

Recent Research Results

# SAR ADC Algorithm with Redundancy

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Supported by STARC

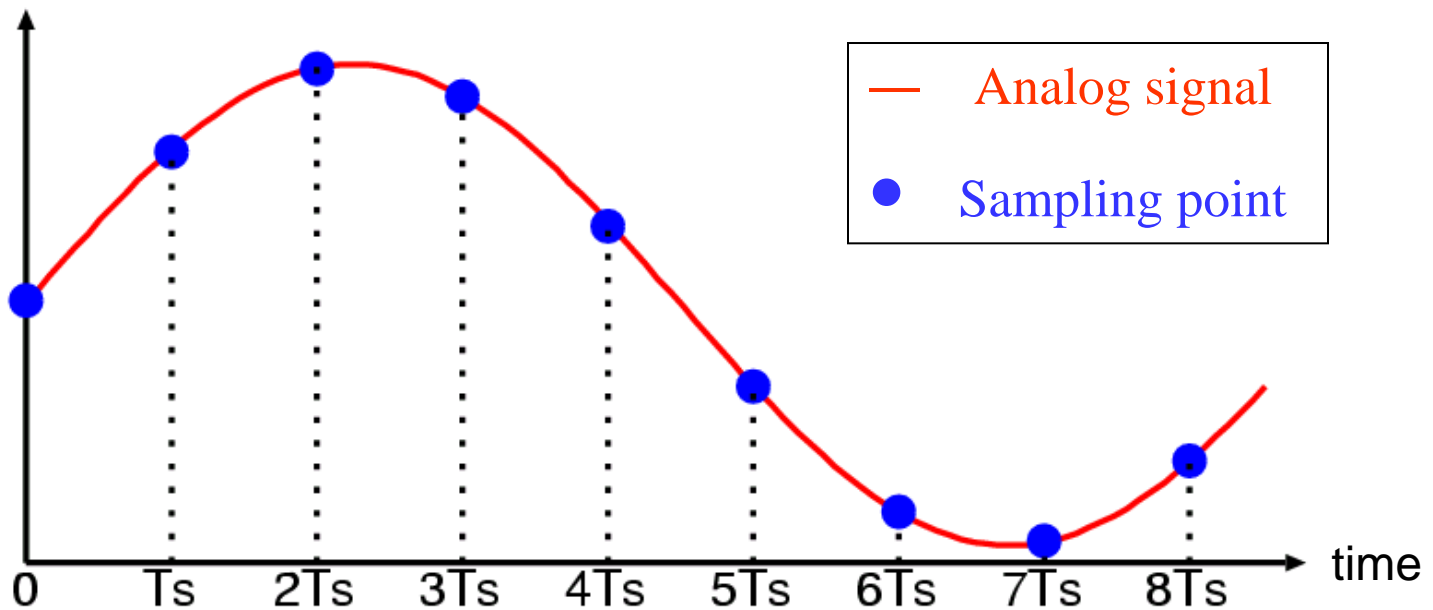
Published in

T. Ogawa, H. Kobayashi, et. al.,  
“SAR ADC Algorithm with Redundancy and Digital Error Correction”,  
IEICE Trans. Fundamentals, vol.E93-A, no.2, (Feb. 2010).

# Sampling

## Quantization in time domain

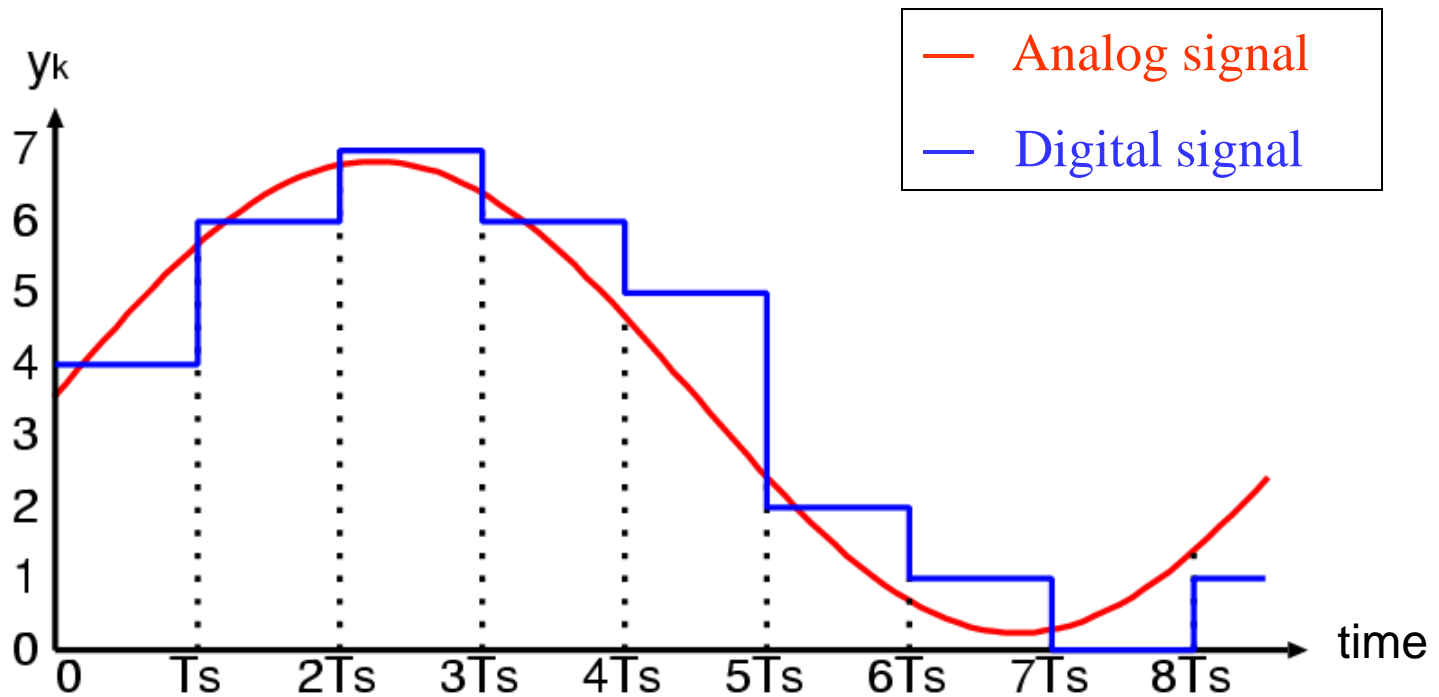
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Take data ● and discard the other data.

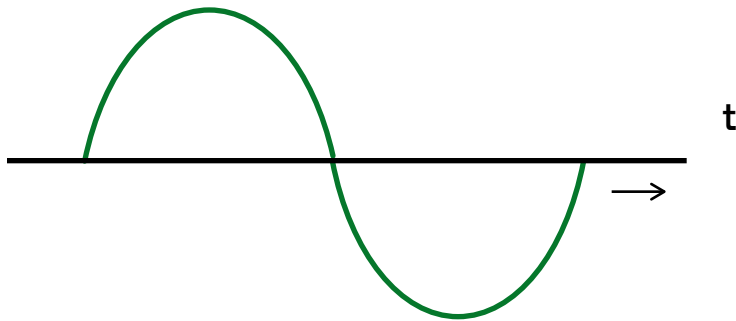
What is digital signal ?

# Quantization of Signal Level

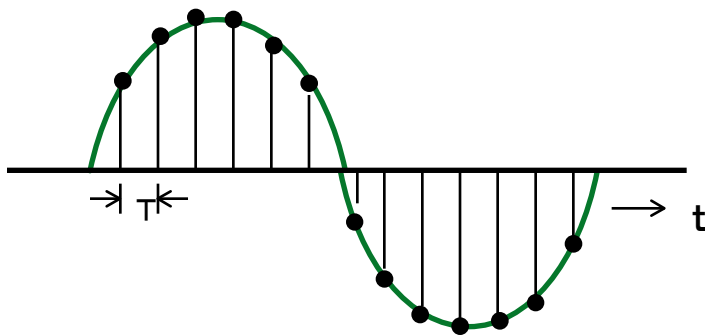


Round the signal level to an integer.

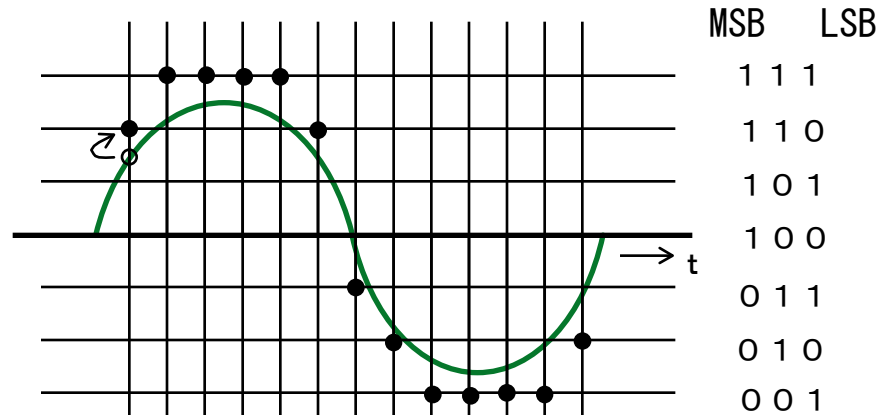
# Analog-to-Digital Conversion



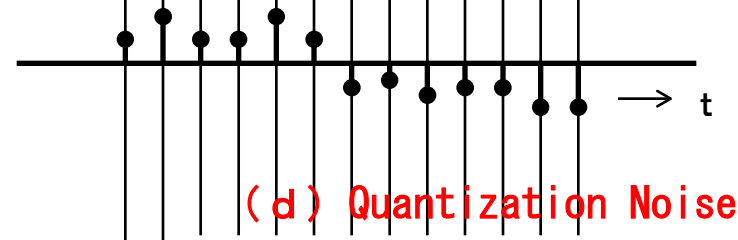
(a) Analog Signal



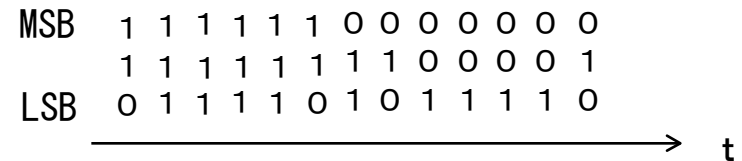
(b) Sampling



(c) Quantization

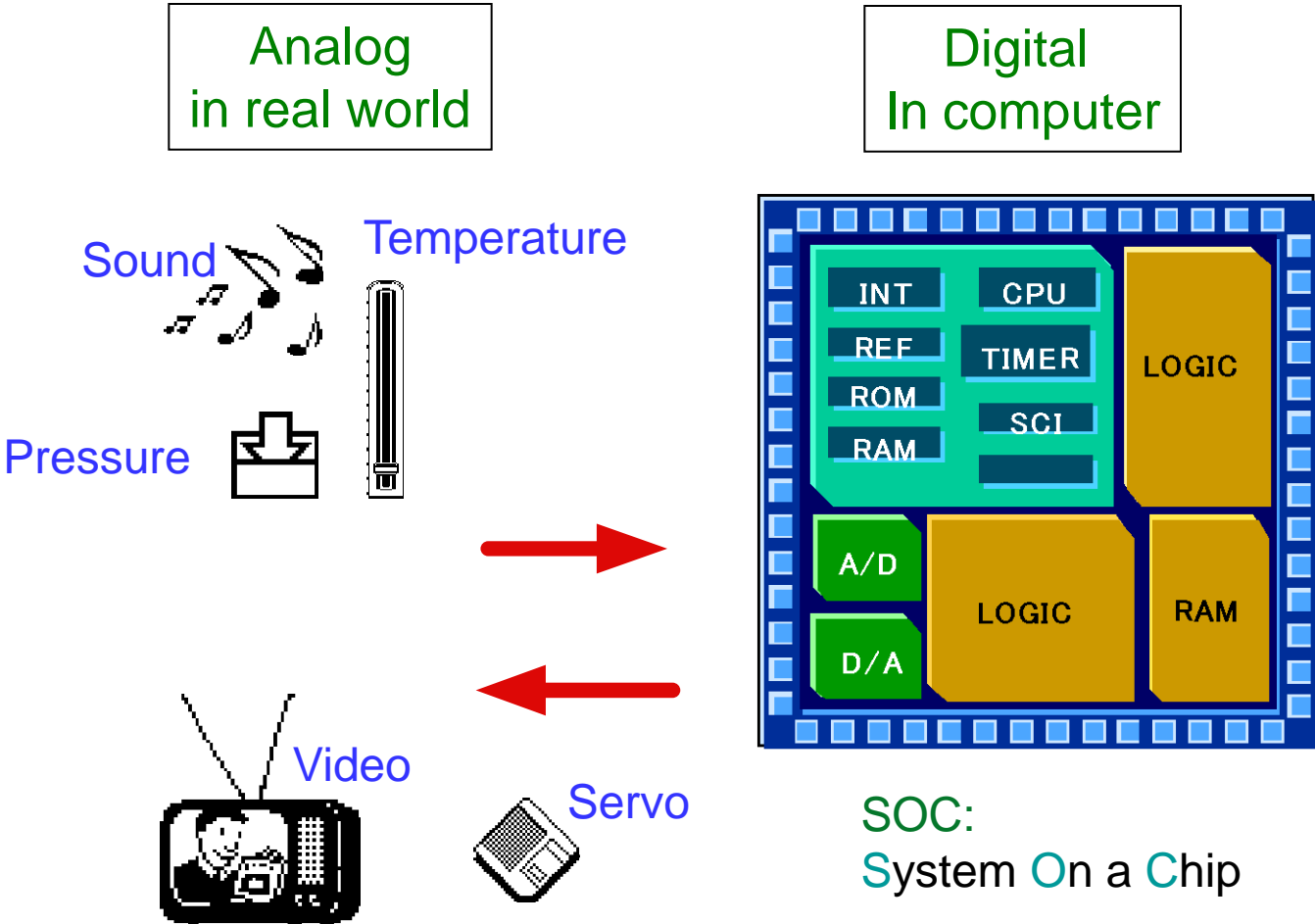


(d) Quantization Noise



(e) Encoding

# ADC is for Digital Signal Processing



# Outline

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- Research purpose
- SAR ADC
- Binary search algorithm
- Non-binary search algorithm
- Proposed non-binary search algorithm
- DAC incomplete settling
- Conclusion

# Outline

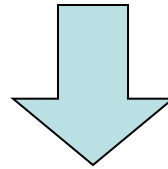
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# Research purpose

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- Automotive electronics is the spotlight now.
- High speed, reliable SAR ADCs in microcontroller are important there.



- Optimal digital error correction algorithm for their realization.



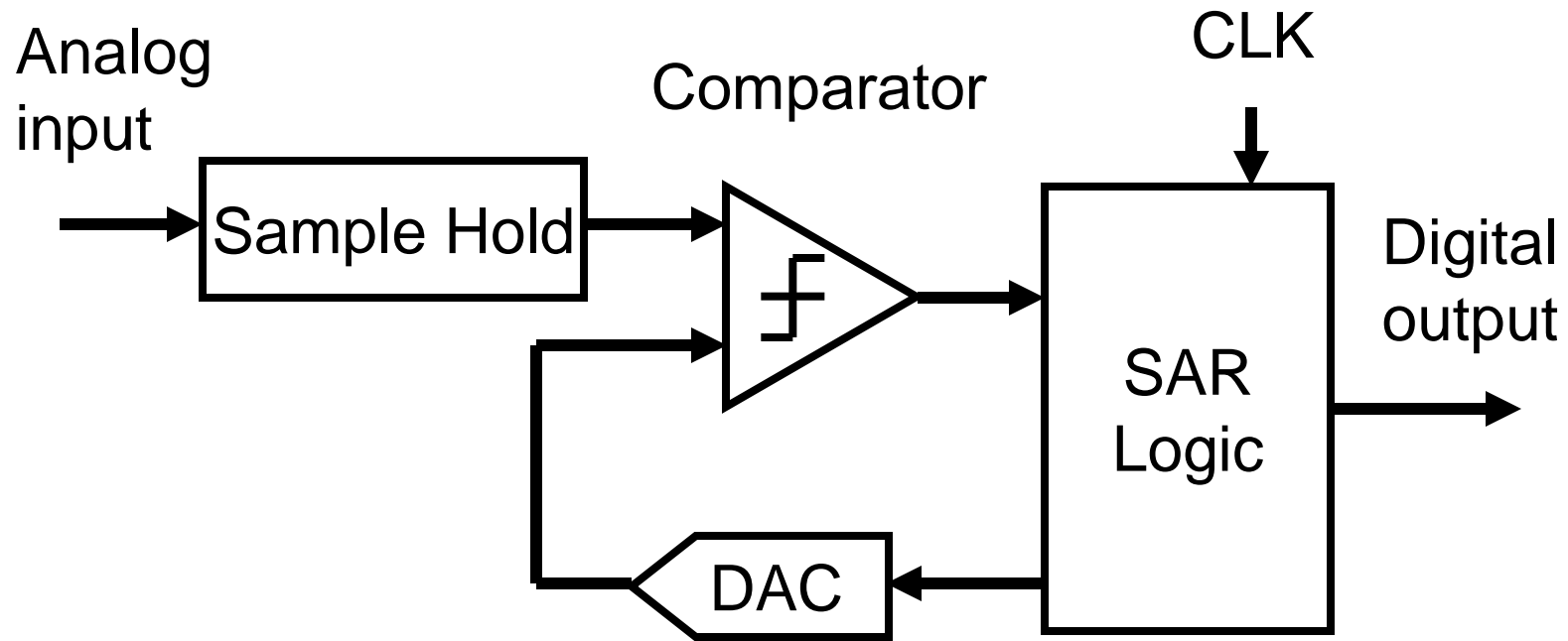
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# SAR ADC Block

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SAR ADC is digital centric.

→ Suitable for fine CMOS implementation.

# SAR ADC Characteristics

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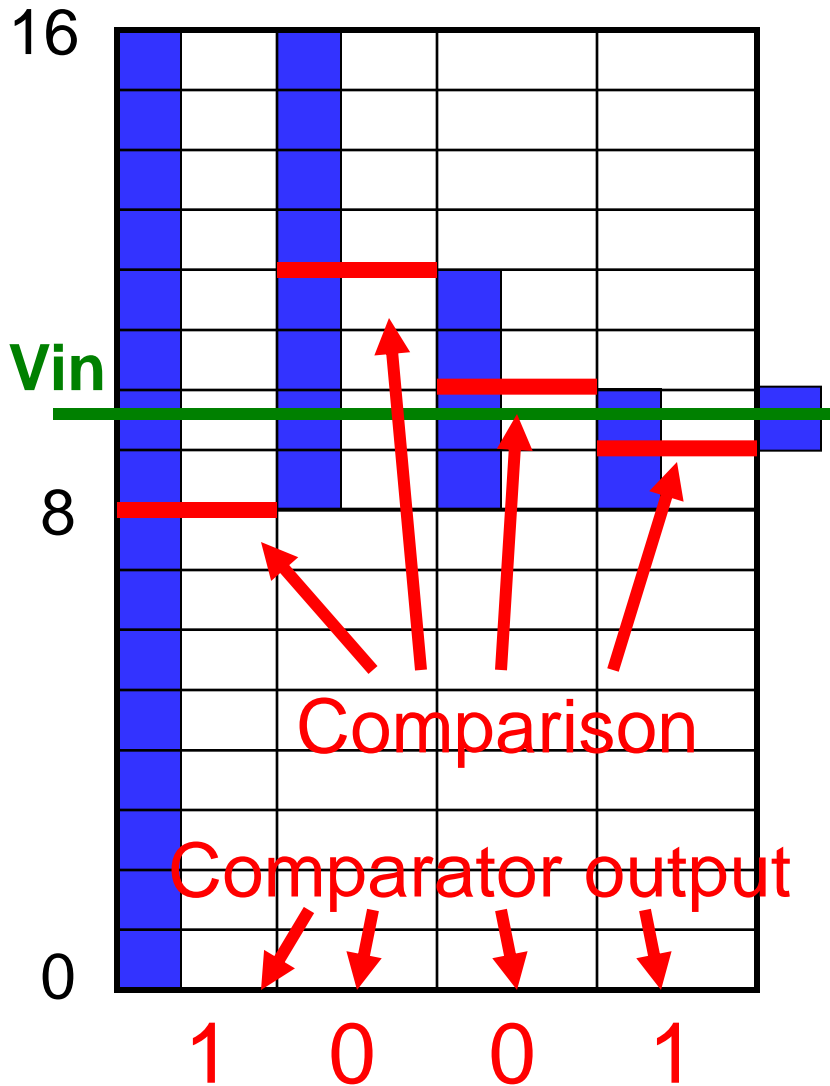
- High resolution (10-14bit)
- Middle sampling speed (10-40 MS/s)
- Small die area
- Low power (a few mW)
- Not use OP-amp

# Outline

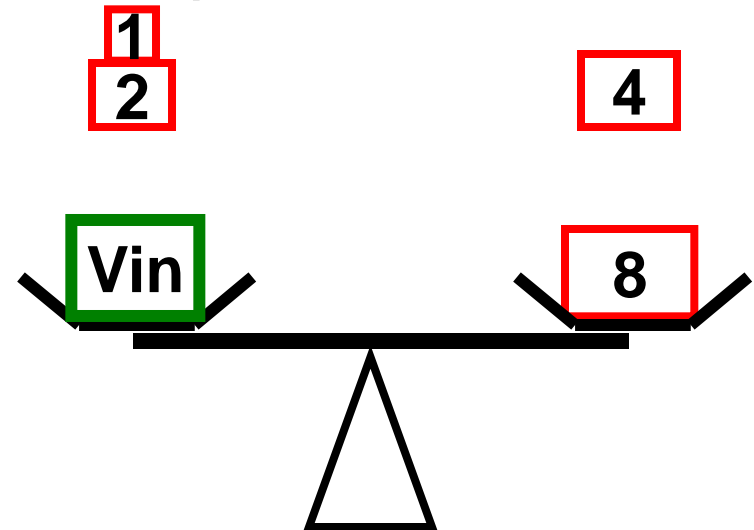
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# Binary search algorithm

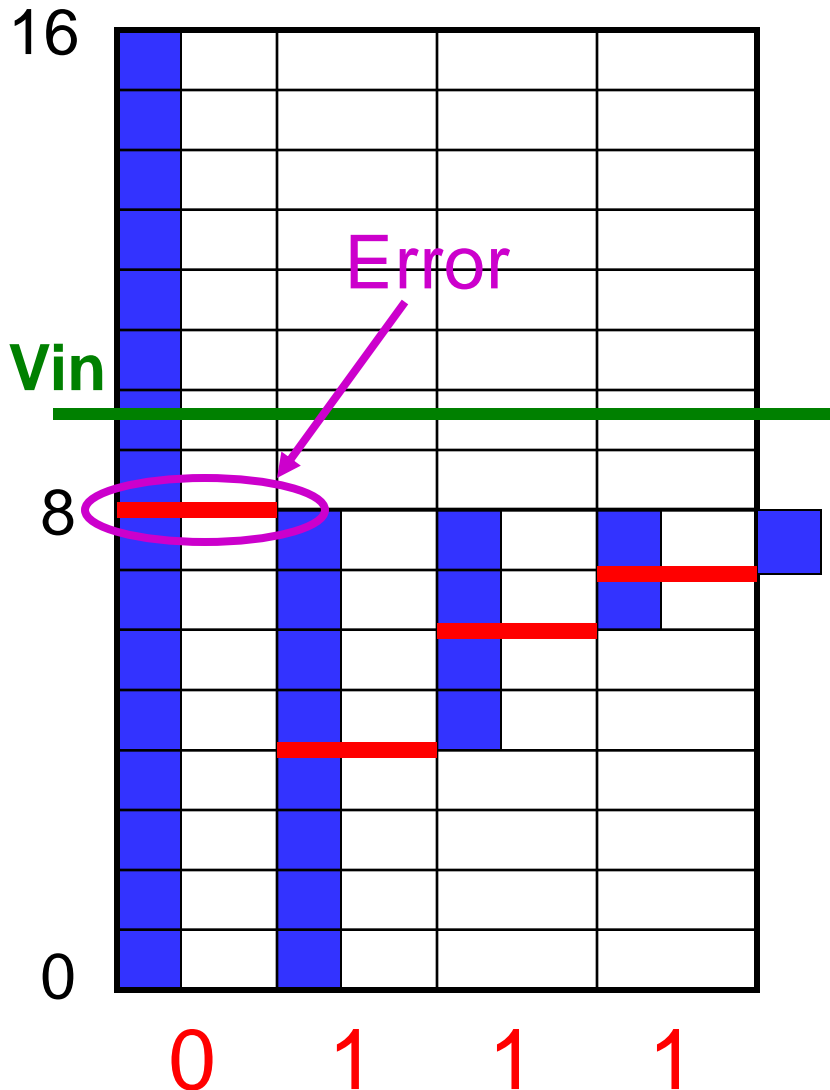


“Principle of a balance”



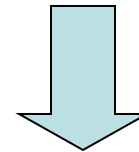
$$\boxed{\text{Vin}} = \begin{array}{c} \boxed{4} \\ \boxed{8} \end{array} - \begin{array}{c} \boxed{1} \\ \boxed{2} \end{array} = 9$$

# Problem of binary search algorithm



No redundancy

Search result has error.



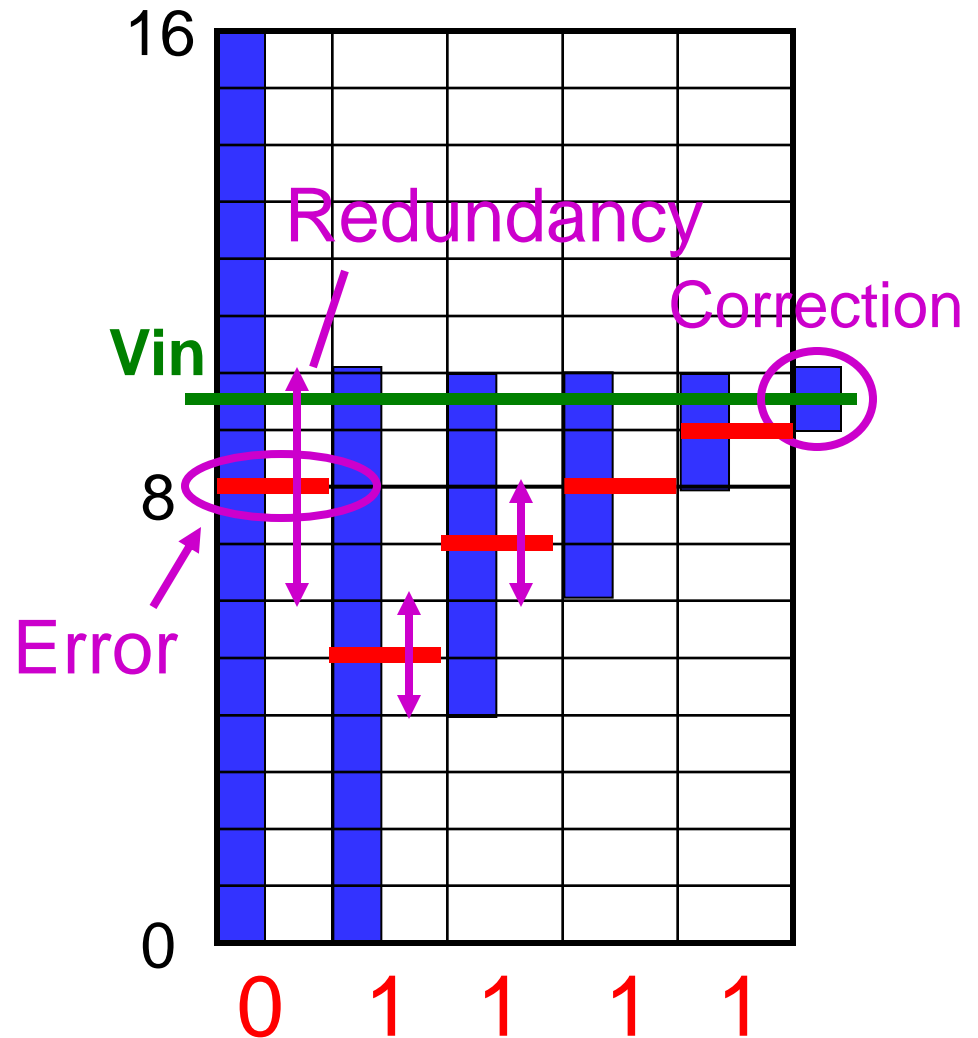
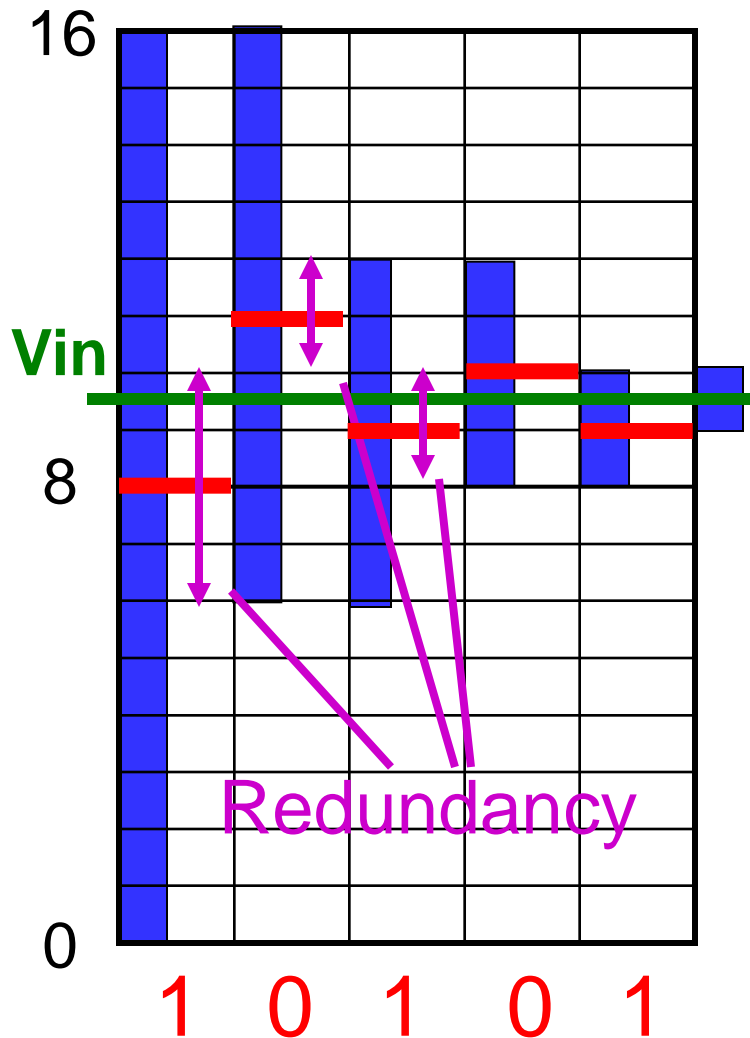
Digital output has error.

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# Non-binary search algorithm





# Non-binary search algorithm

## Binary search algorithm(4-bit 4-step)

$$Dout = 2^3 + \underline{2^2} \cdot d_1 + \underline{2} \cdot d_2 + \underline{1} \cdot d_3 + 0.5 \cdot d_4 - 0.5$$

Binary (Radix :2)

## Conventional non-binary search algorithm (4-bit 5-step)

$$Dout = 2^3 + \underline{\gamma^3} \cdot d_1 + \underline{\gamma^2} \cdot d_2 + \underline{\gamma} \cdot d_3 + \underline{1} \cdot d_4 + 0.5 \cdot d_5 - 0.5$$

Radix :  $\gamma$

$$\gamma = 2^{\frac{3}{4}}$$

$d_k$  : +1 or -1

# Principle of error correction

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## Binary search algorithm

Comparator output : **1 0 0 1** ← Only one

$$\text{Dout} = 8 + 4 - 2 - 1 + 0.5 - 0.5 = 9$$

## Non-binary search algorithm

Comparator output : **1 0 1 0 1** ← Multiple

$$\text{Dout} = 8 + 3 - 2 + 1 - 1 + 0.5 - 0.5 = 9$$

Comparator output : **0 1 1 1 1**

$$\text{Dout} = 8 - 3 + 2 + 1 + 1 + 0.5 - 0.5 = 9$$

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# Proposed non-binary search algorithm

## Conventional non-binary search algorithm

$$D_{out} = 2^3 + \underbrace{\gamma^3}_{\leftarrow} \cdot d_1 + \underbrace{\gamma^2}_{\uparrow} \cdot d_2 + \underbrace{\gamma}_{\leftarrow} \cdot d_3 + \underbrace{1}_{\leftarrow} \cdot d_4 + 0.5 \cdot d_5 - 0.5$$

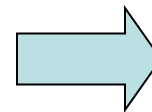
Radix :  $\gamma$

$$\gamma = 2^{\frac{3}{4}}$$

## Proposed Generalized non-binary search algorithm

$$D_{out} = 2^3 + \underbrace{p_2}_{\uparrow} \cdot d_1 + \underbrace{p_3}_{\leftarrow} \cdot d_2 + \underbrace{p_4}_{\leftarrow} \cdot d_3 + \underbrace{p_5}_{\leftarrow} \cdot d_4 + 0.5 \cdot d_5 - 0.5$$

Flexible (not restricted to  $\gamma$ )



Optimal design

$d_k : +1 \text{ or } -1$

# Design method of proposed algorithm

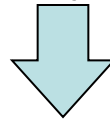
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N-bit, M-step (M>N)

Design redundancy.

$$2^M - 2^N = \left( \sum_{i=1}^{M-1} 2^i q_i \right)$$

$q_k$  : Redundancy at k-th step

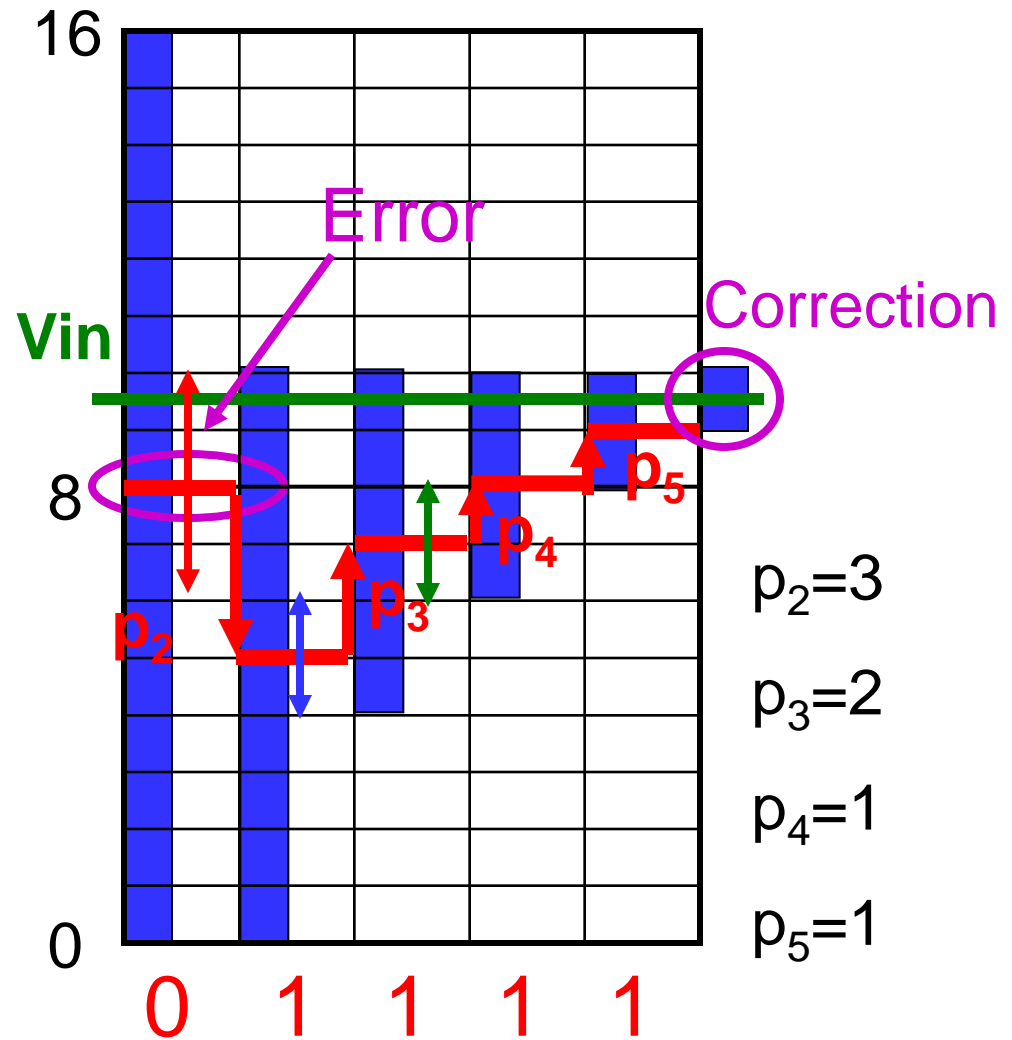
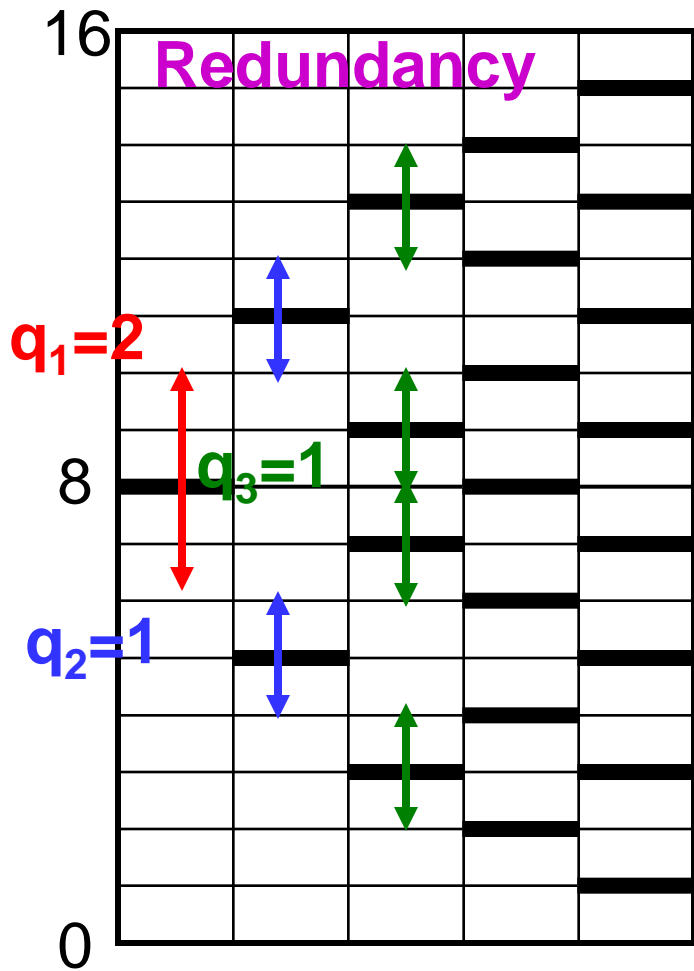


Calculate step of reference voltage.

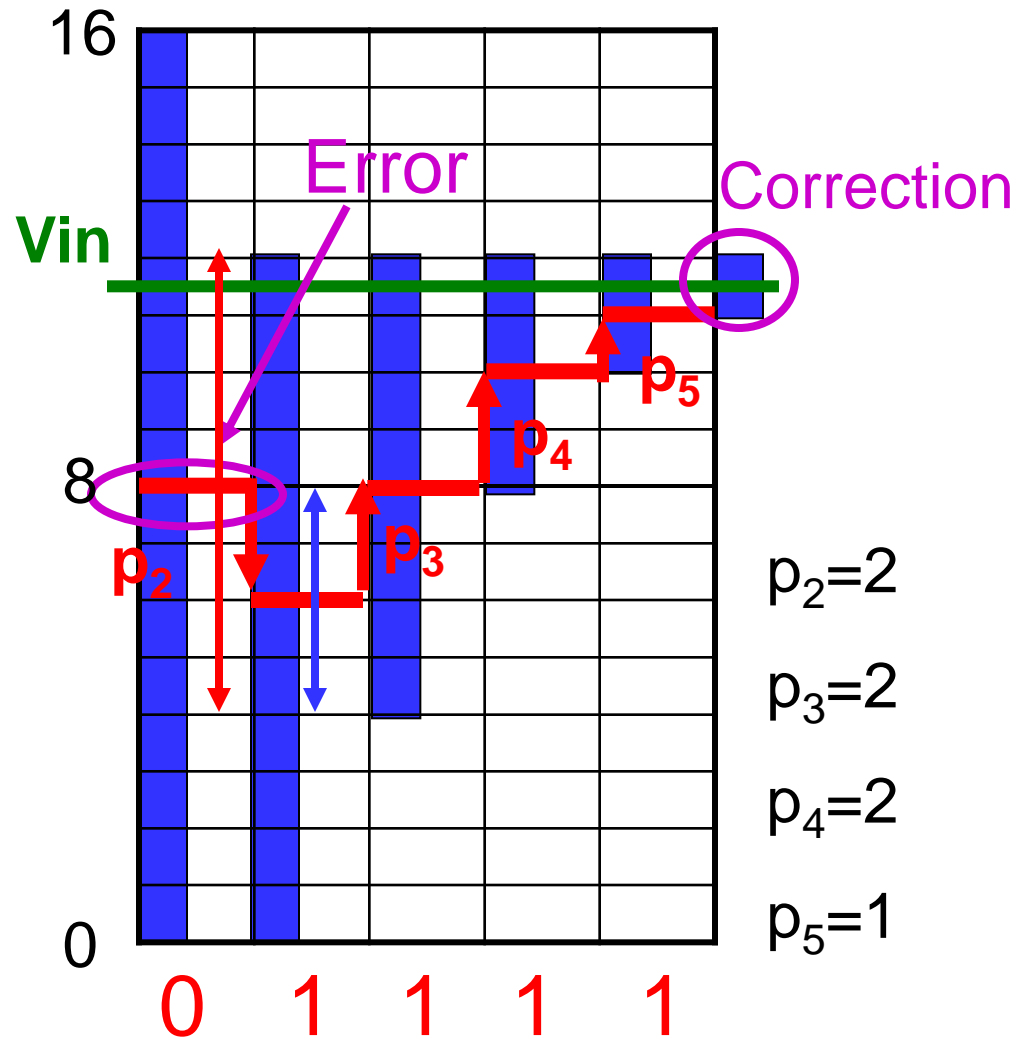
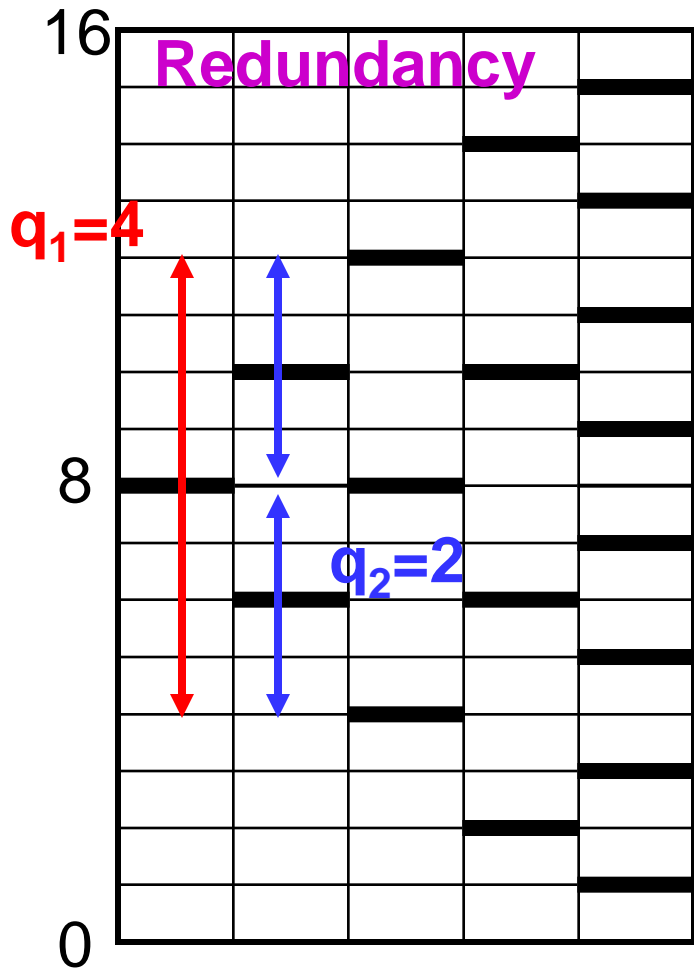
$$p_{k+1} = -q_k + 2^{M-k-1} - \sum_{i=k+1}^{M-1} 2^{i-k-1} q_i$$

$p_k$  : Step of reference voltage at k-th step

# Proposed algorithm Example 1



# Proposed algorithm Example 2



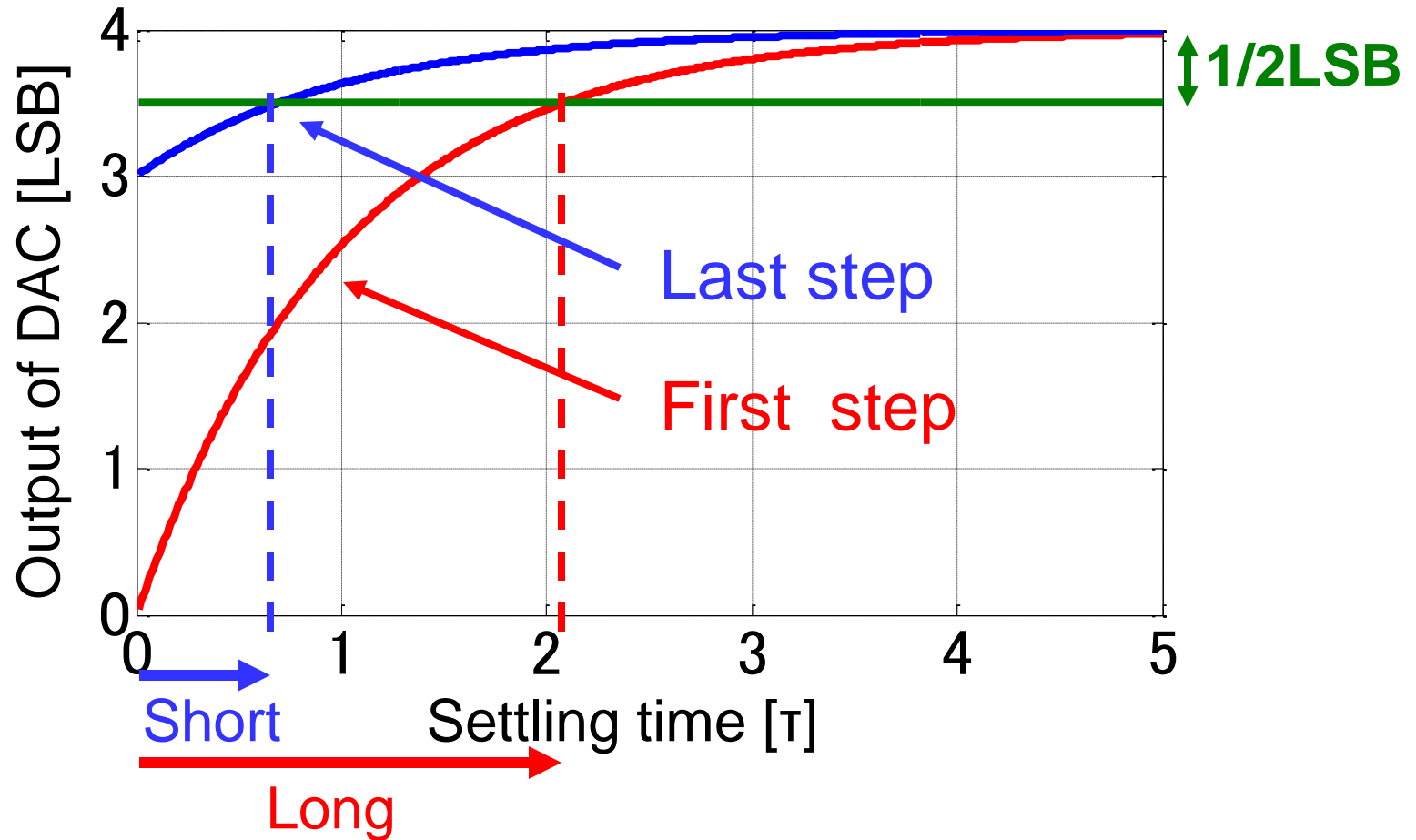
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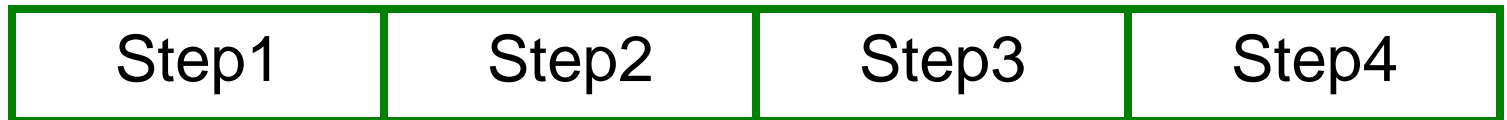
# Settling of DAC output



# Conversion time of each algorithm

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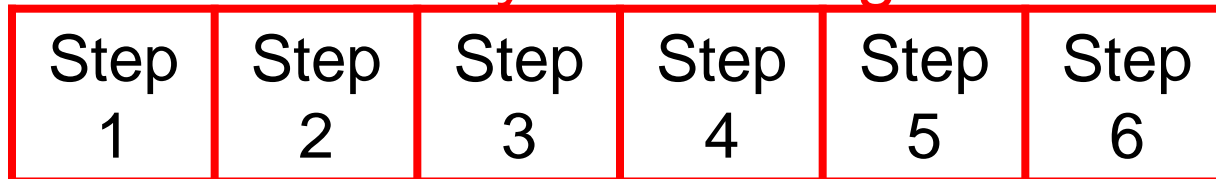
## Binary search algorithm



Exact DAC settling → Long time

A/D conversion time

## Non-binary search algorithm



Correct incomplete settling error.

Incomplete DAC settling → Short time

# Simulation of AD Conversion Ttime

## Binary algorithm

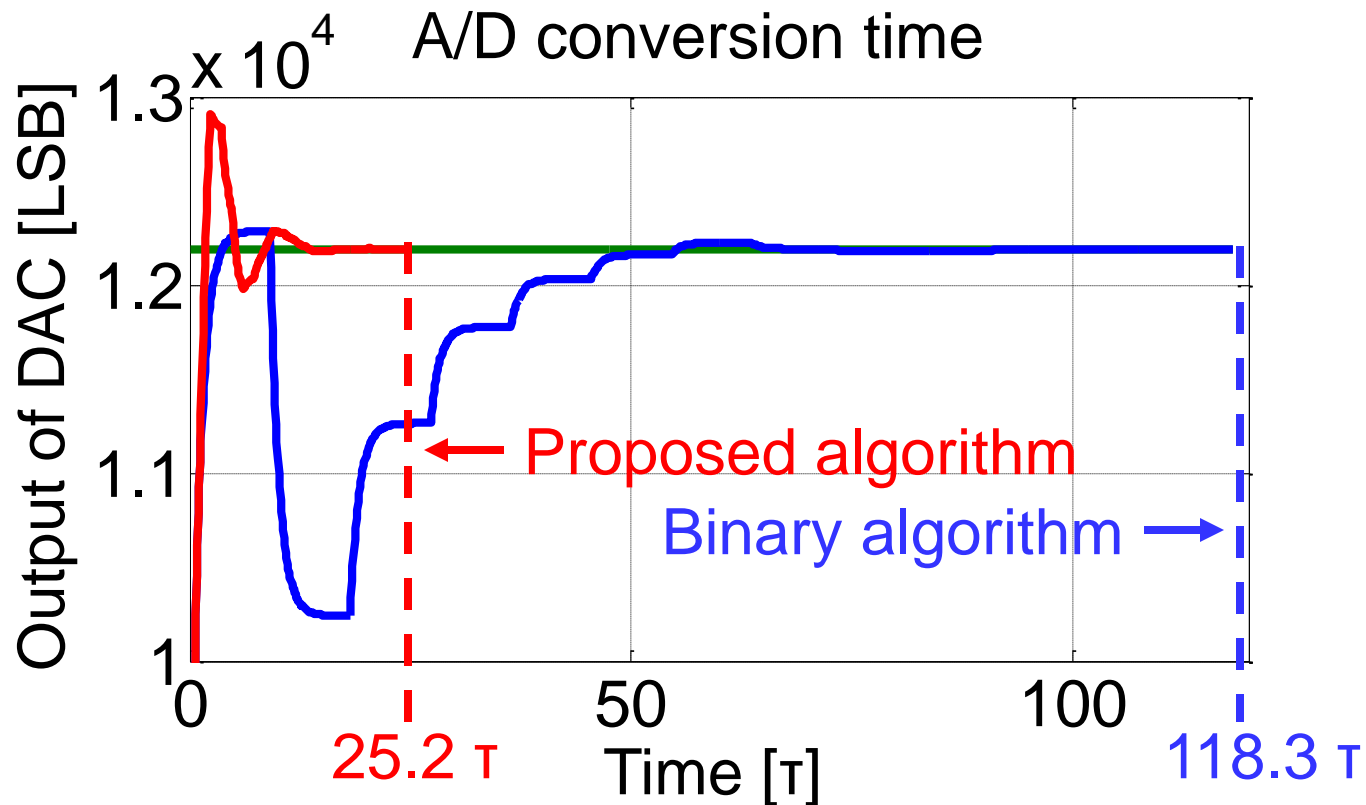
14-bit, 14-step

Step time :  $9.1 \tau$

## Proposed algorithm

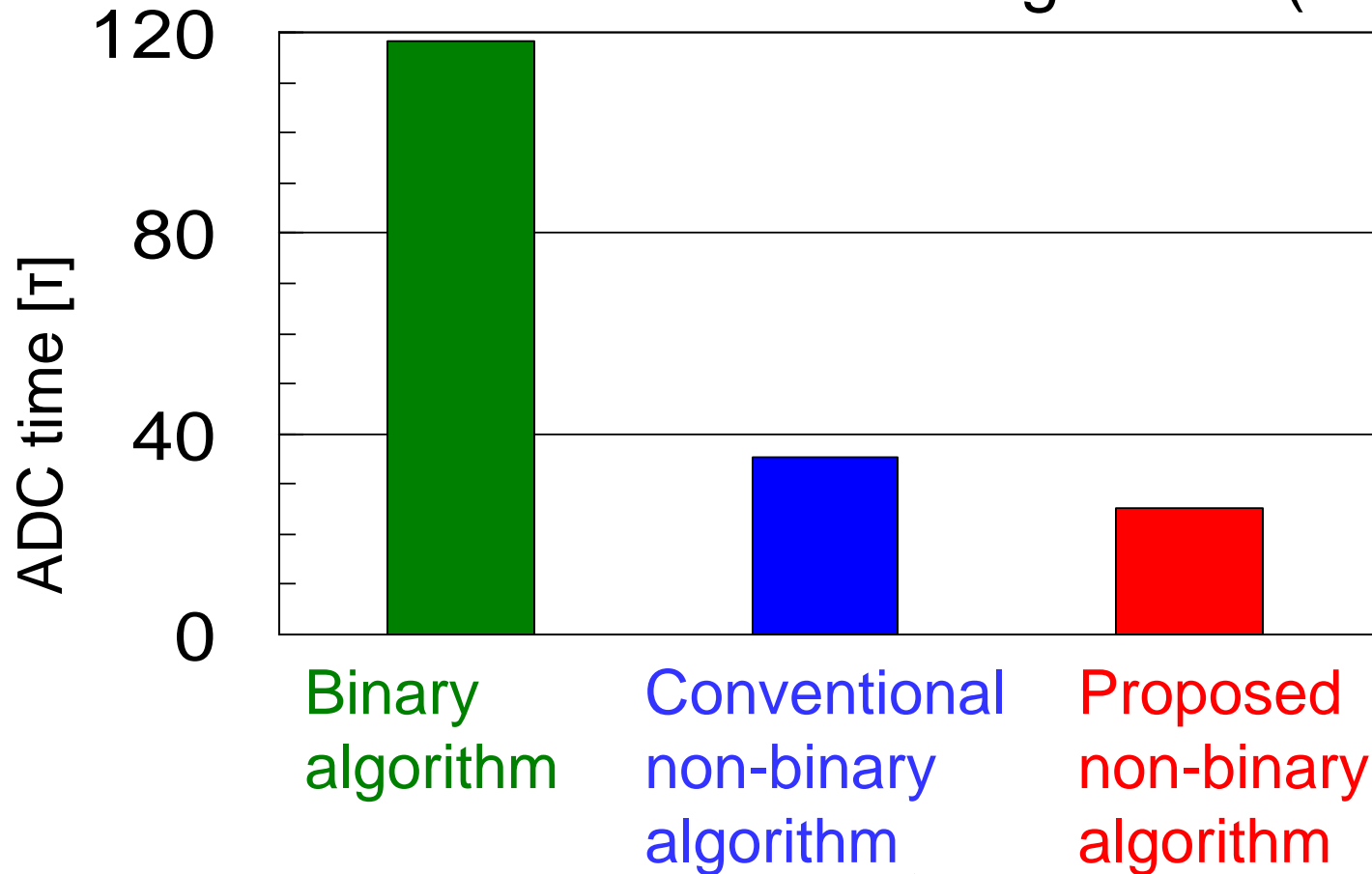
14-bit, 22-step

Step time :  $1.2 \tau$



# Comparison of ADC speed

Conversion time of each algorithm (14-bit)



Proposed algorithm  20% faster

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# Conclusion

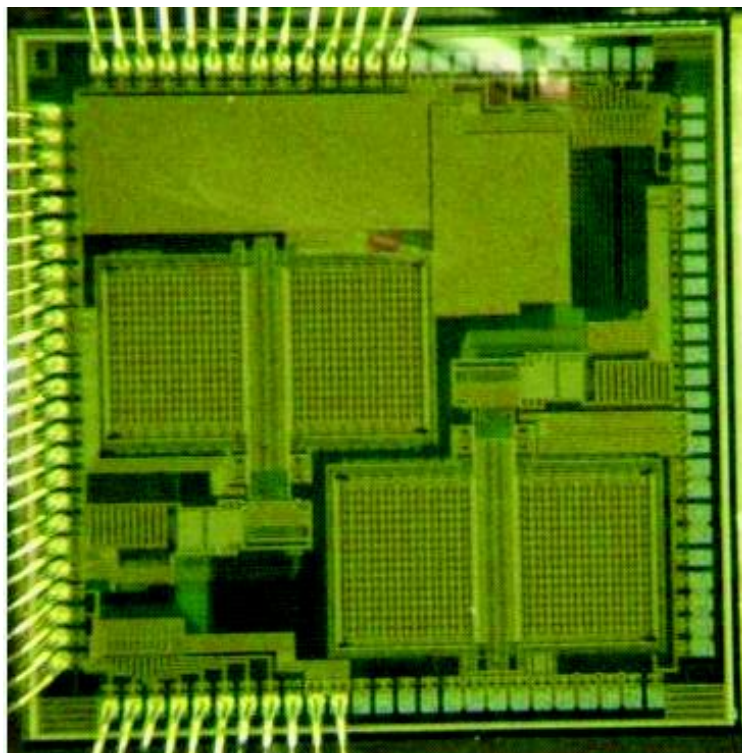
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## SAR ADC for automotive

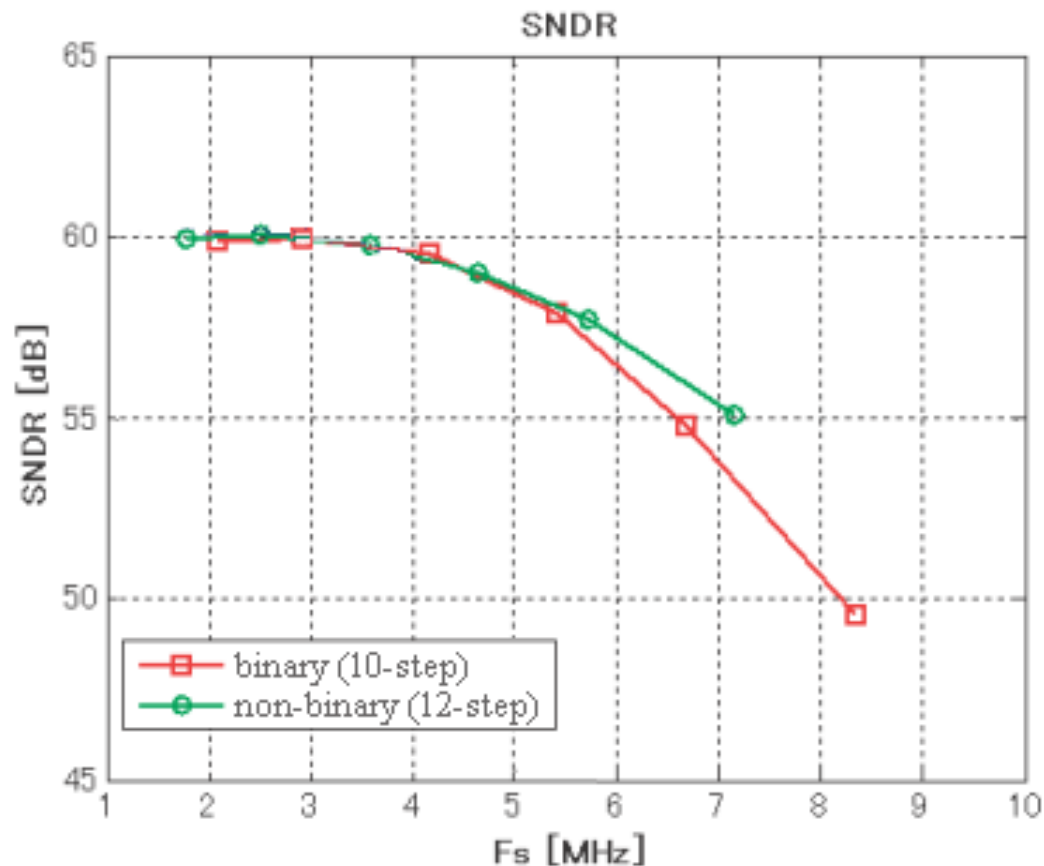
- Generalized non-binary algorithm.
- Optimal redundancy design method.
  - Reliable, Faster SAR ADC
- Digital Error Correction
  - Suitable for fine CMOS implementation.

20% faster than  
conventional non-binary algorithm  
only with ROM contents modification.

# Non-Binary SAR ADC Implementation and Measurement Results



0.18um CMOS  
2.5mm x 2.5mm  
with two SAR ADCs



SNDR comparison of  
10step (binary) and 12step (non-binary)  
F<sub>in</sub>:100kHz

# Lesson from 老子

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Redundancy  
makes ADC performan  
better



「無用」之「用」

Un-useful things are actually useful.

