3rd International Conference on
Advanced Micro-Device Engineering,
AMDE 2011

CONTROL SYSTEM DESIGN CONFORMABLE TO PHYSICAL ACTUALITIES

2011.12.8

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Outline

- Design equation
  - Expression easy to solve design equation
  - Expression easy to measure controlled-object
  - Expression easy to specify desirable control system
- New control scheme: I-PD control
- Smooth extension to design of sampled-data control
- Some examples designed
  - Servo systems, decoupling control
- Smooth extension to nonlinear control

Design Equation

Controlled-object (connection) Compensator/Controller

= Desirable control system

Expression should be easy

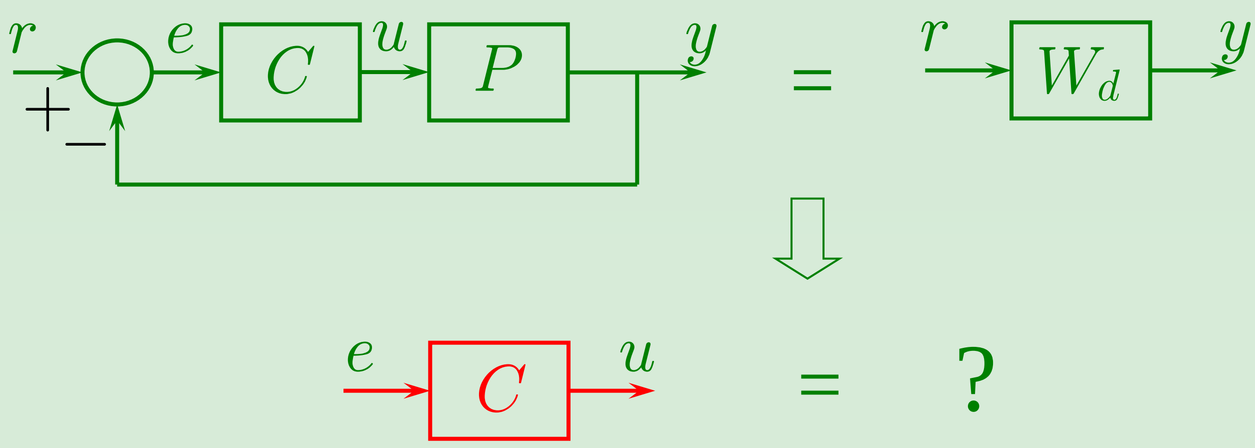
to indentify the controlled-object

to specify the desirable control system

to solve the compensator/controller

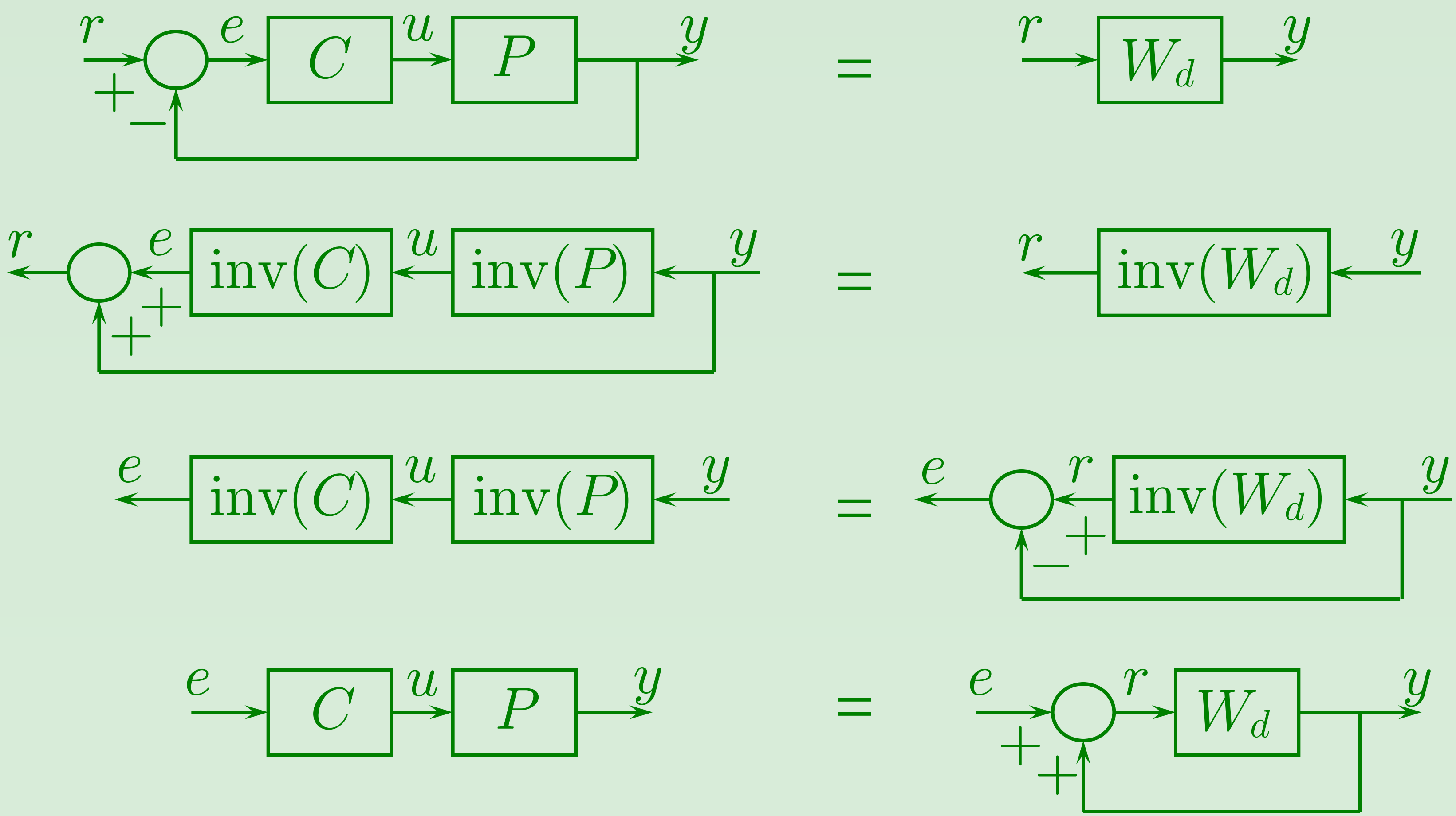
What functions should the expression have for easy solution ?

We can clarify the functions in the process to solve the design equation below:

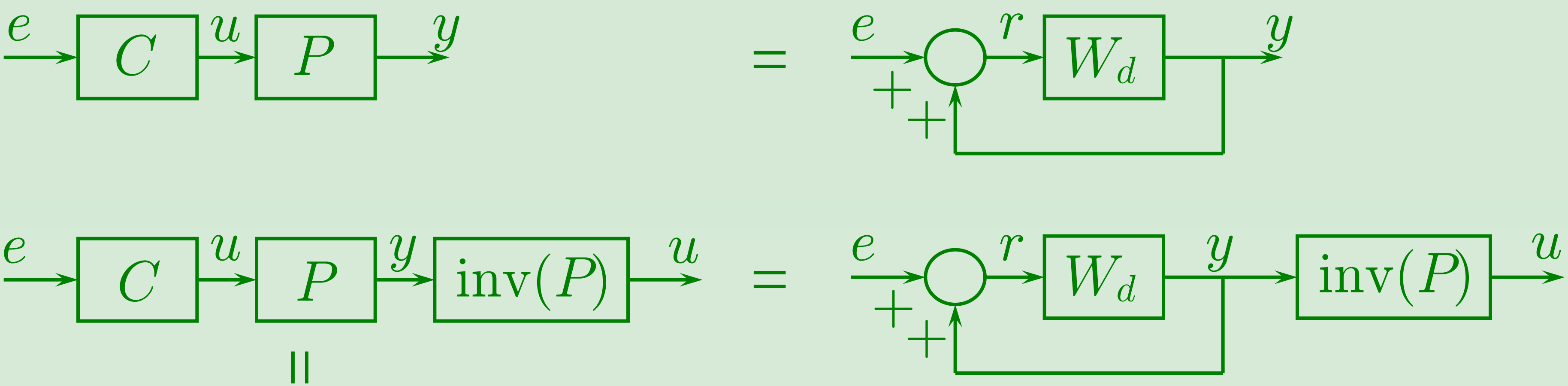


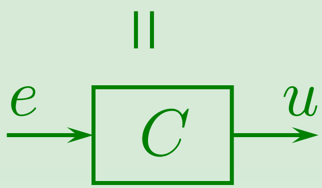
The element is assumed as any expression which can determine the output for the given input.

Solution of design equation



To be continued





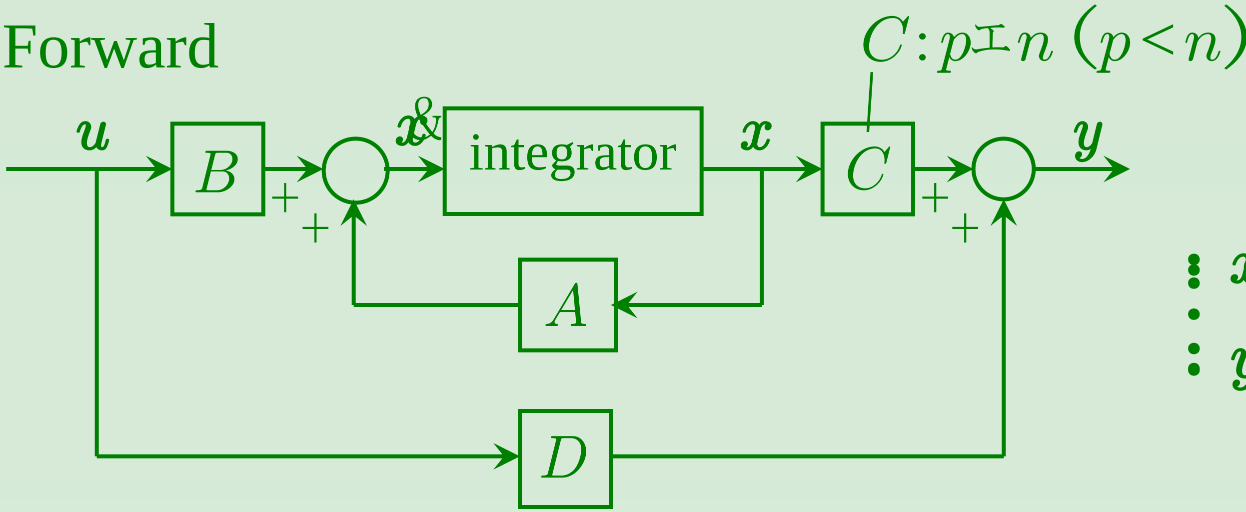
$$C = \mathrm{ser}(\mathrm{inv}(P), \mathrm{inv}(\mathrm{par}(-I, \mathrm{inv}(W_d))))$$

The function turns out to be computability of series connection of two elements, parallel connection of two elements and inverse of each element.

Inverse is an element to give the input from the output.

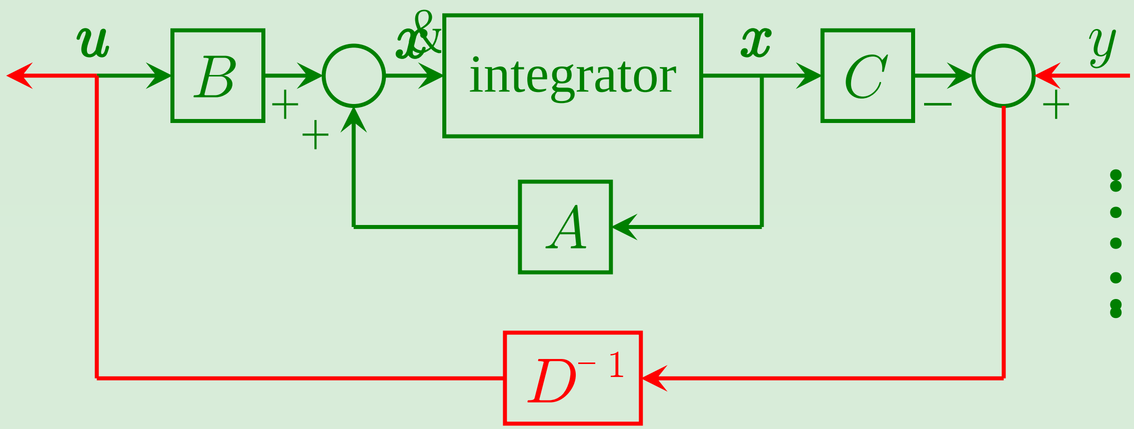
State equation expression is not invertible

Forward



$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Inverse if $|D| \neq 0$



$$\begin{cases} \dot{x} = (A - BD^{-1}C)Ax + BD^{-1}y \\ u = -D^{-1}Cx + D^{-1}y \end{cases}$$

A physical system does not have the inverse because $D = 0$ in general.

What is the expression easy to compute series and parallel connections and inverse ?

- In a static system the output is the same as the input multiplied by a gain for an arbitrary input. For such static systems three computations are straightforward.
- A static system is a special case of a dynamical system. So we should be able to extend the expression smoothly from static to dynamical system.
- In a dynamical system, for an arbitrary input the output is not the same as the input multiplied by any constant.
- We can find, however, a special class of inputs for witch the output is the same as the input multiplied by a constant.
- Using the class of inputs as a key, we can get an expression easy to compute series and parallel connections and inverse after some manipulation. It turns out to be the transfer function expression.

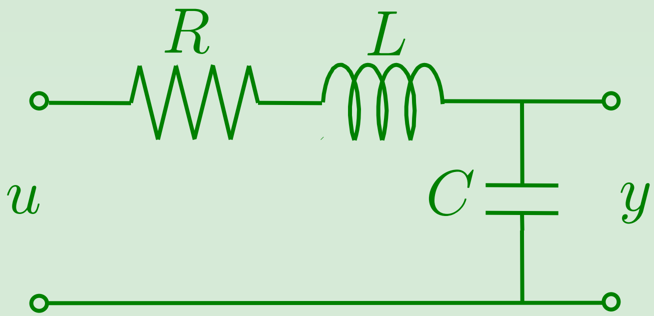
Expression Easy to Measure Controlled Object

- The physical system is, in general, a distributed-parameter system. The order of dynamics of a distributed-parameter system is infinitely high. It is by no means linear. It can be time-variant. In addition, the boundary conditions are very much complicated. Thus any model we can get through measurement is an approximation.
- Yet we know empirically that if time-variation of the system is not so fast, time-invariant approximation is useful, that if the range of operation is narrow enough, linear approximation is useful, and that if dynamical change is not so fast, a lower-order approximation is useful and even static, that is, 0-th order approximation can be useful. Thus we have a lot of approximating expressions for a system, and we naturally think that those approximating expressions are close to each other.

To be continued

- In measurement of dynamic characteristics we gather data from phenomena which the system shows us and we estimate parameters in the model expression. So any expressions approximating the same phenomena should be very close to each other.
- From observations in the following two slides, the transfer function expression is suitable for measurement.

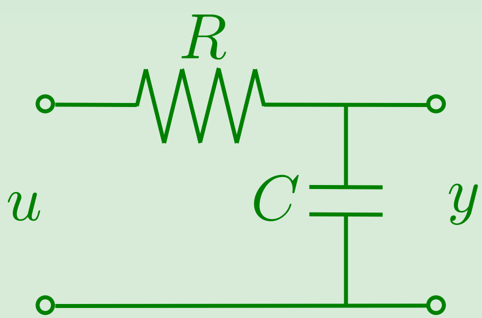
From the step response data two models are assumed and the parameters of transfer functions are estimated.



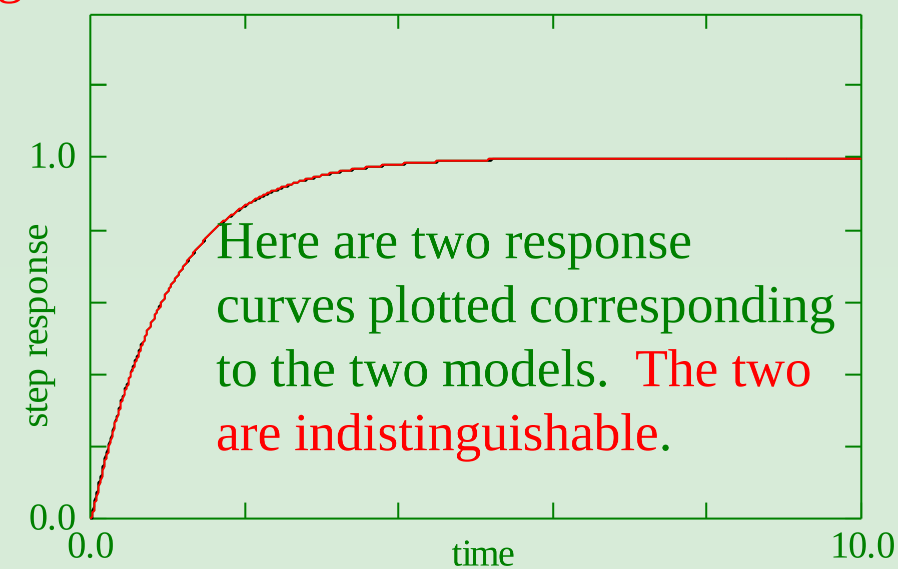
$$\frac{1}{1 + s + 0.01 s^2}$$

Negligible

Close to each other



$$\frac{1}{1 + s}$$

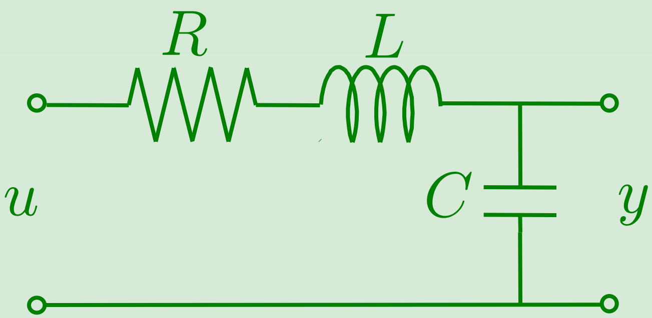


Here are two response curves plotted corresponding to the two models. The two are indistinguishable.

$R = 10^6\,\Omega$, $C = 10^{-6}$F, $L = 10^4$H

The transfer function expressions are close to each other and suitable to be measured.

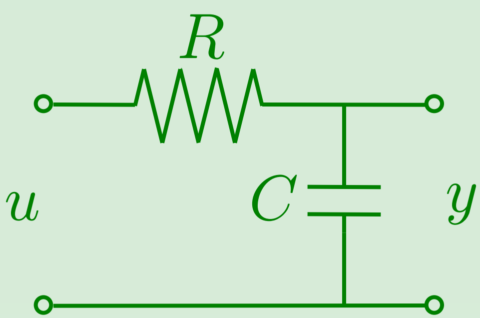
From the step response data two models are assumed and the parameters of state equation are estimated.



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 10^6\\ -10^2 & -10^8\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 10^2\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}u$$

Too big to be neglected



$$[\dot{x}_1] = [-1][x_1] + [1]u$$

$$y = [1][x_1] + [0]u$$

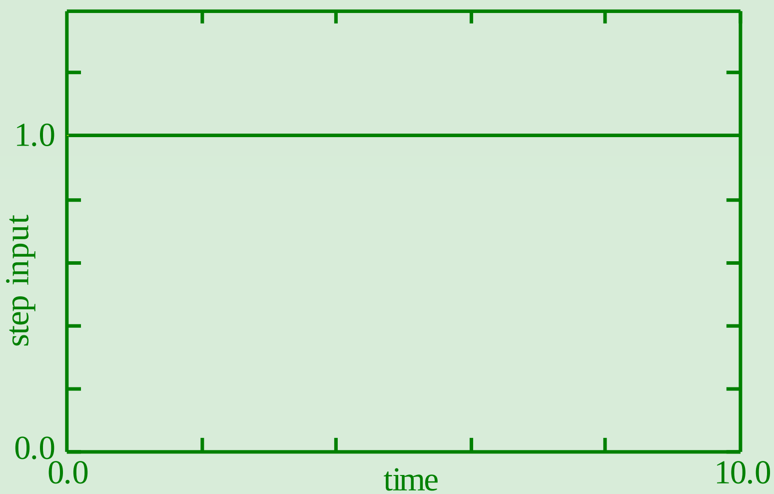
Not close to each other

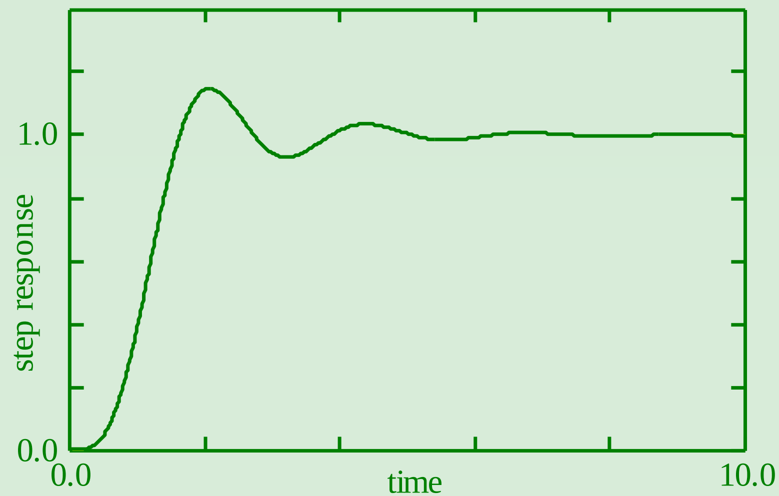
$R = 10^6\,\Omega$, $C = 10^{-6}$F, $L = 10^4$H

The state equation expressions are not close enough in spite of step responses are indistinguishably close to each other.

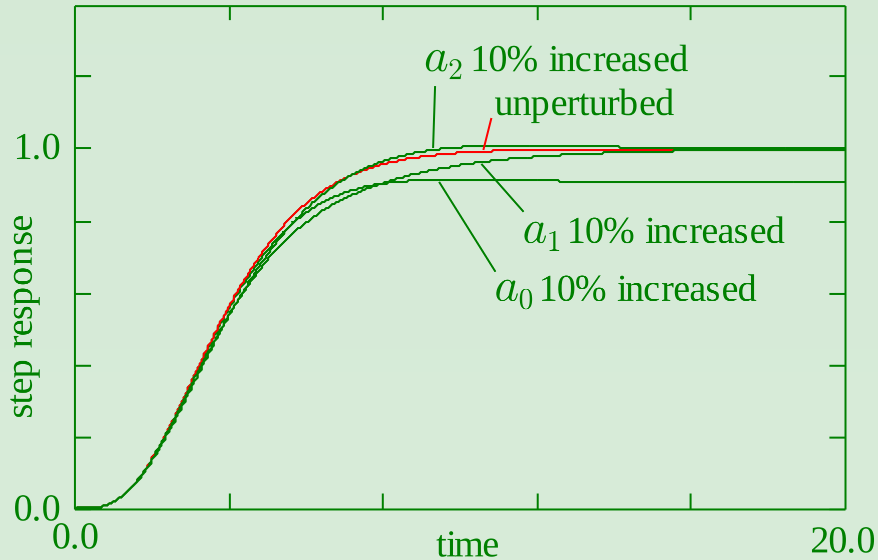
How to specify desirable control system

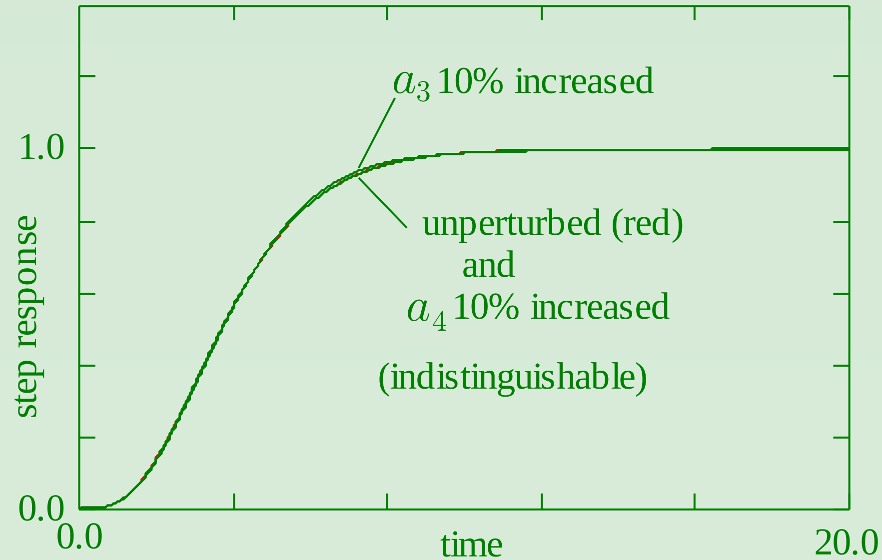
- People intuitively evaluate control performance from the shape of step response rather than performance indices.
  - Steady state error is as small as possible, and is zero if possible.
  - Response time is as short as possible.
  - Adequate damping is desirable.





How do coefficients operate step response shape ?

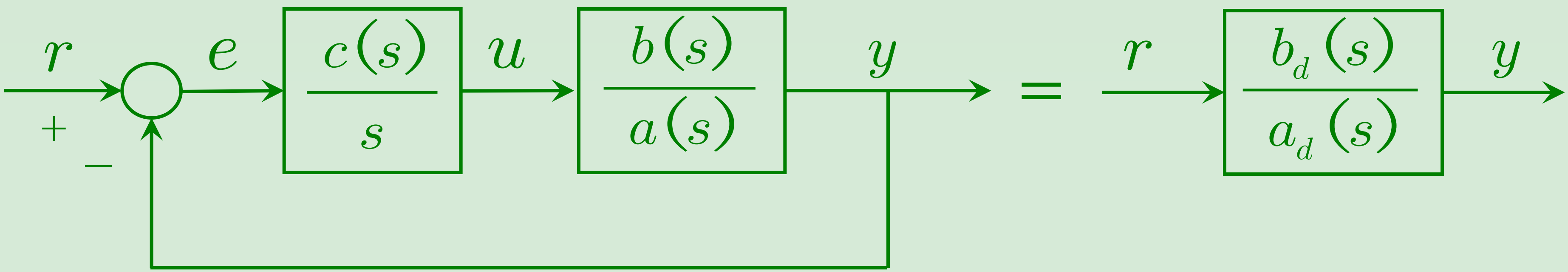




$$\frac{1}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4} = \frac{1}{1 + 4s + 6s^2 + 4s^3 + 1s^4}$$

Lower-order terms are effective to shape the step response,
whereas higher-order terms have little effect on the shape.
This enables us partial compensation of lower-order terms only.

Design equation for PID control



$$\frac{c(s)}{s} = \frac{c_0 + c_1 s + c_2 s^2 + \cdots}{s} = K_P \left(1 + \frac{1}{T_I s} + T_D s \right)$$

$$a(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots$$

$$b(s) = b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots$$

$$a_d(s) = a_{d0} + a_{d1} s + a_{d2} s^2 + a_{d3} s^3 + \cdots$$

$$b_d(s) = b_{d0} + b_{d1} s + b_{d2} s^2 + b_{d3} s^3 + \cdots$$

$$\frac{\dfrac{c(s)}{s}\dfrac{b(s)}{a(s)}}{1 + \dfrac{c(s)}{s}\dfrac{b(s)}{a(s)}} = \frac{b_d(s)}{a_d(s)}$$

DEF is essential for PID design

$$\frac{\dfrac{c(s)}{s}\,\dfrac{b(s)}{a(s)}}{1+\dfrac{c(s)}{s}\,\dfrac{b(s)}{a(s)}}=\frac{b_d(s)}{a_d(s)}$$

$$c(s)=c_0+c_1 s+c_2 s^2+\cdots$$

$$=\frac{a(s)}{b(s)}\,\frac{s}{\dfrac{a_d(s)}{b_d(s)}-1}$$

Inverse of controlled-object

Inverse of desirable control system

Information needed about controlled-object:

$$\frac{a(s)}{b(s)}=a^{\lceil}(s)$$

$$=a_0^{\lceil}+a_1^{\lceil}s+a_2^{\lceil}s^2+\cdots$$

and about desirable control system:

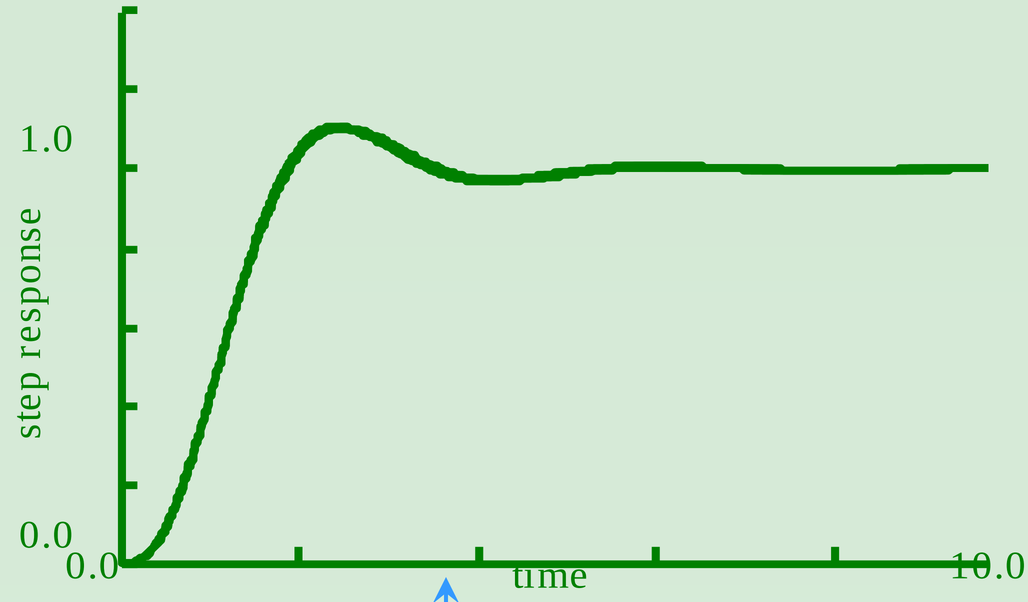
$$\frac{a_d(s)}{b_d(s)}=a_d^{\lceil}(s)$$

$$=a_{d0}^{\lceil}+a_{d1}^{\lceil}s+a_{d2}^{\lceil}s^2+\cdots$$

Denominator expanded form (DEF)

Maclaurin expansion of inverse

Transfer function and step response shape



$$\frac{1}{1+s+0.5s^2+0.15s^3}$$

$$\frac{1+0.25s}{1+1.25s+0.75s^2+0.275s^3+0.0275s^4}$$

$$\frac{1+0.5s}{1+1.5s+s^2+0.4s^3+0.065s^4}$$

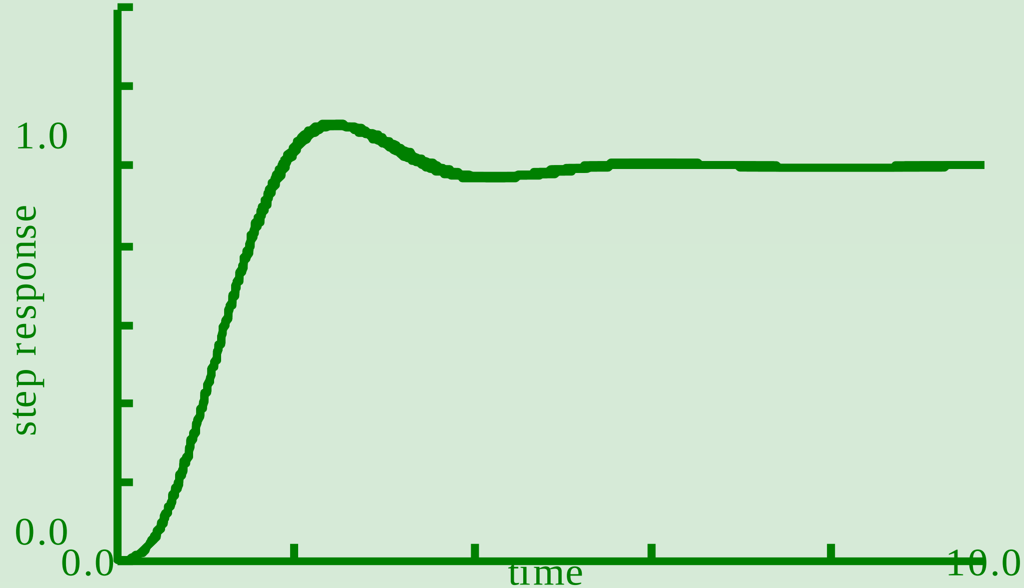
$$\frac{1+s}{1+2s+1.5s^2+0.65s^3+0.14s^4}$$

$$\frac{1+2s}{1+3s+2.5s^2+1.15s^3+0.29s^4}$$

Many to one correspondence.

Transfer function expression is redundant.

DEF is decisive for step response shape



$$\frac{1}{1+s+0.5s^2+0.15s^3}$$

$$\frac{1}{1+s+0.5s^2+0.15s^3-0.01s^4+0.0025s^5-0.000625s^6+\mathrm{L}}$$

$$\frac{1}{1+s+0.5s^2+0.15s^3-0.01s^4+0.005s^5-0.0025s^6+\mathrm{L}}$$

$$\frac{1}{1+s+0.5s^2+0.15s^3-0.01s^4+0.01s^5-0.01s^6+\mathrm{L}}$$

$$\frac{1}{1+s+0.5s^2+0.15s^3-0.01s^4+0.02s^5-0.04s^6+\mathrm{L}}$$

Denominator expanded form

DEF

$$\frac{1}{\dfrac{\text{denominator polynomial}}{\text{numerator polynomial}}}$$

Relations among MacLaurin series, moment series, and numerator expanded form (NEF)

$$G(s) = G(0) + \left.\frac{dG}{ds}\right|_{s=0} s + \frac{1}{2!}\left.\frac{d^2G}{ds^2}\right|_{s=0} s^2 + \frac{1}{3!}\left.\frac{d^3G}{ds^3}\right|_{s=0} s^3 + \cdots$$

$$G(s) = \int_0^\infty g(t)e^{-st}dt = m_0 - m_1 s + \frac{1}{2!}m_2 s^2 - \frac{1}{3!}m_3 s^3 + \cdots$$

where $m_i = \int_0^\infty g(t)\,t^i dt$ is i-th moment

of impulse response around t=0

$$G(s) = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots} = d_0 + d_1 s + d_2 s^2 + d_3 s^3 + \cdots$$

$$d_i = \frac{1}{i!}\left.\frac{d^i G(s)}{ds^i}\right|_{s=0} = (-1)^i \frac{1}{i!} m_i$$

Moment series expression of transfer function

$$G(s)=\int_0^\infty g(t)e^{-st}dt$$

$$=\int_0^\infty g(t)\left[1-st+\frac{1}{2!}s^2t^2-\frac{1}{3!}s^3t^3+\frac{1}{4!}s^4t^4-\cdots\right]dt$$

$$=\int_0^\infty g(t)dt-s\int_0^\infty g(t)\,t\,dt+\frac{1}{2!}s^2\int_0^\infty g(t)\,t^2dt-\frac{1}{3!}s^3\int_0^\infty g(t)\,t^3dt+\cdots$$

$$=m_0-m_1s+\frac{1}{2!}m_2s^2-\frac{1}{3!}m_3s^3+\cdots$$

where $m_i=\int_0^\infty g(t)t^i dt$ is i-th moment of impulse response around t=0

Relation between DEF and NEF

$$G(s) = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \mathrm{L}}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \mathrm{L}} \quad \text{(transfer function)}$$

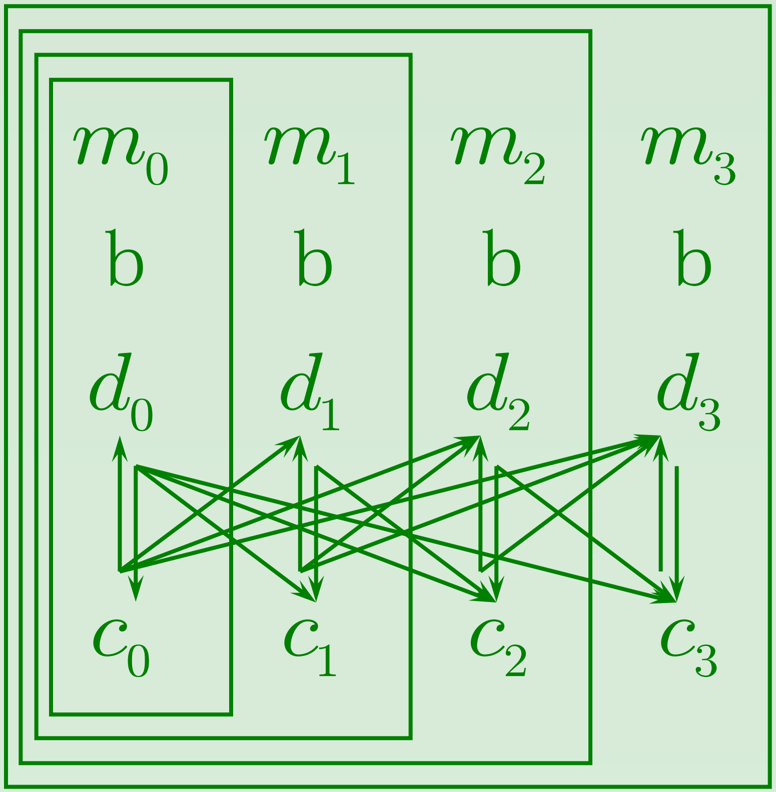
$$= d_0 + d_1 s + d_2 s^2 + d_3 s^3 + \mathrm{L} \quad \text{(NEF)}$$

$$= \frac{1}{c_0 + c_1 s + c_2 s^2 + c_3 s^3 + \mathrm{L}} \quad \text{(DEF)}$$

$$c_0 = \frac{1}{d_0}, \quad c_1 = -\frac{1}{d_0} c_0 d_1, \quad c_2 = -\frac{1}{d_0}(c_0 d_2 + c_1 d_1)$$

$$c_3 = -\frac{1}{d_0}(c_0 d_3 + c_1 d_2 + c_2 d_1), \quad \mathrm{L} \quad \mathrm{L} \quad \mathrm{L}$$

Equivalence relation among expressions



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$$c_0 = \frac{1}{d_0}$$

$$c_1 = -\frac{1}{d_0} c_0 d_1$$

$$c_2 = -\frac{1}{d_0}(c_0 d_2 + c_1 d_1)$$

L L L

Determinable relation independently from successors

(Series can be truncated at any term.)

Independency from successors (IFS)

Time scale normalization of DEF

$$\frac{a_0}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + \cdots}$$

$$= \frac{1}{1 + \frac{a_1}{a_0} s + \frac{a_2}{a_0} s^2 + \frac{a_3}{a_0} s^3 + \frac{a_4}{a_0} s^4 + \cdots}$$

$$\frac{a_1}{a_0} s \equiv s', \qquad s = \frac{a_0}{a_1} s'$$

$$\therefore \quad \frac{1}{1 + s' + \frac{a_2}{a_0}\left(\frac{a_0}{a_1}\right)^2 s'^2 + \frac{a_3}{a_0}\left(\frac{a_0}{a_1}\right)^3 s'^3 + \frac{a_4}{a_0}\left(\frac{a_0}{a_1}\right)^4 s'^4 + \cdots}$$

$$= \frac{1}{1 + s' + a_2 s'^2 + a_3 s'^3 + a_4 s'^4 + \cdots}$$

The first moment (average delay time) is set to 1.

Average delay time set to σ

The average delay time σ of impulse response or the rise-time of step response depends on the controlled-object given and the controller/compensator used. Therefore it cannot be specified beforehand. It is to be determined within the process of design or model matching.

If we put the rise-time equal to s ($s^{\ulcorner}\sqsupset\ s\,s$), we obtain the DEF of desirable control system as

$$W_d(s) = \frac{1}{1 + s\,s + a_2 s^2 s^2 + a_3 s^3 s^3 + a_4 s^4 s^4 + \mathrm{L}}$$

Specification of desirable control system

$$W_d(s) = \frac{1}{1 + s s + a_2 s^2 s^2 + a_3 s^3 s^3 + a_4 s^4 s^4 + a_5 s^5 s^5 + \mathrm{L}}$$

- Zero offset error in step response:

$$W_d(0) = 1$$

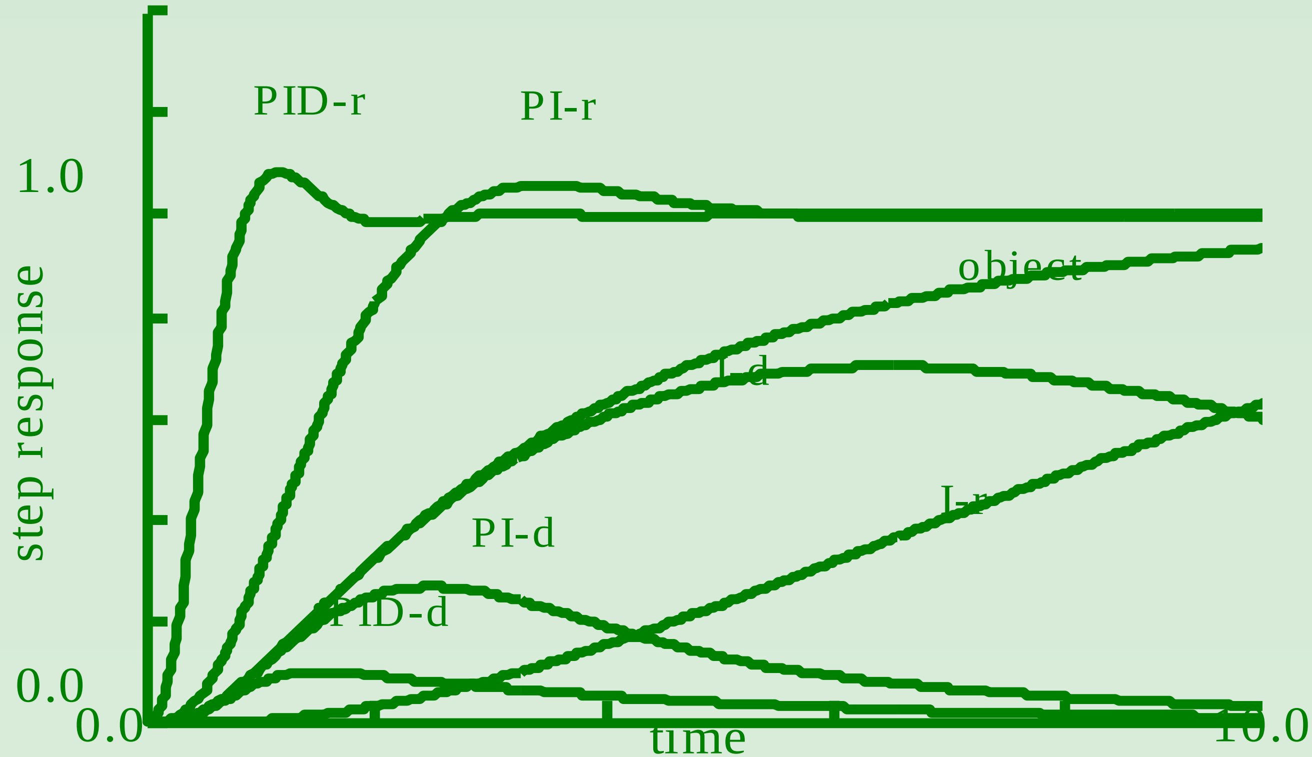
- Adequate damping:

$$\{a_i\} = \{\ 1,\ 1,\ 0.5,\ 0.15,\ 0.03,\ 0.003,\ \mathrm{L}\ \}$$

- Quick response speed:

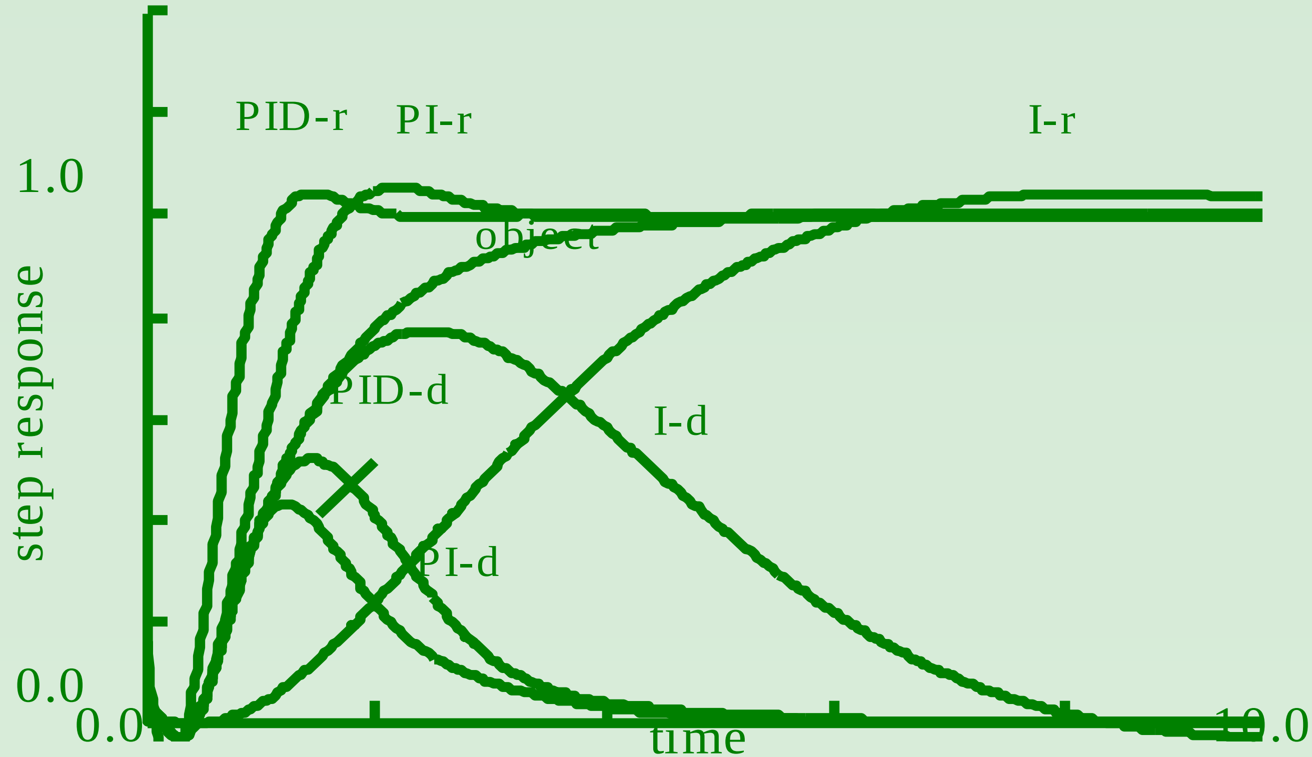
S ∃ as small as possible positive value

PID control for forth-order lag



$$\frac{1}{1+4s+2.4s^2+0.448s^3+0.0256s^4}$$

PID control for object with pure delay



$$\frac{e^{-0.5s}}{1+s} = \frac{1}{1+1.5s+0.625s^2+0.146s^3+0.0234s^4+\cdots}$$

A new control scheme

- Why PID ?
- No better control scheme ?
- Introduction of I-PD control scheme

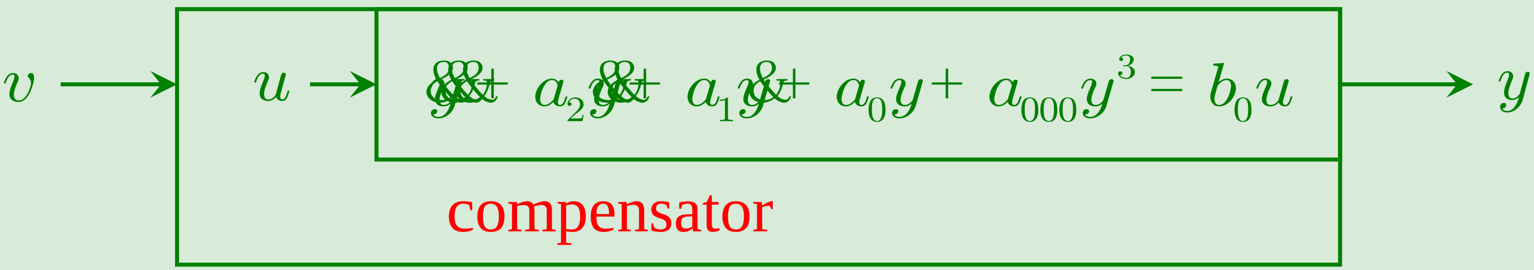
What compensation should we use ?

Given the controlled-object and the desirable system as follows:

controlled-object : $\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y + a_{000} y^3 = b_0 u$

desirable system : $\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y + a_{000} y^3 = b_0 v$

Solve the compensator form the equation below:



Rewriting the desirable system into the form of controlled-object as

$$\dddot{y} + (a_2 + a_2 - a_2)\ddot{y} + (a_1 + a_1 - a_1)\dot{y} + (a_0 + a_0 - a_0)y + (a_{000} + a_{000} - a_{000})y^3 = b_0 v$$

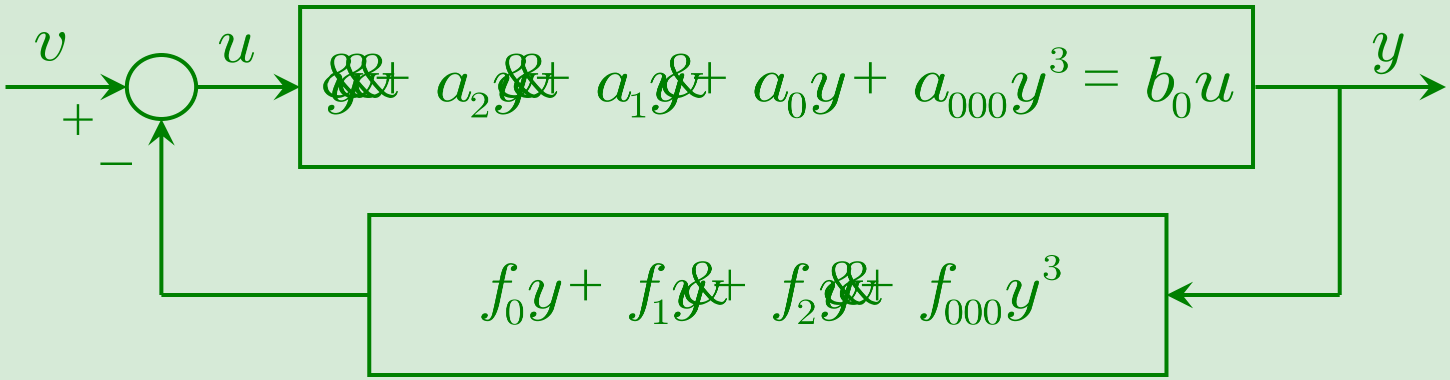
$$\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y + a_{000}y^3 = b_0 \underbrace{\left[v - \frac{a_2 - a_2}{b_0}\ddot{y} - \frac{a_1 - a_1}{b_0}\dot{y} - \frac{a_0 - a_0}{b_0}y - \frac{a_{000} - a_{000}}{b_0}y^3 \right]}_{u}$$

we get the control input u to the object as follows:

$$u = v - \frac{a_0 - a_0}{b_0}y - \frac{a_1 - a_1}{b_0}\dot{y} - \frac{a_2 - a_2}{b_0}\ddot{y} - \frac{a_{000} - a_{000}}{b_0}y^3$$

$$= v - f_0 y - f_1\dot{y} - f_2\ddot{y} - f_{000}y^3$$

Structure of compensation



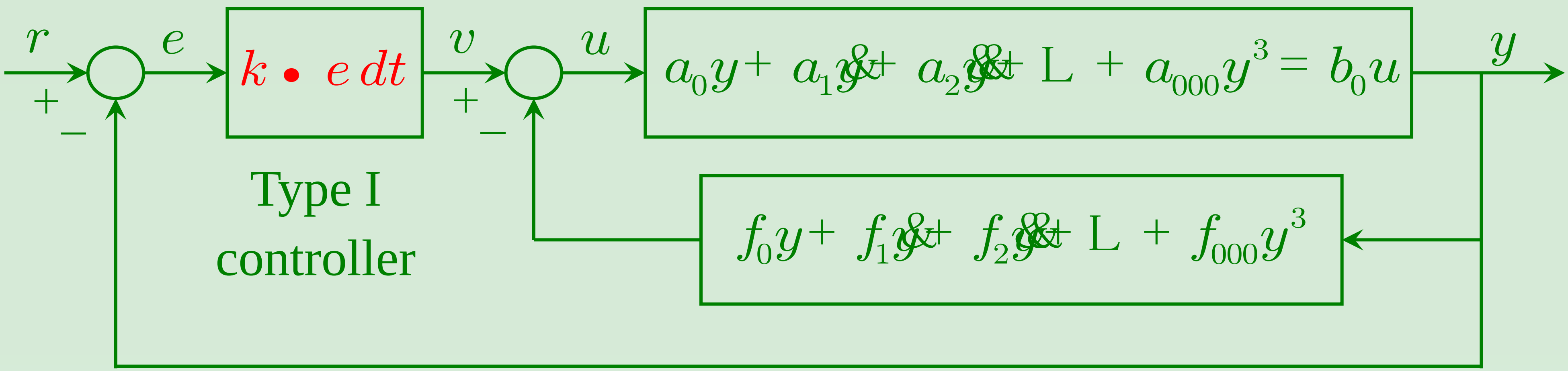
Feedback compensation structure is obtained !

$$\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y + a_{000}y^3 = b_0\left(v - f_0 y - f_1\dot{y} - f_2\ddot{y} - f_{000}y^3\right)$$

$$\dddot{y} + (a_2 + b_0 f_2)\ddot{y} + (a_1 + b_0 f_1)\dot{y} + (a_0 + b_0 f_0)y + (a_{000} + b_0 f_{000})y^3 = b_0 v$$

One-to-one, additive compensation is very easy to adjust each coefficient to any value.

Structure of control : I-PD scheme



$$\dot{v} = ke$$

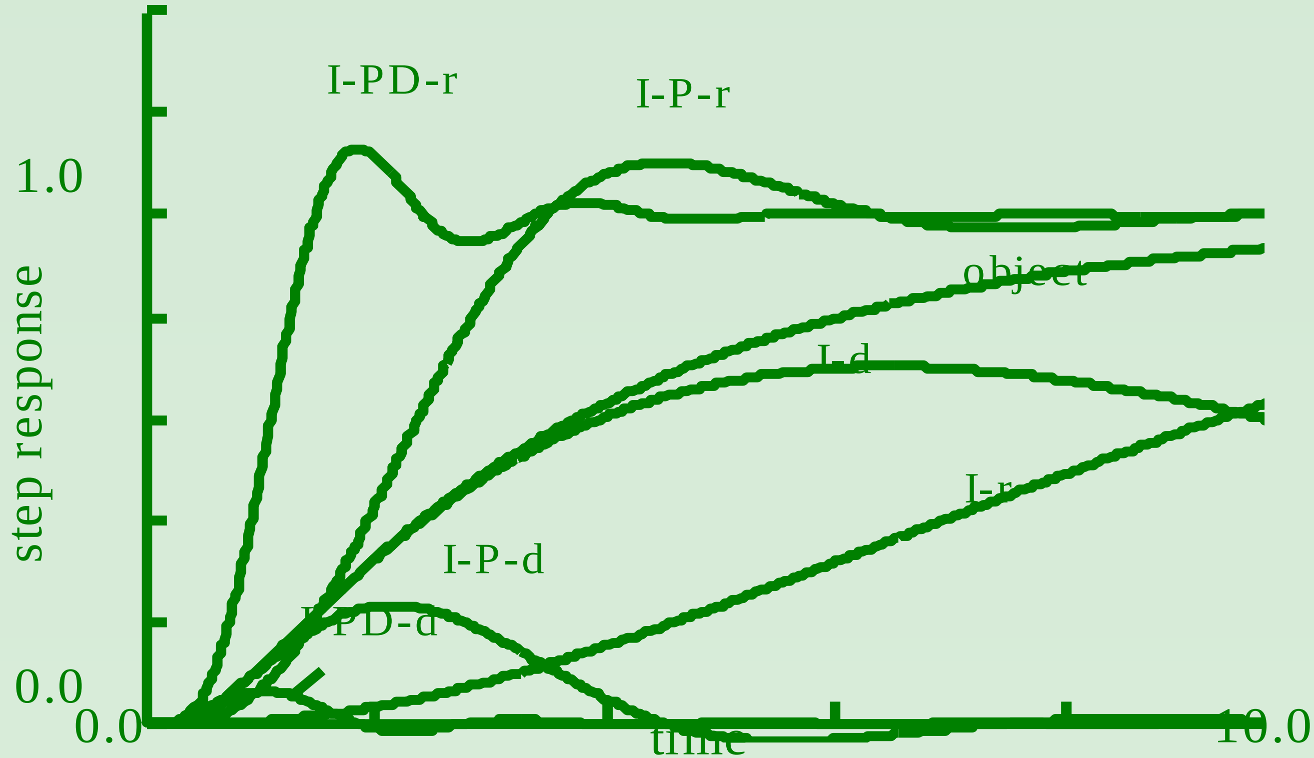
$$\dddot{y} + a_2\ddot{y} + a_1\dot{y} + \cdots + a_0 y + a_{000} y^3 = b_0 v$$

$$e = r - y$$

$$\ddddot{y} + a_2\dddot{y} + a_1\ddot{y} + a_0\dot{y} + 3a_{000} y^2\dot{y} + b_0 k y = b_0 k r$$

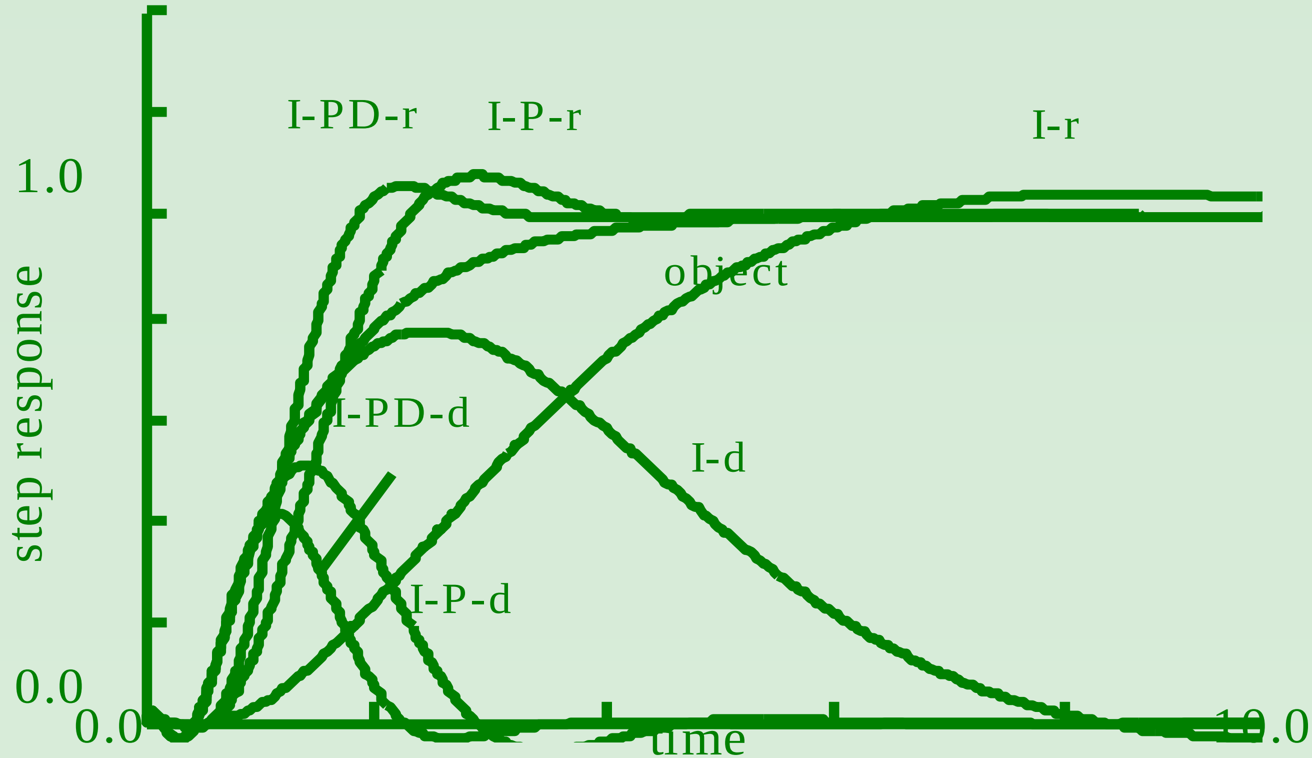
Zero offset in the step response is assured structurally.

I-PD control for forth-order lag



$$\frac{1}{1+4s+2.4s^2+0.448s^3+0.0256s^4}$$

I-PD control for object with pure delay



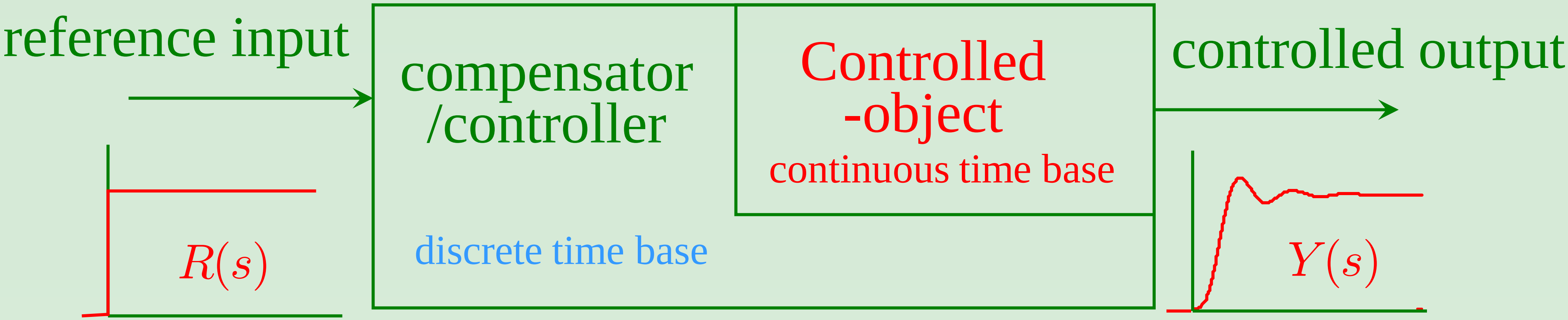
$$\frac{e^{-0.5s}}{1+s} = \frac{1}{1+1.5s+0.625s^2+0.146s^3+0.0234s^4+\text{L}}$$

T. Kitamori

Sampled-data control

- Is it digital or analog ?
- Controlled-object is analog, so sampled-data control system as a whole is analog.
- If the sampling period approaches to zero, the system should become the continuous-time system. (Continuity of sampled-data control and continuous-time control)

Design of sampled-data control



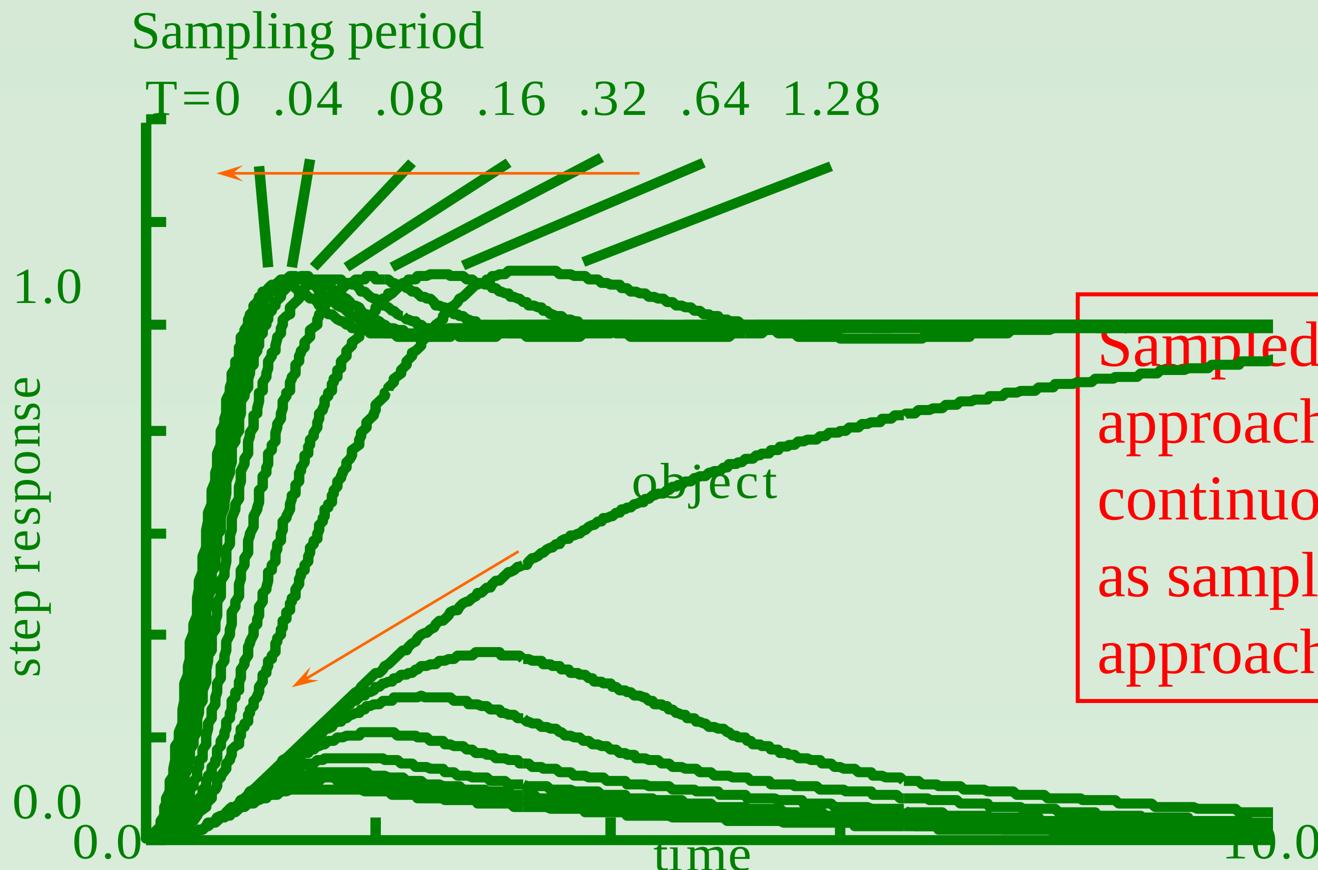
sampled-data control system

continuous time base

Both reference input and controlled output can be described in Laplace transform.
Thus, control system should be treated in Laplace transform.

$$\frac{Y(s)}{R(s)} = W(s)$$

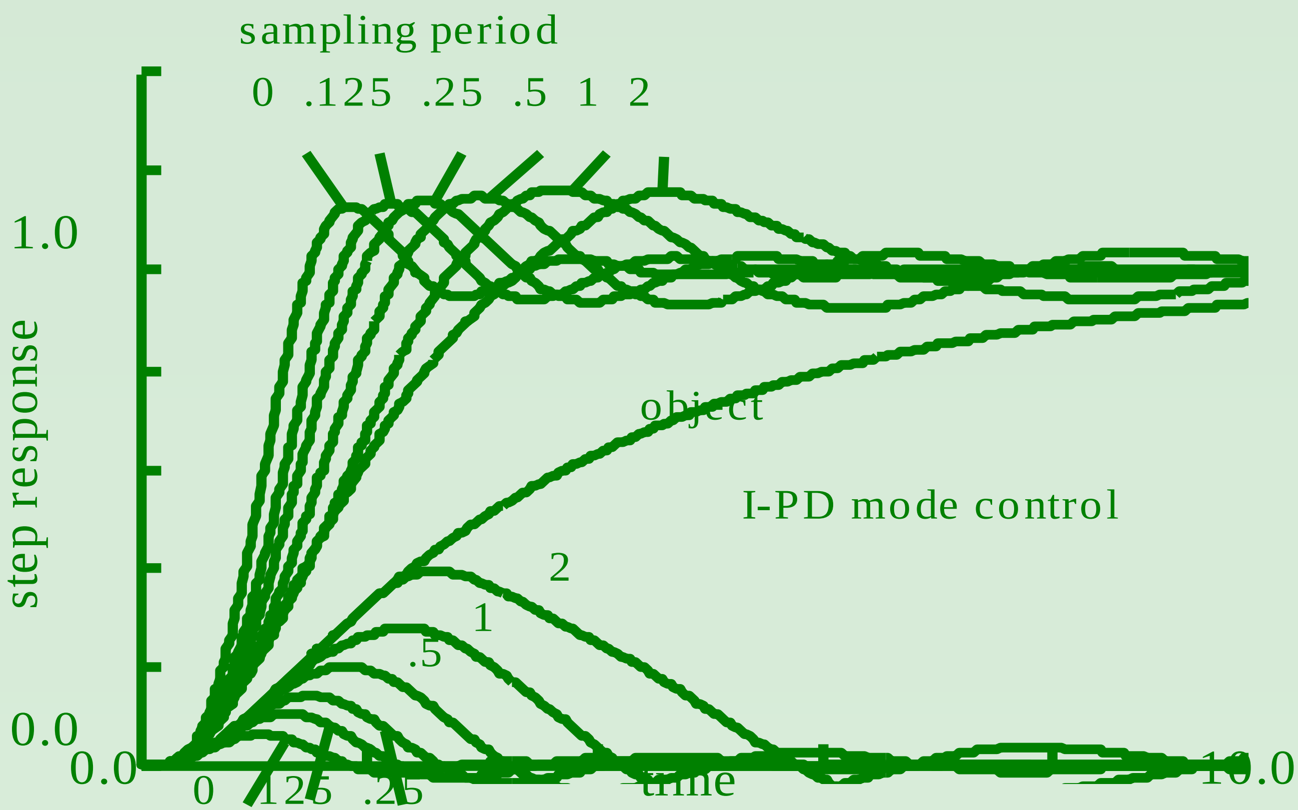
Continuity of sampled data-control and continuous-time control (PID)



Sampled-data control approaches to continuous-time control as sampling period approaches to zero.

controlled object: $\dfrac{1}{1+4s+2.4s^2+0.448s^3+0.0256s^4}$

Sampled-data I-PD control

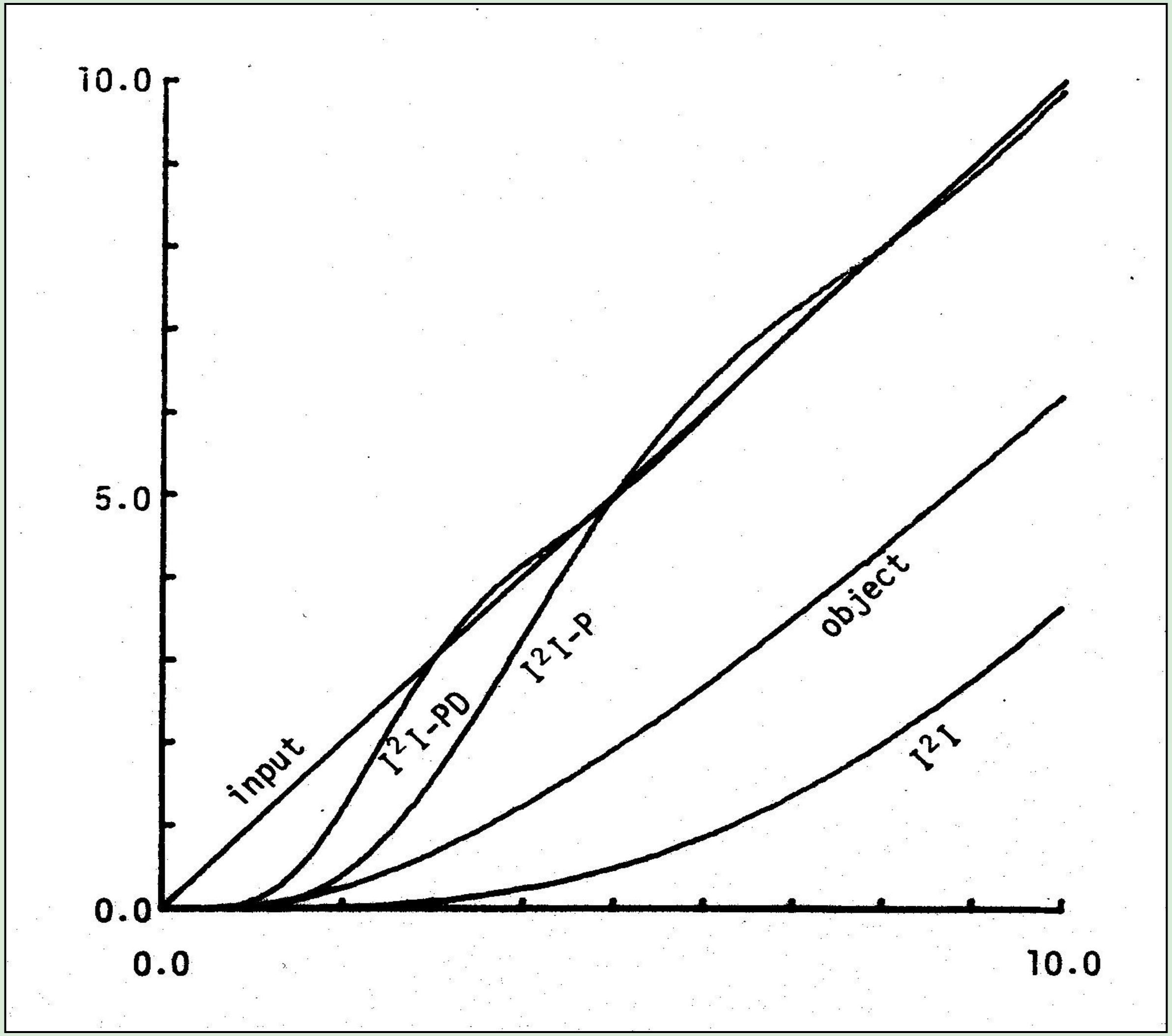


$$\frac{1}{1+4s+2.4s^2+0.448s^3+0.0256s^4}$$

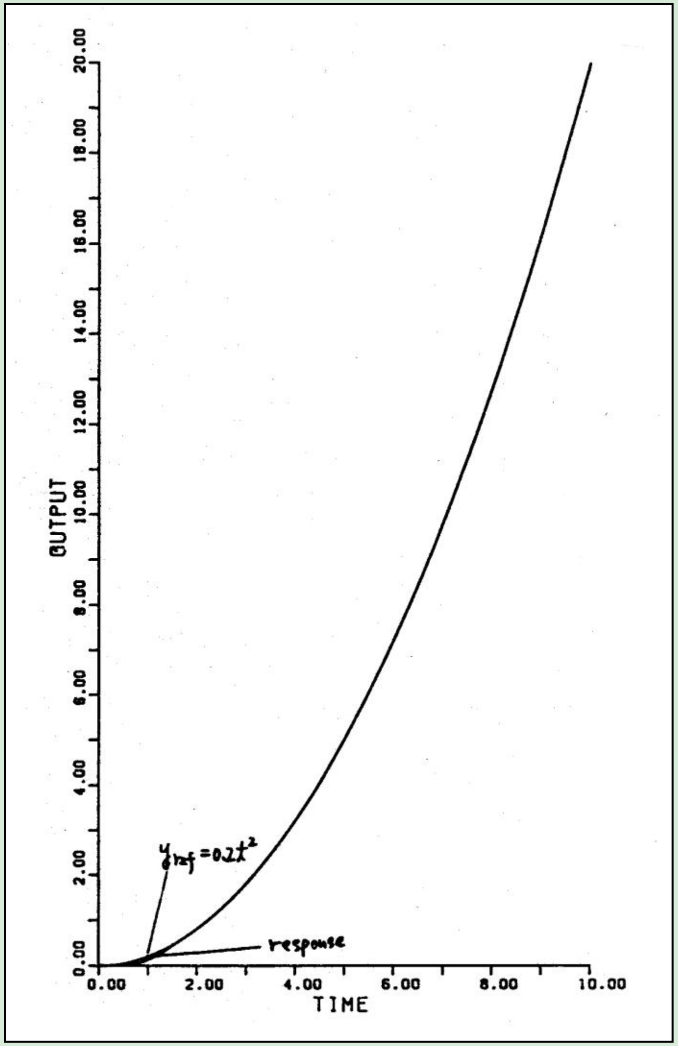
Some examples designed

- Tracking servo for ramp input
- Tracking servo for parabolic input
- Tracking servo for sinusoidal input
- PID and I-PD decoupling control
- Sampled-data decoupling control with two different sampling periods

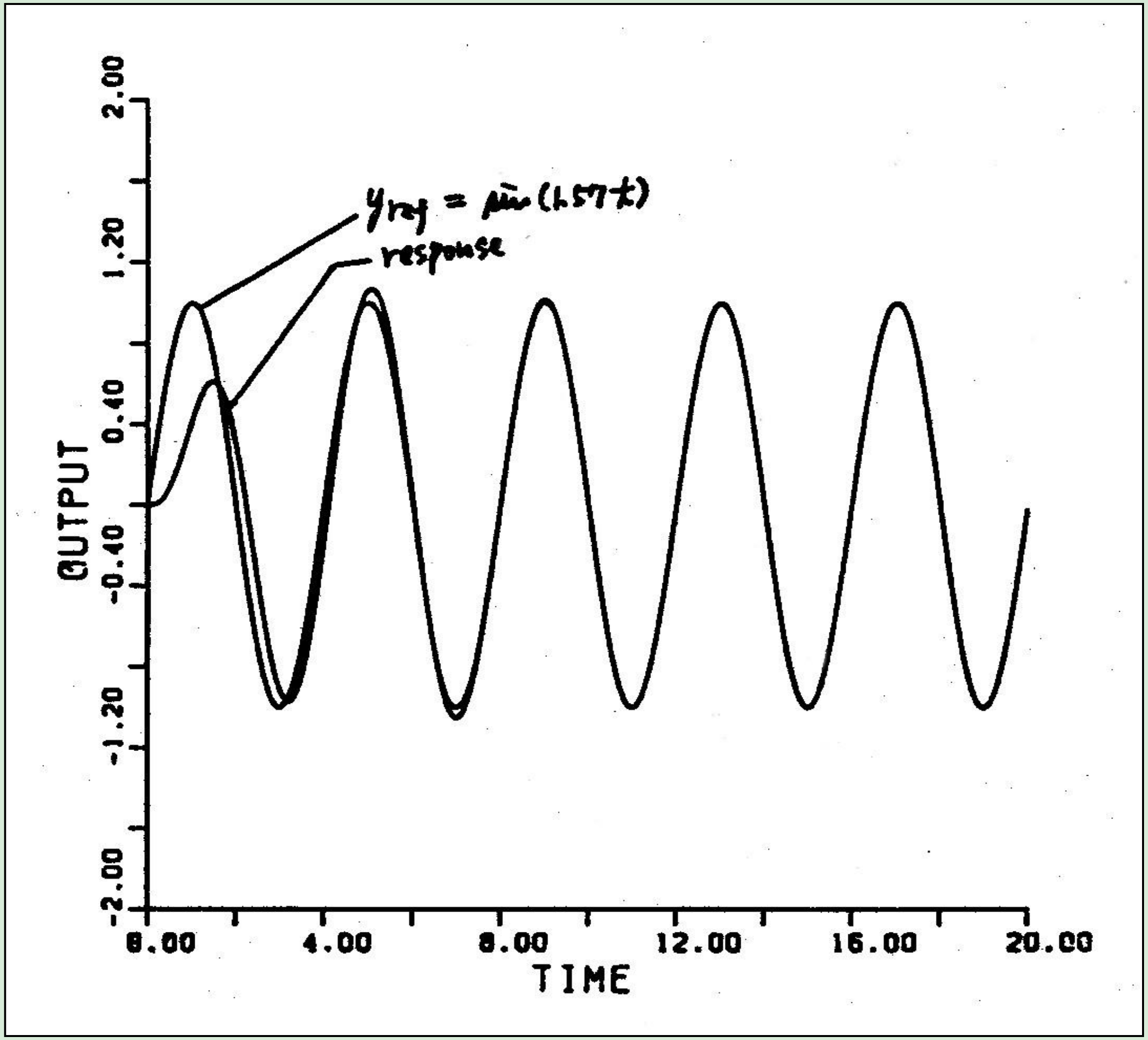
Tracking servo for ramp input



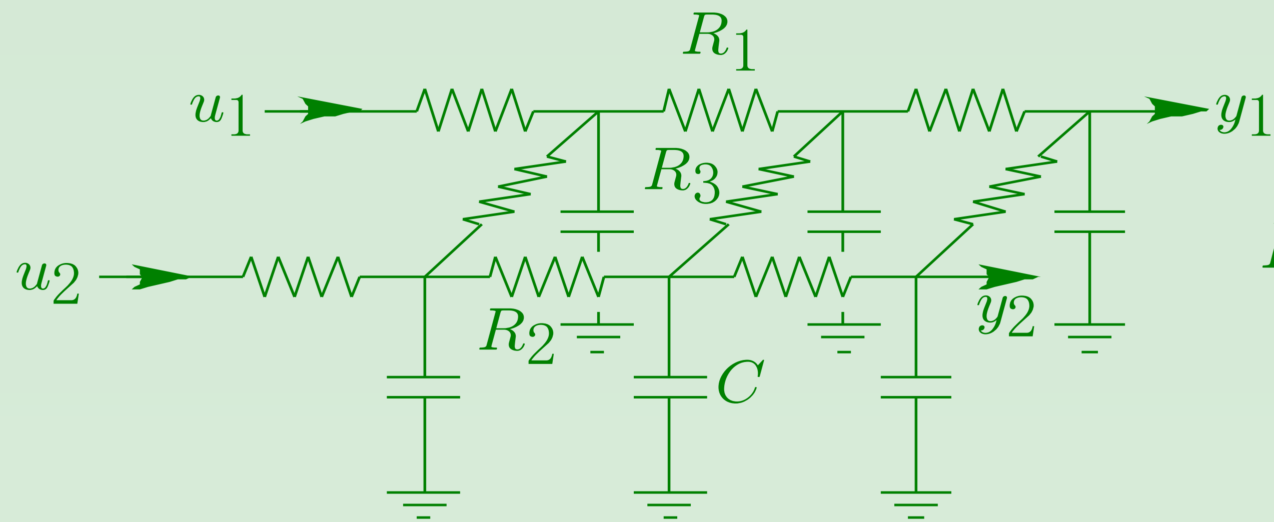
Tracking servo for parabolic input



Tracking servo for sinusoidal input



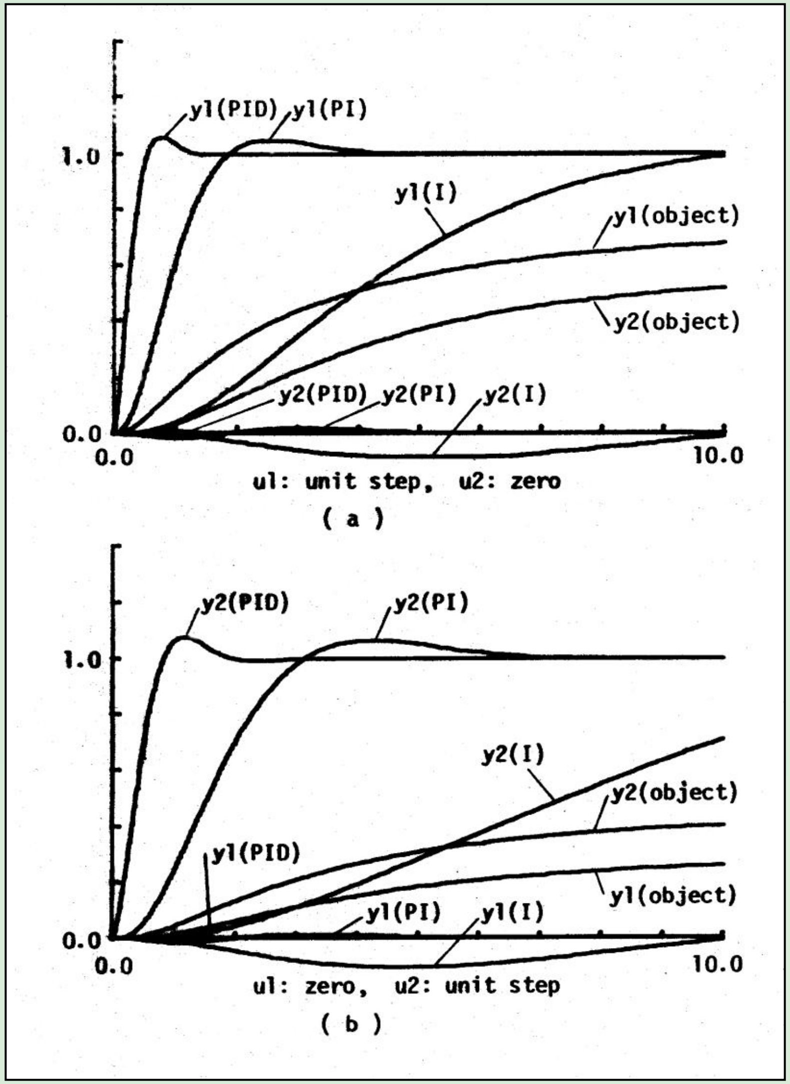
Example of 2-input-2-output controlled-object



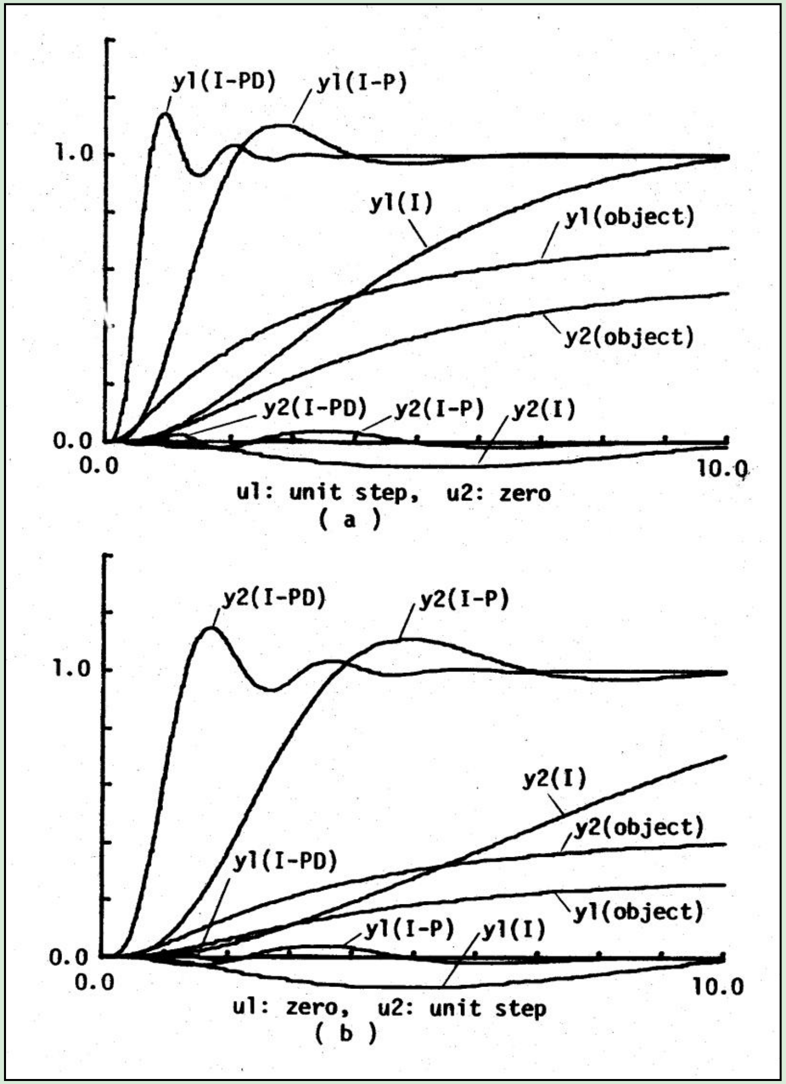
$R_1 : R_2 : R_3 = 1:2:5$

$$\frac{\begin{bmatrix} 37.95 & 14.98 \\ 29.95 & 22.98 \end{bmatrix} + \begin{bmatrix} 85.44 & 14.24 \\ 28.48 & 33.76 \end{bmatrix} s + \begin{bmatrix} 49.6 & 2.8 \\ 5.6 & 11.2 \end{bmatrix} s^2 + \begin{bmatrix} 8.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} s^3}{52.93 + 298.91s + 484.48s^2 + 336.71s^3 + 111.92s^4 + 17.4s^5 + s^6}$$

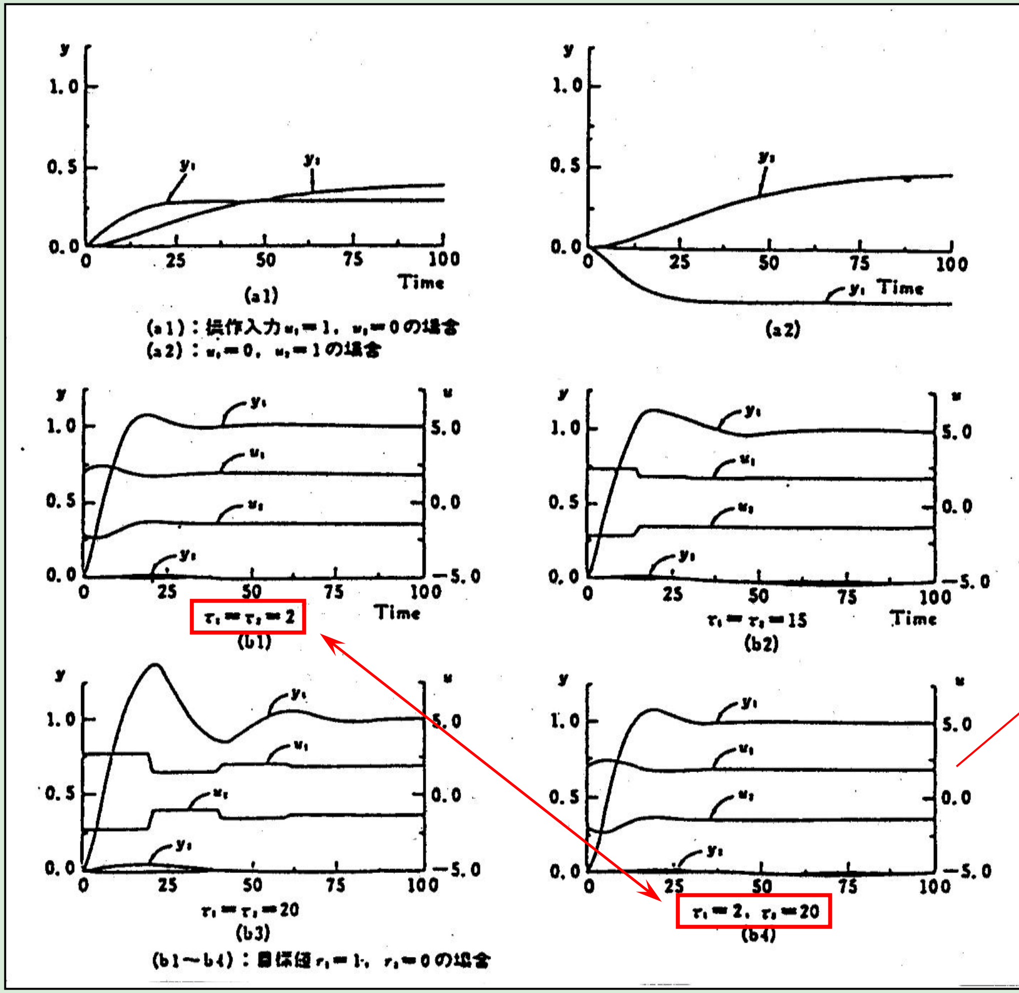
PID decoupling control [4]



I-PD decoupling control [3]



Sampled-data decoupling control with two different sampling periods



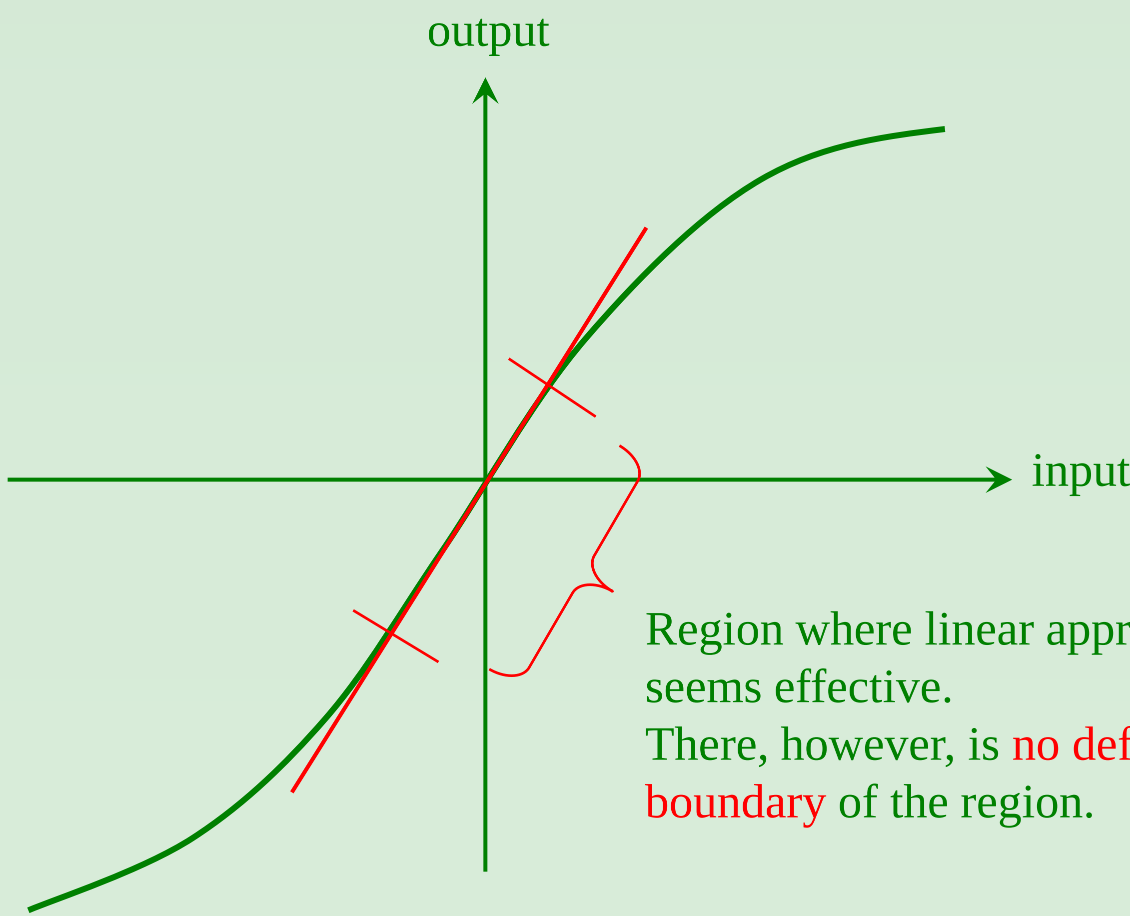
Sampling periods are set as 2s for 1st loop and 20s for 2nd loop. No deterioration is seen.

Mori, *et al.*[8]

Nonlinear control

- If the region of control is narrow enough linear control is sufficient. For wider region nonlinearity becomes unable to be neglected. However, there is no definite boundary of the region.
- Nonlinear control should be smoothly extended from linear control.

Continuity of nonlinearity and linearity



Region where linear approximation seems effective.
There, however, is no definite boundary of the region.

Smooth extension of linear expression to nonlinear

Linear static

Nonlinear static

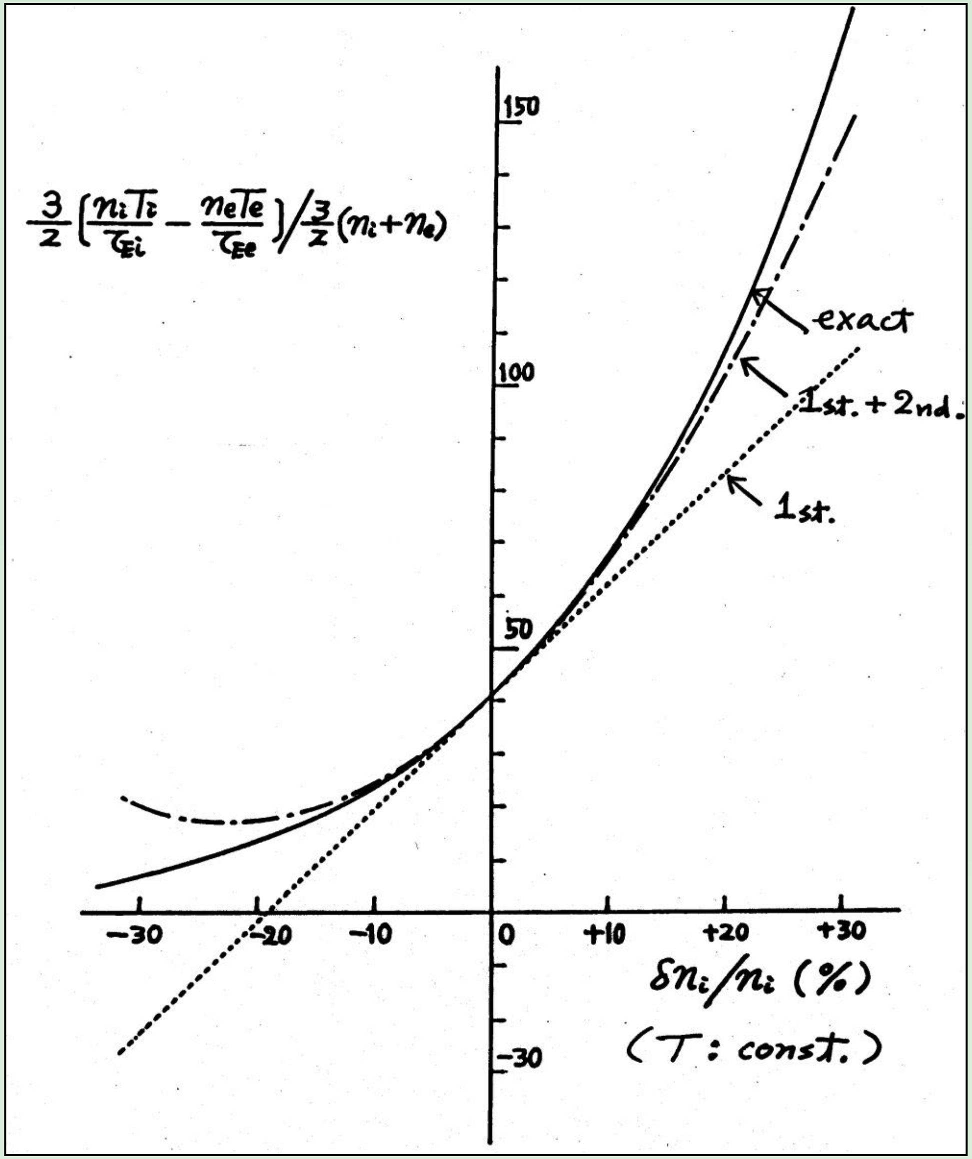
Linear dynamic

Nonlinear dynamic

$$
\begin{aligned}
& a_0 y + a_1 \dot{y} + a_2 \ddot{y} + \cdots \\
& + a_{00} yy + a_{01} y\dot{y} + a_{10} \dot{y}y + a_{02} y\ddot{y} + \cdots \\
& + a_{000} yyy + a_{001} yy\dot{y} + a_{010} y\dot{y}y + a_{100} \dot{y}yy + a_{002} yy\ddot{y} + \cdots \\
& + \cdots \quad \cdots \quad \cdots \\
& = b_0 u + b_0 \dot{u} + b_0 \ddot{u} + \cdots \\
& + b_{00} uu + b_{01} u\dot{u} + b_{10} \dot{u}u + b_{02} u\ddot{u} + \cdots \\
& + b_{000} uuu + b_{001} uu\dot{u} + b_{010} u\dot{u}u + b_{100} \dot{u}uu + b_{002} uu\ddot{u} + \cdots \\
& + \cdots \quad \cdots \quad \cdots
\end{aligned}
$$

Computation of series and parallel connections and inverse is straightforward.

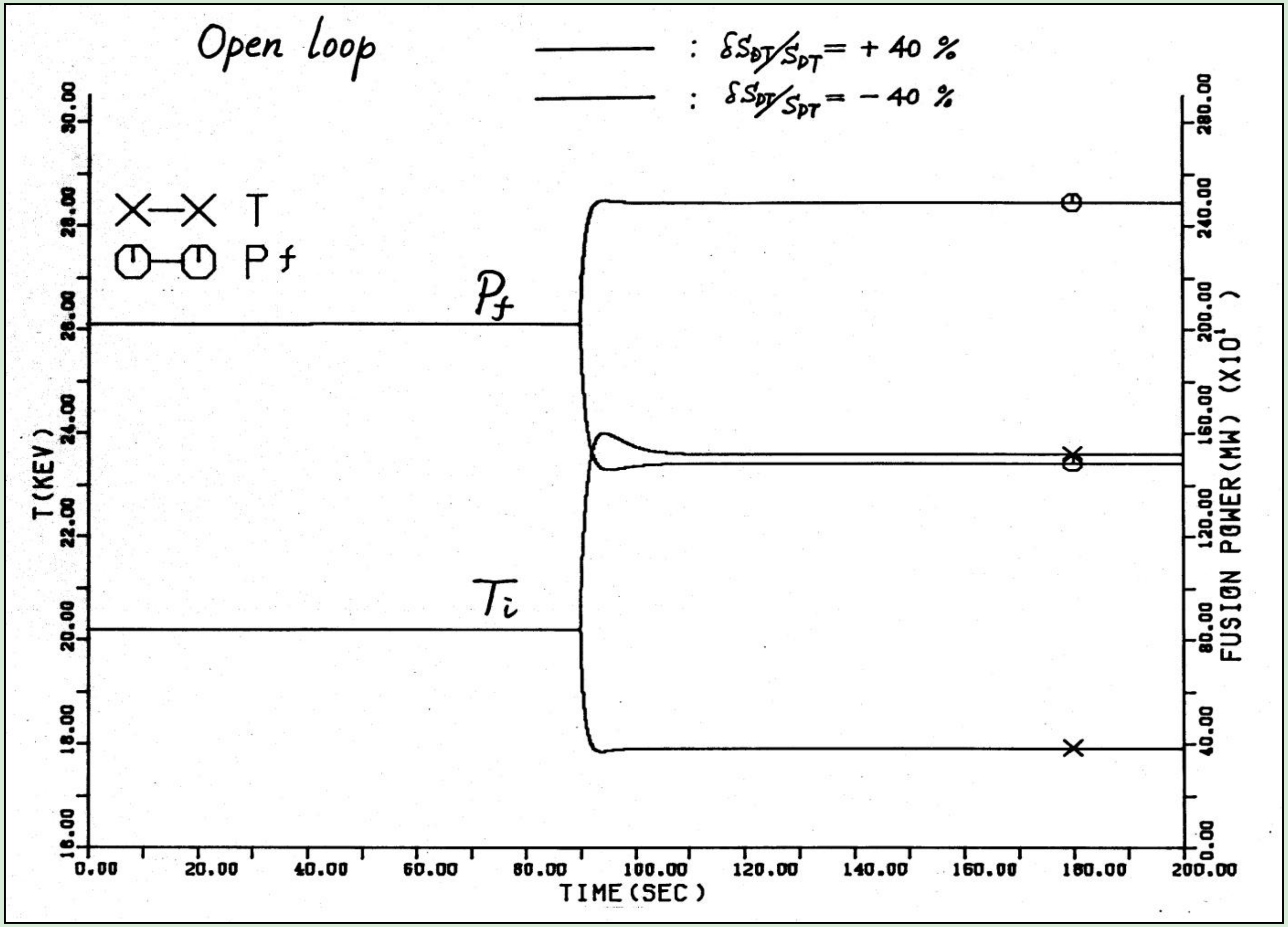
Linear and second degree approximation around the working point of reversed field pinch (RFP) fusion power reactor



The state equation of reactor has nonlinearity as high as eighth degree.

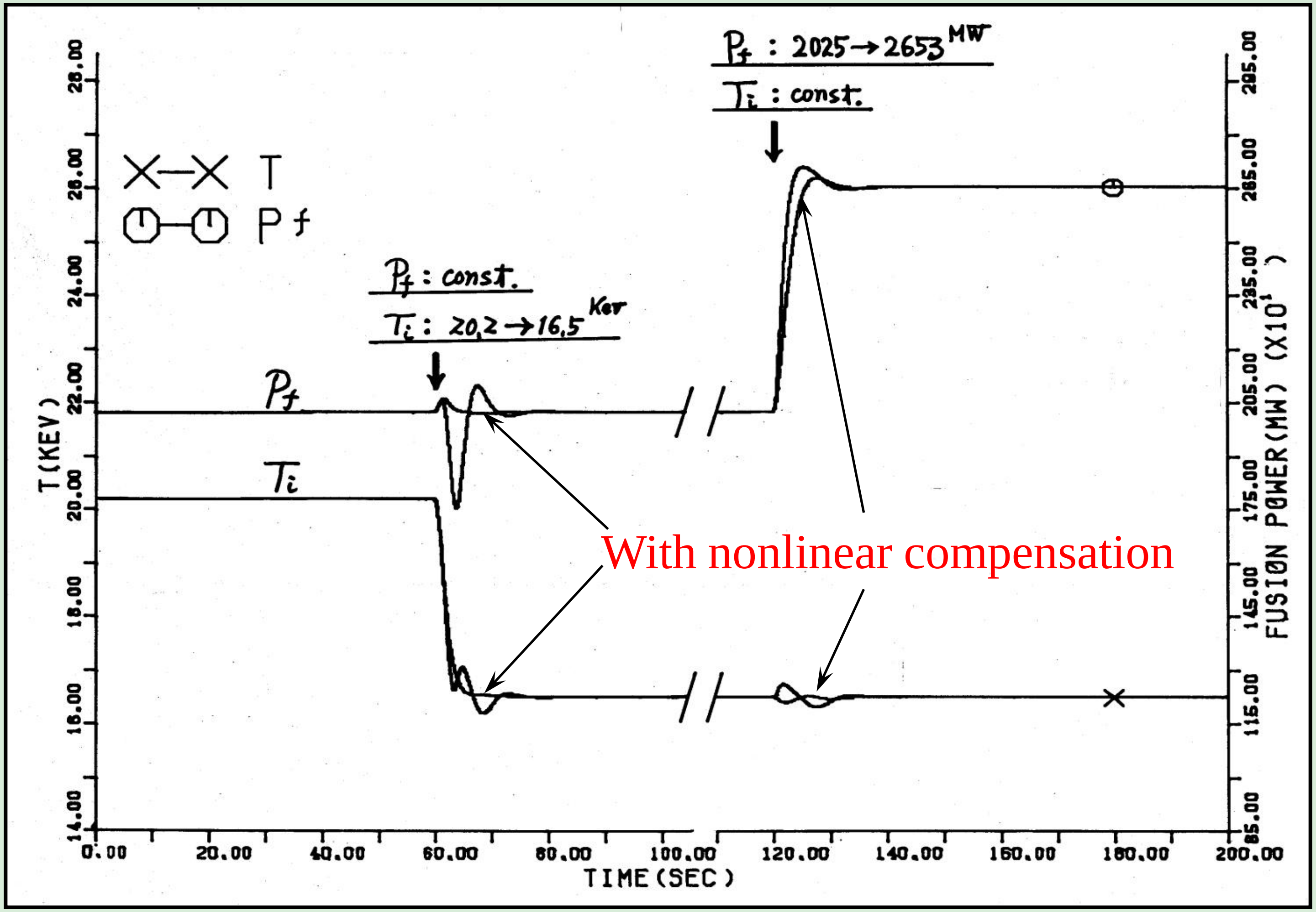
Shimotohno, *et al*. [9]

Interaction from temperature to power output



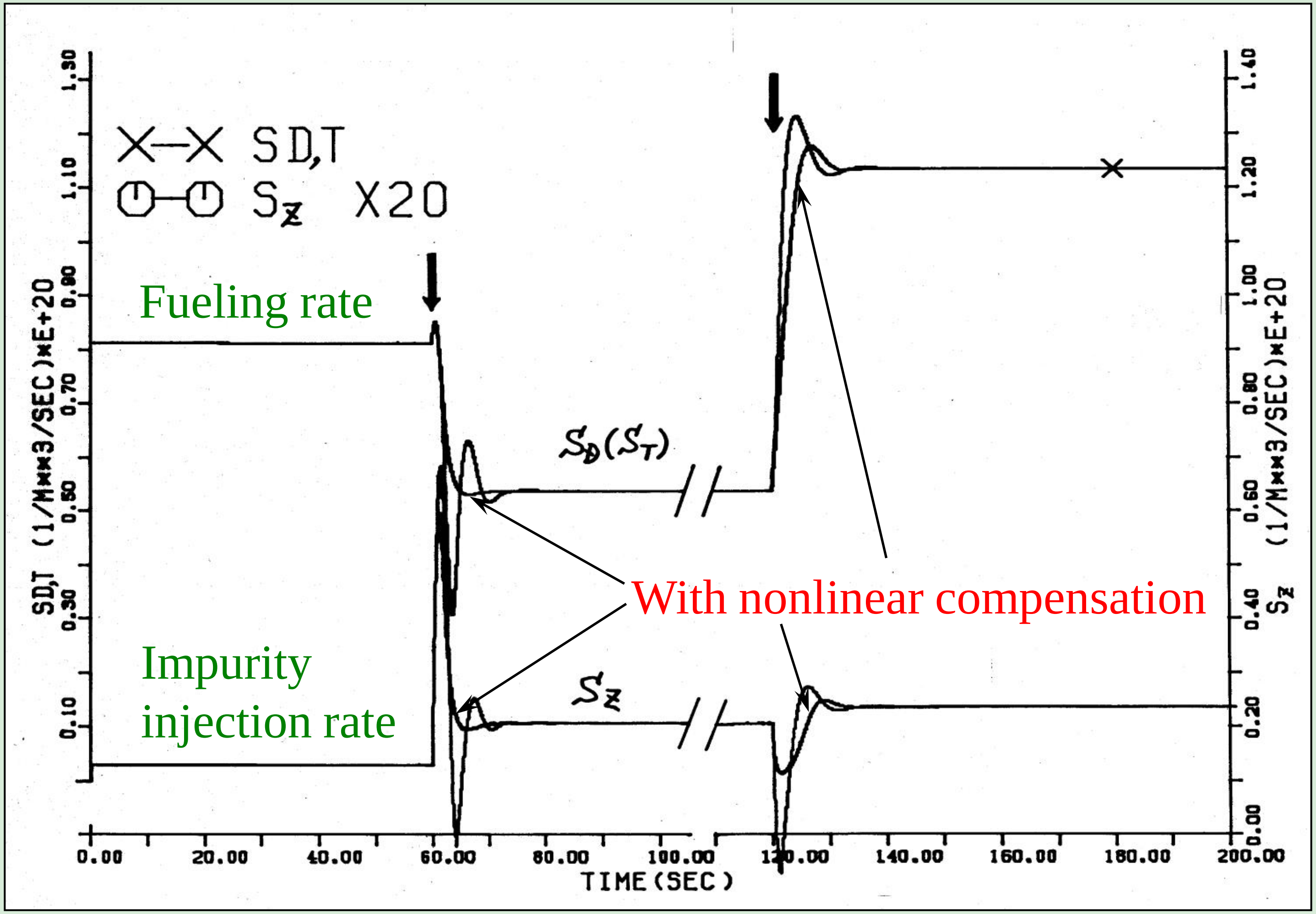
Shimotohno, *et al.* [9]

Decoupling control with I-P and second-degree compensation



Shimotohno, *et al*. [9]

Control inputs



Better control with smaller control input
by I-P and second degree compensation

Shimotohno, *et al*. [9]

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THANK YOU

FOR YOUR ATTENTION