the equivalent inductance in the transient state is equal to L_{eq2} or the equivalent inductance in mode 2. Thus, transient response time is determined only by the value of $L(1 + k)$. In addition, the transient response in a multiphase buck converter with coupled inductors is independent of switching duty cycle and directly proportional to the value of mutual inductance or coupling coefficient. Perfect coupling, with coupling coefficient k equal to -1 (since mutual inductance is negative), is the optimum for fast response.

3. Simulation Verification

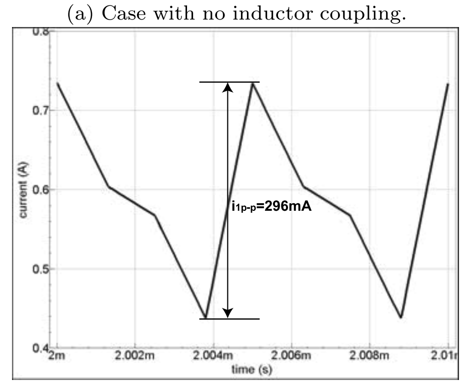
Circuit simulation was performed on SmartSPICE software based on the following specification and circuit parameters.

Vin	Input Voltage	5 V
$Vout$	Output Voltage	1.2 V
Fs	Switching Frequency	200 KHz
$Cout$	Output capacitor	220 uF
L	Inductor	15 uH
k	Coupling Coefficient	−0.2

The circuit operates open loop with constant duty cycle. Coupling coefficient was set to -0.2 so that the buck converter with coupled inductors can achieve both low-ripple and fast-response characteristics, considering that values of coupling coefficient to achieve the lowest current ripple are between -0.128 and -0.225 for duty cycles between 0.2 and 0.3, respectively, as discussed in the steady-state analysis. Simulation results in steady state are shown in Fig. 4, where i1p-p is the peak-to-peak inductor current ripple in the first phase. Approximately 2.6% ripple reduction was obtained, and this agrees well with theoretical design values from Eq.(16). Transients are simulated by load changes from 1.2 A to 3.6 A. Output voltages during load transients are shown in Fig. 5. From these figures, we can see that the buck converter with coupled inductors achieves a lower peak-to-peak per-phase current ripple, and better output voltage undershoot and overshoot characteristics when the load changes, than the converter with no inductor coupling.



(a) Case with no inductor coupling.

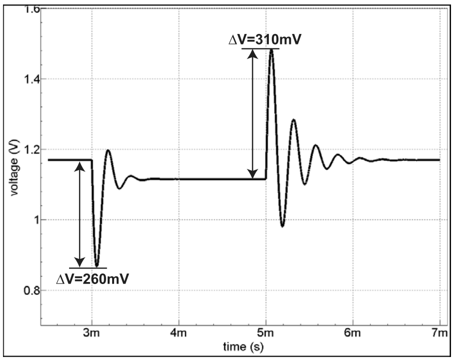


(b) Coupled inductor case with k=-0.2.

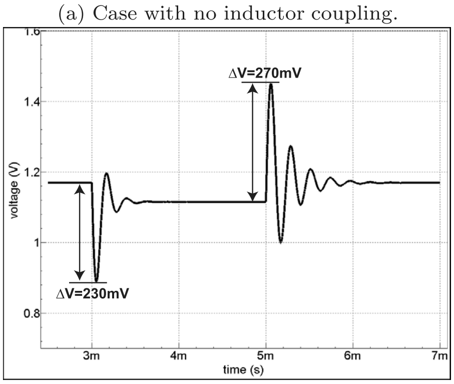
Fig. 4 Simulation results for inductor current in the first phase in steady state.

4. Experimental Verification

A circuit for experimental verification was built with Schottky diodes as asynchronous rectifiers, and with parameter values as close as possible to those used in the circuit simulation. The coupled inductor was made by cleaving together two separate inductor bobbins with 15 uH inductance on each inductor. The coupling coefficient value of -0.2 was measured by an LCR tester. The steady-state experiment results are shown in Fig. 6. Waveforms are plotted on the same scales of 200 mA/div and 5 us/div. Figure 7 shows experimental results for the output voltage (500 mV/div) during load transients. The time scale is 200 us/div. The load

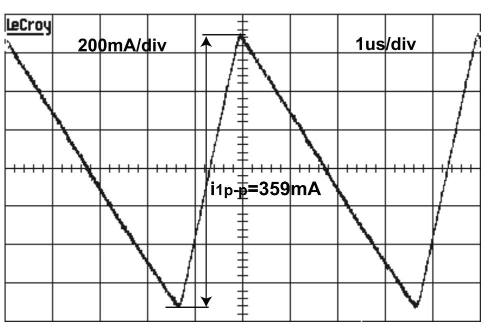


(a) Case with no inductor coupling.

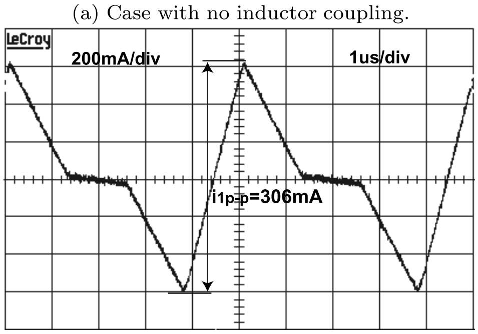


(b) Coupled inductor case with k=-0.2.

Fig. 5 Simulation results for output voltage with load transients.



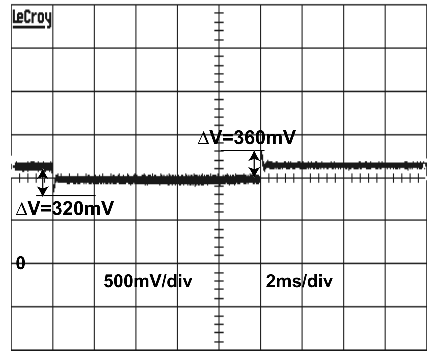
(a) Case with no inductor coupling.



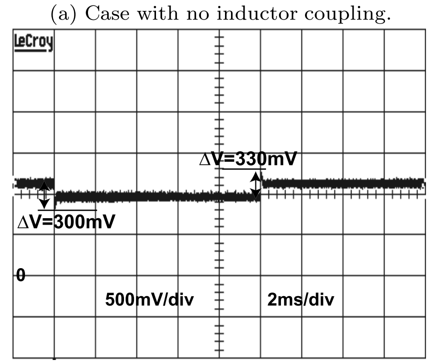
(b) Coupled inductor case with k=-0.2.

Fig. 6 Experimental results for inductor current in the first phase in steady state.

transient response was measured for load changes from 1.2 A to 3.6 A. Because of differences between experimental and simulation circuits, experimental results cannot be com-



(a) Case with no inductor coupling.



(b) Coupled inductor case with k=-0.2.

Fig. 7 Experimental results for output voltage with load transients.

pared directly with simulation results, however results qualitatively show that a coupled-inductor circuit has a low per-phase ripple in the steady state, and improved transient response during step-up and step-down load transients, compared with a circuit with no coupling between inductors.

5. Conclusions

We have presented an analysis of the use of coupled inductors to achieve high-efficiency buck converters. Analysis shows that using coupled inductors results in lower per-phase ripple current, reduced switching losses, and faster transient response. We showed that, in the steady state, average equivalent inductance depends on duty cycle and coupling coefficient. We also found that for low duty cycles this method can reduce current ripple even with small inductor coupling coefficients. The characteristics of the coupled-inductor circuits were verified by circuit simulation and by experiments. Simulation results agreed well with calculated theoretical design values.

Acknowledgment

The authors would like to thank Prof. T. Nabeshima, Prof. T. Sato, Prof. K. Nishijima of Oita University and Prof. T. Tanaka of Kagoshima University for valuable discussions. Thanks are also due to K. Wilkinson for improving the English in the manuscript.

References

[1] S.A. Wibowo, Z. Ting, M. Kono, T. Taura, I. Mori, Y. Kobori, and H. Kobayashi, "Analysis of coupled inductor for fast-response low-ripple buck converter," IEICE Technical Report, EE2007-67, CPM2007-152, Feb. 2008.

[2] J. Gallagher, "Coupled inductors improve multiphase buck efficiency," Power Electronics Technology, pp.36–42, Jan. 2006.

[3] J. Czogalla, J. Li, and C.R. Sullivan, "Automotive application of multi-phase coupled inductor DC-DC converter," IEEE Industry Applications Society Annual Meeting, vol.3, pp.1524–1529, Oct. 2003.

[4] P. Sernia, D. Finn, and G. Walker, "The effects of coupled inductors in parallel interleaved buck converters," Australasian Universities Power Engineering Conference, vol.1, pp.1–5, Sept. 2003.